Partition Of The Primorial Square By Remainder Agreement Counts

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ABSTRACT For the n-th prime P, P# or P primorial is the product of all the primes up to and including P. Let (c, d) be a pair of integers that represents a point in the primorial square, 1 ≤ c, d ≤ P#. For each prime p, 2 ≤ p ≤ P, the remainders of c and d mod p may be the same, opposite (sum to a multiple of p) or neither. Count the number of remainders of (c, d) which have same, opposite or either agreement for any such P. This gives three partitions of the primorial square, by counts for same, opposite and either agreement. Polynomial multiplication is used to find the number of points in each part of these partitions.

The Primorial Square. For the n-th prime P, P# or P primorial is the product of all the primes up to and including P. Let (c, d) be a pair of integers that represents a point in the primorial square, 1 ≤ c, d ≤ P#. One can consider that the n-th primorial square is built by replicating the (n - 1)-st square P by P times.

Same, Opposite or Either Agreement of Remainders. For (c, d) in the n-th primorial square, and any prime p, 2 ≤ p ≤ P, the remainders of c and d mod p may be the same, opposite (sum to a multiple of p) or neither. For example, for (17, 3) in the 30 x 30 square for P = prime(3) = 5, the numbers 17 and 3 have the same remainder, 1, modulo 2 (which, for modulo 2, is also opposite), different remainders, 2 and 0, modulo 3, and opposite remainders, 2 and 3 (= -2), modulo 5.
Thus the point (17, 3) has a same agreement count of 1, an opposite agreement count of 2, and an either agreement count of 2 for P = 5.
Figures 1 - 3 show the remainder agreement counts for each point in the primorial squares for P = 2, P = 3 and P = 5. Note that numbering starts from the upper left of each square.

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Figure 1. P = prime(1) = 2. P# = 2. Same, opposite, and either agreement, 2 x 2 square.

| | | | | | |
|---|---|---|---|---|---|
| 2 | 0 | 1 | 1 | 1 | 0 |
| 0 | 2 | 0 | 1 | 1 | 1 |
| 1 | 0 | 2 | 0 | 1 | 1 |
| 1 | 1 | 0 | 2 | 0 | 1 |
| 1 | 1 | 1 | 0 | 2 | 0 |
| 0 | 1 | 1 | 1 | 0 | 2 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 2 | 0 |
| 1 | 1 | 0 | 2 | 0 | 1 |
| 1 | 0 | 2 | 0 | 1 | 1 |
| 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 2 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 2 | 0 |
| 1 | 2 | 0 | 2 | 1 | 1 |
| 1 | 0 | 2 | 0 | 1 | 1 |
| 1 | 2 | 0 | 2 | 1 | 1 |
| 2 | 1 | 1 | 1 | 2 | 0 |
| 0 | 1 | 1 | 1 | 0 | 2 |

Figure 2. P = prime(2) = 3. P# = 6. Same, opposite, and either agreement, 6 x 6 square.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 |
| 2 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 |
| 3 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 |
| 4 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 |
| 5 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 |
| 6 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 |
| 7 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 |
| 8 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 |
| 9 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 |
| 10 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 |
| 11 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 |
| 12 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 |
| 13 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 |
| 14 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 |
| 15 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 |
| 16 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 |
| 17 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 |
| 18 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 |
| 19 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 |
| 20 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 |
| 21 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 |
| 22 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 |
| 23 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| 24 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 |
| 25 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 |
| 26 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 |
| 27 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 |
| 28 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 |
| 29 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 |
| 30 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 |

Figure 3a. P = prime(3) = 5. P# = 30. Same agreement counts, 30 x 30 square.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 |
| 2 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 |
| 3 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 |
| 4 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 |
| 5 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 |
| 6 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 |
| 7 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| 8 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 |
| 9 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 |
| 10 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 |
| 11 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 |
| 12 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 |
| 13 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 |
| 14 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 |
| 15 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 |
| 16 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 |
| 17 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 |
| 18 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 |
| 19 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 |
| 20 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 |
| 21 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 |
| 22 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 |
| 23 | 2 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 |
| 24 | 1 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 |
| 25 | 1 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 |
| 26 | 1 | 1 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 |
| 27 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 |
| 28 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 |
| 29 | 3 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 |
| 30 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 |

Figure 3b. P = prime(3) = 5. P# = 30. Opposite agreement counts, 30 x 30 square.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 |
| 2 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 |
| 3 | 1 | 1 | 3 | 0 | **1** | 1 | 2 | 1 | 2 | 0 | **1** | 2 | 2 | 0 | 2 | 0 | **2** | 2 | 1 | 0 | 2 | 1 | **2** | 1 | 1 | 0 | 3 | 1 | **1** | 1 |
| 4 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 |
| 5 | 2 | 1 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 2 | 1 | 2 | 1 | 2 | 0 | 2 | 2 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 1 | 2 | 1 |
| 6 | 1 | 1 | 1 | 2 | 0 | 3 | 0 | 1 | 2 | 1 | 1 | 2 | 0 | 2 | 1 | 2 | 0 | 2 | 1 | 1 | 2 | 1 | 0 | 3 | 0 | 2 | 1 | 1 | 1 | 2 |
| 7 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 |
| 8 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 |
| 9 | 2 | 0 | 2 | 1 | **1** | 2 | 1 | 0 | 3 | 0 | **2** | 1 | 1 | 1 | 2 | 1 | **1** | 1 | 2 | 0 | 3 | 0 | **1** | 2 | 1 | 1 | 2 | 0 | **2** | 1 |
| 10 | 1 | 2 | 0 | 2 | 2 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 2 | 1 | 2 |
| 11 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 |
| 12 | 0 | 2 | 2 | 1 | 0 | 2 | 1 | 2 | 1 | 1 | 0 | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 0 | 1 | 1 | 2 | 1 | 2 | 0 | 1 | 2 | 2 | 0 | 2 |
| 13 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 |
| 14 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 |
| 15 | 1 | 0 | 2 | 0 | **2** | 1 | 1 | 0 | 2 | 1 | **1** | 1 | 1 | 0 | 3 | 0 | **1** | 1 | 1 | 1 | 2 | 0 | **1** | 1 | 2 | 0 | 2 | 0 | **1** | 2 |
| 16 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 |
| 17 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 |
| 18 | 0 | 2 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 0 | 1 | 1 | 2 | 1 | 2 | 0 | 1 | 2 | 2 | 0 | 2 |
| 19 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 |
| 20 | 1 | 2 | 0 | 2 | 2 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 2 | 1 | 2 |
| 21 | 2 | 0 | 2 | 1 | **1** | 2 | 1 | 0 | 3 | 0 | **2** | 1 | 1 | 1 | 2 | 1 | **1** | 1 | 2 | 0 | 3 | 0 | **1** | 2 | 1 | 1 | 2 | 0 | **2** | 1 |
| 22 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 |
| 23 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 |
| 24 | 1 | 1 | 1 | 2 | 0 | 3 | 0 | 1 | 2 | 1 | 1 | 2 | 0 | 2 | 1 | 2 | 0 | 2 | 1 | 1 | 2 | 1 | 0 | 3 | 0 | 2 | 1 | 1 | 1 | 2 |
| 25 | 2 | 1 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 2 | 1 | 2 | 1 | 2 | 0 | 2 | 2 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 1 | 2 | 1 |
| 26 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 |
| 27 | 1 | 1 | 3 | 0 | **1** | 1 | 2 | 1 | 2 | 0 | **1** | 2 | 2 | 0 | 2 | 0 | **2** | 2 | 1 | 0 | 2 | 1 | **2** | 1 | 1 | 0 | 3 | 1 | **1** | 1 |
| 28 | 1 | 3 | 1 | 2 | 1 | 1 | 2 | 3 | 0 | 2 | 1 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 |
| 29 | 3 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 0 | 2 | 2 | 1 | 2 | 2 | 0 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 0 |
| 30 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 | 1 | 0 | 2 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 3 |

Figure 3c. P = prime(3) = 5. P# = 30. Either agreement counts, 30 x 30 square.

Remainder Agreement for Replicated Points. In Figures 3a, 3b, and 3c, the 25 points obtained by replicating the point (5, 3) when the 5# square is formed by replicating the 3# square are marked. (These points are (5, 3), (11, 3), (17, 3), (23, 3), (29, 3), (5, 9), (11, 9), (17, 9), (23, 9), (29, 9), (5, 15), (11, 15), (17, 15), (23, 15), (29, 15), (5, 21), (11, 21), (17, 21), (23, 21), (29, 21), (5, 27), (11, 27), (17, 27), (23, 27), (29, 27).) There are 5 x-coordinates separated by 6 and 5 y-coordinates separated by 6, the previous primorial. Therefore, first, the remainders of the coordinates of these points modulo primes smaller than 5, (2 and 3) are all the same, and so the number of remainder agreements are the same for these points for the smaller primes.

Second, for 5 itself, the remainders of the 5 x-coordinates or the 5 y-coordinates are all different, comprising all the remainders modulo 5, because the coordinates are in an arithmetic progression with a difference which is relatively prime to 5. The remainders for a particular point modulo 5 may have no agreement, so its agreement count would stay the same, same agreement, so its same or either agreement count would increase by one (one per line), or opposite agreement, so its opposite or either agreement count would increase by one (also one per line).

Whatever individual point is replicated, the number of replicated points that do not change or that increase their agreement count by one is the same. Therefore the number of points in the P# square with a particular agreement count can be calculated from the counts for the previous primorial square. Multiply the next smaller count for the previous square by the number of increases for each replicated point. Add that to the particular value count for the previous square times the number of unchanged counts per replicated point. This calculation can be done by polynomial multiplication, as shown below.

Number of Points for Each Count For the Same and For the Opposite Agreement of Remainders. Start with the 2 x 2 square for P = prime(1) = 2. There are 2 points with 0 remainders that agree and two points with 1 remainder that agrees.

Consider one particular point of the 2 x 2 square as the square is replicated 3 times horizontally and 3 times vertically to form the 6 x 6 square. On each horizontal line the three copies of the replicated point, which are separated by 2, will have x-coordinates with different remainders modulo 3. One remainder will have same agreement with the remainder for the common y-coordinate and one will have opposite agreement. Thus one point will have an increased agreement count, and two will not.

There are three horizontal lines of replicated points, so the agreement count will increase for 3 points, and not change for six points.

Therefore, replicating the two points with 0 agreement of remainders gives 2 x 6 = 12 points with unchanged zero agreement and 2 x 3 = 6 points with increased agreement of 1 remainder. Also, replicating the 2 points that agree in one remainder gives 2 x 6 = 12 points that are unchanged in agreeing in one remainder and 2 x 3 = 6 points that agree in two remainders.

Thus, in the 6 x 6 primorial square for P = prime(2) = 3, there are 12 points which agree in no remainders, 6 + 12 = 18 points that agree in one remainder, and 6 points that agree in two remainders.

For P = prime(n), consider replicating a point of the primorial square for the preceding prime P times horizontally and P times vertically. The replicated points on a horizontal line are separated by the previous primorial, which is relatively prime to P. Thus the

remainders for the sequence of P x-coordinates are all different modulo P, so one has same agreement, one has opposite agreement with the common y-coordinate modulo P. That is, one point has an agreement count that increases by one and P – 1 have agreement counts that stay the same. For all P horizontal lines of replicated points, (P – 1) x P points have unchanged agreement counts and 1 x P have agreement counts that increase by 1.
A way to find the number of points with each agreement count is to use the coefficients of the polynomial (2 + 2x)((3 - 1)3 + 1(3)x)…((prime(n) – 1)prime(n) + 1(prime(n))x) = 2(3)…(prime(n))(1 + x)((3 – 1) + x)…((prime(n) – 1) + x). Except for the first factor, the constant for the n-th factor is the number of replicated points for the n-th prime that do not change their agreement count modulo that prime. The coefficient of x is the number of replicated points that increase their agreement counts. The k-th coefficient of the product of polynomials gives the number of points in the square that agree for k primes. The first few polynomials are

$2 + 2x$
$(2 + 2x)(6 + 3x) = 2(3)(1 + x)(2 + x) = 12 + 18x + 6x^2$
$(2 + 2x)(6 + 3x)(20 + 5x) =2(3)(5)(1 + x)(2 + x)(4 + x) = 240 + 420x + 210x^2 + 30x^3$
$(2 + 2x)(6 + 3x)(30 + 5x)(42 + 7x) = 2(3)(5)(7)(1 + x)(2 + x)(4 + x)(6 + x) = 10080 + 19320x + 11760x^2 + 2730x^3 + 210x^4$

Referring to the third polynomial, in Figure 3b (or 3a) there are 240 0's, 420 1's, 210 2's and 30 3's.

Number of Points for Each Count For Either (Same or Opposite) Agreement of Remainders. Again, start with the 2 x 2 square for P = prime(1) = 2. There are 2 points with coordinates with 0 remainders that agree and two points with 1 remainder that agrees.
Consider one particular point as the 2 x 2 square is replicated 3 times horizontally and 3 times vertically to form the 6 x 6 square.
On each horizontal line the three copies of the replicated point, which are separated by 2, will have x-coordinates with different remainders modulo 3. One remainder will have same agreement with the remainder for the common y-coordinate and one will have opposite agreement. Thus two points will have an increased agreement count (either type of agreement), and one will not. There are again three horizontal lines of replicated points. However, along one of the three horizontal lines of replicated points the common y-coordinate will have remainder 0. (The y-coordinates of the horizontal lines differ by 2, and so have different remainders modulo 3.) Therefore only one of the points on that line has both same and opposite agreement, and the other two do not. For all nine replicated points the agreement count will increase for 3 x 2 – 1 = 5 of them and stay the same for 2 x 1 + 2 = 2 x 2 = 4 of them.
Thus, replicating the two points with 0 agreement of remainders gives 2 x 4 = 8 points with unchanged zero agreement and 2 x 5 = 10 points with agreement increased to one remainder. Also, replicating the 2 points that agree in one remainder gives 2 x 4 = 8 points that are unchanged in agreeing in one remainder and 2 x 5 = 10 points that agree in two remainders.

So, in the 6 x 6 primorial square for P = prime(2) = 3, there are 8 points which agree in no remainders, 10 + 8 = 18 points that agree in one remainder, and 10 points that agree in two remainders.

For P = prime(n), consider replicating a point of the primorial square for the preceding prime P times horizontally and P times vertically. The replicated points on a horizontal line are separated by the previous primorial, which is relatively prime to P. Thus the remainders for the sequence of P x-coordinates are all different modulo P, so one has same agreement, one has opposite agreement with the common y-coordinate modulo P. That is, two points have an either agreement count that increases by one and P – 2 have agreement counts that stay the same. There are again P lines of replicated points, all of which have different y-coordinates modulo P because they are separated by the previous primorial. One line has y-coordinate 0 modulo P. The x-coordinate for only one point on that line has matching remainder 0 which has both same and opposite agreement. On that one line, only one point has an increased agreement count and P – 1 points have unchanged agreement counts. For all P lines of replicated points in the vertical direction, $(P – 2) \times (P – 1) + (P – 1) = (P – 1)^2$ points have unchanged agreement counts and 2 x P – 1 have agreement counts that increase by 1.

To find the number of points with each agreement count, find the coefficients of the polynomial $(2 + 2x)((3 - 1)^2 + (2(3) – 1)x)\ldots((\text{prime}(n) – 1)^2 + (2\text{prime}(n) – 1)x)$.

The first few of these polynomials are

$2 + 2x$

$(2 + 2x)(2^2 + (2(3) – 1)x) = (2 + 2x)(4 + 5x) = 8 + 18x + 10x^2$

$(2 + 2x)(2^2 + (2(3) – 1)x)(4^2 + (2(5) – 1)x) = (2 + 2x)(4 + 5x)(16 + 9x) = 128 + 360x + 322x^2 + 90x^3$

$(2 + 2x)(2^2 + (2(3) – 1)x)(4^2 + (2(5) – 1)x)(6^2 + (2(7) – 1)x) = (2 + 2x)(4 + 5x)(16 + 9x)(36 + 13x) = 4608 + 14624x + 16272x^2 + 7426x^3 + 1170x^4$

Thus, referring to the third polynomial, in Figure 3c there are 128 0’s, 360 1’s, 322 2’s and 90 3’s as remainder agreement counts.

Note 1. The coefficients of these polynomials can be arranged in remainder agreement count triangles. The coefficients of the binomial that produces the next line are shown in parentheses.

For same or opposite agreement:

| | | | | | |
|---|---|---|---|---|---|
| 2 | 2 | | | | (6 3) |
| 12 | 18 | 6 | | | (20 5) |
| 240 | 420 | 210 | 30 | | (42 7) |
| 10080 | 19320 | 11760 | 2730 | 210 | (110 11) |

For either agreement:

| | | | | | |
|---|---|---|---|---|---|
| 2 | 2 | | | | (4 5) |
| 8 | 18 | 10 | | | (16 9) |
| 128 | 360 | 322 | 90 | | (36 13) |
| 4608 | 14624 | 16272 | 7426 | 1170 | (100 21) |

Note 2. The k-th coefficient of the polynomial (1 + x)(2 + x)(4 + x)…((prime(n) – 1) + x) gives the number of points on the primorial line segment, $1 \leq c \leq P\#$, P = prime(n), which are divisible by k distinct primes from 2 to P.

Note 3. In some cases, for a point (c, d) with 0 either remainder agreement, both c – d and c + d are prime and (c – d) + (c + d) = 2c. If for all c in P#, the smallest d with 0 either remainder agreement were small enough in terms of P, Goldbach's Conjecture could be proved.