SOME CONJECTURES ON INEQUALITIES IN OPERATOR AXIOMS

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Abstract. The Operator axioms have deduced number systems. A complete calculator has been invented as a software calculator to execute complete operations. In this paper, we conjecture some inequalities in Operator axioms. The complete calculator is applied to verify all the conjectures in this paper. The general inequalities show the value of Operator axioms.

1. Introduction

In [3], we define the Operator axioms to extend the classical real number system. The classical real number system only includes special irrational numbers. However, Operator axioms import the general irrational numbers to construct a complete real number system. In this paper, we apply Operator axioms to generalize various inequalities. The general Bernoulli's Inequality show the value of Operator axioms.

In [4, TABLE 2], we have defined some replacements for the notations of the Operator axioms. In this paper, we will apply these concise replacements. For convenience, we define two symbols '$>$' and '$\geq$'. Then we add two axioms as follows into Operator axioms:

$$
\begin{array}{ll}
 & \Psi\{ \\
(OA.116) & (\bar{a} > \bar{b}) \Leftrightarrow (\bar{b} < \bar{a}), \\
(OA.117) & (\bar{a} \geq \bar{b}) \Leftrightarrow (\bar{b} \leq \bar{a}) \\
 & \}.
\end{array}
$$

In [5], we invent a complete calculator as a software calculator to execute complete operations. The experiments on the complete calculator could directly prove such a corollary: Operator axioms are consistent. The complete calculator in [5] is applied to verify all the conjectures in this paper.

In this paper, we suppose that $n \in N$, and we suppose that $a \in [1, +\infty)$, $b \in [1, +\infty)$, $c \in [1, +\infty)$ are real numbers.

The paper is organized as follows. In Section 2, we conjecture some commutative inequalities in Operator axioms. In Section 3, we conjecture some distributive inequalities in Operator axioms. In Section 4, we define the general distance inequality in Operator Axioms and conjecture a general distance inequality in Operator Axioms. In Section 5, we conjecture a general Bernoulli's inequality in Operator axioms.

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2. Commutative Inequalities In Operator Axioms

Conjecture 2.1. *The following inequality holds:*

$$[[a +'_n a] + [b +'_n b]] \geq [[a +'_n b] + [b +'_n a]]$$

Example 2.2.

$$n = 4, a = 1.3, b = 2.2 :$$
$$[[1.3 +'_4 1.3] + [2.2 +'_4 2.2]] \approx 8.102245853$$
$$[[1.3 +'_4 2.2] + [2.2 +'_4 1.3]] \approx 4.109254405$$
$$[[1.3 +'_4 1.3] + [2.2 +'_4 2.2]] \geq [[1.3 +'_4 2.2] + [2.2 +'_4 1.3]]$$

The Conjecture 2.1 holds for the above example.

Example 2.3.

$$n = 5, a = 1.3, b = 2.2 :$$
$$[[1.3 +'_5 1.3] + [2.2 +'_5 2.2]] \approx 8.082276060$$
$$[[1.3 +'_5 2.2] + [2.2 +'_5 1.3]] \approx 3.526901423$$
$$[[1.3 +'_5 1.3] + [2.2 +'_5 2.2]] \geq [[1.3 +'_5 2.2] + [2.2 +'_5 1.3]]$$

The Conjecture 2.1 holds for the above example.

Conjecture 2.4. *The following inequality holds:*

$$[[a +'_n a] + +[b +'_n b]] \geq [[a +'_n b] + +[b +'_n a]]$$

Example 2.5.

$$n = 4, a = 1.3, b = 2.2 :$$
$$[[1.3 +'_4 1.3] + +[2.2 +'_4 2.2]] \approx 8.965070619$$
$$[[1.3 +'_4 2.2] + +[2.2 +'_4 1.3]] \approx 3.806470792$$
$$[[1.3 +'_4 1.3] + +[2.2 +'_4 2.2]] \geq [[1.3 +'_4 2.2] + +[2.2 +'_4 1.3]]$$

The Conjecture 2.4 holds for the above example.

Example 2.6.

$$n = 5, a = 1.3, b = 2.2 :$$
$$[[1.3 +'_5 1.3] + +[2.2 +'_5 2.2]] \approx 8.817037543$$
$$[[1.3 +'_5 2.2] + +[2.2 +'_5 1.3]] \approx 2.915136160$$
$$[[1.3 +'_5 1.3] + +[2.2 +'_5 2.2]] \geq [[1.3 +'_5 2.2] + +[2.2 +'_5 1.3]]$$

The Conjecture 2.4 holds for the above example.

Conjecture 2.7. *The following inequality holds:*

$$[[a -'_n a] + [b -'_n b]] \leq [[a -'_n b] + [b -'_n a]]$$

Example 2.8.

$$n = 4, a = 1.3, b = 2.2 :$$
$$[[1.3 -'_4 1.3] + [2.2 -'_4 2.2]] \approx 2.889521132$$
$$[[1.3 -'_4 2.2] + [2.2 -'_4 1.3]] \approx 3.163781170$$
$$[[1.3 -'_4 1.3] + [2.2 -'_4 2.2]] \leq [[1.3 -'_4 2.2] + [2.2 -'_4 1.3]]$$

The Conjecture 2.7 holds for the above example.

Example 2.9.

$$n = 5, a = 1.3, b = 2.2 :$$
$$[[1.3 -'_5 1.3] + [2.2 -'_5 2.2]] \approx 2.981197458$$
$$[[1.3 -'_5 2.2] + [2.2 -'_5 1.3]] \approx 3.477851237$$
$$[[1.3 -'_5 1.3] + [2.2 -'_5 2.2]] \leq [[1.3 -'_5 2.2] + [2.2 -'_5 1.3]]$$

The Conjecture 2.7 holds for the above example.

Conjecture 2.10. *The following inequality holds:*

$$[[a -'_{[n+1]} a] + +[b -'_{[n+1]} b]] \leq [[a -'_{[n+1]} b] + +[b -'_{[n+1]} a]]$$

Example 2.11.

$$n = 4, a = 1.3, b = 2.2 :$$
$$[[1.3 -'_5 1.3] + +[2.2 -'_5 2.2]] \approx 2.185551227$$
$$[[1.3 -'_5 2.2] + +[2.2 -'_5 1.3]] \approx 2.815436530$$
$$[[1.3 -'_5 1.3] + +[2.2 -'_5 2.2]] \leq [[1.3 -'_5 2.2] + +[2.2 -'_5 1.3]]$$

The Conjecture 2.10 holds for the above example.

Example 2.12.

$$n = 5, a = 1.3, b = 2.2 :$$
$$[[1.3 -'_6 1.3] + +[2.2 -'_6 2.2]] \approx 2.302761691$$
$$[[1.3 -'_6 2.2] + +[2.2 -'_6 1.3]] \approx 2.859968449$$
$$[[1.3 -'_6 1.3] + +[2.2 -'_6 2.2]] \leq [[1.3 -'_6 2.2] + +[2.2 -'_6 1.3]]$$

The Conjecture 2.10 holds for the above example.

Conjecture 2.13. *The following inequality holds:*

$$[[a/'_n a] + [b/'_n b]] \leq [[a/'_n b] + [b/'_n a]]$$

Example 2.14.

$$n = 4, a = 1.8, b = 2.2 :$$
$$[[1.8/'_4 1.8] + [2.2/'_4 2.2]] \approx 2.000000000$$
$$[[1.8/'_4 2.2] + [2.2/'_4 1.8]] \approx 2.064047222$$
$$[[1.8/'_4 1.8] + [2.2/'_4 2.2]] \leq [[1.8/'_4 2.2] + [2.2/'_4 1.8]]$$

The Conjecture 2.13 holds for the above example.

Example 2.15.

$$n = 5, a = 1.8, b = 2.2 :$$
$$[[1.8/'_5 1.8] + [2.2/'_5 2.2]] \approx 2.000000000$$
$$[[1.8/'_5 2.2] + [2.2/'_5 1.8]] \approx 2.519588588$$
$$[[1.8/'_5 1.8] + [2.2/'_5 2.2]] \leq [[1.8/'_5 2.2] + [2.2/'_5 1.8]]$$

The Conjecture 2.13 holds for the above example.

3. Distributive Inequalities In Operator Axioms

Conjecture 3.1. *The following inequality holds:*

$$[[a +'_{[n+2]} [b + c]]] \geq [[a +'_{[n+2]} b] +'_{[n+1]} [a +'_{[n+2]} c]]$$

Example 3.2.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 +'_5 [1.2 + 1.4]]] \approx 75.642908557$$
$$[[2.2 +'_5 1.2] +'_4 [2.2 +'_5 1.4]] \approx 7.324328863$$
$$[[2.2 +'_5 [1.2 + 1.4]]] \geq [[2.2 +'_5 1.2] +'_4 [2.2 +'_5 1.4]]$$

The Conjecture 3.1 holds for the above example.

Conjecture 3.3. *The following inequality holds:*

$$[[a +'_{[n+1]} [b + +c]]] \geq [[a +'_{[n+1]} b] +'_{[n+1]} c]$$

Example 3.4.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 +'_4 [1.2 + +1.4]]] \approx 4.508462027$$
$$[[2.2 +'_4 1.2] +'_4 1.4] \approx 3.790435091$$
$$[[2.2 +'_4 [1.2 + +1.4]]] \geq [[2.2 +'_4 1.2] +'_4 1.4]$$

The Conjecture 3.3 holds for the above example.

Conjecture 3.5. *The following inequality holds:*

$$[[a -'_{[n+2]} [b + c]]] \leq [[a -'_{[n+2]} b] +'_{[n+1]} [a -'_{[n+2]} c]]$$

Example 3.6.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 -'_5 [1.2 + 1.4]]] \approx 1.666746975$$
$$[[2.2 -'_5 1.2] +'_4 [2.2 -'_5 1.4]] \approx 6.403681425$$
$$[[2.2 -'_5 [1.2 + 1.4]]] \leq [[2.2 -'_5 1.2] +'_4 [2.2 -'_5 1.4]]$$

The Conjecture 3.5 holds for the above example.

Conjecture 3.7. *The following inequality holds:*

$$[[a + b] +'_{[n+1]} c] \geq [[a +'_{[n+1]} c] + [b +'_{[n+1]} c]]$$

Example 3.8.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 + 1.2] +'_4 1.4] \approx 9.560317294$$
$$[[2.2 +'_4 1.4] + [1.2 +'_4 1.4]] \approx 4.284962102$$
$$[[2.2 + 1.2] +'_4 1.4] \geq [[2.2 +'_4 1.4] + [1.2 +'_4 1.4]]$$

The Conjecture 3.7 holds for the above example.

Conjecture 3.9. *The following inequality holds:*

$$[[a + +b] +'_{[n+2]} c] \geq [[a +'_{[n+2]} c] + +[b +'_{[n+2]} c]]$$

Example 3.10.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 + +1.2] +'_5 1.4] \approx 2.791867556$$
$$[[2.2 +'_5 1.4] + +[1.2 +'_5 1.4]] \approx 2.688498324$$
$$[[2.2 + +1.2] +'_5 1.4] \geq [[2.2 +'_5 1.4] + +[1.2 +'_5 1.4]]$$

The Conjecture 3.9 holds for the above example.

Conjecture 3.11. *The following inequality holds:*

$$[[a + b] -'_{[n+1]} c] \leq [[a -'_{[n+1]} c] + [b -'_{[n+1]} c]]$$

Example 3.12.

$$n = 3, a = 2.2, b = 1.2, c = 1.4 :$$
$$[[2.2 + 1.2] -'_4 1.4] \approx 2.310492137$$
$$[[2.2 -'_4 1.4] + [1.2 -'_4 1.4]] \approx 3.024845840$$
$$[[2.2 + 1.2] -'_4 1.4] \leq [[2.2 -'_4 1.4] + [1.2 -'_4 1.4]]$$

The Conjecture 3.11 holds for the above example.

Conjecture 3.13. *The following inequality holds:*

$$[[a++b]-'_{[n+1]}c] \leq [[a-'_{[n+1]}c]++[b-'_{[n+1]}c]]$$

Example 3.14.

$$n=3, a=2.2, b=1.2, c=1.4:$$
$$[[2.2++1.2]-'_4 1.4] \approx 2.036559160$$
$$[[2.2-'_4 1.4]++[1.2-'_4 1.4]] \approx 2.181256807$$
$$[[2.2++1.2]-'_4 1.4] \leq [[2.2-'_4 1.4]++[1.2-'_4 1.4]]$$

The Conjecture 3.13 holds for the above example.

Conjecture 3.15. *The following inequality holds:*

$$[[a+b]/'_{[n+1]}c] \geq [[a/'_{[n+1]}c]+[b/'_{[n+1]}c]]$$

Example 3.16.

$$n=3, a=2.2, b=1.2, c=1.8:$$
$$[[2.2+1.2]/'_4 1.8] \approx 2.370089873$$
$$[[2.2/'_4 1.8]+[1.2/'_4 1.8]] \approx 1.777362187$$
$$[[2.2+1.2]/'_4 1.8] \geq [[2.2/'_4 1.8]+[1.2/'_4 1.8]]$$

The Conjecture 3.15 holds for the above example.

4. General Bernoulli's Inequality In Operator Axioms

Conjecture 4.1. *If $d \in [2,+\infty)$, then the following inequality holds:*

$$[[1+a]+'_{[n+1]}d] \geq [1+[a+'_n d]]$$

Example 4.2.

$$n=4, a=1.2, d=2.5:$$
$$[[1+1.2]+'_5 2.5] \approx 11.274544787$$
$$[1+[1.2+'_4 2.5]] \approx 2.249584869$$
$$[[1+1.2]+'_5 2.5] \geq [1+[1.2+'_4 2.5]]$$

The Conjecture 4.1 holds for the above example.

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