RESOLVING THE TENSION BETWEEN PLANCK H_0=66.93[1] AND RIESS ET AL H_0=73.24[2]

Bruce Wallman, PhD, Princeton, New Jersey, brucewallman@gmail.com

ABSTRACT

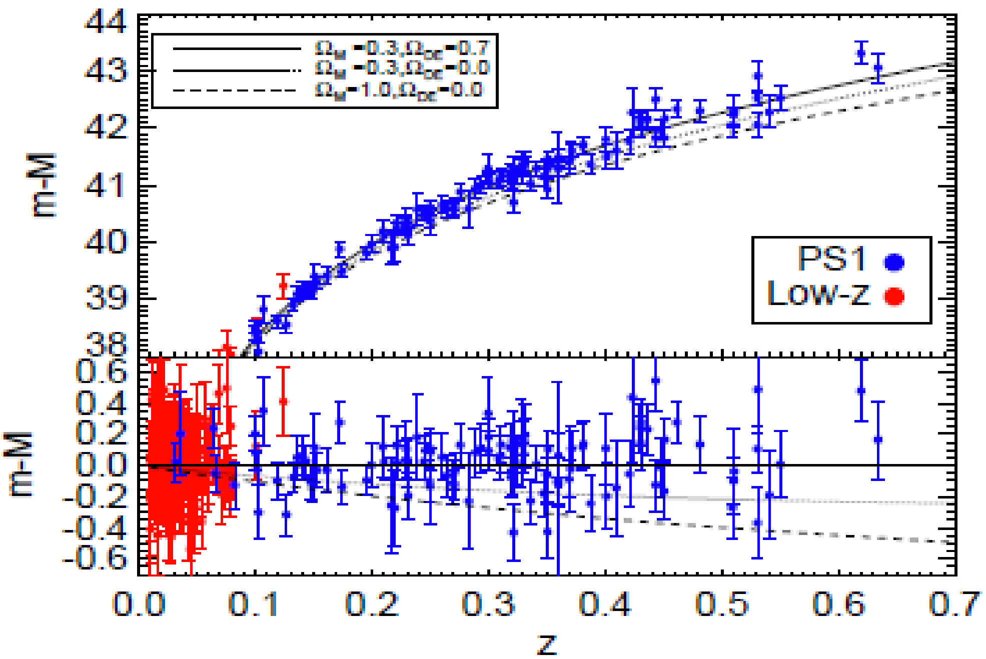
There is an open question in cosmology about the significance of the difference between the 2016 Planck collaboration value H_0=66.93±0.62 km/s/Mpc and the 2016 Riess et al value H_0=73.24±1.74 km/s/Mpc. This paper shows that both the Planck collaboration and Riess et al values are valid. Tracing the cosmological redshift z from SN1a several thousand megaparsecs in the past to those that exploded closer to today describes an increase in our perception of H_v as distance to us lessens. A best fit power function of $v_r=73.227*D_p^{0.9907}$ for recessional velocity versus proper distance (where v_r is recessional velocity and D_p is proper distance) is derived connecting these two values of H_0 using 1836 SN1a in the NED database[3]. Taking the derivative of this formula to get Hubble slope gives the power function $H_v=dv_r/dD_p= 72.546*D_p^{-0.0093}$. When this formula is used to describe an extrapolation of the universe from near the Milky Way galaxy to the Cosmic Microwave Background (CMB), it describes a change of about 7 km/s/Mpc in the Hubble value for the observable universe increasing smoothly from 66.4 for our measures at the CMB to 73.4 km/s/Mpc locally as the universe expands. Most of the increase occurs in the last 1000 Mpc as was stated in Riess, Perlmutter, and subsequent SN1a studies. This is consistent with ΛCDM cosmology[4]. After converting distance in megaparsecs to light-seconds, it is possible to show H_v is an acceleration that varies from 6.45 angstrom/s/lt-s when measuring the CMB to 7.13 angstrom/s/lt-s in the local universe.

1. INTRODUCTION

The starting data points for deriving a Hubble value from SN1a are redshift z and distance modulus m-M. These are used to yield recessional velocity and proper distance for each SN1a. The recessional velocity V_r is an integration using redshift, omega matter, omega Lambda, omega radiation, and an assumption of flatness. Omega matter=0.295, omega lambda=0.704913, omega radiation=0.000087, and omega curvature=0.0 are used to calculate a local density function. These cosmological parameters are similar to those commonly used in SN1a and Planck papers. The tightness of increments used to examine the expansion factor affects the precision of the answer, but the effect is minimal once the incremental value is small enough. The proper distance is calculated from the formula $D_p=10^{-6}x10^{((m-M)/5)+1)}/(1+z)$ using the m-M distance modulus and redshift z. The Hubble values H_v derived are the currently measured Hubble values from our location in space and time, not the value at the time the observed signal was emitted. The local Hubble value is affected by where the Hubble flow is assumed to begin in a space empty of local galaxies and their gravity. 0.30 Mpc is assumed.

A first chart is provided to give context for the format of other charts that follow. Chart 1, showing distance modulus (m-M) versus redshift, is copied from the Rest et al[5] (includes Riess) 2014 paper cited for the 2014ApJ...795...44R SN1a data. Chart 1 provides one validation point for m-M error bars and the cosmological parameters used in this paper.

Chart 1 - Distance modulus (m-M) versus redshift z according to Rest et al in 2014.

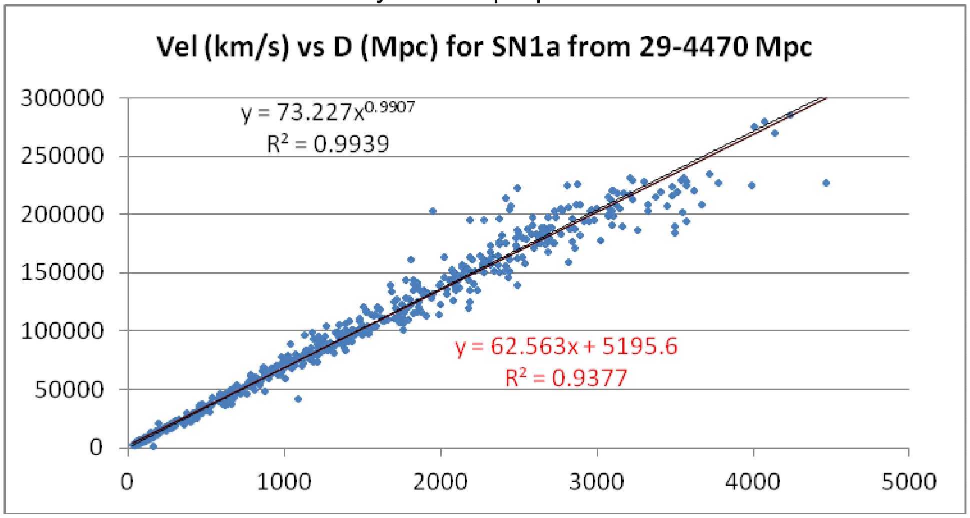


While relying on the data and analysis in the Rest et al paper for its backdrop, this paper changes chart formats from the Rest et al axes in chart 1 in order to help visualize the Hubble value. First, proper distance is calculated from the m-M distance modulus and redshift z for SN1a in the NED data base and shown in charts rather than showing the distance modulus itself. The values calculated are consistent with the distance in Mpc given in the NED database. Second, redshift z is converted to recessional velocity (using an integration derived from a cosmological calculator[6]) because recessional velocity in km/s is the numerator when literally calculating a Hubble value. Third, proper distance in Mpc is plotted along the x axis and recessional velocity in km/s is plotted on the y axis rather than vice versa. These changes allow the recessional velocity charts to visualize Hubble values similarly to Hubble's intuitive work in the 1920's. The derivative of recessional velocity over distance, originally seen as a straight line over a relatively short distance by Hubble, is shown to change over large distances according to a power function computed using current data. The set of 1836 data points from NED SN1a (reference 2014ApJ...795...44R and 2013MNRAS.433.2240G) was chosen because it comprises a low variance spectrographic data set that is consistent with later Riess et al papers.

2. ANALYSIS

The computation of recessional velocity (the rate at which proper distance to source increases per unit time expressed in km/s) used in chart 2 is done via an integration loop taken from the cosmological calculator and coded in Visual Basic for Applications to run in Excel. The loop continues for a large number of cycles looking for the two points where the incremental expansion factor first exceeds the initial expansion factor and the incremental expansion factor reaches 1. Recessional velocity is calculated via the difference between these two comoving points multiplied by the speed of light. The Visual Basic for Applications code with notes is provided in the Appendix. The cosmological values given earlier are used to calculate the density function in the integration loop. These values were selected because they provide a flat universe, are similar to the values given by the Planck collaboration and Rest et al, and provide a satisfactory solution to the problem of connecting the H_0=66.93±0.62 km/s/Mpc and H_0=73.24±1.74 values.

Chart 2 - Recessional velocity versus proper distance.

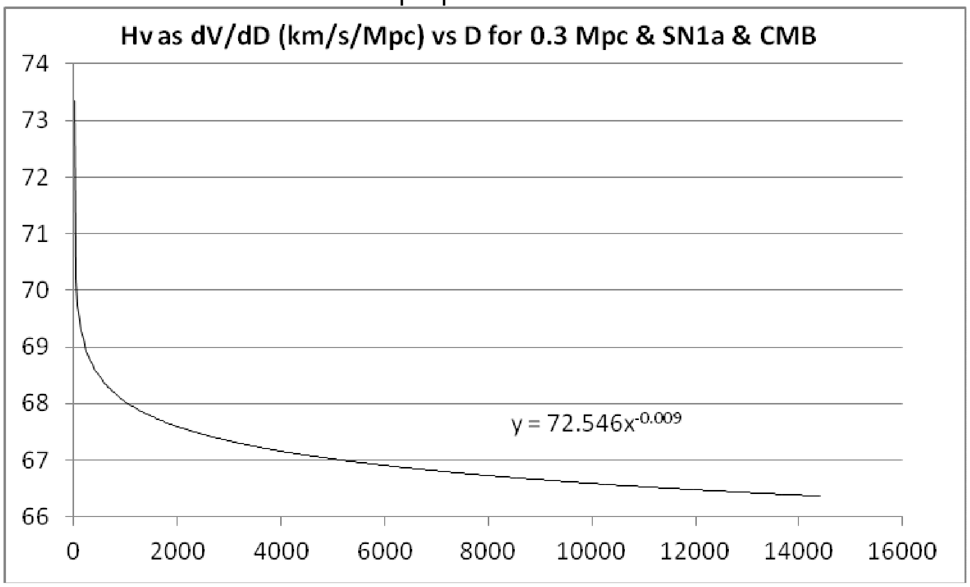


Regression squared for the recessional velocity power function is 0.9939. This value is better than alternatives tested such as linear regression (shown in red). If the linear fit is used, it sets the Hubble value to a constant slope of 62.563 km/s/Mpc. Hubble originally made the assumption of a linear slope for the narrow set of data available in the 1920's. This is not a valid assumption given today's range of NED SN1a data. Accordingly, the analysis that follows uses the better fit power function v_r=73.227*$D_p^{-0.9907}$ and its derivative. The data points on the chart could be shown as short horizontal bars to emphasize that there is an average error measurement of 0.18 m-M in the distance modulus used to calculate proper distance. This is consistent with error bars shown vertically by Rest et al in chart 1. In the 310 SN1a from the 2014 Riess study there are 102 SN1a measurements with err<=0.15 m-M, 232 with err<=0.20, and 283 with err<=0.25 m-M. Overall, there are 306 SN1a from that study with err<0.32 m-M and 4 SN1a with 0.32<=err<=0.64. There is no error for the y axis because the recessional velocity is calculated from observed redshift z and the observed redshift has no error listed in the NED database. This is consistent with the observed redshift being identical for every measurement of the same SN1a listed in the two studies used from the NED database.

Two data points are added in the next chart showing the derivative of recessional velocity. These data points extend the Hubble values to the local and distant universe. One is for the local universe at 0.3 Mpc and the other is for the CMB at 14,400 Mpc. The value for the local universe is envisioned as where the Hubble flow would start if the Milky Way was alone in the universe. These points are added to extrapolate for H_v comparisons to Riess in recent times and Planck at roughly 13.8 billion years ago. The recessional velocity computation for the CMB at 14,400 Mpc using the parameters selected ends up as 959,225 km/sec. This is 3.1996 times the speed of light as expected.

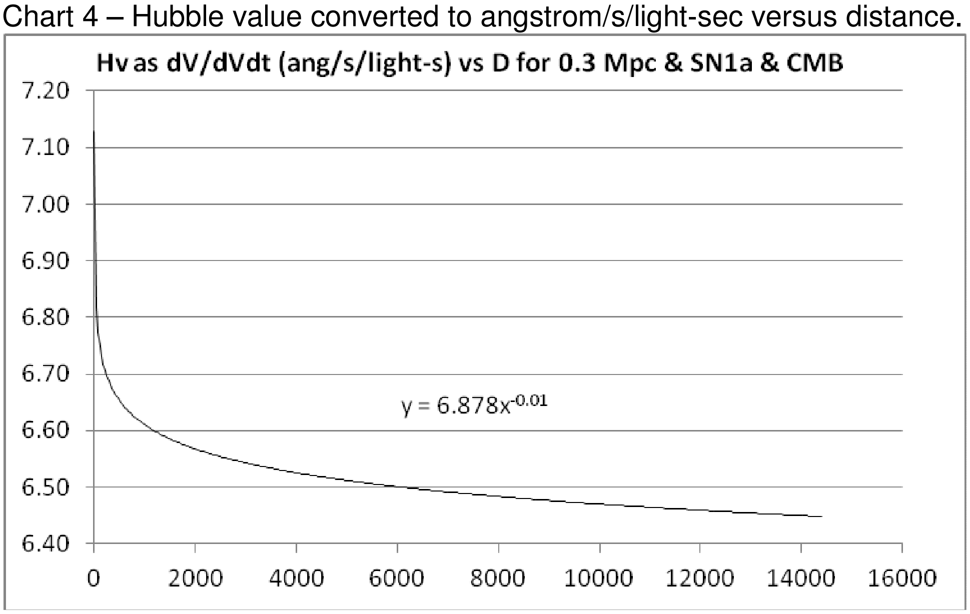
Given the power function best fit for recessional velocity v_r=73.227*$D_p^{-0.9907}$, the derivative of this power function at each proper distance is by definition the Hubble value for that proper distance. That derivative is H_v=dv_r/dD_p= 72.35*$D_p^{-0.009}$. Plotting this derivative formula…

Chart 3 - Hubble value versus proper distance.



The line shown is the derivative at all points of the recessional velocity per unit distance using 0.3 Mpc as the nearest distance and 14,400 Mpc as the farthest. The Hubble value is extrapolated up to 73.4 km/sec/Mpc at 0.3 Mpc and down to 66.4 km/sec/Mpc at 14,400 Mpc using this data. Both of these values are well within the ranges given by the Planck collaboration and Riess et al for their respective distances. These results are consistent with the 1998 Riess and Perlmutter conclusions when the relationship between the Hubble value and the acceleration of the universe over time was recognized. The Hubble value is typically expressed as km/s/Mpc. The Hubble value can be expressed in km/s/light-sec or angstrom/s/light-sec by assuming a constant speed of light.

Chart 4 – Hubble value converted to angstrom/s/light-sec versus distance.



The relationship between the last two diagrams is helpful in understanding why the derivative of the recessional velocity with respect to time gives the Hubble value and why we refer to an increasing acceleration in the expansion of the visible universe rather than an increasing Hubble value.

3. CONCLUSIONS

In order to reach the goal of connecting the Planck value of H_0=66.93±0.62 km/s/Mpc and the Riess et al value of H_0=73.24±1.74 km/s/Mpc, the analysis has used the following concepts:

a. Two low variance data sets from the NED database and omega cosmological parameters consistent with prior studies and a flat universe are used. The selection of low variance data sets with a wide distance range and working with a sizeable number of unique SN1a provide a statistically justifiable result.
b. Hubble acceleration $H_v=dv_r/dD_p= 72.546*D_p^{-0.009}$ is calculated as a derivative of recessional velocity $v_r=73.227*D_p^{-0.9907}$ versus distance. This approach (rather than plotting independent H_v values) smooths the variance seen in the imperfectly measured distance modulus. Plotting independent H_v values instead of the derivative of velocity creates an identical Hubble formula.
c. The use of a best fit power function rather than a lesser fit linear regression allows the slope (derivative of recessional velocity) to connect the range of H_0 values from Riess et al to the Planck collaboration by not forcing the Hubble value to be a constant.
d. Similar analysis after converting to angstroms/s/lt-sec is objectively consistent with the Riess, Perlmutter, et al conclusion that the acceleration of the visible universe's expansion appears to have increased in the last few billion years.

4. FUTURE ANALYSIS

By connecting the dots for several hundred SN1a, this paper shows statistically that the Hubble values derived by the Planck collaboration and Riess et al are each valid for their respective distances. It does not address the physics of why this is so. The chart extrapolates smoothly to the Planck CMB value suggesting that the change reflects a reality that the Hubble value as we see it today increases modestly in the late time, nearby universe. This suggests a systematic change. This interpretation of the Hubble value increasing as we see it for recent times is consistent with what Riess, Perlmutter, et al gave us and leaves us still wondering why this is so.

What happens if we extend this analysis to the far future? There is nothing in the calculation process to suggest a significant change to the formula $v_r=73.227*D_p^{-0.9907}$ and its derivative. As a result there is nothing to suggest that a nearby SN1a a billion years from now will yield a much higher value of H_v than 73.24±1.74 km/s/Mpc. The local value in the far future depends on whether the velocity and acceleration curves shift to the right with time or whether they stretch to the right with time. An interpretation of the curves stretching to the right differs from modern suggestions that the universe is flying apart. It says instead that the nearby visible universe shows and will always show a Hubble value of roughly 73.24 km/s/Mpc and a higher accelerated expansion than the very distant visible universe.

5. ACKNOWLEDGEMENT

I have had two departments (Physics and Astrophysics) mentoring me at Princeton University. The physicists there have taught me much since I started retirement in 2012. This paper acknowedges their open attitudes towards helping me learn. I especially want to acknowledge Alwin Mao, a Princeton astrophysics PhD student, who helped me understand the cosmological calculator and Stephen Boughn, a Princeton and Haverford astrophysics professor emeritus, who kept asking me questions until I trusted the results of this paper.

REFERENCES

[1] Planck collaboration arXiv: 1502.01589v3, Planck 2015 results. XIII. Cosmological parameters selected by Riess et al as LambdaCDM with 3 neutrino flavors having a mass of 0.06 eV and the Planck CMB data (TT,TE,EE+SIMlow; 3.2 sigma for TT+SIMlow), other early universe (CMB) models range from 66.88 to 67.31.
[2] Riess et al arXiv:1604.01424v3, A 2.4% Determination of the Local Value of the Hubble Constant.
[3] NED Extragalactic database NED27.02.1-D-14.1.0-20170227 at https://ned.ipac.caltech.edu/.
[4] Noted in papers such as Paturel et al, Hubble Law: Measure and Interpretation, Foundation of Physics (2007):47: p1214.
[5] Rest et al arXiv: 1310.3828v2 Cosmological Constraints from Measurements of Type Ia Supernovae discovered during the first 1.5 years of the Pan-STARRS1 Survey.
[6] Relativity 4 Engineers cosmological calculator initially used available at http://www.einsteins-theory-of-relativity-4engineers.com/cosmocalc.htm served as basis for the VBA code shown.

APPENDIX - The code used to establish recessional velocities

```
Option Explicit
Public OmegaMatter, OmegaRadiation, OmegaLambda, OmegaCurvature, ExpFactorA, zMeasured
Public RowNumber, zEq, zComovingDist, ExpFactor, ComovingLoopControl, TimeVariableDensityFunction
Public NumberofIterations, ProperDistance, zRel, RecessionalVelocity, zDiff, C
Public Increment, SwOnceExpFactorGTExpFactorA, zComovingDistOnceExpFactorGTExpFactorA, FinalRow
Public Sub Main()
    OmegaMatter = Worksheets("Const").Cells(5, 7).Value
'Observed normal + dark matter fraction of the critical energy density.
    OmegaLambda = Worksheets("Const").Cells(6, 7).Value
'Observed cosmological constant as fraction of the critical energy density.
    zEq = Worksheets("Const").Cells(7, 7).Value
'Deduced redshift where radiation energy density and matter density are equal.
    OmegaRadiation = OmegaMatter / zEq
    Worksheets("Const").Cells(11, 7).Value = OmegaRadiation
    OmegaCurvature = 1 - OmegaMatter - OmegaLambda - OmegaRadiation
    Worksheets("Const").Cells(12, 7).Value = OmegaCurvature
    Worksheets("Const").Cells(13, 7).Value = OmegaMatter + OmegaLambda + OmegaRadiation + OmegaCurvature
'Values of omega chosen to force OmegaCurvature to equal 0.
    NumberofIterations = Worksheets("Const").Cells(3, 7).Value
'Ran with number of iterations set to 100,000. This value balances
'high accuracy in the charts versus a satisfactory computation speed.
    Increment = 1 / NumberofIterations
    For RowNumber = 1 To 2000
        If Worksheets("Data").Cells(RowNumber, 3).Value <> "" And
          IsNumeric(Worksheets("Data").Cells(RowNumber, 2).Value) Then
            zMeasured = Worksheets("Data").Cells(RowNumber, 3).Value
'Factor of increase in wavelength of light emitted from source, due to the
'cosmic expansion; z=1 means that wavelength and all cosmic scale proper
'distances have doubled since that light was emitted.
            ExpFactorA = 1 / (1 + zMeasured)
'ExpFactorA is expansion factor for each SN1a at zMeasured.
            zComovingDist = 0
            zComovingDistOnceExpFactorGTExpFactorA = 0
            TimeVariableDensityFunction = 0
            SwOnceExpFactorGTExpFactorA = 0
            ComovingLoopControl = 0
            Do While ComovingLoopControl < 1
                ExpFactor = ComovingLoopControl + (Increment / 2)
                TimeVariableDensityFunction = ((OmegaMatter / ExpFactor) + OmegaCurvature +
                 (OmegaRadiation / (ExpFactor ^ 2)) + (OmegaLambda * ExpFactor ^ 2)) ^ (1 / 2)
                zComovingDist = zComovingDist + (Increment / (ExpFactor * TimeVariableDensityFunction))
'Comoving proper distance of the source like measuring cosmic distances
'using measuring rods.
                ComovingLoopControl = ComovingLoopControl + Increment
                If ExpFactor > ExpFactorA And SwOnceExpFactorGTExpFactorA = 0 Then
                    zComovingDistOnceExpFactorGTExpFactorA = zComovingDist
                    SwOnceExpFactorGTExpFactorA = 1
                End If
            Loop
            zDiff = zComovingDist - zComovingDistOnceExpFactorGTExpFactorA
            C = Worksheets("Const").Cells(2, 1).Value
            RecessionalVelocity = zDiff * C
'Vr = C *(z@ComovingDistEqualTo1 -
'  z@ComovingDistWhereIncrementalExpansionFactorFirstExceedsOriginalExpansionFactor)
            Worksheets("Data").Cells(RowNumber, 4).Value = RecessionalVelocity
            Worksheets("Data").Cells(RowNumber, 8).Value = zDiff
            FinalRow = RowNumber - 1
            Worksheets("Data").Cells(2, 5).Value = FinalRow
        End If
    Next RowNumber
    Worksheets("Data").Cells(2, 5).Value = FinalRow & " fin"
End Sub
```