Dirac's philosophical principles

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Abstract

This paper examines Paul Dirac's assumptions and principles in the introduction to his *Principles of Quantum Mechanics* as part of a larger logical, philosophical and epistemological reflection on why a realist interpretation of quantum mechanics would or would not be possible.

Contents

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Introduction

Paul Dirac was not known for being wordy. It is said his colleagues in Cambridge jokingly defined the 'dirac' as a new unit, which was defined as one word per hour. However, the first chapter of his *Principles of Quantum Mechanics* – which deals with the principles of superposition and linearity – is surprisingly verbose. While it introduces some of the weirdness of quantum math, it also explores the basic *intuition* behind the theory. It might, therefore, be useful to discuss these intuitions and see if and how Dirac's basic assumptions, principles and intuition might prevent us – or not – from a common-sense or *realist* understanding of quantum mechanics.

In order to do so, this paper will follow the same structure and logical flow of the mentioned text (Chapter I of Dirac's *Principles of Quantum Mechanics*): it consists of six sections, whose headings will also be used as heading in this paper.

1. The Need for a Quantum Theory

The black-box radiation problem

Dirac starts his story with an introduction that is now standard in almost any textbook on physics: the 'necessity for a departure from classical mechanics' is motivated by what is usually referred to as the black-body radiation problem or the Rayleigh-Jeans catastrophe. In fact, Dirac does not mention this problem explicitly but refers to the more general problem of explaining the specific heat of gases using classical laws. To put it simply, classical theory assumes the energy of the gas molecules (monatomic or polyatomic) will be distributed evenly over the various modes of vibration, rotation or whatever other internal motion one can imagine. This theory then yields a *theoretical* value for the specific heat which should *not* vary with temperature but experiments show this theoretical value approaches the *measured* value only at extreme temperatures.[1]

Clearly, another explanation is needed, and we now know that this and other related phenomena are easily explained when applying the Planck-Einstein law: we should think of atoms as atomic oscillators whose possible energy *states* are given by the $E_n = n\cdot h\cdot f = n\cdot\hbar\cdot\omega$ law. In fact, it is often said that the black-body radiation problem (or whatever other manifestation of this so-called failure of classical mechanics) shows that one should *not* apply the classical energy equipartition theorem to explain quantum mechanics but the Planck-Einstein law is, in fact, just another expression of it: the energy states $E_1 = h\cdot f$, $E_2 = 2\cdot h\cdot f$, $E_3 = 3\cdot h\cdot f$,…, $E_n = n\cdot h\cdot f$ are all separated by the same amount of energy: $E_n - E_{n-1} = \hbar\cdot\omega = h\cdot f$.

[1] This is standard physics and, hence, we could give a dozen references here. However, we find Feynman's overview of the matter (*Feynman's Lectures*, Vol. I, Chapter 40) both complete as well as very accessible and so we borrowed his illustration here.

As such, I actually like to think this particular example of the so-called failure of classical mechanics does not quite cut it: the likes of Jeans, Raleigh and Wien were, correctly, assuming evenly spaced energy states or *modes* based on some kind of resonant or natural frequency but they just couldn't figure out their fundamental nature. Hence, I'd say the application of quantum mechanics to the problem *extended* the validity of classical laws rather than invalidating them.

Before we move onto Dirac's next reflection, it is probably useful to add a few words on the idea of a natural or resonant frequency of an atom. The Planck-Einstein relation tells us we should be able to calculate it using the $f = E/h$ equation, but what energy should we use? The mass of a hydrogen atom is pretty huge as compared to the energy of the photons an atom will absorb or emit when transitioning from one energy state to another.

To be precise, the mass of a hydrogen atom is 1.008 *amu*, so that's $1.00784 \cdot 931.5\ \text{MeV}/c^2 \approx 938.8\ \text{MeV}/c^2$. That is – roughly – about 2,000 times the rest mass of an electron ($0.511\ \text{MeV}/c^2$). In turn, the (rest) energy of an electron is of an entirely different order of magnitude of the energy of the photons that the hydrogen atom emits or absorbs when transitioning from one energy state to another. Let us consider, for example, the transition from the second to the first level. The energy *difference* will be given by the Rydberg formula:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \frac{\alpha^2 \mathrm{m} c^2}{2}$$

The $(1/n_1)^2 - (1/n_2)^2$ factor here is equal to $1/1^2 - 1/2^2 = 0.75$. Hence, the photon energy is equal to $(0.75) \cdot E_R \approx 10.2$ eV. Fairly energetic[2] but this is only about 1/50,000 of the energy of the electron itself. Hence, the relevant energy concept in the $E_1 = h \cdot f$, $E_2 = 2 \cdot h \cdot f$, $E_3 = 3 \cdot h \cdot f$,…, $E_n = n \cdot h \cdot f$ is the energy of the *orbital oscillation* itself: this energy concept *excludes* the rest mass of the electron and, of course, the rest mass of the atom as a whole.

At this point, we should correct ourselves and point out that the classical energy equipartition theorem morphs into something that is far more fundamental: the electron orbitals are separated *not* by equal energies $h \cdot f$ but by *equal amounts of physical action*: Planck's quantum of action (not $E = h \cdot f$) is the fundamental unit here and we, therefore, do *not* have one single frequency: the frequency depends on the orbital, as shown in Table 1. This table gives us the classical calculations for all variables based on the assumption that the 1st, 2nd, n^{th} orbital effectively (1) packs an amount of energy that is equal to $E_1 = h \cdot f_1$, $E_2 = 2 \cdot h \cdot f_2$, $E_3 = 3 \cdot h \cdot f_3$,…, $E_n = n \cdot h \cdot f_n$ and (2) is to be associated with an angular moment $L_1 = \hbar$, $L_2 = 2\hbar$, $L_3 = 3\hbar$,…, $L_n = n \cdot \hbar$ or – what amounts to the same – an amount of (physical) action that is equal to $S_1 = h$, $S_2 = 2h$, $S_3 = 3h$,…, $S_n = n \cdot h$.

[2] This is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. The ozone layer of our atmosphere blocks most of it.

Table 1: Classical calculations for the Bohr orbitals

$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v_n = \frac{1}{n}\alpha c$
$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}} n\hbar$
$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

The point is: the Ritz combination principle, the specific heat of gases, the physical properties that we associate with electron orbitals and other presumed 'clashes between classical mechanics and the results of experiment' can all be explained by one simple rule: the quantization of angular momentum. This rule associates the Planck's unit – the quantum of action – with an elementary cycle. All other physical results can then be explained in a direct and straightforward manner. Hence, Dirac's conclusion that these phenomena illustrate "the breakdown of classical mechanics" and that this breakdown is "not merely an inaccuracy in its laws of motion, but *an inadequacy of its concepts to supply us with a description of atomic events*" sounds rash and premature.

Of course, one might still think that Dirac's call for "a new scheme" may be justified for some other reason. The weird wave-particle character of the electron comes to mind here: can classical mechanics deal with that? We believe it can. Dirac himself actually acknowledged this possibility when writing about Schrödinger's discovery of the *Zitterbewegung*, which he stumbled upon while exploring solutions to Dirac's wave equation for free electrons. It's worth quoting Dirac's summary of it:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

David Hestenes is to be credited with the revival of what is now referred to as the *Zitterbewegung* model of an electron[3] and we believe it is a strong contender for a *realist* interpretation of quantum mechanics. We will limit ourselves to noting that the *hybrid* description of an electron as a pointlike charge whizzing around some center at the speed of light may be a viable alternative.

Alternative to what? An alternative to the orthodox interpretation of an electron as some 'black box' that has no internal structure but, at the same time, does have properties such as angular momentum and a magnetic moment which – because of the 'black box' philosophy that is inherent to the Copenhagen interpretation of QM – cannot be explained.[4] We have written about this elsewhere[5] and, hence, we should not repeat ourselves here.

The behavior of light

To be fair, Dirac writes what he writes above as part of a second line of attack: the wave-particle character of *light*. We are much more sympathetic to these arguments because interference and diffraction of *photons* is, effectively, a most remarkable phenomenon and, while it is not that difficult to develop a consistent model of an electron combining wave and particle characteristics, a similar model for a photon raises various questions that cannot be answered easily. Let us quote Dirac's introduction to the matter on hand:

> "We have, on the one hand, the phenomena of interference and diffraction, which can be explained only on the basis of a wave theory; on the other, phenomena such as photo-electric emission and scattering by free electrons, which show that light is composed of small particles. These particles, which are called photons, have each a definite energy and momentum, depending on the frequency of the light, and appear to have just as real and existence as electrons, or any other particles known in physics. *A fraction of a photon is never observed*."

The italics are mine, and I italicized Dirac's because it does adequately summarize the philosophical issue here. The *Zitterbewegung* model of an electron assumes an electron consists of some pointlike charge: a indivisible naked charge that has no properties but its charge and – we should add – its size[6]. We can, therefore, envision an explanation for electron diffraction and interference[7] (think of the

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[3] See: David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics* (http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf) and David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a Research Program* (https://arxiv.org/pdf/0802.2728.pdf).

[4] Dirac boldly states that such things "cannot by investigated by experiment" and that they should, therefore, be regarded as "outside the domain science." In light of more recent experiments involving the weak measurement of quantum-mechanical variables, such statement comes across as ideological and, therefore, non-scientific itself.

[5] Jean Louis Van Belle, *The Electron as a Harmonic Electromagnetic Oscillator* (http://vixra.org/abs/1905.0521), 31 May 2019.

[6] While the *Zitterbewegung* charge is considered to be pointlike, it is *not* dimensionless. We infer its radius from elastic photon scattering experiments: it is equal to the classical or *Thomson* electron radius, which is $\alpha \approx 1/137$ times the *Compton* radius of the electron, which we get from inelastic (Compton) scattering experiments. Inelastic scattering occurs when high-energy photons (the light is X- or gamma-rays, with high frequency and very small wavelength) hit the electron. Their energy is briefly absorbed before the electron comes back to its equilibrium situation by emitting another photon whose energy is *less*, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum. It is because of the interference effect that Compton scattering is referred to as inelastic. In contrast, low-energy photons scatter elastically: the photon seems to bounce off some hard *core*: there is no interference. This picture is consistent with the *Zitterbewegung* model of an electron: the hard core is just the pointlike charge itself, and its radius is equal to a fraction (α) of the Compton radius: $r_e = \alpha \cdot a \approx a/137 \approx 2.818 \times 10^{-15}$ m.

[7] Interference and diffraction do not have a unambiguous description in physics but – for ease of reference – we would associate diffraction with a one-slit experiment, while interference patterns result from two or more slits.

famous two-slit experiment here[8]) that in terms of this hybrid description—a pointlike charge whizzing around some center in an electromagnetic oscillation. The charge will always be *somewhere*: we just don't know where *exactly* because of its phenomenal velocity (***v*** = ***c***). Hence, the idea of the *charge* always hitting the detector as one single lump[9] comes quite naturally to us. At the same time, we can imagine the electromagnetic field would, somehow, travel through two slits simultaneously or, when traveling through one slit only, that its *direction of travel* might change.[10] Hence, the idea of a diffraction or interference pattern forming as we're sending many electrons – mind you: one-by-one! – through the slit makes sense as well.

However, applying such logic to some model of a photon is, perhaps, not so easy. What photon model could we be thinking of? The Bohr orbitals are separated by a amount of *action* that is equal to *h*. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose or gain one unit of *h*. We think the photon that is emitted or absorbed will have to pack that, somehow. It will also have to pack the related energy, which is given by the Rydberg formula. To focus our thinking, we can consider the transition from the second to the first level once again. We calculated the photon energy as 10.2 eV. Now, if the action in this oscillation is equal to *h*, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135\times10^{-15}\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}} \approx 0.4\times10^{-15}\ \mathrm{s}$$

This corresponds to a wave train with a length of $(3\times10^{8}\ \mathrm{m/s})\cdot(0.4\times10^{-15}\ \mathrm{s}) = 122$ nm. That is the size of a large molecule and it is, therefore, much more reasonable than the length of the wave trains we get when thinking of transients using the supposed Q of an atomic oscillator.[11] In fact, this length is *exactly* equal to the wavelength $\lambda = c/f = c\cdot\mathrm{T} = hc/\mathrm{E}$.

What picture of the photon are we getting here? Because of the angular momentum, we will probably want to think of it as a circularly polarized wave, which we may represent by the elementary wavefunction, as shown below.[12] We will call this interpretation of the wavefunction the *one-cycle photon*: the wavefunction represents the rotating field vector itself or, remembering the $F = q_e E$ equation, the force field.

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[8] It is useful to point out that this experiment is not a thought experiment only. Nanotechnology has made it possible to actually *perform* the experiment. For an overview, see: https://en.wikipedia.org/wiki/Double-slit_experiment.

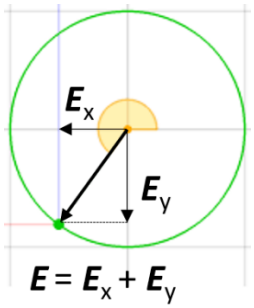
[9] A lump is the term Feynman uses when describing the particle-like character of an electron in the two-slit interference experiment (Feynman's *Lectures*, Vol. III-1)

[10] We like to write the Uncertainty Principle in terms of *vector quantities*. One possibility is to also write Planck's quantum of action as a vector quantity (angular momentum is a vector quantity and, hence, physical action could be too), but our manuscript (http://vixra.org/abs/1901.0105) explores other possibilities as well.

[11] In one of his *Lectures* (I-32-3), Feynman thinks about a sodium atom, which emits and absorbs sodium light, of course. Based on various assumptions – assumptions that make sense in the context of the blackbody radiation model but *not* in the context of the Bohr model – he gets a Q of about 5×10^{7}. Now, the frequency of sodium light is about 500 THz (500×10^{12} oscillations per second). Hence, the *decay time* of the radiation is of the order of 10^{-8} seconds. So that means that, after 5×10^{7} oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm (600×10^{-9} meter), we get a wave train with a considerable length: $(5\times10^{6})\cdot(600\times10^{-9}\ \mathrm{meter}) = 3$ meter. *Surely you're joking, Mr. Feynman!* A photon with a length of 3 meter – or longer? While one might argue that relativity theory saves us here (relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light), this just doesn't feel right – especially when one takes a closer look at the assumptions behind.

[12] Note that the wave could be either left- or right-handed.

Figure 1: The one-cycle photon



It is a delightfully simple model: the photon is just one single cycle traveling through space and time, which packs one unit of angular momentum ($\hbar$) or – which amounts to the same, one unit of physical action (h). This gives us an equally delightful interpretation of the Planck-Einstein relation (f = 1/T = E/h) and we can, of course, do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the E = mc^2 mass-energy equivalence relation can be written as p = mc = E/c for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

Nice but, as Dirac would say, this *interpretation* cannot be investigated by experiment and may, therefore, be regarded as 'outside the domain of science.' At the same time, if we would be able to show such photon model explains diffraction and interference, then we may also introduce some rule that says Nature respects the *integrity* of the photon cycle and that it will, therefore, also arrive as a single *lump* of energy when the detector absorbs it.

Again, we're not saying that we have some consistent hidden variables theory here – let alone that we could *prove* such theory – but we are saying that Dirac's conclusion that wave-particle duality somehow amounts to a total "breakdown of classical mechanics" and that classical concepts cannot possibly supply us with an adequate description of reality just sounds rash and premature. Dirac discusses one-photon interference experiments in the next section of the chapter that we're analyzing here, so we will come back to this. Before we do so, we want to mention another of Dirac's arguments against a realist interpretation of quantum mechanics.

The ultimate structure of matter

Dirac's argument is rather lengthy here and hence, we will try to summarize it by just quoting some essential point in the argument as he develops it[13]:

> "In a classical explanation of the constitution of matter, one would assume it to be made up of a large number of small constituent parts and one would postulate laws for the behavior of these parts, from which the laws of the matter in bulk could be deduced. […] So long as *big* and *small* are merely relative concepts, it is no help to explain the big in terms of the small. It is therefore necessary to modify classical ideas in such a way as to give an absolute meaning to size. […] In order to give an absolute meaning to size, such as is required for any theory of the ultimate structure of matter, we have to assume that *there is a limit to the fineness of our powers of observation and the smallness of the accompanying disturbance—a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer.* [Dirac's italics.] If the object under observation is such that the unavoidable limiting disturbance is negligible, then the object is big in the absolute sense and we may apply classical mechanics to it. If, on the other hand, the limiting disturbance is not negligible, then the object is small in the absolute sense and we require a new theory for dealing with it."

This argument is, obviously, purely philosophical, and Dirac admits so much. It is purely philosophical because it depends on Dirac's definition of a 'classical explanation' and of what would be new in 'a new theory.' As far as we are concerned, there is nothing in classical theory which would prevent it from analyzing "small systems that cannot be observed without producing a serious disturbance" and there is also nothing in classical theory that is inconsistent with the idea of matter having some "ultimate structure." In fact, that's the whole idea of the *Zitterbewegung* model of an electron which – let us remind the reader – is an idea that Dirac himself acknowledged in his 1933 Nobel Prize lecture.

The simplest of models[14] may actually be best suited to show how we should imagine the 'lumpiness' or 'quantization' of matter or – to use Dirac's words – how classical laws give "an absolute meaning to size." We can take Einstein's mass-energy equivalence relation ($E = m \cdot c^2$) and, interpreting c as the tangential velocity of the naked charge (or the toroidal photon, as Burinskii refers to it), we can substitute c for $a \cdot \omega$: the tangential velocity equals the radius times the angular frequency. We then can then use the Planck-Einstein relation ($E = \hbar \cdot \omega$) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{m \cdot c^2} = \frac{\hbar}{m \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – packs (i) the electron's energy ($E = m \cdot c^2$) as well as (ii) one unit of physical action ($S = h$). The idea of an oscillation packing some amount of physical action may not be very familiar but it is quite simple: just re-write the Planck-Einstein relation as $h = E \cdot f = E/T$. The cycle time $T = h/E$ is equal to:

[13] In case the reader would feel there is a risk to us doing injustice to the original, he or she can always examine the original text.

[14] For *zbw* models that are far more advanced than ours, see David Hestenes (1990, 2008) and Alexander Burinskii (2008, 2016, 2017).

$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

Hence, this cycle time T is the time it takes for the *zbw* charge (or the naked charge, if you prefer that term) to go around the loop (λ_C) at the extreme velocity we assume it has ($v = c$):

$$\mathrm{T} = \lambda_C/c = (h/\mathrm{m}c)\cdot(1/c) = h/\mathrm{E}$$

Physical action is the product of force, distance and time and it is, therefore, easy to associate it with the idea of a cycle—and elementary cycle, in this particular case. It is just an interpretation of the general quantum-mechanical rule that angular momentum comes in units of ħ: the $\mathrm{S} = n\cdot\mathrm{h}$ and $\mathrm{L} = n\cdot\hbar$ (n = 1, 2,…) are, therefore, equivalent.

Preliminary conclusions

Dirac rightly points out there is a need for a quantum theory. However, nothing in the introduction so far shows us why the Planck-Einstein relation would not embody such theory—or why it would not *completely* embody such theory. In other words, none of what Dirac writes makes a convincing case *against* a common-sense or *realist* interpretation of the quantum-mechanical equations and formulas we are acquainted with (Schrödinger's equation, wavefunctions, the *de Broglie* relations and the related expressions for the Uncertainty Principle, etcetera).

Hence, we would like to see some more advanced arguments. Dirac may or may not provide such arguments in the second section of his introductory chapter, in which he further explores the weird behavior of photons.

2. The Polarization of Photons

Dirac starts by describing a simple experiment: a light beam goes through a tourmaline crystal. Because it is crystal, it has a crystalline structure, which is defined by a hexagonal lattice. This sounds simple enough but its chemical composition is actually quite complicated, as evidenced by its formula which can be written as: $XY_3Z_6(T_6O_{18})(BO_3)_3V_3W$.[15] It is, therefore, rather remarkable that its optical properties are remarkably simple: tourmaline is a *polaroid* material, which means that it has an *optic axis*: a polaroid will transmit light that is linearly polarized *parallel* to the axis of the polaroid, with very little or no absorption, but light that is polarized in a direction that is *perpendicular* to the axis of the polaroid will be very strongly absorbed.

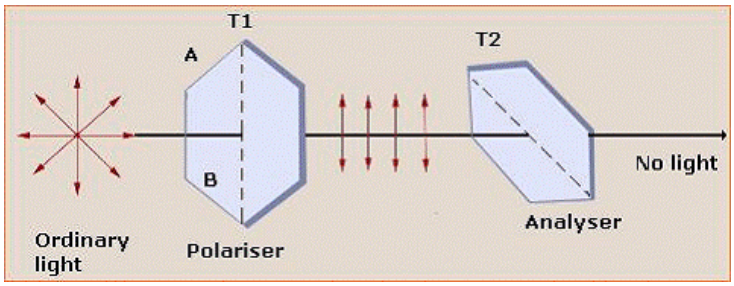
We want to explicitly makes this point so as to make sure we are well understood here: we do *not* doubt the phenomenal power of the quantum-mechanical framework to *reduce* complexity. In fact, we do *not* doubt quantum mechanics gives the right results (that would be *very* foolish in light of all the experimental evidence so far). What we are saying is that the quantum-mechanical rules and dogmas may prevent us from examining what might *actually* be happening. Let us make this point more

[15] There are some placeholder symbols in this formula. The formula can be further explained by noting the following:
X = Ca, Na, K, ☐ = vacancy; Y = Li, Mg, Fe^{2+}, Mn^{2+}, Zn, Al, Cr^{3+}, V^{3+}, Fe^{3+}, Ti^{4+}, vacancy; Z = Mg, Al, Fe^{3+}, Cr^{3+}, V^{3+}; T = Si, Al, B; B = B, vacancy; V = OH, O; W = OH, F, O.

explicitly by further examining the matter at hand here—literally: we're talking some polarizing material here and a light beam that goes through it. What's happening?

When we pass unpolarized light through a sheet of polaroid, only the part of the unpolarized beam which is vibrating parallel to the axis of the polaroid will get through. We, therefore, get a transmitted beam is linearly polarized. This is illustrated in Figure 2. We need to be precise here: 'unpolarized' light may actually be polarized in some other way (circularly or elliptically, for example). 'Unpolarized' here means: not linearly polarized along the optic axis of our crystal.

Figure 2: The polarization of light



So what happens with this picture when we think of photons? As Dirac right notes, in quantum mechanics we will want to think of the *state of polarization* of a photon, and if this *state* of polarization is the parallel state, then the photon should *not* go through. Not at all, that is—because we cannot think in terms of a fraction of a photon. Conversely, if *this* state of polarization is the perpendicular state, then it will go through—again: all of it. Don't even *try* to think of a photon fraction. That is fine. The question is: how do we think of the photons in a beam that is obliquely polarized? We know the *intensity* of the beam will be affected but how should we think of the photons?

Let's first go through the classical argument. As mentioned, what we referred to as unpolarized light above may actually be polarized – including linearly polarized – but the angle between the optical axis and the axis of polarization of the light will not be zero: it is some angle which we will denote as θ. This makes it possible to understand the situation in a different way: we can resolve the incident light – *any* beam, really – into a component that is perpendicular to the optic axis, and a component whose direction is the same as the optic axis. The amplitude which comes out of the polaroid (and we're talking the amplitude of some *real* electromagnetic wave here—not some quantum-mechanical probability amplitude) is only cosθ times the amplitude of the incoming wave. The sinθ component is absorbed.

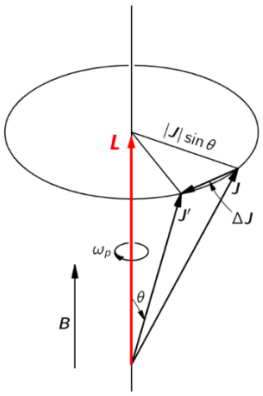
Let me phrase this differently: the (real-valued) amplitude which passes through the polaroid is smaller than the amplitude which entered by a factor cosθ.

Now, because the energy in an oscillation is always proportional to the square of the amplitude of the oscillation, the energy which passes through the polaroid (the *intensity* of the light, in other words) will be proportional to the square of cosθ. Hence, $\cos^2\theta$ is the intensity transmitted, and the absorbed intensity is $\sin^2\theta$. Dirac rightly notes we cannot escape the conclusion that only a fraction $\cos^2\theta$ of the

photons will get through[16], but his reasoning of what this implies for the photons *individually* is somewhat less sound.

Indeed, there is a weird contradiction between (a) the definiteness in his assumption that "each of the photons is obliquely polarized" at that θ angle – *exactly*, that is – and that (b) we can only assign some *probability* (which we calculated to be equal to $\cos^2\theta$) to the photon actually getting through or not. One does not need to have a very advanced understanding of the Uncertainty Principle to question the assumption: why would a photon have some *definite* polarization direction? In the *Zitterbewegung* model of an electron, we wil have a magnetic moment (the *spin* of the electron) but – in a magnetic field – we would think of its magnetic moment as wobbling around because of precession (see Figure 3). Hence, we should not preclude a similar *uncertainty* in the spin of an individual photon.

Figure 3: The precession of a current ring in a magnetic field



As such, we are able "to preserve the individuality of the photon" while, at the same time, "abandoning the determinacy of the classical theory", as required to explain the phenomenon at hand. We would, in fact, establish a complete analogy of the quantum-mechanical rule for an electron or any particle, really—for which we also know that the angular momentum is never "completely" along any direction.[17]

Again, the point is not to present an alternative theory here but to show that Dirac's line of defense – or line of *attach*, I should say – against such theory is not waterproof. Not at all, actually. To show that, we must move on and think about some other experiment: the interference of a photon with itself. Dirac discusses this in the section that follows.

[16] We should note a rather embarrassing mistake in Dirac's *Principles of Mechanics* here: he writes that it's the fraction $\sin^2\theta$ that will actually *get through*, as opposed to being absorbed. We might think it's got to do with the definition of the angle, but no. Dirac's α angle is the same: it's the angle of polarization to the optic axis. The mistake is not significant, but quite embarrassing in light of the many editions that this book has gone through.

[17] For a particularly enlightening and simple approach to essential idea in quantum mechanics, see: Feynman's *Lectures*, Vol. II-34-7, *Angular Momentum in Quantum Mechanics* (http://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S7).

3. Interference of Photons

Without explicitly stating so, Dirac basically discusses the Mach-Zehnder experiment here: the interference of a photon with itself as it – in some kind of inexplicable way – seems to travel along two paths simultaneously. The concept of a wavefunction of a particle gives way to the idea of a probability amplitude which we associate with a possible *path* rather than the particle itself. Dirac refers to this idea as a "translational state". It is good to, once again, quote Dirac here:

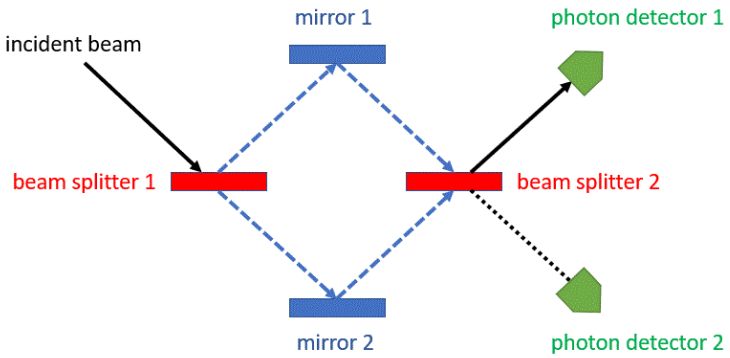
> "Corresponding to the description that we had in the case of the polarization, we must now describe the photon as going partly into each of the two components into which the incident beam is split. *The photon is then, as we may say, in a translational state given by the superposition of the two translational states associated with the two components*."

Let us examine the logic here. Before the photon enters the beam splitter, we have one wavefunction: the photon. When it goes through, we have two probability amplitudes that – somehow – recombine and interfere with each other. Dirac claims this cannot be explained classically. We wonder why, because we think an explanation in terms of some circularly polarized photon being split into two linearly polarized waves might do the trick. Again, the point is not to present anything definite here, but we want just to show you that an alternative explanation – using classical concepts – might, in fact, be possible.

The Mach-Zehnder experiment

The Mach-Zehnder interferometer consists of two beam splitters (BS1 and BS2) and two perfect mirrors (M1 and M2). An incident beam coming from the left is split at BS1 and recombines at BS2, which sends two outgoing beams to the photon detectors D0 and D1. More importantly, the interferometer can be set up to produce a precise interference effect which ensures all the light goes into D0, as shown below. Alternatively, the setup may be altered to ensure all the light goes into D1.

Figure 4: The Mach-Zehnder interferometer



If we have a proper *beam* of light, then we have an easy explanation, which goes like this:

- The first beam splitter (BS1) splits the beam into *two* beams.
- These two beams arrive in phase or, alternatively, out of phase and we, therefore, have constructive or destructive interference that recombines the original beam and makes it go towards D0 or, alternatively, towards D1.

However, when we analyze this in terms of a *single* photon – we now think of the photons going one-by-one through the apparatus – then this classical picture becomes quite complicated. Complicated but – as we argue – not impossible. An alternative theory of what happens in the Mach-Zehnder interferometer might be the following one:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

We will detail this in the next section, because what happens in a Mach-Zehnder interferometer is not all that straightforward. We should note, for example, that there are phase shifts along both paths: classical physics tells us that, on transmission, a wave does not pick up any phase shift, but it does so on reflection. To be precise, it will pick up a phase shift of π on reflection. We will refer to the standard textbook explanations of these subtleties and just integrate them in our more detailed explanation in the next section.[18] Before we do so, we will show the assumption that the two linear waves are orthogonal to each other is quite crucial. If they weren't, we would be in trouble with the energy conservation law. Let us show that before we proceed. Why? Because Dirac also mentions it as an argument *against* any classical explanation of the interference phenomenon.

The energy conservation law

Suppose the beams would be polarized along the same direction. If *x* is the direction of propagation of the wave, then it may be the *y*- or *z*-direction of anything in-between. The magnitude of the electric field vector will then be given by a sinusoid. Now, we assume we have *two* linearly polarized beams, of course, which we will refer to as beam *a* and *b* respectively. These waves are likely to arrive with a phase difference – unless the apparatus has been set up to ensure the distances along both paths are exactly the same. Hence, the general case is that we would describe *a* by $cos(\omega \cdot t - k \cdot x) = cos(\theta)$ and *b* by $cos(\theta + \Delta)$ respectively. In the classical analysis, the difference in phase (Δ) will be there because of a difference of the path lengths[19] and the recombined wavefunction will be equal to the same cosine function, but with argument $\theta + \Delta/2$, multiplied by an *envelope* equal to $2 \cdot cos(\Delta/2)$. We write[20]:

$$cos(\theta) + cos(\theta + \Delta) = 2 \cdot cos(\theta + \Delta/2) \cdot cos(\Delta/2)$$

We always get a recombined beam with the same frequency, but when the phase difference between the two incoming beams is small, its amplitude is going to be much larger. To be precise, it is going to be twice the amplitude of the incoming beams for Δ = 0. In contrast, if the two beams are out of phase, the

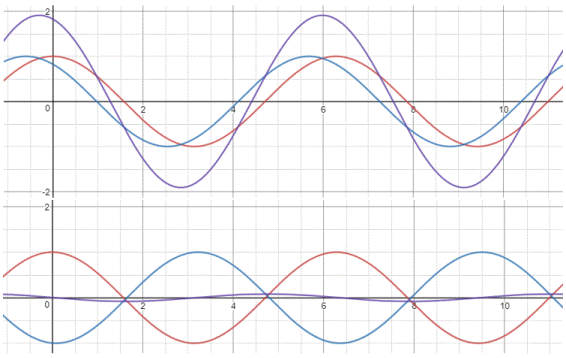
[18] For a good *classical* explanation of the Mach-Zehnder interferometer, see: K.P. Zetie, S.F. Adams and R.M. Tocknell, January 2000, *How does a Mach–Zehnder interferometer work?* (https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf, accessed on 5 November 2018).
For a good quantum-mechanical explanation (interference of single photons), see – for example – the Mach-Zehnder tutorial from the PhysPort website (https://www.physport.org/curricula/QuILTs/, accessed on 5 November 2018).

[19] Feynman's path integral approach to quantum mechanics allows photons (or probability amplitudes, we should say) to travel somewhat slower or faster than *c*, but that should not bother us here.

[20] We are just applying the formula for the sum of two cosines here. If we would add sines, we would get $sin(\theta) + sin(\theta + \Delta) = 2 \cdot sin(\theta + \Delta/2) \cdot cos(\Delta/2)$. Hence, we get the same envelope: $2 \cdot cos(\Delta/2)$.

amplitude is going to be much smaller, and it's going to be zero if the two waves are 180 degrees out of phase ($\Delta = \pi$), as shown below. That does not make sense because twice the amplitude means *four* times the energy, and zero amplitude means zero energy. The energy conservation law is being violated: photons are being multiplied or, conversely, are being destroyed.

Figure 5: Constructive and destructive interference for linearly polarized beams



Let us be explicit about the energy calculation. We assumed that, when the incoming beam splits up at BS1, that the energy of the *a* and *b* beam will be split in half too. We know the energy is given by (or, to be precise, proportional to) the square of the amplitude (let us denote this amplitude by *A*).[21] Hence, if we want the energy of the two individual beams to add up to $A^2 = 1^2 = 1$, then the (maximum) amplitude of the *a* and *b* beams must be $1/\sqrt{2}$ of the amplitude of the original beam, and our formula becomes:

$$(1/\sqrt{2})\cdot\cos(\theta) + (1/\sqrt{2})\cdot\cos(\theta + \Delta) = (2/\sqrt{2})\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

This reduces to $(2/\sqrt{2})\cdot\cos(\theta)$ for $\Delta = 0$. Hence, we still get *twice* the energy – $(2/\sqrt{2})^2$ equals 2 – when the beams are in phase and zero energy when the two beams are 180 degrees out of phase. This doesn't make sense.

Of course, the mistake in the argument is obvious. This is why our assumption that the two linear waves are orthogonal to each other comes in: we cannot just add the amplitudes of the *a* and *b* beams because they have different directions. If the *a* and *b* beams – after being split from the original beam – are linearly polarized, then the angle between the axes of polarization should be equal to 90 degrees to ensure that the two oscillations are independent. We can then add them like we would add the two parts of a complex number. Remembering the geometric interpretation of the imaginary unit as a counterclockwise rotation, we can then write the sum of our a and b beams as:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \Delta) = (1/\sqrt{2})\cdot[\cos(\theta) + i\cdot\cos(\theta + \Delta)]$$

What can we do with this? Not all that much, except noting that we can write the $cos(\theta + \Delta)$ as a sine for $\Delta = \pm\pi/2$. To be precise, we get:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \pi/2) = (1/\sqrt{2})\cdot(\cos\theta - i\cdot\sin\theta) = (1/\sqrt{2})\cdot e^{-i\cdot\theta}$$

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta - \pi/2) = (1/\sqrt{2})\cdot(\cos\theta + i\cdot\cos\theta) = (1/\sqrt{2})\cdot e^{i\cdot\theta}$$

[21] If we would reason in terms of average energies, we would have to apply a 1/2 factor because the average of the $\sin^2\theta$ and $\cos^2\theta$ over a cycle is equal to 1/2.

This gives us the classical explanation we were looking for:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

What about the 1/√2 factor? If the $e^{-i\cdot\theta}$ and $e^{i\cdot\theta}$ wavefunctions can, effectively, be interpreted geometrically as a *physical* oscillation in *two* dimensions – which is, effectively, our interpretation of the wavefunction[22] – then then each of the two (independent) oscillations will pack one half of the energy of the wave. Hence, if such *circularly* polarized wave splits into two *linearly* polarized waves, then the two linearly polarized waves will effectively, pack half of the energy without any need for us to think their (maximum) amplitude should be adjusted. If we now think of the ***x***-direction as the direction of the incident beam in the Mach-Zehnder experiment, and we would want to also think of rotations in the ***xz***-plane, then we need to need to introduce some new convention here. Let us introduce *another* imaginary unit, which we'll denote by *j*, and which will represent a 90-degree counterclockwise rotation in the ***xz***-plane.[23]

A classical explanation for the interference of a photon with itself

We may now advance the following *classical* explanation for the results of the one-photon Mach-Zehnder experiment:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = cos\theta + i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $-j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

[22] We can assign the physical dimension of the electric field (force per unit charge, N/C) to the two perpendicular oscillations.
[23] This convention may make the reader think of the quaternion theory but we are thinking more of simple Euler angles here: *i* is a (counterclockwise) rotation around the *x*-axis, and *j* is a rotation around the *y*-axis.

Of course, we may also set up the apparatus with different path lengths, in which case the two linearly polarized beams will be out of phase when arriving at BS1. Let us assume the phase shift is equal to $\Delta = 180° = \pi$. This amounts to putting a minus sign in front of *either* the sine *or* the cosine function. Why? Because of the $cos(\theta \pm \pi) = -cos\theta$ and $sin(\theta \pm \pi) = -sin\theta$ identities. Let us assume the distance along the upper path is longer and, hence, that the phase shift affects the sine function.[24] In that case, the sequence of events might be like this:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{j\cdot\theta} = cos\theta + j\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $j\cdot\sin(\theta + \pi) = -j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-j\cdot\theta} = cos\theta - j\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $-j\cdot\sin(\theta + \pi) = +j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

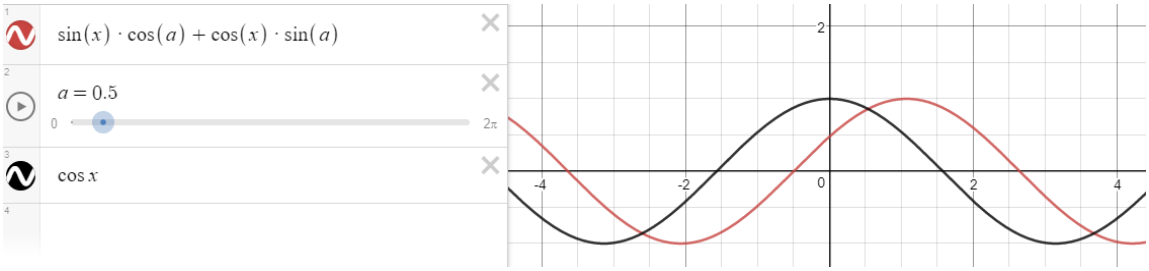
What happens when the difference between the phases of the two beams is not equal to 0 or 180 degrees? What if it is some random value in-between? Do we get an elliptically polarized wave or some other nice result? Denoting the phase shift as Δ, we can write:

$$cos\theta + j\cdot sin(\theta + \Delta) = cos\theta + j\cdot(sin\theta\cdot cos\Delta + cos\theta\cdot sin\Delta)$$

However, this is also just a circularly polarized wave, but with a random phase shift between the horizontal and vertical component of the wave, as shown below. Of course, for the special values $\Delta = 0$ and $\Delta = \pi$, we get $cos\theta + j\cdot sin\theta$ and $cos\theta - j\cdot sin\theta$ once more.

[24] The reader can easily work out the math for the opposite case (longer length of the lower path).

Figure 6: Random phase shift between two waves



Are we done? For the purposes of this paper, yes. Sections 4 (superposition and indeterminacy), 5 (mathematical formulation of the principle) and 6 (bra and ket vectors) are no longer about interpretation but present the basic concepts and theorems of quantum mechanics as you will find them in (almost) any introductory course of quantum mechanics. We do not want to present those – but they are the standard *lore* of physics and that's not what our papers are about.

Conclusions

Mystery solved? Maybe. Maybe not. Some of the alternative explanations (in particular those related to Mach-Zehnder interference) may come across as being artificial. However, such sentiments are *not* to the point here: what we wanted to do here is to show that an alternative explanation – using classical concepts and hypotheses – is, in fact, *possible*. We are *not* saying our alternative theory is *the* explanation.

In other words, all that we are saying is that Dirac's arguments *against* a realist interpretation of quantum mechanics do not quite cut it. Nothing more, nothing less. We think it amounts to showing that Bell's No-Go Theorem should not prevent us from trying to go *every*where: all that it takes is – as Bell himself pointed out – some kind of 'radical conceptual renewal'.

We hope our papers show how that radical conceptual renewal might look like.

Jean Louis Van Belle, 11 June 2019

References

Academics will usually add a long list of books and articles here, but I don't want to do that. I would also advice interested readers to not trust too much in the latest update of this or that textbook. I recommend reading originals such as Dirac's *Principles of Quantum Mechanics*, which is the topic of this paper!

There are several advantages of reading original work. The most obvious advantage is that they are often available online. More importantly, however, they are also widely referenced in various discussion fora. Hence, if you have an issue with this or that interpretation, or some formula, in an original book of one of the founding fathers of quantum mechanics, then you will be able to *google* for help very easily.

Feynman's Lectures is and remains a classic for me (http://www.feynmanlectures.caltech.edu/). Some of his *Lectures* on quantum mechanics – such as chapter 4, on identical particles – suffer from excessive and speculative generalization (see my paper on *Philosophy and Physics* in this regard: http://vixra.org/abs/1906.0082), but even this chapter makes you think for yourself. That is very valuable, in my humble view, because I find more modern textbooks often too confident in their approach: they emphasize what we know, as opposed to what we don't know. Feynman's Lectures also have the advantage that you get the math you need with the physics you study.

However, in case you'd want a good mathematical introduction, Mathews and Walker's *Mathematical Methods of Physics*, is a reference that stands out for me.