Calculating “Speeding to Andromeda” easier

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Abstract: I derive an equation to calculate the constant speed needed by an unpowered rocket, such that its crew ages a given time during a trip of a given distance.

Speeding to Andromeda

See "Speeding to Andromeda" at Chapter 1 of Exploring Black Holes:

> At approximately what constant speed with respect to our Sun must a spaceship travel so that its occupants age only 1 year during a trip from Earth to the Andromeda galaxy?

The method therein to get the answer (v = 0.999999999999875c) requires several steps and assumes the speed is close to c. Here is an equation that works in every case, using the variables defined at The Relativistic Rocket, and in geometric units:

$$v = \frac{d/T}{\sqrt{1 + (d/T)^2}} \quad [1]$$

When d = 2 million light years and T = 1 year, the equation returns v = 0.999999999999875c. The speed required to get to Andromeda while aging 1 year is the speed that length-contracts the distance to Andromeda to that which is traversed in 1 year at that speed. Time dilation and length contraction go hand-in-hand that way.

See also about rapidity, a convenient way to express velocities close to c, at How Do You Add Velocities in Special Relativity?. The velocity above corresponds to a rapidity of atanh(0.999999999999875) = ~15.

The derivation of eq. 1

From basic physics:

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$$t = \frac{d}{v}$$

From Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

From Time Dilation:

$$t = \frac{T}{\sqrt{1 - v^2}}$$

From The Relativistic Rocket:

$$v = \frac{at}{\sqrt{1 + (at)^2}} \quad [2]$$

$$\gamma = \sqrt{1 + (at)^2}$$

Substituting and rearranging:

$$t = T\gamma = \frac{d}{v}$$

$$\frac{d}{T} = v\gamma$$

$$v = \frac{at}{\gamma}$$

$$v\gamma = at = \frac{d}{T}$$

Substituting into eq. 2 gives eq. 1.