On the Equation $2\cosh(2x)\left(\tanh e^{-x}\right)^2-1=0$

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Abstract

We give a formula for Pi

The number Pi "π" is defined by the integral

$$\pi = 2\int_{-1}^{1}\sqrt{1-x^2}\,dx \tag{1}$$

In this note we give a simple formula for π .

Formula

Entry 1. The unique real root of the equation

$$2\cosh(2x)\left(\tanh e^{-x}\right)^2 = 1 \tag{2}$$

Is

$$x = a = 0.171877394140481893 72... \tag{3}$$

Entry 2. Iteration

$$n = 0,1,2,3,...;\ x_0 = 0;\ \ x_{n+1} = -\ln\tanh^{-1}\left(\frac{1}{\sqrt{2\cosh(2x_n)}}\right) \Rightarrow x_n \to a \tag{4}$$

Entry 3.

$$\begin{aligned}\pi &= 4\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-2n}\left(\tanh e^{-a}\right)^{2n+1}\cosh\left((2n+1)a\right)}{2n+1} = \\ &= 4\tanh e^{-a}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-3n}\cosh\left((2n+1)a\right)}{(2n+1)\left(\cosh(2a)\right)^n}\end{aligned} \tag{5}$$

Entry 4. Iteration - Convergence

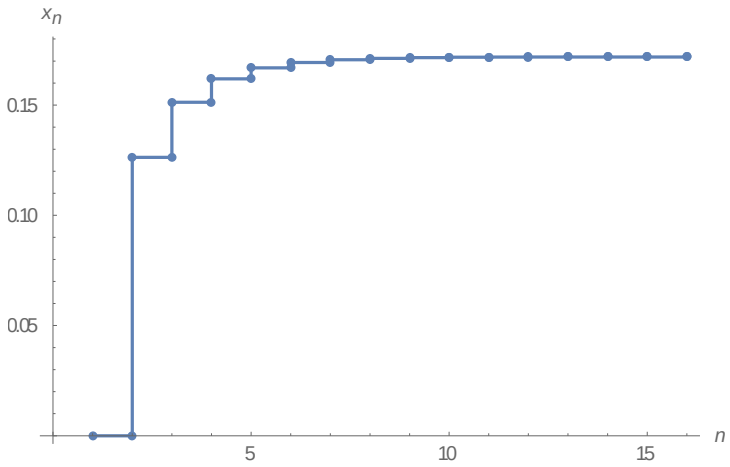

Figure 1. convergence iteration (4) .

References

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