

The Greatest  
Common Divisor  
Function is  
Symmetric

Pedro Hugo García Peláez

Copyright © 2019 by Pedro Hugo García Peláez

All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission of the publisher, except in the case of brief quotations embodied in critical reviews and certain other noncommercial uses permitted by copyright law. For permission requests, write to the publisher, addressed "Attention: Permissions Coordinator," at the address below.

[hugo117711@gmail.com](mailto:hugo117711@gmail.com)

Suppose we have two numbers  $a$  and  $b$  whose great common divisor from now g.c.d. is  $d$

The sum of  $a$  and  $b$  or  $a + b$  and subtraction  $a - b$  have the same g.c.d

The symmetry extends much more since  $a$ ,  $b + (b - a)$ ,  $b + (b - a) * n$  have the same g.c.d being  $(n)$  a natural number.

This gives us a set of numbers whose g.c.d are equal to that of  $a$  and  $b$  that is an arithmetic sequence of ratio  $(b - a)$  either adding it to our number  $(a)$  or subtracting it successively from our number  $(b)$  gives us a set of numbers with the same g.c.d

For example we have two very large numbers  $a$ ,  $b$  and we want to know their g.c.d

We would subtract  $b - a$  from the number  $a$  and subtract  $b - a$  again until we had two more manageable numbers or at least one of them would be as small as we wanted and we could

find the g.c.d of both easily.

The symmetry of g.c.d extends even further.

Suppose we have two numbers  $a$  and  $b$  with  
 $\text{g.c.d.} = d$

Then the numbers that are the sum of the  
previous two as if they were a Fibonacci  
sequence also have the same g.c.d

That is

$$a + b = n$$

$$b + n = n_1$$

$$n + n_1 = n_2$$

... ..

This symmetry explains why two consecutive  
numbers are coprimes to each other.

Since if 1 and 2 have  $\text{g.c.d} = 1$   
the sum of the next number for that difference

they have would be  $2 + 1$  the next  $3 + 1$  or we would be adding all the natural numbers and all the pairs would have  $\text{g.c.d.} = 1$

Change as an explanation you can see that if we take a prime number and compare them with 1 obviously they will have  $\text{g.c.d.} = 1$  and since their subtraction would be  $p-1$  until the number  $p + (p-1)$  they would still have  $\text{g.c.d.} = 1$  but for  $2p$  those two numbers they would already have a  $\text{g.c.d.} = p$

To find the  $\text{g.c.d.}$  of any polynomial we would also follow this procedure

Given this it is obvious that if we want to build an arithmetic sequence of natural numbers and we want it to contain primes we should not do it with a beginning where the first two numbers do not have  $\text{g.c.d.} = 1$

The algorithm can be extended to polynomials and complex numbers.

If we have two polynomials:

$$x^n + x^{(n-1)} + x^{(n-2)} \dots$$

$$x^p + x^{(p-1)} + x^{(p-2)} \dots$$

Its addition and subtraction give us another polynomial whose g.c.d with respect to any of the two initial polynomials is equal to g.c.d of that polynomials.

And on the other hand given two complex numbers:

$$(a + bi)$$

$$(c + di)$$

Its addition and subtraction give us other complex numbers whose g.c.d. respect to the initial numbers is equal to g.c.d. of  $(a + bi)$  and  $(c + di)$