Final Extra Credit Possibilities

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Abstract

The teaching of college algebra or algebra 2 has been influenced a little by calculators, but not nearly enough. Currently algebra texts do not really develop their topics with programming calculators or making spreadsheets. This *article* gives some of the ways students could program calculators. The article is glossed as extra credit possibilities. But really it should be hard to suggest that students, loaded up with their i-phones and computers, should ever think to use pencil and paper to solve quadratics, for example; it is embarrassing to the teaching profession. This is the sense in which algebra pedagogy has not changed nearly enough over the years. Regression analysis and programming calculators to solve real problems should be paramount, not doing algebra the same way it was done fifty or more years ago.

Four points each

Here are eleven programs. You will need to have these programs in your calculator, have a set of five demonstration problems for each program, and show me you know how to use the program. That is: I will give a problem and you will need to do it in front of me using a program on your calculator. You will need to establish that you have written the program yourself. That is: have your own calculator with programs that you have personally entered and tested. You should understand where the program came from. That is be able to show the algebra used to create the program. Guidance is given below.

1 DISC

This program gives the discriminant for a quadratic equation. The discriminant is the value $B^2 - 4AC$ and this value can be found within

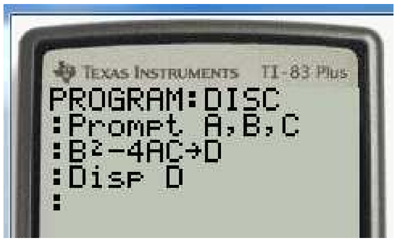


Figure 1: The DISC program gives the discriminant for a quadratic.

the quadratic formula: if $Ax^2 + Bx + C = 0$, then

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

Give five quadratics in the first column of the table; give the discriminant in the second column, and the types of roots in the third. If

Quadratic	Discriminant	OneR, TwoR, TwoC
1: $5x^2 + 2x + 7 = 0$;	2:	3:
4:	5:	6:
7:	8:	9:
10:	11:	12:
13:	14:	15:

Table 1: Make up five quadratics and fill in the table.

the discriminant is equal to zero, there is 16: real root. If the discriminant is greater than zero, there are 17: 18:
If the discriminant is less than zero, there are two complex roots.

Without simplifying, use the discriminant to solve your five quadratics. For example, for the first quadratic, $5x^2 + 2x + 7 = 0$, using the DISC program,

$$x = \frac{-2 \pm \sqrt{-136}}{10}.$$

Quadratic	Roots
19: $5x^2 + 2x + 7 = 0$;	20:
21:	22:
23:	24:
25:	26:
27:	28:

Table 2: Use the DISC program plus the quadratic formula to solve your quadratics.

2 DISCPLUS

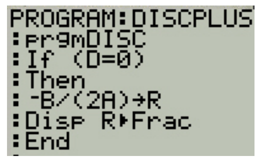


Figure 2: The DISCPLUS program gives a single root.

We can use the DISC program to make another program. The DISCPLUS program filters for DISC values of zero. These quadratics have one rational root. Perfect square quadratics of the form $(ax+b)^2$ have one root. Fill out the table.

Perfect square	Expanded	Root
$(3x+2)^2$	$9x^2+12x+4$	$-2/3$
29:	30:	31:
32:	33:	34:
35:	36:	37:
38:	39:	40:

Table 3: Make up five perfect square quadratics and fill in the table.

3 EVAL

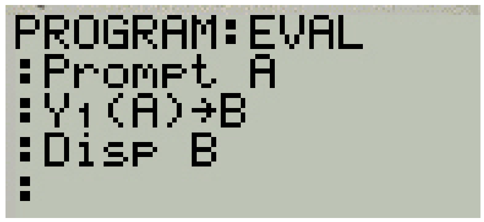


Figure 3: EVAL gives the value of Y_1 at A.

One can find the value of a function by entering it in the Y_1 variable and using the EVAL program. Test the program by making up a linear (L), quadratic (Q), rational (Rt), and radical function (Rd). For example, a radical function is $x^{3/2}$. Give two values that your function is to be evaluated at.

Function	$F(4)$	F(41:)
L 42:	43:	44:
Q 45:	46:	47:
Rt 48:	49:	50:
Rd $x^{3/2}$	8	51:

Table 4: Make up five quadratics and fill in the table.

Be able to demonstrate the table feature of your calculator to find various integer values of your functions. Indicate restrictions on the domain of rational and radical functions and show each function's graph as requested. The table will indicate ERROR when the function is not defined, i.e. a value is not in its domain. How do you show the value displayed as a rational?

4 FACT

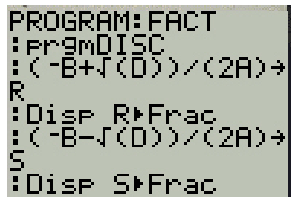


Figure 4: If DISC is a perfect square, roots are rational.

There are several ways to solve a quadratic equation: factoring, using the square root property, completing the square, and the quadratic formula. Factoring only works if the discriminant is a perfect square. We can reverse engineer such quadratics: $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$ can be factored. So, for example, $(2x+3)(5x-7) = 10x^2 + x - 21$ (FOIL it out) and the right side can be factored. Make up five such quadratics and use the FACT program to give their roots:

Quadratic	Expanded	Roots
$(2x+3)(5x-7)$	$10x^2 + x - 21$	$-3/2$, $7/5$
52:	53:	54:
55:	56:	57:
58:	59:	60:
61:	62:	63:

Table 5: Make up five quadratics that can be factored and fill in the table using FOIL and the FACT program.

Extra extra: Add code to test if the discriminant (DISC) is a perfect square or make a program for this test.

5 SIFROMGF

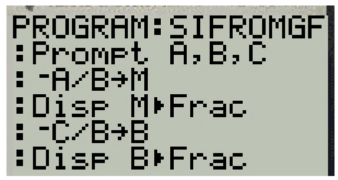


Figure 5: Given GF convert it to SI.

This program converts a line from general form (GF) $Ax+By+C=0$ to slope intercept form $y=Mx+B$ (SI). Make up five lines in general form and convert them to slope intercept using the following table.

GF	SI
$2x+5y-8=0$	$y=-\frac{2}{5}x+8/5$
64:	65:
66:	67:
68:	69:
70:	71:

Table 6: Make up five lines in GF and convert them to SI using the SIFROMGF program.

You should be able to do the derivation of the program. Here it is

$$Ax+By+C=0$$

gives

$$By=-Ax-C$$

which gives

$$y=-A/Bx-C/B.$$

So set $-A/B$ to M and $-C/B$ to B. You can over-ride the variable B with a new value, the y-intercept.

The code to store the result in Y_1 is “$(-A/B)X-C/B$” $\rightarrow Y_1$. Why would you want to store the line in Y_1? 72:

6 SLOPE

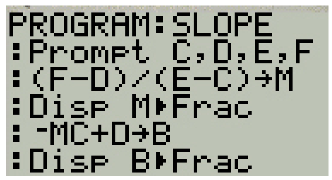


Figure 6: Given two points give SI.

Given two points (C, D) and (E, F) give the SI form of the line going through these points. Give an example of two points that form

Two points	SI
$(3, 8), (5, 6)$	$y = -1x + 11$
73:	74:
75:	76:
77:	78:
79:	80:
H: 81:	82:
V: 83:	84:

Table 7: Make up five pairs of points and give SI for line through the points.

a vertical line (V), form a horizontal line (H). What are the slopes of these types of lines: 85: ; 86: .

Here is the derivation.

$$\frac{(F - D)}{(E - C} = M$$

$$(Y - D) = M(X - C)$$

implies

$$Y = MX - MC + D$$

and this implies $B = -MC + D$. How do you show the resulting line using your calculator? 87:

7 PERPFGFP

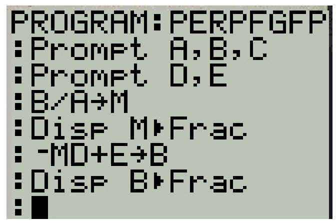


Figure 7: Given a line and a point find perpendicular through point.

GF $Ax + By + C = 0$	Point (D, E)	SI perp
$3x + 5y - 7 = 0$	$(3, 8)$	$y = \frac{5}{3}x + 3$
88:	89:	90:
91:	92:	93:
94:	95:	96:
97:	98:	99:

Table 8: Make up five lines in GF and five points.

Here is the derivation.

$$Ax + By + C = 0$$

implies $y = -A/B \quad x - C/A$, so a perpendicular slope (the negative reciprocal) is B/A. Set this to M. Now, using PS

$$y - E = M(x - D)$$

in SI,

$$y = Mx - MD + E.$$

Set $-MD + E$ to B, the y-intercept.

Extra extra: store the original line in Y_1 and its perpendicular in Y_2 and adjust the window using ZOOM/ZSQUARE. Do they look perpendicular?

Continue...

Be able to do five problems for each of the following programs and give the algebraic derivation for each. That is: make a table on separate sheets that give the five problems in a way similar to previous programs.

8 COMPINT

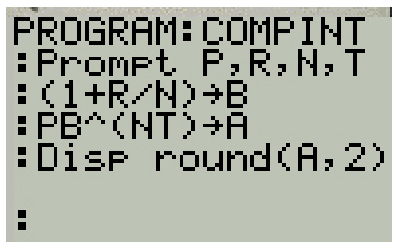


Figure 8: The storage to B is to keep scrolling from creating a new line.

The formula for compound interest is given by

$$A = P(1 + r/n)^{nt},$$

where P is the principal earning r annual interest, compounded n times a year, for t years.

Extra extra: Edit this program so that it displays all typical N values for the given P, T, and R. Typical N values are N = 2,4,12, and 365. That is interest compound semi-annually, quarterly, monthly, and daily. Add continuous compounding too. Use your book to find the formula for continuous compounding.

9 MINMAX

This program gives the coordinates of the vertex of the parabola generated by the quadratic $Ax^2 + Bx + C = 0$. The derivation converts this form of a quadratic to standard form: $A(x - h)^2 + k = 0$. The point (h, k) is the vertex. If $A < 0$, the parabola has a maximum of value k at $x = h$. If $A > 0$, the parabola has a minimum of value k at $x = h$.

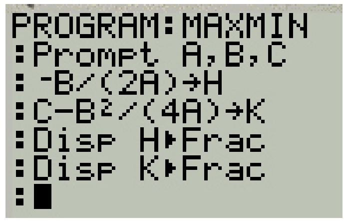


Figure 9: Doing it right one time allows for lots of problems to be solved easily and accurately.

Here's the derivation:

$$Ax^2 + Bx + C = 0$$

gives

$$x^2 + \frac{B}{A}x = -\frac{C}{A}.$$

Completing the square, we need

$$\left(\frac{1}{2}\frac{B}{A}\right)^2 = \frac{B^2}{4A^2}.$$

So:

$$x^2 + \frac{B}{A}x + \frac{B^2}{4A^2} = -\frac{C}{A} + \frac{B^2}{4A^2}$$

and thus

$$\left(x + \frac{B}{2A}\right)^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

and multiplying by A and moving the right side to the left gives

$$A\left(x + \frac{B}{2A}\right)^2 + C - \frac{B^2}{4A} = 0.$$

So

$$H = -\frac{B}{2A} \text{ and } K = C - \frac{B^2}{4A}.$$

Extra extra: Add code that indicates whether the vertex occurs at a MAX or MIN. Also store the quadratic in Y_1 and look at it using GRAPH.

10 EXPMODEL

Figure 10: For credit on this problem, store the model in Y1 (add the code).

The formula for exponential growth ($k > 0$) and decay ($k < 0$) is given by

$$A = A_0 e^{kt}.$$

Using two values of A, one at $t = 0$, call it F (as in first or initial value) and one at some other time t, call it L (as in last or current value) we can calculate k and obtain a model for the phenomenon. Here's the derivation.

$$L = Fe^{KT} \text{ implies } e^{KT} = \frac{L}{F}.$$

Taking ln of both sides gives

$$KT = \ln \frac{L}{F}, \text{ so } K = \frac{1}{T} \ln \frac{L}{F}.$$

Extra extra: Add the code necessary to store the model in Y_1 and use the TABLE feature of your calculator or the EVAL program to find values at various Ts.

11 GETTIME

Figure 11: How could the programs EXPMODEL and GETTIME be combined?

If A_0 and k are known in the model

$$A = A_0 e^{kt},$$

one can find the time it will take for a value A to be reached. Here's the derivation.

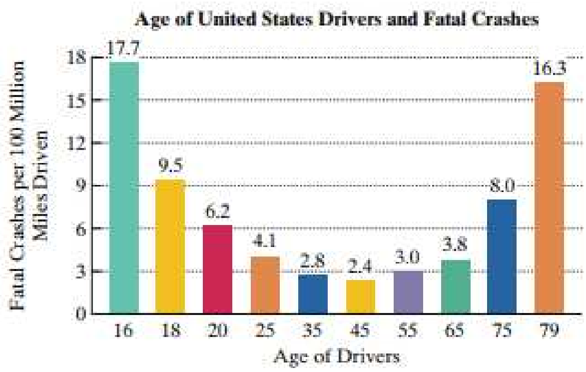
We want to solve for T, using $W = Fe^{KT}$, where, as before F is the first or initial value, W is the wanted value, and K is the constant for the model (computed with EXPMODEL above).

$$W = Fe^{KT} \text{ implies } e^{KT} = \frac{W}{F} \text{ implies } T = \frac{\ln W/F}{K}$$

Extra extra: Write a program (GETTIMES, adds an S) that assumes EXPMODEL is run first to get K and just asks for W. The variable values remain intact until the are assigned different values.

Regression problem (6 points)

A driver's age has something to do with his or her chance of getting into a fatal car crash. The bar graph shows the number of fatal vehicle crashes per 100 million miles driven for drivers of various age groups. For example, 25-year-old drivers are involved in 4.1 fatal crashes per 100 million miles driven. Thus, when a group of 25-year-old Americans have driven a total of 100 million miles, approximately 4 have been in accidents in which someone died.



Source: Insurance Institute for Highway Safety

The number of fatal vehicle crashes per 100 million miles, N, for drivers of age x can be modeled by the formula

$$N = 0.013x^2 - 1.19x + 28.24.$$

Use the formula to solve Exercises 135–136.

Figure 12: An example of finding a quadratic using regression with datapoints.

Input the datapoints for this problem and generate the best fit quadratic using your calculator. Show me the data points and the curve using STATPLOT.

Harder: solving systems of linear equations (12 points)

Here is the problem. Determine whether the lines $Ax + By + C = 0$ and $Dx + Ey + F = 0$ are the same (infinitely many solutions), parallel (no solutions), or have one solution or intersection point.

Next we work on the logic of the program. We put each line into SI form:

$$Ax + By + C = 0 \implies By = -Ax - C \implies y = \frac{-A}{B}x - \frac{C}{B}$$

$$Dx + Ey + F = 0 \implies Ey = -Dx - F \implies y = \frac{-D}{E}x - \frac{F}{E}.$$

We can store these in Y_1 and Y_2 and test the program with some lines. Here is the code for the first progam, SSLINEAR.

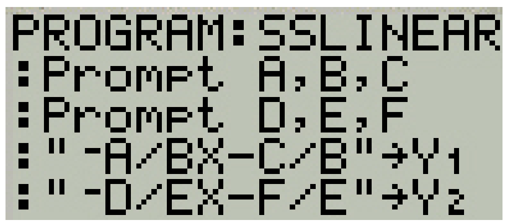


Figure 13: The result of our algebra is stored in the calculator.

Be able to input a system of two lines and use the CALC/INTERSECT feature of your calculator to find the intersection point for one solution systems. Have an example of a system with no solutions and infinitely many solutions.

Next we will do these last tasks with the calculator. The code documents the idea. If the slopes of the two lines are the same, the lines are either the same or our parallel. We know they are parallel if the slopes are the same, but the y-intercept is not the same. This is a nested "if then else end" structure.

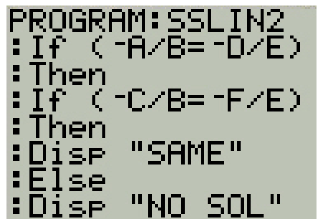


Figure 14: First part of SSLIN2.

Continuing, if they are neither the same nor parallel, they intersect. This program assumes that the first program, SSLINEAR has been run and the variables are still the same. Should we combine the programs? If I ask you to combine the programs, know how to do that.

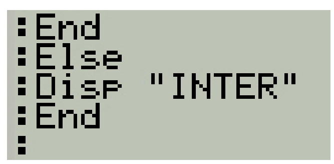


Figure 15: Second part of SSLIN2.

Finally, if the lines do intersect, what is the point of intersection. We can solve general systems for points of intersection using algebra. Here is the derivation. Set $y_1 = y_2$ and solve for x:

$$\frac{-A}{B}x - \frac{C}{B} = \frac{-D}{E}x - \frac{F}{E}$$

gives

$$x = \frac{\frac{C}{B} - \frac{F}{E}}{\frac{C}{B} - \frac{F}{E}} = \frac{CE - BF}{BD - AE}$$

We can feed this point into Y_1 to get the y-value of the intersection point. Here's the final program. It assumes SSLINEAR has been run.

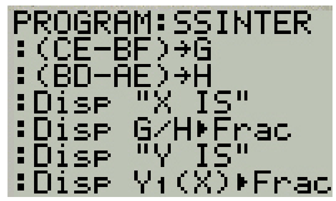


Figure 16: Caption for sslinear3

Have five systems ready for each of the cases: same, no solution, one intersection point.