The Greatest Common Divisor Function is Symmetric

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Suppose we have two numbers a and b whose great common divisor from now g.c.d. is d

The sum of a and b or a + b and subtraction a-b have the same g.c.d

The symmetry extends much more since a, b + (b-a), b + (b-a) * n have the same g.c.d
being (n) a natural number.

This gives us a set of numbers whose g.c.d are equal to that of a and b that is an arithmetic sequence of ratio (b-a) either adding it to our number (a) or subtracting it successively from our number (b) gives us a set of numbers with the same g.c.d

For example we have two very large numbers a, b and we want to know their g.c.d

We would subtract b-a from the number a and subtract b-a again until we had two more manageable numbers or at least one of them would be as small as we wanted and we could find the g.c.d of both easily.

The symmetry of g.c.d extends even further.

Suppose we have two numbers a and b with g.c.d. = d

Then the numbers that are the sum of the previous two as if they were a Fibonacci sequence also have the same g.c.d

That is

$a + b = n$
$b + n = n1$
$n + n1 = n2$
... ..

This symmetry explains why two consecutive numbers are coprimes to each other.

Since if 1 and 2 have g.c.d = 1
the sum of the next number for that difference

they have would be 2 + 1 the next 3 + 1 or we would be adding all the natural numbers and all the pairs would have g.c.d. = 1

Change as an explanation you can see that if we take a prime number and compare them with 1 obviously they will have g.c.d. = 1 and since their subtraction would be p-1 until the number p + (p-1) they would still have g.c.d = 1 but for 2p those two numbers they would already have a g.c.d. = p

To find the g.c.d. of any polynomial we would also follow this procedure

Given this it is obvious that if we want to build an arithmetic sequence of natural numbers and we want it to contain primes we should not do it with a beginning where the first two numbers do not have g.c.d = 1

The algorithm can be extended to polynomials and complex numbers.

If we have two polynomials:

$x \wedge n + x \wedge (n-1) + x \wedge (n-2)$ ....
$x \wedge p + x \wedge (p-1) + x \wedge (p-2)$ ....

Its addition and subtraction give us another polynomial whose g.c.d with respect to any of the two initial polynomials is equal to g.c.d of that polynomials.

And on the other hand given two complex numbers:

$(a + bi)$
$(c + di)$

Its addition and subtraction give us other complex numbers whose g.c.d. respect to the initial numbers is equal to g.c.d. of$(a + bi)$ and $(c + di)$