On the possible mathematical connections between various Ramanujan's equations and some sectors of Particle Physics, String Theory and Physics of Black Holes

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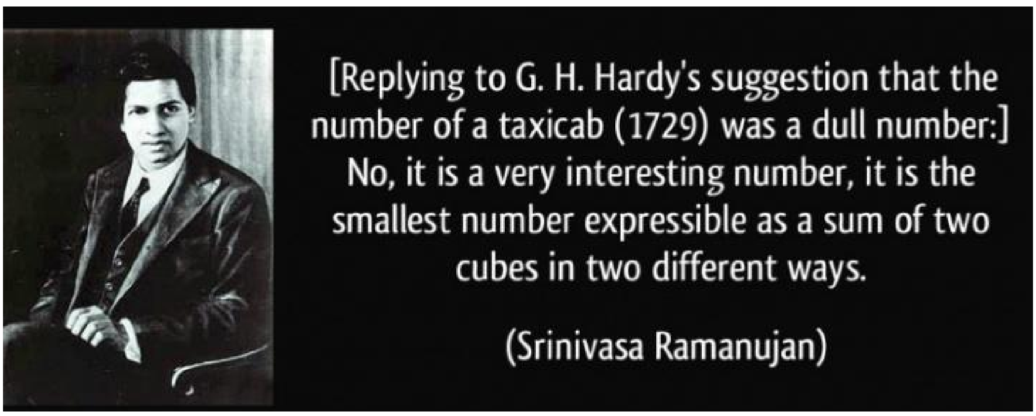
Abstract

In this research paper, we have described and analyzed the possible mathematical connections between various Ramanujan's equations and some sectors of Particle Physics (rest mass of meson $f_0(1710)$, mass of proton, electric charge of positron, mass of Higgs boson), String Theory and Physics of Black Holes (entropy)



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https://www.indiatimes.com/entertainment/7-facts-about-mathematician-srinivasa-ramanujan-the-man-who-knew-infinity-245643.html

1,729 is the smallest number which can be represented in two different ways as the sum of two cubes:

$$\begin{aligned} 1729 &= 1^3 + 12^3 \\ &= 9^3 + 10^3 \end{aligned}$$

It is also incidentally the product of 3 prime numbers:

$$1729 = 7 \times 13 \times 19$$

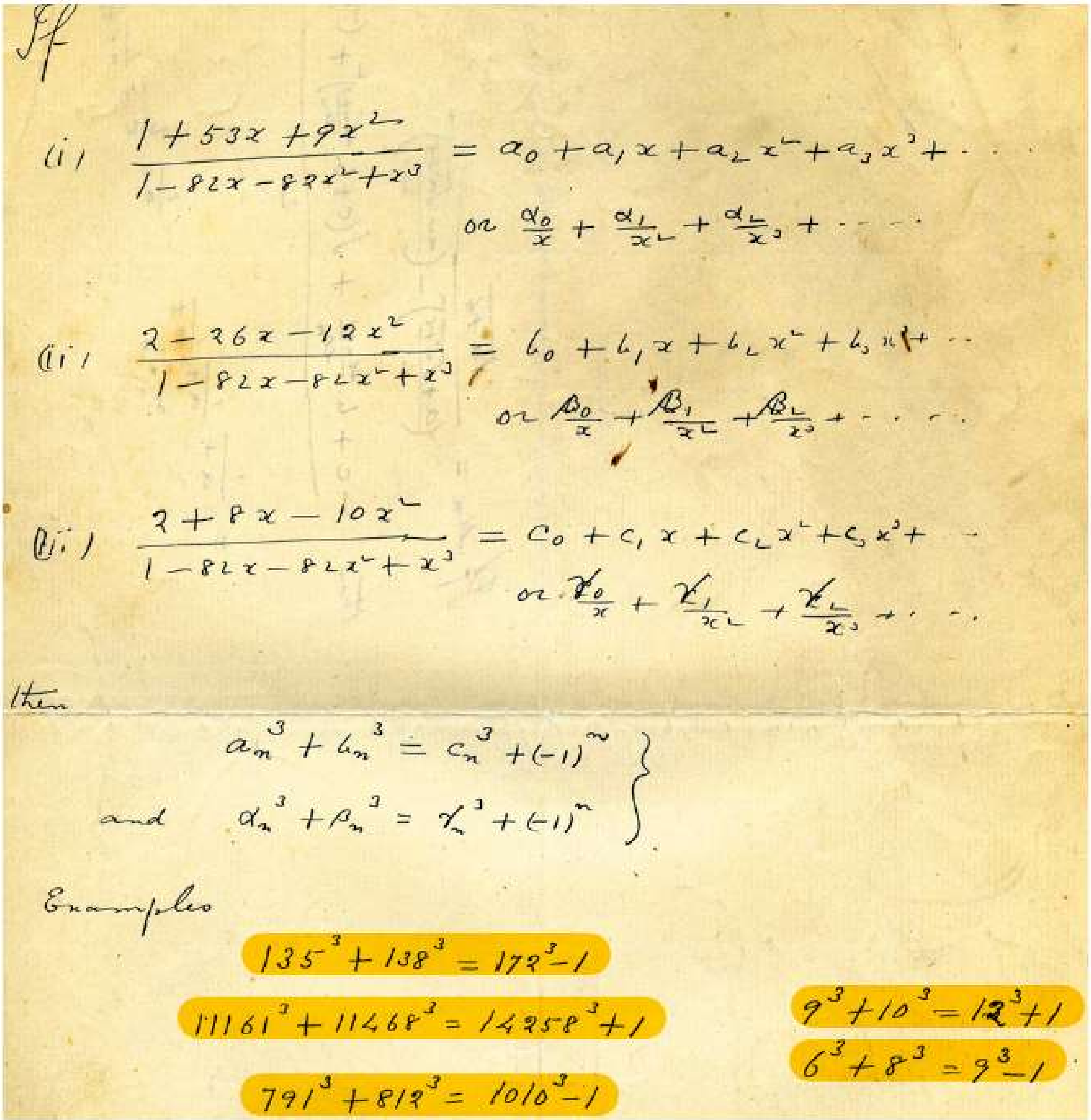
The largest known similar number is:

$$\begin{aligned} 885623890831 &= 7511^3 + 7730^3 \\ &= 8759^3 + 5978^3 \\ &= 3943 \times 14737 \times 15241 \end{aligned}$$

https://www.science20.com/news_articles/divine_patterns_ramanujans_magical_mind_gets_math_formula-99186

$$\begin{aligned}
\mathrm{Ta}(1) &= 2 = 1^3 + 1^3 \\
\mathrm{Ta}(2) &= 1729 = 1^3 + 12^3 \\
&= 9^3 + 10^3 \\
\mathrm{Ta}(3) &= 87539319 = 167^3 + 436^3 \\
&= 228^3 + 423^3 \\
&= 255^3 + 414^3 \\
\mathrm{Ta}(4) &= 6963472309248 = 2421^3 + 19083^3 \\
&= 5436^3 + 18948^3 \\
&= 10200^3 + 18072^3 \\
&= 13322^3 + 16630^3 \\
\mathrm{Ta}(5) &= 48988659276962496 = 38787^3 + 365757^3 \\
&= 107839^3 + 362753^3 \\
&= 205292^3 + 342952^3 \\
&= 221424^3 + 336588^3 \\
&= 231518^3 + 331954^3
\end{aligned}$$

https://awesci.com/the-taxicab-number/



If

(i) $$\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$
or $$\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$$

(ii) $$\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$$
or $$\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$$

(iii) $$\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$
or $$\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$$

then

$$a_n^3 + b_n^3 = c_n^3 + (-1)^n$$
and
$$\alpha_n^3 + \beta_n^3 = \gamma_n^3 + (-1)^n$$

Examples

$135^3 + 138^3 = 172^3 - 1$

$11161^3 + 11468^3 = 14258^3 + 1$

$791^3 + 812^3 = 1010^3 - 1$

$9^3 + 10^3 = 12^3 + 1$

$6^3 + 8^3 = 9^3 - 1$

Page from Ramanujan's Lost Notebook. Image credit: Trinity College Cambridge. Reproduced from Ono, 2015.

We will see that several formulas (numbers which can be represented in two different ways as the algebraic sum of two cubes) written by Ramanujan on the page above, gives us useful and new mathematical connections with some sectors of Particle Physics, String Theory, and Black Hole physics.

We see that:

135 and 138 are numbers very near to the values of the pion rest mass
139.57018±0.00035   134.9766±0.0006.

11468^3 = 1508214295232 and that $(1508214295232)^{1/4}$ = 1108,1939202 value very near to the rest mass of Lambda baryon 1115.683±0.006

14258^3 = 2898516861512 and that $(2898516861512)^{1/4}$ = 1304,8 value very near to the rest mass of Xi baryon 1314.86±0.20

791 and 1010 are numbers very near to the rest mass of Omega meson and Phi meson
782.65±0.12   1019.445±0.020

9^3 = 729 and $12^3 + 1$ = 1729 are numbers very near to the values of mass of candidate glueball, the scalar meson $f_0(1710)$ that is 1723 (+6,-5) and practically equal at the nonperturbative contribution (727,39) to the mass of a 1S quarkonium for m_q = 4.78 MeV/c^2 = 0.00478 GeV/c^2 , that is the mass of quark down is 4.8±0.5±0.3 = 4.78 MeV/c^2.

Indeed:

Input:

$$\frac{\pi^2 \times 0.00478 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.00478\right)^4}$$

Result:

727.392...

$14258^3 + 1$ = 2898516861513;   2898516861513 / 1729 = 1676412297 =

= 1,676412297 * 10^9; (2898516861513 / 1729) * 10^{-6} = 1676,412297 that is very near to the rest mass of Omega baryon 1672.45±0.29

$1010^3 - 1$ = 1030300999; 1030300999 / 1728 = 596239,004; √596239,004 =

= 772,16514 that is very near to the rest mass of Charged rho meson 775.4±0.4

$172^3 - 1$ = 5088447; 5088447 / 1729 = 2943 that is a good approximations of the rest mass of Charmed eta meson 2980.3±1.2 . (Note that 2943 + 36 = 2979, practically equal to the above Charmed eta meson value).

$f_0(1710)$

$I^G(J^{PC}) = 0^+(0^{++})$

See our mini-review in the 2004 edition of this *Review*, Physics Letters **B592** 1 (2004). See also the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

f_0(1710) MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1723^{+6}_{-5} OUR AVERAGE		Error includes scale factor of 1.6. See the ideogram below.			
$1759 \pm 6\ ^{+14}_{-25}$	5.5k	[1] ABLIKIM	13N	BES3	$e^+e^- \to J/\psi \to \gamma\eta\eta$
$1750^{+6}_{-7}\ ^{+29}_{-18}$		UEHARA	13	BELL	$\gamma\gamma \to K^0_S K^0_S$
$1701 \pm 5\ ^{+9}_{-2}$	4k	[2] CHEKANOV	08	ZEUS	$e p \to K^0_S K^0_S X$
$1765^{+4}_{-3} \pm 13$		ABLIKIM	06V	BES2	$e^+e^- \to J/\psi \to \gamma\pi^+\pi^-$
$1760 \pm 15\ ^{+15}_{-10}$		[3] ABLIKIM	05Q	BES2	$\psi(2S) \to \gamma\pi^+\pi^- K^+K^-$
1738 ± 30		ABLIKIM	04E	BES2	$J/\psi \to \omega K^+K^-$
$1740 \pm 4\ ^{+10}_{-25}$		[4] BAI	03G	BES	$J/\psi \to \gamma K\overline{K}$
1740^{+30}_{-25}		[4] BAI	00A	BES	$J/\psi \to \gamma(\pi^+\pi^-\pi^+\pi^-)$
1698 ± 18		[5] BARBERIS	00E		450 $pp \to p_f \eta\eta p_s$
$1710 \pm 12 \pm 11$		[6] BARBERIS	99D	OMEG	450 $pp \to K^+K^-, \pi^+\pi^-$
1710 ± 25		[7] FRENCH	99		300 $pp \to p_f(K^+K^-)p_s$
1707 ± 10		[8] AUGUSTIN	88	DM2	$J/\psi \to \gamma K^+K^-, K^0_S K^0_S$
1698 ± 15		[8] AUGUSTIN	87	DM2	$J/\psi \to \gamma\pi^+\pi^-$
$1720 \pm 10 \pm 10$		[9] BALTRUSAIT...	87	MRK3	$J/\psi \to \gamma K^+K^-$
1742 ± 15		[8] WILLIAMS	84	MPSF	200 $\pi^- N \to 2K^0_S X$
1670 ± 50		BLOOM	83	CBAL	$J/\psi \to \gamma 2\eta$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$1744 \pm 7 \pm 5$	381	[10,11] DOBBS	15		$J/\psi \to \gamma\pi^+\pi^-$
$1705 \pm 11 \pm 5$	237	[10,11] DOBBS	15		$\psi(2S) \to \gamma\pi^+\pi^-$
$1706 \pm 4 \pm 5$	1.0k	[10,11] DOBBS	15		$J/\psi \to \gamma K^+K^-$
$1690 \pm 8 \pm 3$	349	[10,11] DOBBS	15		$\psi(2S) \to \gamma K^+K^-$
1750 ± 13		AMSLER	06	CBAR	1.64 $\overline{p}p \to K^+K^-\pi^0$
1747 ± 5	80k	[12,13] UMAN	06	E835	5.2 $\overline{p}p \to \eta\eta\pi^0$
1776 ± 15		VLADIMIRSK...	06	SPEC	40 $\pi^- p \to K^0_S K^0_S n$
1790^{+40}_{-30}		[3] ABLIKIM	05	BES2	$J/\psi \to \phi\pi^+\pi^-$
1670 ± 20		[12] BINON	05	GAMS	33 $\pi^- p \to \eta\eta n$
1726 ± 7	74	[13] CHEKANOV	04	ZEUS	$e p \to K^0_S K^0_S X$
$1732+15$		[14] ANISOVICH	03	RVUE	
1682 ± 16		TIKHOMIROV	03	SPEC	40.0 $\pi^- C \to K^0_S K^0_S K^0_L X$
1670 ± 26	3.6k	[4,15] NICHITIU	02	OBLX	
1770 ± 12		[16,17] ANISOVICH	99B	SPEC	0.6–1.2 $p\overline{p} \to \eta\eta\pi^0$

1730 ± 15		[4] BARBERIS	99	OMEG	450 $pp \to p_s p_f K^+ K^-$
1750 ± 20		[4] BARBERIS	99B	OMEG	450 $pp \to p_s p_f \pi^+ \pi^-$
1750 ± 30		[18] ANISOVICH	98B	RVUE	Compilation
1720 ± 39		BAI	98H	BES	$J/\psi \to \gamma \pi^0 \pi^0$
1775 ± 1.5	57	[19] BARKOV	98		$\pi^- p \to K^0_S K^0_S n$
1690 ± 11		[20] ABREU	96C	DLPH	$Z^0 \to K^+ K^- + X$
$1696 \pm 5 \, ^{+9}_{-34}$		[9] BAI	96C	BES	$J/\psi \to \gamma K^+ K^-$
$1781 \pm 8 \, ^{+10}_{-31}$		[4] BAI	96C	BES	$J/\psi \to \gamma K^+ K^-$
1768 ± 14		BALOSHIN	95	SPEC	40 $\pi^- C \to K^0_S K^0_S X$
1750 ± 15		[21] BUGG	95	MRK3	$J/\psi \to \gamma \pi^+ \pi^- \pi^+ \pi^-$
1620 ± 16		[9] BUGG	95	MRK3	$J/\psi \to \gamma \pi^+ \pi^- \pi^+ \pi^-$
1748 ± 10		[8] ARMSTRONG	93C	E760	$\overline{p} p \to \pi^0 \eta \eta \to 6\gamma$
~ 1750		BREAKSTONE	93	SFM	$pp \to pp \pi^+ \pi^- \pi^+ \pi^-$
1744 ± 15		[22] ALDE	92D	GAM2	38 $\pi^- p \to \eta \eta n$
1713 ± 10		[23] ARMSTRONG	89D	OMEG	300 $pp \to pp K^+ K^-$
1706 ± 10		[23] ARMSTRONG	89D	OMEG	300 $pp \to pp K^0_S K^0_S$
1700 ± 15		[9] BOLONKIN	88	SPEC	40 $\pi^- p \to K^0_S K^0_S n$
1720 ± 60		[4] BOLONKIN	88	SPEC	40 $\pi^- p \to K^0_S K^0_S n$
1638 ± 10		[24] FALVARD	88	DM2	$J/\psi \to \phi K^+ K^-, K^0_S K^0_S$
1690 ± 4		[25] FALVARD	88	DM2	$J/\psi \to \phi K^+ K^-, K^0_S K^0_S$
1755 ± 8		[26] ALDE	86C	GAM2	38 $\pi^- p \to n 2\eta$
1730^{+2}_{-10}		[27] LONGACRE	86	RVUE	22 $\pi^- p \to n 2K^0_S$
1650 ± 50		BURKE	82	MRK2	$J/\psi \to \gamma 2\rho$
1640 ± 50		[28,29] EDWARDS	82D	CBAL	$J/\psi \to \gamma 2\eta$
$1730 \pm 10 \pm 20$		[30] ETKIN	82C	MPS	23 $\pi^- p \to n 2K^0_S$

Nonperturbative contribution [1032, 1033]‖

$$\Delta E^{\rm np} \;=\; \frac{\pi^2 m_q}{(C_F \alpha_s m_q)^4} \frac{624}{425} \left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \right| 0 \right\rangle ,$$

where the gluon condensate is evaluated as [1034, 1035, 1036]

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \right| 0 \right\rangle \;\approx\; 0.012\,{\rm GeV}^4 .$$

and

These quantities are evaluated at a hadronic energy scale $\mu_{\rm H} \sim 1$ GeV. In Ref. [582, 584, 576], $\mu_{\rm H}$ is chosen such that the strong coupling constant satisfies $g_3(\mu_{\rm H}) = 4\pi/\sqrt{6}$. See Sec. 9.3.6 for the QCD correction factors due to the running effect between the EW scale and $\mu_{\rm H}$.

$\beta^{(k)}$ are evaluated with $n_f = n_l$ in (9.1.1). The SU(3) color factors are

$$C_A = 3, \quad C_F = \frac{4}{3}, \quad T_F = \frac{1}{2}. \tag{9.1.60}$$

For $C_F = 4/3$ $\alpha_s = 4\pi/\sqrt{6} = 5.130199 = 5.13$ and $m_q = 4.8$ MeV/c^2 = 0.0048 GeV/c^2 (the mass of quark down is 4.8±0.5±0.3 = 4.8 MeV/c^2), we obtain:

[Pi^2*(0.0048)*624*0.012]/[425*(((4/3)*5.13*(0.0048))^4))]

Input:

$$\frac{\pi^2 \times 0.0048 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.0048\right)^4}$$

Result:

718.337...

For $C_F = 4/3$ $\alpha_s = 5.13$ and $m_q = 4.776483$ MeV/c^2 = 0.004776483 GeV/c^2 (the mass of quark down is 4.8±0.5±0.3 = 4.776483 MeV/c^2), we obtain:

[Pi^2*(0.004776483)*624*0.012]/[425*(((4/3)*5.13*(0.004776483))^4))]

Input interpretation:

$$\frac{\pi^2 \times 0.004776483 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^4}$$

Result:

729.000...

Alternative representations

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = \frac{0.0359424\,(180\,°)^2}{425 \times 0.032832^4}$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = \frac{0.0359424\,(-i \log(-1))^2}{425 \times 0.032832^4}$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = \frac{0.215654\,\zeta(2)}{425 \times 0.032832^4}$$

Series representations:

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 1164.52 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}\right)^2$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 291.131 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 72.7828 \left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}\right)^2$$

Integral representations:

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 291.131 \left(\int_0^{\infty} \frac{1}{1 + t^2} \, dt\right)^2$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 1164.52 \left(\int_0^1 \sqrt{1 - t^2} \, dt\right)^2$$

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 291.131 \left(\int_0^{\infty} \frac{\sin(t)}{t} \, dt\right)^2$$

We note that in the integral representations, we have:

$$\frac{\pi^2 (0.0048 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.0048}{3}\right)^4} = 1164.52 \left(\int_0^1 \sqrt{1 - t^2} \, dt\right)^2$$

from which we can be obtained 1164.52 that is a value very near to the fundamental Ramanujan class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696$

$$\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3 = 1164,269601267364$$

From:

Srinivasa Ramanujan - **Modular equations and approximations to π**
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

5. Since G_n and g_n can be expressed as roots of algebraical equations with rational coefficients, the same is true of G_n^{24} or g_n^{24}. So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \cdots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \cdots.$$

But we know that

$$\begin{aligned} 64e^{-\pi\sqrt{n}}g_n^{24} &= 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \cdots, \\ 64g_n^{24} &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \\ 64a - 64bg_n^{-24} + \cdots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \\ 64a - 4096be^{-\pi\sqrt{n}} + \cdots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \end{aligned}$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \tag{13}$$

Similarly, if

$$1 = aG_n^{-24} - bG_n^{-48} + \cdots,$$

then

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \tag{14}$$

From (13) and (14) we can find whether $e^{\pi\sqrt{n}}$ is very nearly an integer for given values of n, and ascertain also the number of 9's or 0's in the decimal part. But if G_n and g_n be simple quadratic surds we may work independently as follows. We have, for example,

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^6 + (6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

The sum of the expressions highlighted in yellow, gives us useful and new mathematical connections with some sectors of Particle Physics, String Theory, Cosmology and Black Hole physics.

We have that:

64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]

Input:

$$64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)$$

Result:

24792915328

Decimal approximation:

$2.4792915328000000000000000000000000000000000000000... \times 10^{10}$

or:

(([e^(Pi*sqrt(22)))]+[e^(Pi*sqrt(37)))]+[e^(Pi*sqrt(58)))]))

Input:

$$e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}$$

Decimal approximation:

$2.4792915351998235293232263765271237917764189072010905... \times 10^{10}$

From the value of new measurement of Hubble constant, we have that for 73.9

LMC AND THE HUBBLE CONSTANT

Table 5. Best Estimates of H_0 Including Systematics

Anchor(s)	Value [km s^{-1} Mpc^{-1}]	Δ Planck*+ ΛCDM (σ)
LMC	74.22 ± 1.82	3.6
Two anchors		
LMC + NGC 4258	73.40 ± 1.55	3.7
LMC + MW	74.47 ± 1.45	4.6
NGC 4258 + MW	73.94 ± 1.58	3.9
Three anchors (preferred)		
NGC 4258 + MW + LMC	**74.03 ± 1.42**	4.4

NOTE * : $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} (Planck Collaboration et al. 2018)

73.9/(2e) = 13.5931454

And

$$\sqrt[13.5931454]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}680965497649\ldots$$

1,680965497649 * 2.47929153519 = 4,167603

From:

$$t_H \equiv \frac{1}{H_0} = \frac{1}{67.8(\mathrm{km/s})/\mathrm{Mpc}} = 4.55 \cdot 10^{17}\mathrm{s} = 14.4 \text{ billion years.}$$

We have that:

$$\frac{1}{\frac{74.03}{3.08567758130573 \times 10^{19}}}$$

or:

$$\frac{1}{74.03} \times 3.08567758130573 \times 10^{19}$$

$4.168144780907375388356071862758341213021747940024314 4\ldots \times 10^{17}$

416814478090737538 sec = 13.217.099.976,967 billion years

For 416760300000000000 sec * 31536000 = 13.215.382.420,0913 billion years

From the sum of the various exp, multiplied twice the square of golden ratio and divided by π^2 , we obtain:

2/Pi^2 *((sqrt(5)+1)/2))^2)
ln(((([e^(Pi*sqrt(22)))]+[e^(Pi*sqrt(37)))]+[e^(Pi*sqrt(58)))])))

$$\frac{2}{\pi^2}\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)$$

Exact result:

$$\frac{\left(1+\sqrt{5}\right)^2 \log\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)}{2\,\pi^2}$$

Decimal approximation:

12.697482408652163204858670677086684854489638162359791270 95...

Alternate form:

$$\frac{3\log\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)}{\pi^2}+\frac{\sqrt{5}\,\log\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)}{\pi^2}$$

Alternative representations:

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2}=\frac{2\log_e\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}{\pi^2}$$

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2}=\frac{2\log(a)\log_a\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}{\pi^2}$$

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2}=-\frac{2\,\mathrm{Li}_1\left(1-e^{\pi\sqrt{22}}-e^{\pi\sqrt{37}}-e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}{\pi^2}$$

Series representations:

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2}=\frac{\left(3+\sqrt{5}\right)\left(\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}\right)}{\pi^2}$$

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2}=\frac{\left(1+\sqrt{5}\right)^2\left(\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}\right)}{2\pi^2}$$

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2} = \frac{1}{2\pi^2}\left(1+\sqrt{5}\right)^2 \left(2 i \pi \left\lfloor \frac{\arg\left(e^{\sqrt{22}\pi}+e^{\sqrt{37}\pi}+e^{\sqrt{58}\pi}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{\sqrt{22}\pi}+e^{\sqrt{37}\pi}+e^{\sqrt{58}\pi}-x\right)^k x^{-k}}{k}\right) \quad \text{for } x < 0$$

Integral representations:

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2} = \frac{3+\sqrt{5}}{\pi^2} \int_1^{e^{\sqrt{22}\pi}+e^{\sqrt{37}\pi}+e^{\sqrt{58}\pi}} \frac{1}{t}\,dt$$

$$\frac{\left(\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)\right)2}{\pi^2} = -\frac{i\left(3+\sqrt{5}\right)}{2\pi^3} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+e^{\sqrt{22}\pi}+e^{\sqrt{37}\pi}+e^{\sqrt{58}\pi}\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

We note that the value 12,69748 is a good approximation to the value of black hole entropy 12,57.

(From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten_Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the J-function itself, so

$$Z(q) = q^{-1} + 196884q + \ldots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is 4π=12.57... So we are off by just a few percent.)

Now:

72 * ln(((([e^(Pi*sqrt(22))]+[e^(Pi*sqrt(37))]+[e^(Pi*sqrt(58))])))

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)$$

Decimal approximation:

1723.235312017089945798736303347869789862522460911750748201...

Alternative representations:

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=72\log_e\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)$$

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=72\log(a)\log_a\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)$$

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=-72\,\text{Li}_1\left(1-e^{\pi\sqrt{22}}-e^{\pi\sqrt{37}}-e^{\pi\sqrt{58}}\right)$$

Series representations:

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=$$
$$72\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-72\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}$$

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=144\,i\,\pi\left\lfloor\frac{\arg\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)}{2\,\pi}\right\rfloor+$$
$$72\log(x)-72\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=144\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+$$
$$72\log(z_0)-72\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$72\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)=72\int_1^{e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\frac{1}{t}\,dt$$

$$72 \log\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right) = -\frac{36 i}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{\sqrt{22}\pi} + e^{\sqrt{37}\pi} + e^{\sqrt{58}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \, ds \quad \text{for } -1 < \gamma < 0$$

The result 1723,235 is practically equal to the value of mass of candidate glueball, the scalar meson $f_0(1710)$ that is 1723 (+6,-5)

Now:

2*((((sqrt(5)+1))/2)^2))) * ln(((([e^(Pi*sqrt(22)))]+[e^(Pi*sqrt(37)))]+[e^(Pi*sqrt(58)))])))

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right)$$

Exact result:

$$\frac{1}{2}\left(1+\sqrt{5}\right)^2 \log\left(e^{\sqrt{22}\pi} + e^{\sqrt{37}\pi} + e^{\sqrt{58}\pi}\right)$$

Decimal approximation:

125.319128263188099935555804035386999396924409052432595265...

Alternate form:

$$3 \log\left(e^{\sqrt{22}\pi} + e^{\sqrt{37}\pi} + e^{\sqrt{58}\pi}\right) + \sqrt{5} \log\left(e^{\sqrt{22}\pi} + e^{\sqrt{37}\pi} + e^{\sqrt{58}\pi}\right)$$

Alternative representations:

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right) = 2 \log_e\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right) = 2 \log(a) \log_a\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}} + e^{\pi\sqrt{37}} + e^{\pi\sqrt{58}}\right) = -2 \operatorname{Li}_1\left(1 - e^{\pi\sqrt{22}} - e^{\pi\sqrt{37}} - e^{\pi\sqrt{58}}\right)\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

Series representations:

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right) = \left(3+\sqrt{5}\right)\left(\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}\right)$$

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right) = \frac{1}{2}\left(1+\sqrt{5}\right)^2\left(\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}\right)$$

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right) = \frac{1}{2}\left(1+\sqrt{5}\right)^2\left(2\,i\,\pi\left\lfloor\frac{\arg\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)^k x^{-k}}{k}\right) \quad \text{for } x<0$$

Integral representations:

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right) = 3+\sqrt{5}\int_1^{e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\frac{1}{t}\,dt$$

$$2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right) = -\frac{i\left(3+\sqrt{5}\right)}{2\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

The result 125,319 is practically equal to the value of Higgs boson’s mass 125,09

Now:

((((ln(((([e^(Pi*sqrt(22)))]+[e^(Pi*sqrt(37)))]+[e^(Pi*sqrt(58)))])))))^1/(2Pi)

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}$$

Decimal approximation:

1.6575879867368474839402933369982398395737030595289228172 44...

Alternative representations:

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\sqrt[2\pi]{\log_e\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}$$

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\sqrt[2\pi]{\log(a)\log_a\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}$$

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\sqrt[2\pi]{-\text{Li}_1\left(1-e^{\pi\sqrt{22}}-e^{\pi\sqrt{37}}-e^{\pi\sqrt{58}}\right)}$$

Series representations:

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\sqrt[2\pi]{\log\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}}\right)^k}{k}}$$

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-z_0\right)^k z_0^{-k}}{k}\right)^\wedge\left(\frac{1}{2\pi}\right)$$

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)}=\left(2i\pi\left\lfloor\frac{\arg\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}-x\right)^k x^{-k}}{k}\right)^\wedge\left(\frac{1}{2\pi}\right)\quad\text{for } x<0$$

Integral representations:

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)} = \sqrt[2\pi]{\int_1^{e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}} \frac{1}{t}\,dt}$$

$$\sqrt[2\pi]{\log\left(e^{\pi\sqrt{22}}+e^{\pi\sqrt{37}}+e^{\pi\sqrt{58}}\right)} = (2\pi)^{-1/(2\pi)} \sqrt[2\pi]{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\left(-1+e^{\sqrt{22}\,\pi}+e^{\sqrt{37}\,\pi}+e^{\sqrt{58}\,\pi}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

The result 1.657587 is very near to the value of the fourteenth root of Ramanujan's class invariant 1164.2696 and to the mass of the proton.
Indeed:

We have the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 =$ 1164,2696

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164{,}269601267364$$

and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

From:

Critical Exponents from AdS/CFT with Flavor

Andreas Karch, Andy O'Bannon, and Laurence G. Yaffe https://arxiv.org/abs/0906.4959v2

ABSTRACT: We use the AdS/CFT correspondence to study the thermodynamics of massive $\mathcal{N}=2$ supersymmetric hypermultiplet flavor fields coupled to $\mathcal{N}=4$ supersymmetric $SU(N_c)$ Yang-Mills theory, formulated on curved four-manifolds, in the limits of large N_c and large 't Hooft coupling. The gravitational duals are probe D-branes in global thermal AdS. These D-branes may undergo a topology-changing transition in the bulk. The D-brane embeddings near the point of the topology change exhibit a scaling symmetry. The associated scaling exponents can be either real- or complex-valued. Which regime applies depends on the dimensionality of a collapsing submanifold in the critical embedding. When the scaling exponents are complex-valued, a first-order transition associated with the flavor fields appears in the dual field theory. Real scaling exponents are expected to be associated with a continuous transition in the dual field theory. For one example with real exponents, the D7-brane, we study the transition in detail. We find two field theory observables that diverge at the critical point, and we compute the associated critical exponents. We also present analytic and numerical evidence that the transition expresses itself in the meson spectrum as a non-analyticity at the critical point. We argue that the transition we study is a true phase transition only when the 't Hooft coupling is strictly infinite.

We will use a global thermal AdS_5 metric

$$ds^2 = d\rho^2 + \cosh^2\rho \, d\tau^2 + \sinh^2\rho \, d\Omega_3^2 \,, \tag{2.1}$$

where we have set the curvature radius of AdS_5 equal to one. We will work in these units throughout. In these units, we convert between string theory and SYM quantities using the relation $\alpha'^{-2} = 4\pi g_s N_c = g_{YM}^2 N_c = \lambda$, where α' is the square of the string length: $\alpha' \equiv \ell_s^2$.

We may write the AdS_5 metric for these various slicings as

$$ds^2 = d\rho^2 + \cosh^2\rho \, ds^2_{AdS_{4-l}} + \sinh^2\rho \, d\Omega_l^2 \tag{2.3}$$

with $d\Omega_l^2$ the metric of the unit l-sphere S^l and $l = 0, \ldots 4$. When $l \leq 2$, the slicing includes $ds^2_{AdS_{4-l}}$, which denotes the AdS_{4-l} metric. We adopt a form for the AdS_{4-l} metric that is identical to Eq. (2.1), except for the replacement of the S^3 metric, $d\Omega_3^2$, by an S^{2-l} metric, $d\Omega_{2-l}^2$. When $l = 3$, we adopt a convention that $AdS_1 \equiv S^1$. Notice that the S^l collapses to zero volume at the center of AdS.

we have:

$$ds^2_{Dp} = \left[1 + \theta'(\rho)^2\right] d\rho^2 + \cosh^2\rho \, d\tau^2 + \sinh^2\rho \, d\Omega_{i-2}^2 + \cos^2\theta(\rho) \, d\Omega_j^2 \tag{2.5}$$

and

$$\tilde{S}_{Dp} \equiv \mathcal{N}_{Dp} \int d\rho \, \sqrt{\det(g_{ab})} \,. \tag{2.7}$$

We will now explain in detail how the mass m of the flavor fields in the SYM theory, and the thermal expectation value of (the supersymmetric completion of) their mass operator, may be extracted from the Dp-brane geometry. To be concrete, consider the low-temperature phase. Inserting the worldvolume metric Eq. (2.5) into the action Eq. (2.7), we find

$$\tilde{S}_{Dp} = \mathcal{N}_{Dp} \int d\rho \, \cosh\rho \, (\sinh\rho)^{i-2} (\cos\theta)^j \sqrt{1+\theta'^2}. \tag{2.8}$$

We now turn to the scaling analysis. We focus on the region near the center of AdS, and expand the metric to leading nontrivial order using $\sinh\rho \approx \rho$, $\cosh\rho \approx 1$. We first consider the critical solution, that, is, fluctuations of the form $\theta(\rho) = \pi/2 + \delta\theta(\rho)$ with $\delta\theta(\rho)$ small. We may then use $\cos\theta \approx -\delta\theta$. The critical solution has the boundary condition $\delta\theta(0) = 0$. The Dp-brane action, Eq. (2.8), becomes

$$\tilde{S}_{Dp} = \mathcal{N}_{Dp} \int d\rho \, \rho^{i-2} \, \delta\theta^j \sqrt{1+\delta\theta'^2}\,, \tag{3.1}$$

with, once again, $i + j = p + 1$. The resulting equation of motion for $\delta\theta(\rho)$ is

$$\rho\, \delta\theta\, \delta\theta'' + \left[(i-2)\delta\theta\, \delta\theta' - j\rho\right] (1+\delta\theta'^2) = 0\,. \tag{3.2}$$

Using the $AdS_{4-l} \times S^l$ slicing of AdS_5, the induced D7-brane metric is

$$ds^2_{D7} = \left[1+\theta'(\rho)^2\right] d\rho^2 + \cosh^2\!\rho \, ds^2_{AdS_{4-l}} + \sinh^2\!\rho \, d\Omega_l^2 + \cos^2\theta(\rho)\, d\Omega_3^2\,, \tag{3.13}$$

and the D7-brane action is

$$\tilde{S}_{D7} = \mathcal{N}_{D7} \int d\rho \, (\cosh\rho)^{4-l} (\sinh\rho)^l (\cos\theta)^3 \sqrt{1+\theta'^2}\,. \tag{3.14}$$

In the near-center limit this becomes

$$\tilde{S}_{D7} = \mathcal{N}_{D7} \int d\rho \, \rho^l \, \delta\theta^3 \sqrt{1+\delta\theta'^2} \tag{3.15}$$

which is the same as Eq. (3.1), but with $i-2 \to l$ and $j = 3$, so $a = l+3$ and we will have complex exponents $\beta_\perp$ for $l = 0, 1, 2$ and real exponents $\beta_\perp$ for $l = 3, 4$.

As an example of complex exponents, consider the $l = 2$ case, $AdS_2 \times S^2$ slicing, with $a = 5$ and complex exponents $\beta_\pm = -2 \pm i$. We expect a spiral in the $(\theta_{(0)}, \theta_{(2)})$ plane. Figure 6 shows the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$, which indeed exhibits a spiral, so we again have a first-order transition. In this case, the transition occurs at the critical value $\theta_{(0)}^{crit} = m/(2\pi\alpha') = 1.6557$ or equivalently $m = \sqrt{\lambda}\,\theta_{(0)}^{crit}/(2\pi) = 0.2635\sqrt{\lambda}$ times the inverse of the AdS curvature radius.

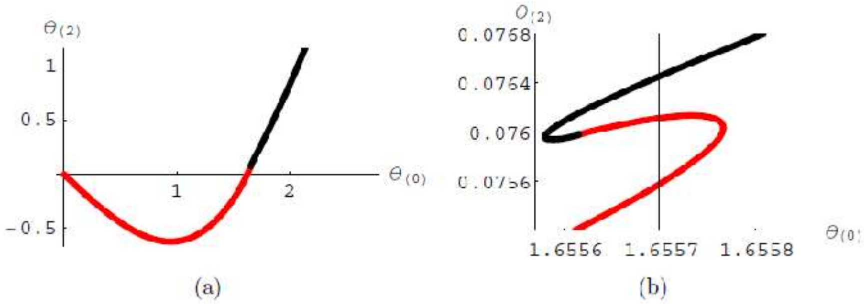


Figure 6: (a.) $\theta_{(2)}$ as a function of $\theta_{(0)}$ for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5. (b.) Close-up of (a.) near the critical solution. The vertical line indicates where the transition occurs, at the critical value $\theta_{(0)}^{crit} \approx 1.6557$.

3.3 Probe D7-brane in Other Slicings: Low-Temperature Phase

By using different slicings, leading to different boundary geometries, we can change the dimension of the critical solution's collapsing submanifold, as different slicings lead to spheres of different dimension, $S^l \subset AdS_5$ for $l = 0, \ldots, 4$, that collapse to zero volume at the center of AdS_5 (see Eq. (2.3)). We will focus on the D7-brane and the low-temperature phase because lower-dimensional Dp-branes, or Dp-branes in the high-temperature phase, will not be able to reach $a \geq 6$. For the D7-brane, the dual SYM theory is $\mathcal{N} = 4$ SYM theory coupled to massive $\mathcal{N} = 2$ hypermultiplets, formulated on different four-manifolds, in the low-temperature, confining phase.

We note that 1,6557 (practically the 14-th root of Ramanujan class invariant 1164,2696) concerning the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$, which indeed exhibits a spiral, so we again have a first-order transition. In this case, the transition occurs at the critical value $\theta^{crit}_{(0)} = m/(2\pi\alpha') = 1,6557$. Note that $\theta_{(2)}$ as a function of $\theta_{(0)}$ for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5

We have the same results also utilizing the other form. Indeed, for example:

72 * ln ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))

Input:

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+ 64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)$$

- $\log(x)$ is the natural logarithm

Exact result:

$$72 \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+ 64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Decimal approximation:

1723.235311947397740690243221856653651096411713027632557692...

1723,23531

Property:

$$72 \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$ is a transcendental number

Also 1723,23531… is a transcendental number and is practically equal to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5)

Alternate forms:

$$72 \log(24792915328)$$

$$504 \log(2) + 72 \log(193694651)$$
$$72 (7 \log(2) + \log(17) + \log(349) + \log(32647))$$

Continued fraction:

[1723; 4, 4, 193, 1, 5, 39, 1, 6, 1, 4, 4, 2, 6, 2, 10, 2, 1, 2, 3, 1, 30, 5, 1, 2, 1, 6, 1, ...]

Alternative representations:

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$72 \log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$72 \log(a) \log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$-72 \operatorname{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

- $\log_b(x)$ is the base- b logarithm
- $\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=72 \log(24792915327)-72 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{24792915327}\right)^{k}}{k}$$

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=144 i \pi \left\lfloor \frac{\arg(24792915328-x)}{2 \pi}\right\rfloor+72 \log(x)-72 \sum_{k=1}^{\infty} \frac{(-1)^{k}(24792915328-x)^{k} x^{-k}}{k} \quad \text{for } x<0$$

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=72\left\lfloor \frac{\arg(24792915328-z_0)}{2 \pi}\right\rfloor \log\left(\frac{1}{z_0}\right)+72 \log(z_0)+72\left\lfloor \frac{\arg(24792915328-z_0)}{2 \pi}\right\rfloor \log(z_0)-72 \sum_{k=1}^{\infty} \frac{(-1)^{k}(24792915328-z_0)^{k} z_0^{-k}}{k}$$

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

Integral representations:

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=72 \int_{1}^{24792915328} \frac{1}{t}\, dt$$

$$72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=-\frac{36 i}{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{24792915327^{-s}\, \Gamma(-s)^{2}\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1<\gamma<0$$

- $\Gamma(x)$ is the gamma function

We note that from this expression, we can to obtain π, that in addition to being irrational, more strongly π is a transcendental number, which means that it is not the solution of any non-constant polynomial equation with rational coefficients:

1/(4*137.1307090028)*72*ln((((64[(1+sqrt(2))^12+(1-sqrt(2))^12]+ 64[(6+sqrt(37))^6+(6-sqrt(37))^6]+64[((5+sqrt(29))/2)^12+((5-sqrt(29))/2)^12]))))

$$\frac{1}{4\times 137.1307090028}\times 72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)$$

Result:

3.141592653605...

Alternative representations:

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000) = \left(72\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)/548.52283601120000$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000) = \left(72\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)/548.52283601120000$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000) = -\left(\left(72\,\mathrm{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)/548.52283601120000\right)$$

Series representations:

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000) = 0.13126162717960182\log\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-0.13126162717960182\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000) = 0.13126162717960182\log\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-0.13126162717960182\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000)=$$

$$0.13126162717960182\log\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-$$

$$0.13126162717960182\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

Integral representations:

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000)=0.13126162717960182\int_1^{64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}+\frac{\left(-5+\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}+\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)}\frac{1}{t}\,dt$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)/(4\times 137.13070900280000)=\frac{0.065630813589800908}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{\Gamma(1-s)}\Gamma(-s)^2\,\Gamma(1+s)\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-s}ds \quad \text{for } -1<\gamma<0$$

The result, π is a transcendental number.

We have also:

((((((((72 * ln ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))))^1/15

Input:

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)\text{^}(1/15)$$

Exact result:

$$\sqrt[5]{2}\ 3^{2/15} \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15)$$

Decimal approximation:

1.643449280886601879682780942551378805700103655268560436783...

Property:

$$\sqrt[5]{2}\ 3^{2/15} \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15) \text{ is a transcendental number}$$

Alternate forms:

$$\sqrt[5]{2}\ 3^{2/15} \sqrt[15]{\log(24792915328)}$$

$$\sqrt[5]{2}\ 3^{2/15} \sqrt[15]{7\log(2)+\log(193694651)}$$

All 15th roots of 72 log(64 ((1-sqrt(2))^12+(1+sqrt(2))^12)+64 ((5-sqrt(29))^12/4096+(5+sqrt(29))^12/4096)+64 ((6-sqrt(37))^6+(6+sqrt(37))^6)):

- Polar form

$$\sqrt[5]{2}\ 3^{2/15} e^{0} \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15) \approx 1.64345 \text{ (real, principal root)}$$

$$\sqrt[5]{2}\ 3^{2/15} e^{(2 i \pi)/15} \log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15) \approx 1.5014+0.6685 i$$

$$\sqrt[5]{2}\ 3^{2/15}\ e^{(4 i \pi)/15}$$

$$\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15)\approx 1.0997+1.2213\,i$$

$$\sqrt[5]{2}\ 3^{2/15}\ e^{(2 i \pi)/5}$$

$$\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15)\approx 0.5079+1.5630\,i$$

$$\sqrt[5]{2}\ 3^{2/15}\ e^{(8 i \pi)/15}$$

$$\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\text{^}(1/15)\approx -0.17179+1.63445\,i$$

Alternative representations:

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)\text{^}(1/15)=\left(72\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)\text{^}(1/15)$$

$$\left(72\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)\text{^}(1/15)=\left(72\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)\text{^}(1/15)$$

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right) \wedge (1/15) = \left(-72 \operatorname{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right) \wedge (1/15)$$

Series representations:

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right) \wedge (1/15) = \sqrt[5]{2}\ 3^{2/15} \sqrt[15]{\log(24792915327)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{24792915327}\right)^k}{k}}$$

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right) \wedge (1/15) = \sqrt[5]{2}\ 3^{2/15} \sqrt[15]{2 i \pi \left\lfloor \frac{\arg(24792915328-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (24792915328-x)^k x^{-k}}{k}}$$

for $x < 0$

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right) \wedge (1/15) = \sqrt[5]{2}\ 3^{2/15} \left(\log(z_0) + \left\lfloor \frac{\arg(24792915328-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (24792915328-z_0)^k z_0^{-k}}{k}\right) \wedge (1/15)$$

Integral representations:

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right) \wedge (1/15) = \sqrt[5]{2}\ 3^{2/15} \sqrt[15]{\int_1^{24792915328} \frac{1}{t}\, dt}$$

$$\left(72 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)\text{^}(1/15) = \frac{6^{2/15}\sqrt[15]{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{24792915327^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}}{\sqrt[15]{\pi}} \quad \text{for } -1<\gamma<0$$

Also the result 1,64344928... is a transcendental number, and is a good approximation to the fourteenth root of Ramanujan's class invariant 1164.2696 (1,65578), to the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$ 1,6557 for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5 and a good approximation to the mass of the proton (1,67262192369(51) × 10^{-27} k).

-((sqrt(5)+1)/2))^2 + ln((((1729 *728 / ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))

Input:

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 + \log\left(1729\times 728\Big/\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)$$

- $\log(x)$ is the natural logarithm

Exact result:

$$\log\left(1258712\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)-\frac{1}{4}\left(1+\sqrt{5}\right)^2$$

Decimal approximation:

−12.5062582319171092205462274160953366948218868476810727505...

Property:

$$-\frac{1}{4}\left(1+\sqrt{5}\right)^2 + \log\left(1258712\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{(5-\sqrt{29})^{12}}{4096}+\frac{(5+\sqrt{29})^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right) \text{ is a transcendental number}$$

Also -12,5062582... is a transcendental number and is very near to the value of black hole entropy 12,57 with minus sign.

Alternate forms:

$$\frac{1}{2}\left(-3-\sqrt{5}-2\log\left(\frac{3099114416}{157339}\right)\right)$$

$$\frac{1}{2}\left(-3-\sqrt{5}\right)-\log\left(\frac{3099114416}{157339}\right)$$

$$-\frac{1}{4}\left(1+\sqrt{5}\right)^2-4\log(2)+2\log(91)-\log\left(\frac{193694651}{19}\right)$$

Continued fraction:

− [12; 1, 1, 39, 2, 4, 3, 1, 40, 1, 13, 1, 3, 1, 6, 14, 20, 1, 6, 1, 12, 3, 1, 2, 1, 1, 3, 3, ...]

Alternative representations:

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\left((1729\times 728)\Big/\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)=\log_e\left(1258712\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6\right)\right)\right)-\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\left((1729\times 728)\Big/\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)=\log(a)\log_a\left(1258712\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6\right)\right)\right)-\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\Big((1729\times 728)\Big/\Big(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\Big)\Big)=$$
$$-\text{Li}_1\Big(1-1258712\Big/\Big(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6\right)\Big)\Big)-\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2$$

Series representations:

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\Big((1729\times 728)\Big/\Big(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\Big)\Big)=$$
$$-\frac{3}{2}-\frac{\sqrt{5}}{2}-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{3098957077}{3099114416}\right)^k}{k}$$

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\Big((1729\times 728)\Big/\Big(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\Big)\Big)=$$
$$-\frac{3}{2}-\frac{\sqrt{5}}{2}+2i\pi\left\lfloor\frac{\arg\left(\frac{157339}{3099114416}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{157339}{3099114416}-x\right)^k x^{-k}}{k}$$
for $x<0$

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\Big((1729\times 728)\Big/\Big(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\Big)\Big)=$$
$$-\frac{3}{2}-\frac{\sqrt{5}}{2}+\left\lfloor\frac{\arg\left(\frac{157339}{3099114416}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{157339}{3099114416}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{157339}{3099114416}-z_0\right)^k z_0^{-k}}{k}$$

Integral representation:

$$-\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\log\left((1729\times 728)\Big/\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)=$$

$$-\frac{3}{2}-\frac{\sqrt{5}}{2}+\int_1^{\frac{157339}{3099114416}}\frac{1}{t}\,dt$$

We calculate the above integral

$$-\frac{3}{2}-\frac{\sqrt{5}}{2}+\int_1^{\frac{157339}{3099114416}}\frac{1}{t}dt$$

We know that:

$$\int\frac{1}{x}dx=\log|x|+c$$

and that:

$$\int_a^b c\,dx=c(b-a)$$

thence:

$$-\frac{3}{2}-\frac{\sqrt{5}}{2}+\int_1^{\frac{157339}{3099114416}}\frac{1}{t}dt=$$

= -2,6180339887498948482045868343656+ln((5,07690194294523910213710547949e-5)-1) =
= -2,6180339887498948482045868343656-9,8882242431672143723416405817297-0 =
= -12,506258231917109220546227416095

Practically we have used the Fundamental Theorem of Calculus.

https://calculushowto.com/fundamental-theorem-of-calculus/

The **fundamental theorem of calculus** explains how to find definite integrals of functions that have indefinite integrals. It bridges the concept of an antiderivative with the area problem. When you figure out definite integrals (which you can think of as a limit of Riemann sums), you might be aware of the fact that the definite integral is just the area under the curve between two points (upper and lower bounds. You are finding an antiderivative at the upper and lower limits of integration and taking the difference. The Fundamental Theorem of Calculus justifies this procedure.

The technical formula is:

$$\int_a^b f(x)dx = F(b) - F(a)$$

With regard the bosonic string, a string in 26-dimensional spacetime can wiggle in all 24 directions perpendicular to its 2-dimensional surface. So, its ground state energy is simply −1

From the above expression, we can to obtain:

-(Pi/2)^2 [ln((((1711.2/2)*1728 / ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12])))))]

$$-\left(\frac{\pi}{2}\right)^2 \log\left(\frac{1711.2}{2} \times 1728 \Big/ \left(64\left(\left(1+\sqrt{2}\right)^{12} + \left(1-\sqrt{2}\right)^{12}\right) + 64\left(\left(6+\sqrt{37}\right)^6 + \left(6-\sqrt{37}\right)^6\right) + 64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12} + \left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)\right)$$

Result:

24.0012...

Alternative representations:

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728 \times 1711.2) \Big/ \left(\left(64\left(\left(1+\sqrt{2}\right)^{12} + \left(1-\sqrt{2}\right)^{12}\right) + 64\left(\left(6+\sqrt{37}\right)^6 + \left(6-\sqrt{37}\right)^6\right) + 64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12} + \left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right) 2\right)\right) =$$

$$-\log_e\left(1.47848 \times 10^6 \Big/ \left(64\left(\left(1-\sqrt{2}\right)^{12} + \left(1+\sqrt{2}\right)^{12}\right) + 64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12} + \left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right) + 64\left(\left(6-\sqrt{37}\right)^6 + \left(6+\sqrt{37}\right)^6\right)\right)\right)\left(\frac{\pi}{2}\right)^2$$

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728\times 1711.2)\Big/\left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) = -\log(a)\log_a\left(1.47848\times 10^6\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)\left(\frac{\pi}{2}\right)^2$$

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728\times 1711.2)\Big/\left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) = \mathrm{Li}_1\left(1-1.47848\times 10^6\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)\left(\frac{\pi}{2}\right)^2$$

Series representations:

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728\times 1711.2)\Big/\left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) = \frac{1}{4}\pi^2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{23\,101.2}{\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}+\frac{\left(-5+\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}+\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}}\right)^k}{k}$$

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728\times 1711.2)\Big/\left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) = \frac{1}{4}\pi^2\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+1.47848\times 10^6\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)\right)^k$$

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728 \times 1711.2) \Big/ \left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) =$$

$$-\frac{1}{4}\pi^2\left(\log(z_0)+\left\lfloor\frac{1}{2\pi}\arg\left(1.47848\times10^6\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6\right)\right)-z_0\right)\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(1.47848\times10^6\Big/\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6\right)\right)-z_0\right)^k z_0^{-k}\right)$$

Integral representation:

$$-\left(\frac{\pi}{2}\right)^2 \log\left((1728 \times 1711.2) \Big/ \left(\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)2\right)\right) =$$

$$-\frac{\pi^2}{4}\int_1^{\frac{23101.2}{\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}+\frac{\left(-5+\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}+\left(-6+\sqrt{37}\right)^6+\left(6+\sqrt{37}\right)^6}}\frac{1}{t}\,dt$$

The result, practically 24, are the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

Also from the following simple expression, we obtain about the some result:

ln((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))]+0.065578

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^6+\left(6-\sqrt{37}\right)^6\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578$$

Result:

23.9994018...

Alternative representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 = 0.065578+\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 = 0.065578+\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 = 0.065578-\mathrm{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Series representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 = 0.065578+\log\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 =$$
$$0.065578+\log\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 =$$
$$0.065578+\log\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+\frac{1}{64}\left(\left(-5+\sqrt{29}\right)^{12}+\left(5+\sqrt{29}\right)^{12}\right)+64\left(\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-k}$$

Integral representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 =$$
$$0.065578+\int_1^{64\left(\left(-1+\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}+\frac{\left(-5+\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}+\left(-6+\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)}\frac{1}{t}\,dt$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)+0.065578 =$$
$$0.065578+\frac{1}{2i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{\Gamma(1-s)}\Gamma(-s)^2\,\Gamma(1+s)\left(-1+64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)^{-s}ds \quad \text{for } -1<\gamma<0$$

In conclusion we have the following expression:

(24+2Pi) * ln((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))]

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)$$

Exact result:

$$(24+2\pi)\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Decimal approximation:

724.7924205497009246714118091919275693353510720243403022746...

Alternate forms:

$$2(12+\pi)\log(24792915328)$$

$$(24+2\pi)\log(24792915328)$$

$$2(12+\pi)(7\log(2)+\log(193694651))$$

Alternative representations:

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=(24+2\pi)\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=(24+2\pi)\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$-(24+2\pi)\,\text{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Series representations:

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$2(12+\pi)\left(\log(24792915327)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{24792915327}\right)^k}{k}\right)$$

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$2(12+\pi)\left(2i\pi\left\lfloor\frac{\arg(24792915328-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(24792915328-x)^k x^{-k}}{k}\right)\quad\text{for } x<0$$

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$
$$2(12+\pi)\left(\log(z_0)+\left\lfloor\frac{\arg(24792915328-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(24792915328-z_0)^k z_0^{-k}}{k}\right)$$

Integral representations:

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=2(12+\pi)\int_1^{24792915328}\frac{1}{t}\,dt$$

$$(24+2\pi)\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=$$

$$-\frac{i(12+\pi)}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{24\,792\,915\,327^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

The result 724,7924 is very near to the value 728 (Ramanujan's number)

We have also that:

32 * ln ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))))

Input:

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)$$

- $\log(x)$ is the natural logarithm

Exact result:

$$32\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Decimal approximation:

765.8823608655101069734414319362905115984052057900589145300...

Property:

$$32\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$ is a transcendental number

Alternate forms:

$$32\log(24\,792\,915\,328)$$

$$224\log(2)+32\log(193\,694\,651)$$

$$32\,(7\log(2)+\log(17)+\log(349)+\log(32\,647))$$

Continued fraction:

[765; 1, 7, 1, 1, 436, 7, 1, 9, 7, 2, 2, 1, 9, 1, 3, 1, 6, 3, 5, 1, 2, 3, 2, 1, 2, 1, 2, 1, 6, ...]

Alternative representations:

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=32\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=32\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=-32\,\mathrm{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Series representations:

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=32\log(24\,792\,915\,327)-32\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{24\,792\,915\,327}\right)^k}{k}$$

$$32\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=64\,i\,\pi\left\lfloor\frac{\arg(24\,792\,915\,328-x)}{2\,\pi}\right\rfloor+32\log(x)-32\sum_{k=1}^{\infty}\frac{(-1)^k\,(24\,792\,915\,328-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$32 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+ 64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right) = 32\left\lfloor\frac{\arg(24\,792\,915\,328-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+32\log(z_0)+ 32\left\lfloor\frac{\arg(24\,792\,915\,328-z_0)}{2\pi}\right\rfloor\log(z_0)-32\sum_{k=1}^{\infty}\frac{(-1)^k\,(24\,792\,915\,328-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$32 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+ 64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right) = 32\int_1^{24792915328}\frac{1}{t}\,dt$$

$$32 \log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+ 64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right) = -\frac{16\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{24\,792\,915\,327^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

The result 765,882360 is a transcendental number and practically equal at the nonperturbative contribution to the mass of a 1S quarkonium for $m_q = 4.7$ MeV/c^2 = 0.0047 GeV/c^2 , (765,171) that is the mass of quark down is 4.8±0.5±0.3 = 4.7 MeV/c^2.

In conclusion, we have that:

ln ((((64[(1+sqrt(2))^12 + (1-sqrt(2))^12] + 64[(6+sqrt(37))^6 + (6-sqrt(37))^6] + 64[((5+sqrt(29))/2)^12 + ((5-sqrt(29))/2)^12]))))))

Input:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+ 64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)$$

- $\log(x)$ is the natural logarithm

Exact result:

$$\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+ 64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Decimal approximation:

23.93382377704719084292004474800907848745016268093934107906...

Property:

$$\log\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$ is a transcendental number

Alternate forms:

$\log(24\,792\,915\,328)$

$7\log(2)+\log(193\,694\,651)$

$7\log(2)+\log(17)+\log(349)+\log(32\,647)$

Continued fraction:

[23; 1, 14, 8, 1, 217, 2, 3, 1, 1, 4, 2, 3, 1, 5, 20, 1, 5, 4, 1, 21, 1, 1, 1, 10, 8, 1, 1, 4, ...]

Alternative representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=\log_e\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=\log(a)\log_a\left(64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)+64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)+64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=-\mathrm{Li}_1\left(1-64\left(\left(1-\sqrt{2}\right)^{12}+\left(1+\sqrt{2}\right)^{12}\right)-64\left(\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}\right)-64\left(\left(6-\sqrt{37}\right)^{6}+\left(6+\sqrt{37}\right)^{6}\right)\right)$$

Series representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=\log(24792915327)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{24792915327}\right)^{k}}{k}$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=2 i \pi\left\lfloor\frac{\arg(24792915328-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^{k}(24792915328-x)^{k}x^{-k}}{k}\quad\text{for } x<0$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=\left\lfloor\frac{\arg(24792915328-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(24792915328-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^{k}(24792915328-z_0)^{k}z_0^{-k}}{k}$$

Integral representations:

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=\int_{1}^{24792915328}\frac{1}{t}\,dt$$

$$\log\left(64\left(\left(1+\sqrt{2}\right)^{12}+\left(1-\sqrt{2}\right)^{12}\right)+64\left(\left(6+\sqrt{37}\right)^{6}+\left(6-\sqrt{37}\right)^{6}\right)+64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)\right)=-\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{24792915327^{-s}\,\Gamma(-s)^{2}\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

The result 23,9338237770... ≈ 24 is a transcendental number that represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

The set of transcendental numbers is uncountably infinite. Since the polynomials with rational coefficients are countable, and since each such polynomial has a finite number of zeroes, the algebraic numbers must also be countable. However, Cantor's

diagonal argument proves that the real numbers (and therefore also the complex numbers) are uncountable. Since the real numbers are the union of algebraic and transcendental numbers, they cannot both be countable. This makes the transcendental numbers uncountable.
In set theory, the **cardinality of the continuum** is the cardinality or "size" of the set of real numbers R, sometimes called the continuum. It is an infinite cardinal number and is denoted by | R | or c.
The smallest infinite cardinal number is $\aleph_0$ (aleph-null). The second smallest is $\aleph_1$ (aleph-one). The continuum hypothesis, which asserts that there are no sets whose cardinality is strictly between $\aleph_0$ and c implies that $c = \aleph_1$.
A great many sets studied in mathematics have cardinality equal to c the transcendental numbers We note that the set of real algebraic numbers is countably infinite (assign to each formula its Gödel number.) So the cardinality of the real algebraic numbers is $\aleph_0$. Furthermore, the real algebraic numbers and the real transcendental numbers are disjoint sets whose union is R. Thus, since the cardinality of R is c, the cardinality of the real transcendental numbers is $c - \aleph_0 = c$. A similar result follows for complex transcendental numbers, once we have proved that | C | = c

Now, we remember that (from: **Formulae for Supersymmetry | MSSM and more |** - Toru Goto - KEK Theory Center, IPNS, KEK - Tsukuba, Ibaraki, 305-0801 JAPAN - Last Modified: March 31, 2019):

the nonperturbative contribution to the mass of a 1S quarkonium, is equal to:

Nonperturbative contribution [1032, 1033]

$$\Delta E^{\rm np} = \frac{\pi^2 m_q}{(C_F\alpha_s m_q)^4}\frac{624}{425}\left\langle 0\left|\frac{\alpha_s}{\pi}G^{\mu\nu a}G^a_{\mu\nu}\right|0\right\rangle, \tag{9.1.65}$$

where the gluon condensate is evaluated as [1034, 1035, 1036]

$$\left\langle 0\left|\frac{\alpha_s}{\pi}G^{\mu\nu a}G^a_{\mu\nu}\right|0\right\rangle \approx 0.012\,{\rm GeV}^4. \tag{9.1.66}$$

and

These quantities are evaluated at a hadronic energy scale $\mu_{\rm H} \sim 1$ GeV. In Ref. [582, 584, 576], $\mu_{\rm H}$ is chosen such that the strong coupling constant satisfies $g_3(\mu_{\rm H}) = 4\pi/\sqrt{6}$. See Sec. 9.3.6 for the QCD correction factors due to the running effect between the EW scale and $\mu_{\rm H}$.

$\beta^{(k)}$ are evaluated with $n_f = n_l$ in (9.1.1). The SU(3) color factors are

$$C_A = 3, \quad C_F = \frac{4}{3}, \quad T_F = \frac{1}{2}. \tag{9.1.60}$$

$\alpha_s = 4\pi/\sqrt{6} = 5.130199 = 5.13$

For $C_F = 4/3$ $\alpha_s = 5.13$ and $m_q = 4.7\ MeV/c^2 = 0.0047\ GeV/c^2$ (the mass of quark down is $4.8\pm0.5\pm0.3 = 4.7\ MeV/c^2$), we obtain:

we have:

[Pi^2*(0.0047)*624*0.012]/[425*(((4/3)*5.13*(0.0047))^4))]

Input:

$$\frac{\pi^2 \times 0.0047 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.0047\right)^4}$$

Result:

765.171...

Alternative representations:

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = \frac{0.0351936 (180°)^2}{425 \times 0.032148^4}$$

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = \frac{0.0351936 (-i \log(-1))^2}{425 \times 0.032148^4}$$

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = \frac{0.211162\, \zeta(2)}{425 \times 0.032148^4}$$

Series representations:

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = 1240.45 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2$$

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = 310.112 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2$$

$$\frac{\pi^2 (0.0047 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.0047}{3}\right)^4} = 77.5281 \left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^2$$

- $\binom{n}{m}$ is the binomial coefficient

Integral representations:

$$\frac{\pi^2 (0.0047\times 624\times 0.012)}{425\left(\frac{4\times 5.13\times 0.0047}{3}\right)^4} = 310.112\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{\pi^2 (0.0047\times 624\times 0.012)}{425\left(\frac{4\times 5.13\times 0.0047}{3}\right)^4} = 1240.45\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2$$

$$\frac{\pi^2 (0.0047\times 624\times 0.012)}{425\left(\frac{4\times 5.13\times 0.0047}{3}\right)^4} = 310.112\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2$$

The result is 765,171

From:

V Theorems on summations of series; e.g.

(1) $\frac{1}{1^3}\cdot\frac{1}{2} + \frac{1}{2^3}\cdot\frac{1}{2^2} + \frac{1}{3^3}\cdot\frac{1}{2^3} + \frac{1}{4^3}\cdot\frac{1}{2^4} + \&c$

$= \frac{1}{6}(\log 2)^3 - \frac{\pi^2}{12}\log 2 + \left(\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \&c\right).$

(2) $1 + 9\cdot\left(\frac{1}{4}\right)^4 + 17\cdot\left(\frac{1.5}{4.8}\right)^4 + 25\cdot\left(\frac{1.5.9}{4.8.12}\right)^4 + \&c = \frac{2\sqrt{2}}{\sqrt{\pi}.\left\{\Gamma\left(\frac{3}{4}\right)\right\}^2}$

(3) $1 - 5\cdot\left(\frac{1}{2}\right)^3 + 9\cdot\left(\frac{1.3}{2.4}\right)^3 - \&c = \frac{2}{\pi}.$

(4) $\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \&c = \frac{1}{24}.$

(5) $\frac{\coth\pi}{1^7} + \frac{\coth 2\pi}{2^7} + \frac{\coth 3\pi}{3^7} + \&c = \frac{19\pi^7}{56700}.$

(6) $\frac{1}{1^5\cosh\frac{\pi}{2}} - \frac{1}{3^5\cosh\frac{3\pi}{2}} + \frac{1}{5^5\cosh\frac{5\pi}{2}} - \&c = \frac{\pi^5}{768}.$

(7) $\frac{1}{(1^2+2^2)(\sinh 3\pi - \sinh\pi)} + \frac{1}{(2^2+3^2)(\sinh 5\pi - \sinh\pi)}$

$+ \frac{1}{(3^2+4^2)(\sinh 7\pi - \sinh\pi)} + \&c$

$= \left(\frac{1}{\pi} + \coth\pi - \frac{\pi}{2}\tanh^2\frac{\pi}{2}\right) \Big/ 2\sinh\pi.$

(8) $\frac{1}{(25+\frac{1^4}{100})(e^{\pi}+1)} + \frac{3}{(25+\frac{3^4}{100})(e^{3\pi}+1)} + \frac{5}{(25+\frac{5^4}{100})(e^{5\pi}+}$

$+ \&c = \frac{\pi}{8}\coth^2\frac{5\pi}{2} - \frac{4689}{11890}.$

(9) $\frac{1}{1^7\cosh\frac{\pi\sqrt{3}}{2}} - \frac{1}{3^7\cosh\frac{3\pi\sqrt{3}}{2}} + \&c = \frac{\pi^7}{23040}$

(10) $\left\{1+\left(\frac{n}{1}\right)^3\right\}\left\{1+\left(\frac{n}{2}\right)^3\right\}\left\{1+\left(\frac{n}{3}\right)^3\right\}$ &c can always be exactly found if n is any integer positive or negative.

(11). $\frac{2}{3}\int_0^1 \frac{\tan^{-1}x}{x}dx - \int_0^{2-\sqrt{3}}\frac{\tan^{-1}x}{x}dx = \frac{\pi}{12}\log(2+\sqrt{3}).$

Various formulas written by Ramanaujan on the page above, gives us useful and new mathematical connections with some sectors of Particle Physics, String Theory, and Black Hole physics.

Now, we note that, from (6) :

((1/(1 cosh (Pi/2)) - (1/(3^5 cosh (3Pi)/2)) + (1/(5^5 cosh (5Pi)/2)) - 1/(7^5 cosh ((7Pi)/2)) + 1/(9^5 cosh ((9Pi)/2))

Input:

$$\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{3^5\left(\frac{1}{2}\cosh(3\pi)\right)}+\frac{1}{5^5\left(\frac{1}{2}\cosh(5\pi)\right)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}$$

Exact result:

$$\operatorname{sech}\left(\frac{\pi}{2}\right)-\frac{2}{243}\operatorname{sech}(3\pi)-\frac{\operatorname{sech}\left(\frac{7\pi}{2}\right)}{16807}+\frac{\operatorname{sech}\left(\frac{9\pi}{2}\right)}{59049}+\frac{2\operatorname{sech}(5\pi)}{3125}$$

- $\cosh(x)$ is the hyperbolic cosine function
- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.398535485172442072453827704095687818298246056848195074414...

and:

Input:

$$\frac{\pi^5}{768}$$

Decimal approximation:

0.398463131230835225602527747452390112604558211755305977283...

Property:

$\frac{\pi^5}{768}$ is a transcendental number

Alternative representations:

$$\frac{\pi^5}{768}=\frac{1}{768}(180°)^5$$

$$\frac{\pi^5}{768}=\frac{1}{768}(-i\log(-1))^5$$

$$\frac{\pi^5}{768}=\frac{1}{768}\cos^{-1}(-1)^5$$

- $\log(x)$ is the natural logarithm
- i is the imaginary unit
- $\cos^{-1}(x)$ is the inverse cosine function

Series representations:

$$\frac{\pi^5}{768} = \frac{4}{3}\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^5$$

$$\frac{\pi^5}{768} = \frac{4}{3}\left(\sum_{k=0}^{\infty} \frac{(-1)^{1+k}\, 1195^{-1-2k}\left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}\right)^5$$

$$\frac{\pi^5}{768} = \frac{1}{768}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^5$$

Integral representations:

$$\frac{\pi^5}{768} = \frac{4}{3}\left(\int_0^1 \sqrt{1-t^2}\, dt\right)^5$$

$$\frac{\pi^5}{768} = \frac{1}{24}\left(\int_0^\infty \frac{1}{1+t^2}\, dt\right)^5$$

$$\frac{\pi^5}{768} = \frac{1}{24}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\, dt\right)^5$$

Now:

Pi^5 / [[((1/(1 cosh (Pi/2)) - (1/(3^5 cosh (3Pi)/2)) + (1/(5^5 cosh (5Pi)/2)) - 1/(7^5 cosh ((7Pi)/2)) + 1/(9^5 cosh ((9Pi)/2))]]

Input:

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{3^5\left(\frac{1}{2}\cosh(3\pi)\right)} + \frac{1}{5^5\left(\frac{1}{2}\cosh(5\pi)\right)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}}$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{\pi^5}{\operatorname{sech}\left(\frac{\pi}{2}\right) - \frac{2}{243}\operatorname{sech}(3\pi) - \frac{\operatorname{sech}\left(\frac{7\pi}{2}\right)}{16807} + \frac{\operatorname{sech}\left(\frac{9\pi}{2}\right)}{59049} + \frac{2\operatorname{sech}(5\pi)}{3125}}$$

- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

767.86056993863416008475612430743500877501045494860720768 91...

Alternate forms:

$$(3\,101\,364\,196\,875\,\pi^5)\Big/\left(3\,101\,364\,196\,875\,\operatorname{sech}\left(\frac{\pi}{2}\right) - 25\,525\,631\,250\,\operatorname{sech}(3\pi) - 184\,528\,125\,\operatorname{sech}\left(\frac{7\pi}{2}\right) + 52\,521\,875\,\operatorname{sech}\left(\frac{9\pi}{2}\right) + 1\,984\,873\,086\,\operatorname{sech}(5\pi)\right)$$

$$\frac{\pi^5}{\frac{2\cosh\left(\frac{\pi}{2}\right)}{1+\cosh(\pi)} - \frac{4\cosh(3\pi)}{243\,(1+\cosh(6\pi))} - \frac{2\cosh\left(\frac{7\pi}{2}\right)}{16807\,(1+\cosh(7\pi))} + \frac{2\cosh\left(\frac{9\pi}{2}\right)}{59049\,(1+\cosh(9\pi))} + \frac{4\cosh(5\pi)}{3125\,(1+\cosh(10\pi))}}$$

$$\pi^5\Big/\left(\frac{2}{e^{-\pi/2}+e^{\pi/2}} - \frac{4}{243\,(e^{-3\pi}+e^{3\pi})} - \frac{2}{16\,807\,(e^{-(7\pi)/2}+e^{(7\pi)/2})} + \frac{2}{59\,049\,(e^{-(9\pi)/2}+e^{(9\pi)/2})} + \frac{4}{3125\,(e^{-5\pi}+e^{5\pi})}\right)$$

Alternative representations:

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \frac{\pi^5}{\frac{1}{\cos\left(\frac{i\pi}{2}\right)} - \frac{1}{\frac{1}{2}\cos(3i\pi)3^5} + \frac{1}{\frac{1}{2}\cos(5i\pi)5^5} - \frac{1}{\cos\left(\frac{7i\pi}{2}\right)7^5} + \frac{1}{\cos\left(\frac{9i\pi}{2}\right)9^5}}$$

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \frac{\pi^5}{\frac{1}{\cos\left(-\frac{i\pi}{2}\right)} - \frac{1}{\frac{1}{2}\cos(-3i\pi)3^5} + \frac{1}{\frac{1}{2}\cos(-5i\pi)5^5} - \frac{1}{\cos\left(-\frac{7i\pi}{2}\right)7^5} + \frac{1}{\cos\left(-\frac{9i\pi}{2}\right)9^5}}$$

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \frac{\pi^5}{\frac{1}{\frac{1}{\sec\left(\frac{i\pi}{2}\right)}} - \frac{1}{\frac{3^5}{2\sec(3i\pi)}} - \frac{1}{\frac{7^5}{\sec\left(\frac{7i\pi}{2}\right)}} + \frac{1}{\frac{9^5}{\sec\left(\frac{9i\pi}{2}\right)}} + \frac{1}{\frac{5^5}{2\sec(5i\pi)}}}$$

Series representations:

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \pi^5 \Big/ \left(\sum_{k=0}^{\infty}\left(\frac{4(-1)^k e^{-5\pi-10k\pi}}{3125} + \frac{2(-1)^k e^{-(9\pi)/2-9k\pi}}{59\,049} - \frac{2(-1)^k e^{-(7\pi)/2-7k\pi}}{16\,807} - \frac{4}{243}(-1)^k e^{-3\pi-6k\pi} + 2(-1)^k e^{-\pi/2-k\pi}\right)\right)$$

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \pi^5 \Big/ \left(\sum_{k=0}^{\infty}\left(i\left(\text{Li}_{-k}(-i e^{z_0}) - \text{Li}_{-k}(i e^{z_0})\right)\left(3\,101\,364\,196\,875\left(\frac{\pi}{2}-z_0\right)^k - 25\,525\,631\,250\,(3\pi-z_0)^k - 184\,528\,125\left(\frac{7\pi}{2}-z_0\right)^k + 52\,521\,875\left(\frac{9\pi}{2}-z_0\right)^k + 1\,984\,873\,086\,(5\pi-z_0)^k\right)\right) \Big/ (3\,101\,364\,196\,875\,k!)\right) \quad \text{for } \frac{1}{2}+\frac{i z_0}{\pi} \notin \mathbb{Z}$$

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \pi^5 \Big/ \left(\sum_{k=0}^{\infty}\left(\frac{(-1)^k(1+2k)\pi}{\frac{\pi^2}{4}+\left(\frac{1}{2}+k\right)^2\pi^2} - \frac{2(-1)^k(1+2k)\pi}{243\left(9\pi^2+\left(\frac{1}{2}+k\right)^2\pi^2\right)} - \frac{(-1)^k(1+2k)\pi}{16\,807\left(\frac{49\pi^2}{4}+\left(\frac{1}{2}+k\right)^2\pi^2\right)} + \frac{(-1)^k(1+2k)\pi}{59\,049\left(\frac{81\pi^2}{4}+\left(\frac{1}{2}+k\right)^2\pi^2\right)} + \frac{2(-1)^k(1+2k)\pi}{3125\left(25\pi^2+\left(\frac{1}{2}+k\right)^2\pi^2\right)}\right)\right)$$

Integral representation:

$$\frac{\pi^5}{\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)} - \frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}} = \frac{\pi^5}{\int_0^\infty \frac{2\left(3\,101\,364\,196\,875 - 25\,525\,631\,250\,t^{5i} - 184\,528\,125\,t^{6i} + 52\,521\,875\,t^{8i} + 1\,984\,873\,086\,t^{9i}\right)t^i}{3\,101\,364\,196\,875\,\pi\,(1+t^2)}\,dt}$$

The result is 767,8605.

The two results 765.171 and 767,8605 are very similar and are practically equal at the nonperturbative contribution to the mass of a 1S quarkonium for $m_q = 4.7$ MeV/c^2 = 0.0047 GeV/c^2 , that is the mass of quark down is 4.8±0.5±0.3 = 4.7 MeV/c^2.
We note that:

32 * ((1/(1 cosh (Pi/2)) - (1/(3^5 cosh (3Pi)/2)) + (1/(5^5 cosh (5Pi)/2)) - 1/(7^5 cosh ((7Pi)/2)) + 1/(9^5 cosh ((9Pi)/2))

Input:

$$32\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{3^5\left(\frac{1}{2}\cosh(3\pi)\right)}+\frac{1}{5^5\left(\frac{1}{2}\cosh(5\pi)\right)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)$$

- cosh(x) is the hyperbolic cosine function

Exact result:

$$32\left(\operatorname{sech}\left(\frac{\pi}{2}\right)-\frac{2}{243}\operatorname{sech}(3\pi)-\frac{\operatorname{sech}\left(\frac{7\pi}{2}\right)}{16807}+\frac{\operatorname{sech}\left(\frac{9\pi}{2}\right)}{59049}+\frac{2\operatorname{sech}(5\pi)}{3125}\right)$$

- sech(x) is the hyperbolic secant function

Decimal approximation:

12.75313552551814631852248653106201018554387381914224238125...

Property:

$$32\left(\operatorname{sech}\left(\frac{\pi}{2}\right)-\frac{2}{243}\operatorname{sech}(3\pi)-\frac{\operatorname{sech}\left(\frac{7\pi}{2}\right)}{16807}+\frac{\operatorname{sech}\left(\frac{9\pi}{2}\right)}{59049}+\frac{2\operatorname{sech}(5\pi)}{3125}\right)$$

is a transcendental number

The result 12.75 is very near to the value of the black hole entropy 12.57

In conclusion, we have:

(2Pi)) * (3.142988/2)^5 ((1/(1 cosh (Pi/2)) - (1/(3^5 cosh (3Pi)/2)) + (1/(5^5 cosh (5Pi)/2)) - 1/(7^5 cosh ((7Pi)/2)) + 1/(9^5 cosh ((9Pi)/2))

Input interpretation:

$$(2\pi)\left(\frac{3.142988}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{3^5\left(\frac{1}{2}\cosh(3\pi)\right)}+\frac{1}{5^5\left(\frac{1}{2}\cosh(5\pi)\right)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)$$

- cosh(x) is the hyperbolic cosine function

Result:

24.0000...

Alternative representations:

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi = 2\pi\,1.57149^5\left(\frac{1}{\cos\left(\frac{i\pi}{2}\right)}-\frac{1}{\frac{1}{2}\cos(3i\pi)\,3^5}+\frac{1}{\frac{1}{2}\cos(5i\pi)\,5^5}-\frac{1}{\cos\left(\frac{7i\pi}{2}\right)7^5}+\frac{1}{\cos\left(\frac{9i\pi}{2}\right)9^5}\right)$$

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi = 2\pi\,1.57149^5\left(\frac{1}{\cos\left(-\frac{i\pi}{2}\right)}-\frac{1}{\frac{1}{2}\cos(-3i\pi)\,3^5}+\frac{1}{\frac{1}{2}\cos(-5i\pi)\,5^5}-\frac{1}{\cos\left(-\frac{7i\pi}{2}\right)7^5}+\frac{1}{\cos\left(-\frac{9i\pi}{2}\right)9^5}\right)$$

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi = 2\pi\,1.57149^5\left(\frac{1}{\frac{1}{\sec\left(\frac{i\pi}{2}\right)}}-\frac{1}{\frac{3^5}{2\sec(3i\pi)}}-\frac{1}{\frac{7^5}{\sec\left(\frac{7i\pi}{2}\right)}}+\frac{1}{\frac{9^5}{\sec\left(\frac{9i\pi}{2}\right)}}+\frac{1}{\frac{5^5}{2\sec(5i\pi)}}\right)$$

- i is the imaginary unit
- $\sec(x)$ is the secant function

Series representations:

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi = \frac{19.1687\pi}{\sum_{k=0}^{\infty}\frac{4^{-k}\pi^{2k}}{(2k)!}}-\frac{0.157767\pi}{\sum_{k=0}^{\infty}\frac{9^k\pi^{2k}}{(2k)!}}-\frac{0.00114052\pi}{\sum_{k=0}^{\infty}\frac{\left(\frac{49}{4}\right)^k\pi^{2k}}{(2k)!}}+\frac{0.000324624\pi}{\sum_{k=0}^{\infty}\frac{\left(\frac{81}{4}\right)^k\pi^{2k}}{(2k)!}}+\frac{0.012268\pi}{\sum_{k=0}^{\infty}\frac{25^k\pi^{2k}}{(2k)!}}$$

$$\left(\left(\frac{3.14299}{2}\right)^5 \left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5 \cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5 \cosh(5\pi)} - \frac{1}{7^5 \cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5 \cosh\left(\frac{9\pi}{2}\right)}\right)\right) 2\pi =$$

$$-\frac{19.1687\pi}{\sum_{k=0}^{\infty} I_{2k}\left(\frac{1}{2}\right) T_{2k}(\pi)(-2+\delta_k)} + \frac{0.157767\pi}{\sum_{k=0}^{\infty} I_{2k}(3) T_{2k}(\pi)(-2+\delta_k)} + \frac{0.00114052\pi}{\sum_{k=0}^{\infty} I_{2k}\left(\frac{7}{2}\right) T_{2k}(\pi)(-2+\delta_k)} - \frac{0.000324624\pi}{\sum_{k=0}^{\infty} I_{2k}\left(\frac{9}{2}\right) T_{2k}(\pi)(-2+\delta_k)} - \frac{0.012268\pi}{\sum_{k=0}^{\infty} I_{2k}(5) T_{2k}(\pi)(-2+\delta_k)}$$

$$\left(\left(\frac{3.14299}{2}\right)^5 \left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5 \cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5 \cosh(5\pi)} - \frac{1}{7^5 \cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5 \cosh\left(\frac{9\pi}{2}\right)}\right)\right) 2\pi =$$

$$\frac{19.1687\pi}{i\sum_{k=0}^{\infty} \frac{\left(\frac{\pi}{2}-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}} - \frac{0.157767\pi}{i\sum_{k=0}^{\infty} \frac{\left(3\pi-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}} - \frac{0.00114052\pi}{i\sum_{k=0}^{\infty} \frac{\left(\frac{7\pi}{2}-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}} + \frac{0.000324624\pi}{i\sum_{k=0}^{\infty} \frac{\left(\frac{9\pi}{2}-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}} + \frac{0.012268\pi}{i\sum_{k=0}^{\infty} \frac{\left(5\pi-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}}$$

- $n!$ is the factorial function
- $I_n(z)$ is the modified Bessel function of the first kind
- $T_n(x)$ is the Chebyshev polynomial of the first kind
- δ_{n_1,n_2} is the Kronecker delta function

- Integral representations:

$$\left(\left(\frac{3.14299}{2}\right)^5 \left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)} - \frac{1}{\frac{1}{2}\times 3^5 \cosh(3\pi)} + \frac{1}{\frac{1}{2}\times 5^5 \cosh(5\pi)} - \frac{1}{7^5 \cosh\left(\frac{7\pi}{2}\right)} + \frac{1}{9^5 \cosh\left(\frac{9\pi}{2}\right)}\right)\right) 2\pi =$$

$$\frac{19.1687\pi}{\int_{\frac{i\pi}{2}}^{\frac{\pi}{2}} \sinh(t)\,dt} - \frac{0.157767\pi}{\int_{\frac{i\pi}{2}}^{3\pi} \sinh(t)\,dt} - \frac{0.00114052\pi}{\int_{\frac{i\pi}{2}}^{\frac{7\pi}{2}} \sinh(t)\,dt} + \frac{0.000324624\pi}{\int_{\frac{i\pi}{2}}^{\frac{9\pi}{2}} \sinh(t)\,dt} + \frac{0.012268\pi}{\int_{\frac{i\pi}{2}}^{5\pi} \sinh(t)\,dt}$$

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi =$$

$$\frac{19.1687\pi}{1+\frac{\pi}{2}\int_0^1\sinh\left(\frac{\pi t}{2}\right)dt}-\frac{0.157767\pi}{1+3\pi\int_0^1\sinh(3\pi t)\,dt}-\frac{0.00114052\pi}{1+\frac{7\pi}{2}\int_0^1\sinh\left(\frac{7\pi t}{2}\right)dt}+\frac{0.000324624\pi}{1+\frac{9\pi}{2}\int_0^1\sinh\left(\frac{9\pi t}{2}\right)dt}+\frac{0.012268\pi}{1+5\pi\int_0^1\sinh(5\pi t)\,dt}$$

$$\left(\left(\frac{3.14299}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{\frac{1}{2}\times 3^5\cosh(3\pi)}+\frac{1}{\frac{1}{2}\times 5^5\cosh(5\pi)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)\right)2\pi =$$

$$\frac{38.3375\,i\pi^2}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(16s)+s}}{\sqrt{s}}ds}-\frac{0.315535\,i\pi^2}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(9\pi^2)/(4s)+s}}{\sqrt{s}}ds}-\frac{0.00228104\,i\pi^2}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(49\pi^2)/(16s)+s}}{\sqrt{s}}ds}+\frac{0.000649249\,i\pi^2}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(81\pi^2)/(16s)+s}}{\sqrt{s}}ds}+\frac{0.024536\,i\pi^2}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(25\pi^2)/(4s)+s}}{\sqrt{s}}ds}\quad \text{for } \gamma>0$$

$\sinh(x)$ is the hyperbolic sine function

The result 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates. We have that the result is a length of a circle of radius equal to:

Input interpretation:

$$\left(\frac{3.142988}{2}\right)^5\left(\frac{1}{1\cosh\left(\frac{\pi}{2}\right)}-\frac{1}{3^5\left(\frac{1}{2}\cosh(3\pi)\right)}+\frac{1}{5^5\left(\frac{1}{2}\cosh(5\pi)\right)}-\frac{1}{7^5\cosh\left(\frac{7\pi}{2}\right)}+\frac{1}{9^5\cosh\left(\frac{9\pi}{2}\right)}\right)$$

- $\cosh(x)$ is the hyperbolic cosine function

Result:

3.81971...

Indeed: C = 2πr = 6,283185307... * 3,81971... = 24

Now, we have, from (8):

1/((25+1/100)(e^Pi+1)) + 3/((25+3^4/100)((e^(3Pi)+1))) + 5/((25+5^4/100)((e^(5Pi)+1))) + 7/((25+7^4/100)(e^(7Pi)+1)))

Input:

$$\frac{1}{\left(25+\frac{1}{100}\right)(e^{\pi}+1)}+\frac{3}{\left(25+\frac{3^4}{100}\right)(e^{3\pi}+1)}+\frac{5}{\left(25+\frac{5^4}{100}\right)(e^{5\pi}+1)}+\frac{7}{\left(25+\frac{7^4}{100}\right)(e^{7\pi}+1)}$$

Exact result:

$$\frac{100}{2501\,(1+e^{\pi})}+\frac{300}{2581\,(1+e^{3\pi})}+\frac{4}{25\,(1+e^{5\pi})}+\frac{700}{4901\,(1+e^{7\pi})}$$

Decimal approximation:

0.001665694195075570224836070056797018137393783827926220428...

Property:

$$\frac{100}{2501\,(1+e^{\pi})}+\frac{300}{2581\,(1+e^{3\pi})}+\frac{4}{25\,(1+e^{5\pi})}+\frac{700}{4901\,(1+e^{7\pi})}$$ is a transcendental number

Note that:

[1/(0.001665694195075570224836070056797018137393783827926220428)]^1/13

Input interpretation:

$$\sqrt[13]{\frac{1}{0.001665694195075570224836070056797018137393783827926220428}}$$

Result:

1.635774525925176106165337156866647955962634167808587082 1...

$1.6357745259251761061653371568666479559626341678085870821\, e^{0} \approx 1.6358$

(real, principal root)

The result is 1.6358 value very near to the fourteenth root of Ramanujan's class invariant 1164.2696 and a good approximation to the mass of the proton

We have:

(Pi/8) coth^2 (5Pi/2) - (4689/11890)

Input:

$$\frac{\pi}{8}\coth^2\left(5\times\frac{\pi}{2}\right)-\frac{4689}{11\,890}$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$\frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right)-\frac{4689}{11\,890}$$

Decimal approximation:

−0.001665694195127834328346934326933649713197872426167604 42...

Note that:

[-1/(((Pi/8) coth^2 (5Pi/2) - (4689/11890)))]^1/13

Input:

$$\sqrt[13]{-\frac{1}{\frac{\pi}{8}\coth^2\left(5\times\frac{\pi}{2}\right)-\frac{4689}{11\,890}}}$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$\frac{1}{\sqrt[13]{\frac{4689}{11\,890}-\frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right)}}$$

Decimal approximation:

1.6357745259212280045549468248867675362517666589334848 09744...

$$\frac{e^0}{\sqrt[13]{\frac{4689}{11\,890}-\frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right)}}\approx 1.636 \text{ (real, principal root)}$$

The result is 1.636 value very near to the fourteenth root of Ramanujan's class invariant 1164.2696 and a good approximation to the proton mass.

We have, from the integral representation:

Integral representation:

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi-\frac{4689}{11\,890}=-\frac{4689}{11\,890}+\frac{1}{8}\pi\left(\int_{\frac{i\pi}{2}}^{\frac{5\pi}{2}}\operatorname{csch}^2(t)\,dt\right)^2$$

(4689/11890) – 0.001665694195127834328 = 0,3943650126156433978 – 0.001665694195127834328 = 0,39269931842....

0,39269931842 * 8 = 3,14159454736412450 78....;

8 = 3,1415945473641245078 / 0,39269931842;

(3,1415945473641245078 / 0,39269931842)* 27 * 8 = 1728 or

(2*3,1415945473641245078) * 108) / 0,39269931842 = 1728

Indeed:

2Pi * 108 / [((4689/11890) + (Pi/8) coth^2 (5Pi/2) - (4689/11890)]

Input:

$$2\pi \times \frac{108}{\frac{4689}{11890} + \frac{\pi}{8}\coth^2\left(5 \times \frac{\pi}{2}\right) - \frac{4689}{11890}}$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$1728 \tanh^2\left(\frac{5\pi}{2}\right)$$

- $\tanh(x)$ is the hyperbolic tangent function

Decimal approximation:

1727.998958349973207328055391843712995833540664764345677453...

Property:

$1728 \tanh^2\left(\frac{5\pi}{2}\right)$ is a transcendental number

Integral representation:

$$\frac{(2\pi)\,108}{\frac{4689}{11890} + \frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}} = 1728\left(\int_0^{\frac{5\pi}{2}} \operatorname{sech}^2(t)\,dt\right)^2$$

Practically the result 1728 is the length of a circle C = 2πr , with r equal to 275,019575879. Indeed, we have that:

Input:

$$\frac{108}{\frac{4689}{11890} + \frac{\pi}{8}\coth^2\left(5 \times \frac{\pi}{2}\right) - \frac{4689}{11890}}$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$\frac{864\tanh^2\left(\frac{5\pi}{2}\right)}{\pi}$$

- $\tanh(x)$ is the hyperbolic tangent function

Decimal approximation:

275.0195758790444043496760155180770120578061545274840380973...

Note that:

1/144 * [[[2Pi * 108 / [((4689/11890) + (Pi/8) coth^2 (5Pi/2) - (4689/11890)]]]]

Input:

$$\frac{1}{144}\left(2\pi\times\frac{108}{\frac{4689}{11890}+\frac{\pi}{8}\coth^2\left(5\times\frac{\pi}{2}\right)-\frac{4689}{11890}}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$12\tanh^2\left(\frac{5\pi}{2}\right)$$

- $\tanh(x)$ is the hyperbolic tangent function

Decimal approximation:

11.99999276631925838422260688780356247106625461641906720453...

Property:

$12\tanh^2\left(\frac{5\pi}{2}\right)$ is a transcendental number

Integral representation:

$$\frac{2\pi\,108}{\left(\frac{4689}{11890}+\frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right)-\frac{4689}{11890}\right)144}=12\left(\int_0^{\frac{5\pi}{2}}\operatorname{sech}^2(t)\,dt\right)^2$$

The two results 1728 and 12 are very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5) and a good approximation to the value of black hole entropy 12,19.

Furthermore, we have that:

2/144 * [[[2Pi * 108 / [((4689/11890) + (Pi/8) coth^2 (5Pi/2) - (4689/11890)]]]]

$$\frac{2}{144}\left(2\pi\times\frac{108}{\frac{4689}{11890}+\frac{\pi}{8}\coth^2\left(5\times\frac{\pi}{2}\right)-\frac{4689}{11890}}\right)$$

Exact result:

$$24 \tanh^2\left(\frac{5\pi}{2}\right)$$

Decimal approximation:

23.99998553263851676844521377560712494213250923283813440907...

Property:

$24 \tanh^2\left(\frac{5\pi}{2}\right)$ is a transcendental number

Alternate forms:

$$\frac{24 (\cosh(5\pi) - 1)}{1 + \cosh(5\pi)}$$

$$\frac{24 \sinh^2(5\pi)}{(1 + \cosh(5\pi))^2}$$

$$\frac{24 \sinh^2\left(\frac{5\pi}{2}\right)}{\cosh^2\left(\frac{5\pi}{2}\right)}$$

Alternative representations:

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = \frac{432\pi}{\frac{144}{8}\left(\pi\left(1 + \frac{2}{-1+e^{5\pi}}\right)^2\right)}$$

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = \frac{432\pi}{\frac{144}{8}\left(\pi\left(i \cot\left(\frac{5 i\pi}{2}\right)\right)^2\right)}$$

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = \frac{432\pi}{\frac{144}{8}\left(\pi\left(-i \cot\left(-\frac{5 i\pi}{2}\right)\right)^2\right)}$$

Series representations:

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = 24\left(1 - 2\sum_{k=0}^{\infty} e^{(-5-(5-i)k)\pi}\right)^2$$

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = \frac{9600\left(\sum_{k=1}^{\infty} \frac{1}{25+(1-2k)^2}\right)^2}{\pi^2}$$

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = 24\left(1 + 2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2 \quad \text{for } q = e^{(5\pi)/2}$$

Integral representation:

$$\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144} = 24\left(\int_0^{\frac{5\pi}{2}} \operatorname{sech}^2(t)\, dt\right)^2$$

The result 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

In conclusion, we have that:

[((((((((((((((2/144 *((2Pi * 108 / (((4689/11890) + (Pi/8) coth^2 (5Pi/2) - (4689/11890))))))))))]^1/(2Pi)

$$\sqrt[2\pi]{\frac{2}{144}\left(2\pi \times \frac{108}{\frac{4689}{11890} + \frac{\pi}{8}\coth^2\left(5 \times \frac{\pi}{2}\right) - \frac{4689}{11890}}\right)}$$

Exact result:

$$2^{3/(2\pi)}\, \sqrt[2\pi]{3}\, \sqrt[\pi]{\tanh\left(\frac{5\pi}{2}\right)}$$

Decimal approximation:

1.6583164161474762001332927535838235196604640231560658007 74...

Alternate forms:

$$2^{3/(2\pi)}\, \sqrt[2\pi]{3}\, \sqrt[\pi]{\frac{\sinh(5\pi)}{1+\cosh(5\pi)}}$$

$$2^{3/(2\pi)}\, \sqrt[2\pi]{3}\, \sqrt[\pi]{\frac{\sinh\left(\frac{5\pi}{2}\right)}{\cosh\left(\frac{5\pi}{2}\right)}}$$

$$2^{3/(2\pi)}\, \sqrt[2\pi]{3}\, \sqrt[\pi]{\frac{e^{(5\pi)/2} - e^{-(5\pi)/2}}{e^{-(5\pi)/2} + e^{(5\pi)/2}}}$$

Alternative representations:

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = \sqrt[2\pi]{\frac{432\pi}{\frac{144}{8}\left(\pi\left(i \cot\left(\frac{5 i \pi}{2}\right)\right)^2\right)}}$$

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = \sqrt[2\pi]{\frac{432\pi}{\frac{144}{8}\left(\pi\left(1 + \frac{2}{-1+e^{5\pi}}\right)^2\right)}}$$

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = \sqrt[2\pi]{\frac{432\pi}{\frac{144}{8}\left(\pi\left(-i \cot\left(-\frac{5 i \pi}{2}\right)\right)^2\right)}}$$

Series representations:

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = 2^{3/(2\pi)}\sqrt[2\pi]{3}\sqrt[\pi]{1 - 2\sum_{k=0}^{\infty} e^{(-5-(5-i)k)\pi}}$$

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = 2^{3/(2\pi)}\sqrt[2\pi]{3}\sqrt[\pi]{-1 - 2\sum_{k=1}^{\infty} (-1)^k q^{2k}}$$

for $q = e^{(5\pi)/2}$

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = 2^{7/(2\pi)}\sqrt[2\pi]{3}\sqrt[\pi]{\frac{5}{\pi}}\sqrt[\pi]{\sum_{k=1}^{\infty}\frac{1}{25 + (1-2k)^2}}$$

Integral representation:

$$\sqrt[2\pi]{\frac{(2\pi\, 108)\, 2}{\left(\frac{4689}{11890} + \frac{1}{8}\pi \coth^2\left(\frac{5\pi}{2}\right) - \frac{4689}{11890}\right) 144}} = 2^{3/(2\pi)}\sqrt[2\pi]{3}\sqrt[\pi]{\int_0^{\frac{5\pi}{2}} \operatorname{sech}^2(t)\, dt}$$

The result 1,6583164 is very near to the value 1,65578 (14-th root of Ramanujan class invariant 1164,2696) and near to the value of the proton mass.

Now, we have from (11):

Pi/12 ln(2+sqrt(3))

Input:

$$\frac{\pi}{12}\log\left(2+\sqrt{3}\right)$$

- $\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{12}\pi\log\left(2+\sqrt{3}\right)$$

Decimal approximation:

0.3447787711721723606778605902244687195399824255459757567 97...

Continued fraction:

[0; 2, 1, 9, 24, 3, 1, 11, 1, 1, 1, 3, 1, 2, 4, 5, 6, 1, 3, 1, 2, 4, 19, 3, 1, 3, 5, 2, 1, 1, 2, ...]

Alternative representations:

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = \frac{1}{12}\pi\log_e\left(2+\sqrt{3}\right)$$

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = \frac{1}{12}\pi\log(a)\log_a\left(2+\sqrt{3}\right)$$

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = -\frac{1}{12}\pi\,\text{Li}_1\left(-1-\sqrt{3}\right)$$

Series representations:

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = \frac{1}{12}\pi\log\left(1+\sqrt{3}\right)-\frac{1}{12}\pi\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{1+\sqrt{3}}\right)^k}{k}$$

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = \frac{1}{6}i\pi^2\left\lfloor\frac{\arg\left(2+\sqrt{3}-x\right)}{2\pi}\right\rfloor+\frac{1}{12}\pi\log(x)-\frac{1}{12}\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(2+\sqrt{3}-x\right)^k x^{-k}}{k} \quad \text{for } x<0$$

$$\frac{1}{12}\log\left(2+\sqrt{3}\right)\pi = \frac{1}{6}i\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\frac{1}{12}\pi\log(z_0)-\frac{1}{12}\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(2+\sqrt{3}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{12} \log\left(2+\sqrt{3}\right)\pi = \frac{\pi}{12} \int_1^{2+\sqrt{3}} \frac{1}{t}\, dt$$

$$\frac{1}{12} \log\left(2+\sqrt{3}\right)\pi = -\frac{i}{24} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\left(1+\sqrt{3}\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1<\gamma<0$$

We have that:

Pi * ln(2+sqrt(3)) * (1/0.34477877117217236)

$$\pi \log\left(2+\sqrt{3}\right) \times \frac{1}{0.34477877117217236}$$

Result:

12.000000000000000...

Alternative representations:

$$\frac{\pi \log(2+\sqrt{3})}{0.344778771172172360000} = \frac{\pi \log_e(2+\sqrt{3})}{0.344778771172172360000}$$

$$\frac{\pi \log(2+\sqrt{3})}{0.344778771172172360000} = \frac{\pi \log(a) \log_a(2+\sqrt{3})}{0.344778771172172360000}$$

$$\frac{\pi \log(2+\sqrt{3})}{0.344778771172172360000} = -\frac{\pi\, \mathrm{Li}_1(-1-\sqrt{3})}{0.344778771172172360000}$$

Series representations:

$$\frac{\pi \log(2+\sqrt{3})}{0.344778771172172360000} = 2.90041059256699265660\, \pi \log\left(2+\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\pi \log(2+\sqrt{3})}{0.344778771172172360000} = 2.90041059256699265660\, \pi \log\left(1+\sqrt{3}\right) - 2.90041059256699265660\, \pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(1+\sqrt{3}\right)^{-k}}{k}$$

$$\frac{\pi\log(2+\sqrt{3})}{0.3447787711721723600000} =$$

$$2.9004105925669926566\,\pi\log\left(2+\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

Integral representations:

$$\frac{\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 2.9004105925669926566\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

$$\frac{\pi\log(2+\sqrt{3})}{0.3447787711721723600000} =$$

$$\frac{1.4502052962834963283}{i}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(1+\sqrt{3}\right)^{-s}}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

The result 12, is a good approximation to the value of black hole entropy 12,19.

From:

$$\frac{\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 2.9004105925669926566\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

We have that for

$$\int\frac{1}{x}dx = \log|x| + c$$

2.900410592566... * π * ln(2+√3) = 2.900410592566 * 4,137345254066 = 12

and

2 * Pi * ln(2+sqrt(3)) * (1/0.34477877117217236)

Input interpretation:

$$2\,\pi\log\left(2+\sqrt{3}\right)\times\frac{1}{0.34477877117217236}$$

Result:

24.000000000000000...

Alternative representations:

$$\frac{2\,\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = \frac{2\,\pi\log_e(2+\sqrt{3})}{0.3447787711721723600000}$$

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = \frac{2\pi\log(a)\log_a(2+\sqrt{3})}{0.3447787711721723600000}$$

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = -\frac{2\pi\operatorname{Li}_1(-1-\sqrt{3})}{0.3447787711721723600000}$$

Series representations:

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 5.800821185133985313 20\,\pi\log\left(2+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 5.80082118513398531320\,\pi\log\left(1+\sqrt{3}\right) - 5.80082118513398531320\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{3}\right)^{-k}}{k}$$

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 5.80082118513398531320\,\pi\log\left(2+\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

Integral representations:

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 5.80082118513398531320\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = \frac{2.90041059256699265660}{i}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(1+\sqrt{3}\right)^{-s}}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

From:

$$\frac{2\pi\log(2+\sqrt{3})}{0.3447787711721723600000} = 5.80082118513398531320\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

We have that for

$$\int \frac{1}{x} dx = \log|x| + c$$

5.80082118513398 * π * ln(2+√3) = 5.80082118513398 * 4,137345254066 = 24

The result 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

and:

144 * Pi * ln(2+sqrt(3)) * (1/0.34477877117217236)

Input interpretation:

$$144\,\pi \log\left(2+\sqrt{3}\right) \times \frac{1}{0.34477877117217236}$$

- $\log(x)$ is the natural logarithm

Result:

1728.0000000000000...

Alternative representations:

$$\frac{144\,\pi \log(2+\sqrt{3})}{0.34477877117217236000} = \frac{144\,\pi \log_e(2+\sqrt{3})}{0.34477877117217236000}$$

$$\frac{144\,\pi \log(2+\sqrt{3})}{0.34477877117217236000} = \frac{144\,\pi \log(a) \log_a(2+\sqrt{3})}{0.34477877117217236000}$$

$$\frac{144\,\pi \log(2+\sqrt{3})}{0.34477877117217236000} = -\frac{144\,\pi \operatorname{Li}_1(-1-\sqrt{3})}{0.34477877117217236000}$$

Series representations:

$$\frac{144\,\pi \log(2+\sqrt{3})}{0.34477877117217236000} = 417.65912532964694255\,\pi \log\left(2+\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{144\,\pi \log(2+\sqrt{3})}{0.34477877117217236000} = 417.65912532964694255\,\pi \log\left(1+\sqrt{3}\right) - 417.65912532964694255\,\pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(1+\sqrt{3}\right)^{-k}}{k}$$

$$\frac{144\pi\log(2+\sqrt{3})}{0.344778771172172360000} =$$

$$417.659125329646942550\,\pi\log\left(2+\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

Integral representations:

$$\frac{144\pi\log(2+\sqrt{3})}{0.344778771172172360000} = 417.659125329646942550\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

$$\frac{144\pi\log(2+\sqrt{3})}{0.344778771172172360000} =$$

$$\frac{208.829562664823471275}{i}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(1+\sqrt{3}\right)^{-s}}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

From:

$$\frac{144\pi\log(2+\sqrt{3})}{0.344778771172172360000} = 417.659125329646942550\,\pi\int_1^{2+\sqrt{3}}\frac{1}{t}\,dt$$

We have that for

$$\int\frac{1}{x}dx = \log|x| + c$$

417.65912532964 * π * ln(2+√3) = 417,65912532964 * 4,137345254066 = 1728

The result 1728 is very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5)

Now, we have from (4):

[1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}$$

Decimal approximation:

0.041638381585443662182517651348977915286722918403784080981...

This is equal to $\frac{1}{24} = 0,041666 \ldots$

We have:

1 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}$$

Exact result:

$$\frac{1}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}$$

Decimal approximation:

24.016303274130842385703000769652617296434417106114152349 68...

Property:

$$\frac{1}{\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}+\frac{1}{2}(\coth(\pi)-1)}$$

$$-8193+e^{2\pi}+\left(562952336339+562883633152\,e^{2\pi}+562950758400\,e^{4\pi}+13060710400\,e^{6\pi}+65530925\,e^{8\pi}\right)/\left(68711380+68711381\,e^{2\pi}+68719574\,e^{4\pi}+1602518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}\right)$$

$$\frac{(e^{\pi}-1)(1+e^{\pi})(1+e^{2\pi})(1-e^{\pi}+e^{2\pi})(1+e^{\pi}+e^{2\pi})(1+e^{4\pi})}{68711380+68711381\,e^{2\pi}+68719574\,e^{4\pi}+1602518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}}$$

Continued fraction:

[24; 61, 2, 1, 26, 1, 5, 3, 2, 2, 1, 22, 1, 1, 1, 1, 3, 3, 1, 3, 7, 1, 1, 2, 3, 1, 1, 1, 2, 1, ...]

Alternative representations:

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{1^{13}}{-1+e^{360^\circ}}+\frac{2^{13}}{-1+e^{720^\circ}}+\frac{3^{13}}{-1+e^{1080^\circ}}+\frac{4^{13}}{-1+e^{1440^\circ}}}$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}} \quad \text{for } z = 1$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{4^{13}}{-1+e^{-8 i \log(-1)}}+\frac{3^{13}}{-1+e^{-6 i \log(-1)}}+\frac{2^{13}}{-1+e^{-4 i \log(-1)}}+\frac{1^{13}}{-1+e^{-2 i \log(-1)}}}$$

Series representations:

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 1\Bigg/\left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 1\Bigg/\left(\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)\right) \Big/ \left(68711380+68711381\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+68719574\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+1602518\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+8194\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

Integral representations:

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{1}{-1+e^{4\int_0^\infty \sin(t)/t\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin(t)/t\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin(t)/t\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin(t)/t\,dt}}}$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{1}{-1+e^{4\int_0^\infty 1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty 1/(1+t^2)\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty 1/(1+t^2)\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty 1/(1+t^2)\,dt}}}$$

$$\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{\frac{1}{-1+e^{4\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin^2(t)/t^2\,dt}}}$$

The result, about 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

72 * 1 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$72 \times \frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}$$

Decimal approximation:

1729.173835737420651770616055414988445343278031640218969177...

Property:

$$\frac{72}{\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}}$$ is a transcendental number

Alternate forms:

$$\frac{72}{\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}+\frac{1}{2}(\coth(\pi)-1)}$$

$$-589896+72e^{2\pi}+\left(72\left(562952336339+562883633152e^{2\pi}+562950758400e^{4\pi}+13060710400e^{6\pi}+65530925e^{8\pi}\right)\right)/\left(68711380+68711381e^{2\pi}+68719574e^{4\pi}+1602518e^{6\pi}+8194e^{8\pi}+e^{10\pi}\right)$$

$$\frac{72(e^{\pi}-1)(1+e^{\pi})(1+e^{2\pi})(1-e^{\pi}+e^{2\pi})(1+e^{\pi}+e^{2\pi})(1+e^{4\pi})}{68711380+68711381e^{2\pi}+68719574e^{4\pi}+1602518e^{6\pi}+8194e^{8\pi}+e^{10\pi}}$$

Continued fraction:

[1729; 5, 1, 3, 24, 5, 4, 1, 5, 26, 16, 4, 2, 10, 1, 2, 1, 1, 3, 152, 1, 14, 2, 3, 3, 1, 1, ...]

Alternative representations:

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}=\frac{72}{\frac{1^{13}}{-1+e^{360^\circ}}+\frac{2^{13}}{-1+e^{720^\circ}}+\frac{3^{13}}{-1+e^{1080^\circ}}+\frac{4^{13}}{-1+e^{1440^\circ}}}$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}=\frac{72}{\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}} \text{ for } z=1$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{72}{\frac{4^{13}}{-1+e^{-8 i \log(-1)}}+\frac{3^{13}}{-1+e^{-6 i \log(-1)}}+\frac{2^{13}}{-1+e^{-4 i \log(-1)}}+\frac{1^{13}}{-1+e^{-2 i \log(-1)}}}$$

Series representations:

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 72 \Big/ \left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 72 \Big/ \left(\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \left(72\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)\right) \Big/ \left(68711380+68711381\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+68719574\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+1602518\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+8194\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

Integral representations:

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{72}{\frac{1}{-1+e^{4\int_0^\infty \sin(t)/t\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin(t)/t\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin(t)/t\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin(t)/t\,dt}}}$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{72}{\frac{1}{-1+e^{4\int_0^\infty 1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty 1/(1+t^2)\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty 1/(1+t^2)\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty 1/(1+t^2)\,dt}}}$$

$$\frac{72}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{72}{\frac{1}{-1+e^{4\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin^2(t)/t^2\,dt}}}$$

The result, 1729,17 is very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5).

Now:

(1/2) / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$\frac{\frac{1}{2}}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}$$

Decimal approximation:

12.008151637065421192851500384826308648217208553057076174 84...

Property:

$$\frac{1}{2\left(\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{1}{2\left(\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}+\frac{1}{2}(\coth(\pi)-1)\right)}$$

$$-\frac{8193}{2}+\frac{e^{2\pi}}{2}+\left(562\,952\,336\,339+562\,883\,633\,152\,e^{2\pi}+562\,950\,758\,400\,e^{4\pi}+13\,060\,710\,400\,e^{6\pi}+65\,530\,925\,e^{8\pi}\right)\Big/\left(2\left(68\,711\,380+68\,711\,381\,e^{2\pi}+68\,719\,574\,e^{4\pi}+1\,602\,518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}\right)\right)$$

$$\frac{(e^{\pi}-1)(1+e^{\pi})(1+e^{2\pi})(1-e^{\pi}+e^{2\pi})(1+e^{\pi}+e^{2\pi})(1+e^{4\pi})}{2\left(68\,711\,380+68\,711\,381\,e^{2\pi}+68\,719\,574\,e^{4\pi}+1\,602\,518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}\right)}$$

Continued fraction:

[12; 122, 1, 2, 13, 2, 2, 1, 1, 1, 4, 1, 2, 11, 3, 3, 1, 1, 1, 1, 2, 1, 1, 1, 3, 3, 2, 1, 1, 3, ...]

Alternative representations:

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2}=\frac{1}{2\left(\frac{1^{13}}{-1+e^{360\,°}}+\frac{2^{13}}{-1+e^{720\,°}}+\frac{3^{13}}{-1+e^{1080\,°}}+\frac{4^{13}}{-1+e^{1440\,°}}\right)}$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2}=\frac{1}{\left(\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}\right)2}\quad\text{for } z=1$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2}=\frac{1}{2\left(\frac{4^{13}}{-1+e^{-8i\log(-1)}}+\frac{3^{13}}{-1+e^{-6i\log(-1)}}+\frac{2^{13}}{-1+e^{-4i\log(-1)}}+\frac{1^{13}}{-1+e^{-2i\log(-1)}}\right)}$$

Series representations:

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2}=1\Big/\left(2\left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)\right)$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2} =$$

$$1\Big/\left(2\left(\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)\right)$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2} =$$

$$\left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)\right)\Big/\left(2\left(68711380+68711381\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+68719574\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+1602518\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+8194\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)\right)$$

Integral representations:

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2} = \frac{1}{2\left(\frac{1}{-1+e^{4\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{1594323}{-1+e^{12\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{67108864}{-1+e^{16\int_0^{\infty}1/(1+t^2)\,dt}}\right)}$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2} = \frac{1}{2\left(\frac{1}{-1+e^{4\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{1594323}{-1+e^{12\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{67108864}{-1+e^{16\int_0^{\infty}\sin^2(t)/t^2\,dt}}\right)}$$

$$\frac{1}{\left(\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}\right)2} =$$

$$1\Big/\left(2\left(\frac{1}{-1+e^{6\int_0^\infty \sin^4(t)/t^4\,dt}}+\frac{8192}{-1+e^{12\int_0^\infty \sin^4(t)/t^4\,dt}}+\frac{1594323}{-1+e^{18\int_0^\infty \sin^4(t)/t^4\,dt}}+\frac{67108864}{-1+e^{24\int_0^\infty \sin^4(t)/t^4\,dt}}\right)\right)$$

The result 12,0081 is a good approximation to the value of black hole entropy 12,19.

32 * 1 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$32\times\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}$$

Decimal approximation:

768.52170477218695634249602462888375348590134739565287518 99...

Property:

$$\frac{32}{\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}}$$ is a transcendental number

Alternate forms:

$$\frac{32}{\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}+\frac{1}{2}(\coth(\pi)-1)}$$

$$-262176+32e^{2\pi}+(32(562952336339+562883633152e^{2\pi}+562950758400e^{4\pi}+13060710400e^{6\pi}+65530925e^{8\pi}))/(68711380+68711381e^{2\pi}+68719574e^{4\pi}+1602518e^{6\pi}+8194e^{8\pi}+e^{10\pi})$$

$$\frac{32(e^\pi-1)(1+e^\pi)(1+e^{2\pi})(1-e^\pi+e^{2\pi})(1+e^\pi+e^{2\pi})(1+e^{4\pi})}{68711380+68711381e^{2\pi}+68719574e^{4\pi}+1602518e^{6\pi}+8194e^{8\pi}+e^{10\pi}}$$

Continued fraction:

[768; 1, 1, 11, 54, 1, 13, 1, 2, 1, 2, 1, 4, 1, 3, 9, 1, 8, 3, 1, 1, 1, 128, 4, 4, 34, 2, 10, ...]

Alternative representations:

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{1^{13}}{-1+e^{360^\circ}}+\frac{2^{13}}{-1+e^{720^\circ}}+\frac{3^{13}}{-1+e^{1080^\circ}}+\frac{4^{13}}{-1+e^{1440^\circ}}}$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}} \quad \text{for } z=1$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{4^{13}}{-1+e^{-8i\log(-1)}}+\frac{3^{13}}{-1+e^{-6i\log(-1)}}+\frac{2^{13}}{-1+e^{-4i\log(-1)}}+\frac{1^{13}}{-1+e^{-2i\log(-1)}}}$$

Series representations:

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 32\Bigg/\left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = 32\Bigg/\left(\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} =$$

$$\left(32\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)\right) / \left(68\,711\,380+68\,711\,381\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+68\,719\,574\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+1\,602\,518\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+8194\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

Integral representations:

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{1}{-1+e^{4\int_0^{\infty}\sin(t)/t\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}\sin(t)/t\,dt}}+\frac{1\,594\,323}{-1+e^{12\int_0^{\infty}\sin(t)/t\,dt}}+\frac{67\,108\,864}{-1+e^{16\int_0^{\infty}\sin(t)/t\,dt}}}$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{1}{-1+e^{4\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{1\,594\,323}{-1+e^{12\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{67\,108\,864}{-1+e^{16\int_0^{\infty}1/(1+t^2)\,dt}}}$$

$$\frac{32}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{32}{\frac{1}{-1+e^{4\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{1\,594\,323}{-1+e^{12\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{67\,108\,864}{-1+e^{16\int_0^{\infty}\sin^2(t)/t^2\,dt}}}$$

The result 768,52 is practically equal at the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for $m_q = 4.7\ MeV/c^2 = 0.0047\ GeV/c^2$, that is the mass of quark down is 4.8±0.5±0.3 = 4.7 MeV/c^2.

((((((12 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))])))))^1/11

Input:

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}}$$

Exact result:

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}$$

Decimal approximation:

1.673431523947580000084264747070453574235412637360666293354...

Property:

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}}}$$ is a transcendental number

Alternate form:

$$2^{2/11}\sqrt[11]{\frac{3\left(e^{2\pi}-1\right)\left(1+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}\right)}{68711380+68711381\,e^{2\pi}+68719574\,e^{4\pi}+1602518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}}}$$

Continued fraction:

[1; 1, 2, 16, 10, 1, 15, 3, 1, 2, 9, 2, 2, 19, 1, 43, 1, 11, 2, 1, 2, 1, 2, 2, 1, 1, 2, 1, 1, ...]

All 11th roots of 12/(1/(e^2 π)-1)+8192/(e^4 π)-1)+1594323/(e^6 π)-1)+67108864/(e^8 π)-1)):

- Polar form

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\;e^{0}\approx 1.6734 \quad \text{(real, principal root)}$$

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\;e^{(2i\pi)/11}\approx 1.4078+0.9047\,i$$

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\;e^{(4i\pi)/11}\approx 0.6952+1.5222\,i$$

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\;e^{(6i\pi)/11}\approx -0.2382+1.6564\,i$$

$$2^{2/11}\sqrt[11]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\; e^{(8i\pi)/11} \approx -1.0959+1.2647\,i$$

Alternative representations:

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[11]{\frac{12}{\frac{1^{13}}{-1+e^{360^\circ}}+\frac{2^{13}}{-1+e^{720^\circ}}+\frac{3^{13}}{-1+e^{1080^\circ}}+\frac{4^{13}}{-1+e^{1440^\circ}}}}$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[11]{\frac{12}{\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}}} \quad \text{for } z=1$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[11]{\frac{12}{\frac{4^{13}}{-1+e^{-8i\log(-1)}}+\frac{3^{13}}{-1+e^{-6i\log(-1)}}+\frac{2^{13}}{-1+e^{-4i\log(-1)}}+\frac{1^{13}}{-1+e^{-2i\log(-1)}}}}$$

Series representations:

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = 2^{2/11}\sqrt[11]{3}\sqrt[11]{\frac{1}{\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}}}}$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = 2^{2/11}\sqrt[11]{3}\left(1\Big/\left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)\right)^\wedge(1/11)$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = 2^{2/11}\sqrt[11]{3}$$
$$\sqrt[11]{\frac{1}{\frac{1}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}}+\frac{8192}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}}+\frac{1594323}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}}+\frac{67108864}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}}}}$$

Integral representations:

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = 2^{2/11}\sqrt[11]{3}$$
$$\sqrt[11]{\frac{1}{\frac{1}{-1+e^{4\int_0^\infty \sin(t)/t\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin(t)/t\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin(t)/t\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin(t)/t\,dt}}}}$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = 2^{2/11}\sqrt[11]{3}$$
$$\sqrt[11]{\frac{1}{\frac{1}{-1+e^{4\int_0^\infty 1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty 1/(1+t^2)\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty 1/(1+t^2)\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty 1/(1+t^2)\,dt}}}}$$

$$\sqrt[11]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} =$$
$$2^{2/11}\sqrt[11]{3}\left(1\Big/\left(\frac{1}{-1+e^{4\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{1594323}{-1+e^{12\int_0^\infty \sin^2(t)/t^2\,dt}}+\frac{67108864}{-1+e^{16\int_0^\infty \sin^2(t)/t^2\,dt}}\right)\right)\hat{}\,(1/11)$$

The value 1.6734315 is very near to the 14-th root of Ramanujan class invariant 1164.2696, to the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$ 1,6557 for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5 and to the mass of neutron.

While for:

((((((12 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))])))))^1/12

Input:

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}}$$

Exact result:

$$\sqrt[6]{2}\sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}$$

Decimal approximation:

1.6031492287561228828777099998891227628501759679886734670 14...

Property:

$$\sqrt[6]{2}\sqrt[12]{\frac{3}{\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}}}$$ is a transcendental number

Alternate form:

$$\sqrt[6]{2}\sqrt[12]{\frac{3\left(e^{2\pi}-1\right)\left(1+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}\right)}{68711380+68711381\,e^{2\pi}+68719574\,e^{4\pi}+1602518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}}}$$

Continued fraction:

[1; 1, 1, 1, 1, 12, 9, 1, 5, 1, 2, 82, 4, 3, 1, 2, 2, 5, 1, 3, 1, 1, 2, 1, 3, 1, 19, 1, 1, 6, 1, ...]

All 12th roots of 12/(1/(e^2 π)-1)+8192/(e^4 π)-1)+1594323/(e^6 π)-1)+67108864/(e^8 π)-1)):

- Polar form

$$\sqrt[6]{2}\sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\ e^{0}\approx 1.6031$$ (real, principal root)

$$\sqrt[6]{2}\sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\ e^{(i\pi)/6}\approx 1.3884+0.8016\,i$$

$$\sqrt[6]{2}\sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\ e^{(i\pi)/3}\approx 0.8016+1.3884\,i$$

$$\sqrt[6]{2}\ \sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\ e^{(i\pi)/2} \approx 1.6031\, i$$

$$\sqrt[6]{2}\ \sqrt[12]{\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}}\ e^{(2i\pi)/3} \approx -0.8016 + 1.3884\, i$$

Alternative representations:

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[12]{\frac{12}{\frac{1^{13}}{-1+e^{360^\circ}}+\frac{2^{13}}{-1+e^{720^\circ}}+\frac{3^{13}}{-1+e^{1080^\circ}}+\frac{4^{13}}{-1+e^{1440^\circ}}}}$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[12]{\frac{12}{\frac{1^{13}}{\exp^{2\pi}(z)-1}+\frac{2^{13}}{\exp^{4\pi}(z)-1}+\frac{3^{13}}{\exp^{6\pi}(z)-1}+\frac{4^{13}}{\exp^{8\pi}(z)-1}}} \quad \text{for } z = 1$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[12]{\frac{12}{\frac{4^{13}}{-1+e^{-8i\log(-1)}}+\frac{3^{13}}{-1+e^{-6i\log(-1)}}+\frac{2^{13}}{-1+e^{-4i\log(-1)}}+\frac{1^{13}}{-1+e^{-2i\log(-1)}}}}$$

Series representations:

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\ \sqrt[12]{3}\ \sqrt[12]{\frac{1}{\frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}}+\frac{1594323}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}}}}$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\,\sqrt[12]{3}\left(1\Big/\left(\frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{8192}{-1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{1594323}{-1+e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{67108864}{-1+e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}\right)\right)\text{^}(1/12)$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\,\sqrt[12]{3}\,\sqrt[12]{\frac{1}{\frac{1}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}}+\frac{8192}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}}+\frac{1594323}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}}+\frac{67108864}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}}}}$$

Integral representations:

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\,\sqrt[12]{3}\,\sqrt[12]{\frac{1}{\frac{1}{-1+e^{4\int_0^{\infty}\sin(t)/t\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}\sin(t)/t\,dt}}+\frac{1594323}{-1+e^{12\int_0^{\infty}\sin(t)/t\,dt}}+\frac{67108864}{-1+e^{16\int_0^{\infty}\sin(t)/t\,dt}}}}$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\,\sqrt[12]{3}\,\sqrt[12]{\frac{1}{\frac{1}{-1+e^{4\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{1594323}{-1+e^{12\int_0^{\infty}1/(1+t^2)\,dt}}+\frac{67108864}{-1+e^{16\int_0^{\infty}1/(1+t^2)\,dt}}}}$$

$$\sqrt[12]{\frac{12}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}} = \sqrt[6]{2}\,\sqrt[12]{3}\left(1\Big/\left(\frac{1}{-1+e^{4\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{8192}{-1+e^{8\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{1594323}{-1+e^{12\int_0^{\infty}\sin^2(t)/t^2\,dt}}+\frac{67108864}{-1+e^{16\int_0^{\infty}\sin^2(t)/t^2\,dt}}\right)\right)\text{^}(1/12)$$

The value 1,6031492 is practically equal to the electric charge of the positron.

[2((sqrt(5)+1)/2)^2)] * 1 / [1^13/(e^(2Pi) - 1))+2^13/(e^(4Pi) - 1))+3^13/(e^(6Pi) - 1))+4^13/(e^(8Pi) - 1))]

Input:

$$\left(2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\right)\times\frac{1}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}$$

Exact result:

$$\frac{\left(1+\sqrt{5}\right)^2}{2\left(\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}\right)}$$

Decimal approximation:

125.75099651159985724606226226299791847944358236747596214 74...

Property:

$$\frac{\left(1+\sqrt{5}\right)^2}{2\left(\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{3+\sqrt{5}}{\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}+\frac{1}{2}\left(\coth(\pi)-1\right)}$$

$$\frac{\left(1+\sqrt{5}\right)^2\left(e^{\pi}-1\right)\left(1+e^{\pi}\right)\left(1+e^{2\pi}\right)\left(1-e^{\pi}+e^{2\pi}\right)\left(1+e^{\pi}+e^{2\pi}\right)\left(1+e^{4\pi}\right)}{2\left(68711380+68711381\,e^{2\pi}+68719574\,e^{4\pi}+1602518\,e^{6\pi}+8194\,e^{8\pi}+e^{10\pi}\right)}$$

$$\frac{3}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}+\frac{\sqrt{5}}{\frac{1}{e^{2\pi}-1}+\frac{8192}{e^{4\pi}-1}+\frac{1594323}{e^{6\pi}-1}+\frac{67108864}{e^{8\pi}-1}}$$

Continued fraction:

[125; 1, 3, 62, 2, 7, 1, 1, 25, 75, 3, 9, 22, 2, 2, 1, 2, 3, 5, 12, 2, 5, 1, 248, 2, 2, 9, 1, ...]

Series representations:

$$\frac{2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}}=\frac{\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}{2\left(\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}\right)}$$

$$\frac{2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}{2\left(\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}\right)}$$

$$\frac{2\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2}{\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\frac{4^{13}}{e^{8\pi}-1}} = \frac{\left(1+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2}{2\left(\frac{1}{-1+e^{2\pi}}+\frac{8192}{-1+e^{4\pi}}+\frac{1594323}{-1+e^{6\pi}}+\frac{67108864}{-1+e^{8\pi}}\right)}$$

We note that the result 125.7509 is practically equal to the value of Higgs boson's mass 125,09.

Now, we have from (9):

[1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

Input:

$$\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}$$

- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.1310891157889232571228126271483108007891569052600355083 75...

Property:

$$\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}$$
is a transcendental number

Alternate forms:

$$\frac{1}{140\,710\,042\,265\,625}\left(140\,710\,042\,265\,625\ \text{sech}\left(\frac{\sqrt{3}\ \pi}{2}\right) - 64\,339\,296\,875\ \text{sech}\left(\frac{3\sqrt{3}\ \pi}{2}\right) + 1\,801\,088\,541\ \text{sech}\left(\frac{5\sqrt{3}\ \pi}{2}\right) - 170\,859\,375\ \text{sech}\left(\frac{7\sqrt{3}\ \pi}{2}\right)\right)$$

$$\frac{2\cosh\left(\frac{\sqrt{3}\ \pi}{2}\right)}{1+\cosh\left(\sqrt{3}\ \pi\right)} - \frac{2\cosh\left(\frac{3\sqrt{3}\ \pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\ \pi\right)\right)} + \frac{2\cosh\left(\frac{5\sqrt{3}\ \pi}{2}\right)}{78\,125\left(1+\cosh\left(5\sqrt{3}\ \pi\right)\right)} - \frac{2\cosh\left(\frac{7\sqrt{3}\ \pi}{2}\right)}{823\,543\left(1+\cosh\left(7\sqrt{3}\ \pi\right)\right)}$$

$$\frac{2}{e^{-\left(\sqrt{3}\ \pi\right)/2}+e^{\left(\sqrt{3}\ \pi\right)/2}} - \frac{2}{2187\left(e^{-\left(3\sqrt{3}\ \pi\right)/2}+e^{\left(3\sqrt{3}\ \pi\right)/2}\right)} + \frac{2}{78\,125\left(e^{-\left(5\sqrt{3}\ \pi\right)/2}+e^{\left(5\sqrt{3}\ \pi\right)/2}\right)} - \frac{2}{823\,543\left(e^{-\left(7\sqrt{3}\ \pi\right)/2}+e^{\left(7\sqrt{3}\ \pi\right)/2}\right)}$$

Continued fraction:

[0; 7, 1, 1, 1, 2, 4, 4, 2, 22, 2, 1, 4, 30, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 59, 1, 17, 12, 3, 8, ...]

Alternative representations:

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} = \frac{1}{\cos\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7} - \frac{1}{\cos\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7} + \frac{1}{\cos\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7} - \frac{1}{\cos\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}$$

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} = \frac{1}{\cos\left(-\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7} - \frac{1}{\cos\left(-\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7} + \frac{1}{\cos\left(-\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7} - \frac{1}{\cos\left(-\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}$$

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} = \frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)}} - \frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)}} + \frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)}} - \frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)}}$$

Series representations:

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}= \sum_{k=0}^{\infty}\left(\frac{(-1)^k(1+2k)}{(1+k+k^2)\pi}-\frac{(-1)^k(1+2k)}{2187(7+k+k^2)\pi}+\frac{(-1)^k(1+2k)}{78125(19+k+k^2)\pi}-\frac{(-1)^k(1+2k)}{823543(37+k+k^2)\pi}\right)$$

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}= \sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823543}+\frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78125}-\frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187}+2(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)$$

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}= \sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-\left(7\sqrt{3}\pi\right)/2-7\sqrt{3}k\pi}}{823543}+\frac{2(-1)^k e^{-\left(5\sqrt{3}\pi\right)/2-5\sqrt{3}k\pi}}{78125}-\frac{2(-1)^k e^{-\left(3\sqrt{3}\pi\right)/2-3\sqrt{3}k\pi}}{2187}+2(-1)^k e^{-\left(\sqrt{3}\pi\right)/2-\sqrt{3}k\pi}\right)$$

Integral representation:

$$\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}= \int_0^{\infty}\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2t^{5i\sqrt{3}}}{78125\pi(1+t^2)}-\frac{2t^{7i\sqrt{3}}}{823543\pi(1+t^2)}\right)dt$$

Pi^7/960 /[1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

Input:

$$\frac{\frac{\pi^7}{960}}{\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}}$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{\pi^7}{960\left(\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}\right)}$$

- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

24.000000000000001775228456699385260372365264924853705687 07...

Alternate forms:

$$(9\,380\,669\,484\,375\,\pi^7)\Big/\left(64\left(140\,710\,042\,265\,625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64\,339\,296\,875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1\,801\,088\,541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170\,859\,375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)$$

$$-\left((9\,380\,669\,484\,375\,\pi^7)\Big/\left(64\left(-140\,710\,042\,265\,625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)+64\,339\,296\,875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)-1\,801\,088\,541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)+170\,859\,375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)\right)$$

$$\frac{\pi^7}{960\left(\frac{2\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh\left(\sqrt{3}\,\pi\right)}-\frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\,\pi\right)\right)}+\frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125\left(1+\cosh\left(5\sqrt{3}\,\pi\right)\right)}-\frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543\left(1+\cosh\left(7\sqrt{3}\,\pi\right)\right)}\right)}$$

Alternative representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{\pi^7}{960\left(\frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} = \frac{\pi^7}{960\left(\frac{1}{\cos\left(-\frac{1}{2} i \pi \sqrt{3}\right)1^7} - \frac{1}{\cos\left(-\frac{3}{2} i \pi \sqrt{3}\right)3^7} + \frac{1}{\cos\left(-\frac{5}{2} i \pi \sqrt{3}\right)5^7} - \frac{1}{\cos\left(-\frac{7}{2} i \pi \sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} = \frac{\pi^7}{960\left(\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2} i \pi \sqrt{3}\right)}} - \frac{1}{\frac{3^7}{\sec\left(\frac{3}{2} i \pi \sqrt{3}\right)}} + \frac{1}{\frac{5^7}{\sec\left(\frac{5}{2} i \pi \sqrt{3}\right)}} - \frac{1}{\frac{7^7}{\sec\left(\frac{7}{2} i \pi \sqrt{3}\right)}}\right)}$$

Series representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} = \frac{9\,380\,669\,484\,375\,\pi^6}{64 \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)\left(\frac{140\,710\,042\,265\,625}{1+k+k^2} - \frac{64\,339\,296\,875}{7+k+k^2} + \frac{4374\left(14\,493\,358+372\,709\,k+372\,709\,k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)}{\pi^2}}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} = \left(9\,380\,669\,484\,375\,\pi^6\right) \Bigg/ \left(64 \sum_{k=0}^{\infty} \left(2187\left(\frac{823\,543\,(-1)^k\,(1+2k)}{\left(19+k+k^2\right)\pi^2} - \frac{78\,125\,(-1)^k\,(1+2k)}{\left(37+k+k^2\right)\pi^2}\right) + \frac{140\,710\,042\,265\,625\,(-1)^k\,(1+2k)}{\left(1+k+k^2\right)\pi^2} - \frac{64\,339\,296\,875\,(-1)^k\,(1+2k)}{\left(7+k+k^2\right)\pi^2}\right)\right)$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\pi^7\Bigg/\left(960\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823543}+\frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78125}-\frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187}+2(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)\right)$$

Integral representation:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{\pi^7}{960\int_0^\infty\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2t^{5i\sqrt{3}}}{78125\pi(1+t^2)}-\frac{2t^{7i\sqrt{3}}}{823543\pi(1+t^2)}\right)dt}$$

The result 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

(3Pi^7/40) / [1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

Input:

$$\frac{3\times\frac{\pi^7}{40}}{\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}}$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{3\pi^7}{40\left(\operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\pi}{2}\right)}{823543}\right)}$$

- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1728.000000000001278164488823557387468102990745894668094 69...

Alternate forms:

$$(84426025359375\,\pi^7)\Big/\left(8\left(140710042265625\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64339296875\,\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1801088541\,\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170859375\,\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)$$

$$-\left((84426025359375\,\pi^7)\Big/\left(8\left(-140710042265625\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)+64339296875\,\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)-1801088541\,\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)+170859375\,\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)\right)$$

$$\frac{3\pi^7}{40\left(\frac{2\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh\left(\sqrt{3}\,\pi\right)}-\frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\,\pi\right)\right)}+\frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125\left(1+\cosh\left(5\sqrt{3}\,\pi\right)\right)}-\frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543\left(1+\cosh\left(7\sqrt{3}\,\pi\right)\right)}\right)}$$

Alternative representations:

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\frac{3\pi^7}{40\left(\frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\frac{3\pi^7}{40\left(\frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\frac{3\pi^7}{40\left(\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}}\right)}$$

Series representations:

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\frac{84426025359375\,\pi^6}{8\sum_{k=0}^{\infty}\frac{(-1)^k(1+2k)\left(\frac{140710042265625}{1+k+k^2}-\frac{64339296875}{7+k+k^2}+\frac{4374\left(14493358+372709\,k+372709\,k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)}{\pi^2}}$$

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\left(84426025359375\,\pi^6\right)\Big/\left(8\sum_{k=0}^{\infty}\left(2187\left(\frac{823543(-1)^k(1+2k)}{\left(19+k+k^2\right)\pi^2}-\frac{78125(-1)^k(1+2k)}{\left(37+k+k^2\right)\pi^2}\right)+\frac{140710042265625(-1)^k(1+2k)}{\left(1+k+k^2\right)\pi^2}-\frac{64339296875(-1)^k(1+2k)}{\left(7+k+k^2\right)\pi^2}\right)\right)$$

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40}=\left(3\pi^7\right)\Big/\left(40\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823543}+\frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78125}-\frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187}+2(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)\right)$$

Integral representation:

$$\frac{3\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)40} = \frac{3\pi^7}{40\int_0^\infty\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2t^{5i\sqrt{3}}}{78125\pi(1+t^2)}-\frac{2t^{7i\sqrt{3}}}{823543\pi(1+t^2)}\right)dt}$$

The result 1728, is very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5).

(Pi^7/1920) / [1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

Input:

$$\frac{\frac{\pi^7}{1920}}{\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}}$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{\pi^7}{1920\left(\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}\right)}$$

- $\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

12.00000000000000088761422834969263018618263246242685284353...

Alternate forms:

$$(9380669484375\pi^7)\Big/\left(128\left(140710042265625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64339296875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1801088541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170859375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)$$

$$-\left((9380669484375\pi^7)\Big/\left(128\left(-140710042265625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)+64339296875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)-1801088541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)+170859375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)\right)$$

$$\frac{\pi^7}{1920\left(\frac{2\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh\left(\sqrt{3}\,\pi\right)} - \frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\,\pi\right)\right)} + \frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125\left(1+\cosh\left(5\sqrt{3}\,\pi\right)\right)} - \frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543\left(1+\cosh\left(7\sqrt{3}\,\pi\right)\right)}\right)}$$

Alternative representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920} = \frac{\pi^7}{1920\left(\frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)1^7} - \frac{1}{\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)3^7} + \frac{1}{\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)5^7} - \frac{1}{\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920} = \frac{\pi^7}{1920\left(\frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)1^7} - \frac{1}{\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)3^7} + \frac{1}{\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)5^7} - \frac{1}{\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920} = \frac{\pi^7}{1920\left(\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)}} - \frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)}} + \frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)}} - \frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}}\right)}$$

Series representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920} = \frac{9\,380\,669\,484\,375\,\pi^6}{128\sum_{k=0}^{\infty}\frac{(-1)^k(1+2k)\left(\frac{140\,710\,042\,265\,625}{1+k+k^2} - \frac{64\,339\,296\,875}{7+k+k^2} + \frac{4374\left(14\,493\,358+372\,709\,k+372\,709\,k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)}{\pi^2}}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920}=$$

$$\left(9\,380\,669\,484\,375\,\pi^6\right)\Big/\left(128\sum_{k=0}^{\infty}\left(2187\left(\frac{823\,543\,(-1)^k\,(1+2k)}{(19+k+k^2)\,\pi^2}-\frac{78\,125\,(-1)^k\,(1+2k)}{(37+k+k^2)\,\pi^2}\right)+\frac{140\,710\,042\,265\,625\,(-1)^k\,(1+2k)}{(1+k+k^2)\,\pi^2}-\frac{64\,339\,296\,875\,(-1)^k\,(1+2k)}{(7+k+k^2)\,\pi^2}\right)\right)$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920}=$$

$$\pi^7\Big/\left(1920\sum_{k=0}^{\infty}\left(-\frac{2\,(-1)^k\,e^{-7/2\sqrt{3}\,(1+2k)\pi}}{823\,543}+\frac{2\,(-1)^k\,e^{-5/2\sqrt{3}\,(1+2k)\pi}}{78\,125}-\frac{2\,(-1)^k\,e^{-3/2\sqrt{3}\,(1+2k)\pi}}{2187}+2\,(-1)^k\,e^{-1/2\sqrt{3}\,(1+2k)\pi}\right)\right)$$

Integral representation:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)1920}=$$

$$\frac{\pi^7}{1920\int_0^{\infty}\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2t^{5i\sqrt{3}}}{78\,125\,\pi(1+t^2)}-\frac{2t^{7i\sqrt{3}}}{823\,543\,\pi(1+t^2)}\right)dt}$$

The result, about 12, is a good approximation to the value of black hole entropy 12,19.

(Pi^7/30) / [1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2)))
+1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

$$\frac{\frac{\pi^7}{30}}{\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}}$$

Exact result:

$$\frac{\pi^7}{30\left(\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}\right)}$$

Decimal approximation:

768.00000000000005680731061438032833191568847759531858l9864...

Alternate forms:

$$(9\,380\,669\,484\,375\,\pi^7)\Big/\left(2\left(140\,710\,042\,265\,625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64\,339\,296\,875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1\,801\,088\,541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170\,859\,375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)$$

$$-\left((9\,380\,669\,484\,375\,\pi^7)\Big/\left(2\left(-140\,710\,042\,265\,625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)+64\,339\,296\,875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)-1\,801\,088\,541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)+170\,859\,375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\right)\right)$$

$$\frac{\pi^7}{30\left(\frac{2\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh\left(\sqrt{3}\,\pi\right)}-\frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\,\pi\right)\right)}+\frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125\left(1+\cosh\left(5\sqrt{3}\,\pi\right)\right)}-\frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543\left(1+\cosh\left(7\sqrt{3}\,\pi\right)\right)}\right)}$$

Alternative representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30}=$$

$$\frac{\pi^7}{30\left(\frac{1}{\cos\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30}=$$

$$\frac{\pi^7}{30\left(\frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30}=$$

$$\frac{\pi^7}{30\left(\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}}\right)}$$

Series representations:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30}=$$

$$\frac{9\,380\,669\,484\,375\,\pi^6}{\sum_{k=0}^{\infty}\frac{2(-1)^k(1+2k)\left(\frac{140\,710\,042\,265\,625}{1+k+k^2}-\frac{64\,339\,296\,875}{7+k+k^2}+\frac{4374\left(14\,493\,358+372\,709\,k+372\,709\,k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)}{\pi^2}}$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30}=$$

$$\left(9\,380\,669\,484\,375\,\pi^6\right)\Big/\left(\sum_{k=0}^{\infty}\left(4374\left(\frac{823\,543\,(-1)^k\,(1+2k)}{\left(19+k+k^2\right)\pi^2}-\frac{78\,125\,(-1)^k\,(1+2k)}{\left(37+k+k^2\right)\pi^2}\right)+\frac{281\,420\,084\,531\,250\,(-1)^k\,(1+2k)}{\left(1+k+k^2\right)\pi^2}-\frac{128\,678\,593\,750\,(-1)^k\,(1+2k)}{\left(7+k+k^2\right)\pi^2}\right)\right)$$

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30} = \pi^7 \Big/ \left(30\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823543} + \frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78125} - \frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187} + 2(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)\right)$$

Integral representation:

$$\frac{\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)30} = \frac{\pi^7}{30\int_0^\infty\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)} - \frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)} + \frac{2t^{5i\sqrt{3}}}{78125\pi(1+t^2)} - \frac{2t^{7i\sqrt{3}}}{823543\pi(1+t^2)}\right)dt}$$

The result 768 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for $m_q = 4.7\ MeV/c^2 = 0.0047\ GeV/c^2$, that is the mass of quark down is $4.8 \pm 0.5 \pm 0.3 = 4.7\ MeV/c^2$.

[(2*2.61803398*Pi^7)/960] / [1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]

where 2,61803398 is the square of the golden ratio 1,61803398...

$$\frac{\frac{1}{960}(2\times 2.61803398\,\pi^7)}{\frac{1}{1^7\cosh\left(\frac{1}{2}(\pi\sqrt{3})\right)} - \frac{1}{3^7\cosh\left(\frac{1}{2}(3\pi\sqrt{3})\right)} + \frac{1}{5^7\cosh\left(\frac{1}{2}(5\pi\sqrt{3})\right)} - \frac{1}{7^7\cosh\left(\frac{1}{2}(7\pi\sqrt{3})\right)}}$$

Result:

125.665631...

Alternative representations:

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{5.23607\,\pi^7}{960\left(\frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{5.23607\,\pi^7}{960\left(\frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)7^7}\right)}$$

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{5.23607\,\pi^7}{960\left(\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}}\right)}$$

Series representations:

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$(0.00545424\,\pi^7)\Bigg/\left(-\frac{1}{2187\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}+\frac{1}{\sum_{k=0}^{\infty}\frac{4^{-k}\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}+\frac{1}{78\,125\sum_{k=0}^{\infty}\frac{\left(\frac{25}{4}\right)^k\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}-\frac{1}{823\,543\sum_{k=0}^{\infty}\frac{\left(\frac{49}{4}\right)^k\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}\right)$$

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$(0.00545424\,\pi^7)\Big/\left(-\frac{1}{2187\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}\right)^{2k}\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}+\frac{1}{\sum_{k=0}^{\infty}\frac{2^{-2k}\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}+\frac{1}{78\,125\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}\right)^{2k}\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}-\frac{1}{823\,543\sum_{k=0}^{\infty}\frac{\left(\frac{7}{2}\right)^{2k}\left(\pi\sqrt{3}\right)^{2k}}{(2k)!}}\right)$$

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$(0.00545424\,\pi^7)\Big/\left(\frac{1}{i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{\pi\sqrt{3}}{2}\right)^{1+2k}}{(1+2k)!}}-\frac{1}{2187\,i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\pi\sqrt{3}}{2}\right)^{1+2k}}{(1+2k)!}}+\frac{1}{78\,125\,i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{5\pi\sqrt{3}}{2}\right)^{1+2k}}{(1+2k)!}}-\frac{1}{823\,543\,i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{7\pi\sqrt{3}}{2}\right)^{1+2k}}{(1+2k)!}}\right)$$

Integral representations:

$$\frac{2\times 2.61803\,\pi^7}{\left(\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5\pi\sqrt{3}}{2}\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960}=$$

$$\frac{0.00545424\,\pi^7}{\frac{1}{\int_{\frac{i\pi}{2}}^{\frac{\pi\sqrt{3}}{2}}\sinh(t)\,dt}-\frac{1}{2187\int_{\frac{i\pi}{2}}^{\frac{3\pi\sqrt{3}}{2}}\sinh(t)\,dt}+\frac{1}{78\,125\int_{\frac{i\pi}{2}}^{\frac{5\pi\sqrt{3}}{2}}\sinh(t)\,dt}-\frac{1}{823\,543\int_{\frac{i\pi}{2}}^{\frac{7\pi\sqrt{3}}{2}}\sinh(t)\,dt}}$$

$$\frac{2 \times 2.61803\,\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} =$$

$$(0.00545424\,\pi^7) \Big/ \left(\frac{1}{1+\frac{\pi\sqrt{3}}{2}\int_0^1 \sinh\left(\frac{1}{2}\pi t\sqrt{3}\right)dt} - \frac{1}{2187\left(1+\frac{3\pi\sqrt{3}}{2}\int_0^1 \sinh\left(\frac{3}{2}\pi t\sqrt{3}\right)dt\right)} + \frac{1}{78\,125\left(1+\frac{5\pi\sqrt{3}}{2}\int_0^1 \sinh\left(\frac{5}{2}\pi t\sqrt{3}\right)dt\right)} - \frac{1}{823\,543\left(1+\frac{7\pi\sqrt{3}}{2}\int_0^1 \sinh\left(\frac{7}{2}\pi t\sqrt{3}\right)dt\right)}\right)$$

$$\frac{2 \times 2.61803\,\pi^7}{\left(\frac{1}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5\pi\sqrt{3}}{2}\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}\right)960} =$$

$$(0.00545424\,\pi^7) \Big/ \left(\frac{2 i \pi}{\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s+(\pi^2\sqrt{3}^2)/(16 s)}}{\sqrt{s}}\,ds} - \frac{2 i \pi}{2187\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s+(9\pi^2\sqrt{3}^2)/(16 s)}}{\sqrt{s}}\,ds} + \frac{2 i \pi}{78\,125\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s+(25\pi^2\sqrt{3}^2)/(16 s)}}{\sqrt{s}}\,ds} - \frac{2 i \pi}{823\,543\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s+(49\pi^2\sqrt{3}^2)/(16 s)}}{\sqrt{s}}\,ds}\right) \quad \text{for } \gamma > 0$$

We note that the result 125,665 is practically equal to the value of Higgs boson's mass 125,09.

[[[[sqrt[sqrt(1/[1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2)))
+1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2))]]]]]

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}}}}$$

Exact result:

$$\frac{1}{\sqrt[4]{\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}}}$$

Decimal approximation:

1.661913216004292392825052107092679325354169411563268828448...

Property:

$$\frac{1}{\sqrt[4]{\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}}}$$ is a transcendental number

Alternate forms:

$$\left(105\times 105^{3/4}\right)\Big/\left(\left(140\,710\,042\,265\,625\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64\,339\,296\,875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1\,801\,088\,541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170\,859\,375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\text{^}(1/4)\right)$$

$$\frac{1}{\sqrt[4]{\frac{2\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh\left(\sqrt{3}\,\pi\right)}-\frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\left(1+\cosh\left(3\sqrt{3}\,\pi\right)\right)}+\frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125\left(1+\cosh\left(5\sqrt{3}\,\pi\right)\right)}-\frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543\left(1+\cosh\left(7\sqrt{3}\,\pi\right)\right)}}}$$

$$1\Big/\left(\left(\frac{2}{e^{-\left(\sqrt{3}\,\pi\right)/2}+e^{\left(\sqrt{3}\,\pi\right)/2}}-\frac{2}{2187\left(e^{-\left(3\sqrt{3}\,\pi\right)/2}+e^{\left(3\sqrt{3}\,\pi\right)/2}\right)}+\frac{2}{78\,125\left(e^{-\left(5\sqrt{3}\,\pi\right)/2}+e^{\left(5\sqrt{3}\,\pi\right)/2}\right)}-\frac{2}{823\,543\left(e^{-\left(7\sqrt{3}\,\pi\right)/2}+e^{\left(7\sqrt{3}\,\pi\right)/2}\right)}\right)\text{^}(1/4)\right)$$

All 2nd roots of 1/sqrt(sech((sqrt(3) π)/2)-sech((3 sqrt(3) π)/2)/2187+sech((5 sqrt(3) π)/2)/78125-sech((7 sqrt(3) π)/2)/823543):

- Polar form

$$\frac{e^0}{\sqrt[4]{\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}}} \approx 1.6619 \text{ (real, principal root)}$$

$$\frac{e^{i\pi}}{\sqrt[4]{\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}}} \approx -1.6619 \text{ (real root)}$$

Alternative representations:

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} = \sqrt{\sqrt{\frac{1}{\frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)7^7}}}}$$

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} = \sqrt{\sqrt{\frac{1}{\frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)7^7}}}}$$

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} = \sqrt{\sqrt{\frac{1}{\frac{1}{\frac{1^7}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}}}}}$$

Series representations:

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} =$$

$$\left(105\times 105^{3/4}\right)\Big/\left(\sqrt[4]{\pi}\left(\sum_{k=0}^{\infty}\frac{1}{\pi^2}(-1)^k\,(1+2k)\left(\frac{140\,710\,042\,265\,625}{1+k+k^2}-\frac{64\,339\,296\,875}{7+k+k^2}+\frac{4374\,(14\,493\,358+372\,709\,k+372\,709\,k^2)}{(19+k+k^2)(37+k+k^2)}\right)\right)\verb|^|(1/4)\right)$$

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} =$$

$$1\Big/\left(\left(\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}\,(1+2k)\pi}}{823\,543}+\frac{2(-1)^k e^{-5/2\sqrt{3}\,(1+2k)\pi}}{78\,125}-\frac{2(-1)^k e^{-3/2\sqrt{3}\,(1+2k)\pi}}{2187}+2(-1)^k e^{-1/2\sqrt{3}\,(1+2k)\pi}\right)\right)\verb|^|(1/4)\right)$$

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} =$$

$$1\Big/\left(\left(\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-\left(7\sqrt{3}\,\pi\right)/2-7\sqrt{3}\,k\pi}}{823\,543}+\frac{2(-1)^k e^{-\left(5\sqrt{3}\,\pi\right)/2-5\sqrt{3}\,k\pi}}{78\,125}-\frac{2(-1)^k e^{-\left(3\sqrt{3}\,\pi\right)/2-3\sqrt{3}\,k\pi}}{2187}+2(-1)^k e^{-\left(\sqrt{3}\,\pi\right)/2-\sqrt{3}\,k\pi}\right)\right)\verb|^|(1/4)\right)$$

Integral representation:

$$\sqrt{\sqrt{\frac{1}{\frac{1}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}}} = \frac{1}{\sqrt[4]{\int_0^{\infty}\left(\frac{2t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2t^{5i\sqrt{3}}}{78\,125\,\pi(1+t^2)}-\frac{2t^{7i\sqrt{3}}}{823\,543\,\pi(1+t^2)}\right)dt}}$$

The result is 1,66191 value very near to the fourteenth root of Ramanujan's class invariant 1164.2696, to the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$ 1,6557 for the

D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5 and a good approximation to the mass of the proton.

((((((((([(23040 * 1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2))) +1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2)]))))))))))))^1/7

$$\left(23040\times\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}\right)\text{^}(1/7)$$

Exact result:

$$\sqrt[7]{23040\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

Decimal approximation:

3.141593546567757984777156050877551777374023642098884676675...

Property:

$$\sqrt[7]{23040\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$ is a transcendental number

Alternate forms:

$$\frac{1}{105}\left(3241959373800000000\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-64339296875\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)+1801088541\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)-170859375\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\text{^}(1/7)$$

$$\left(\frac{46080\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh(\sqrt{3}\,\pi)}-\frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\,(1+\cosh(3\sqrt{3}\,\pi))}+\frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125\,(1+\cosh(5\sqrt{3}\,\pi))}-\frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543\,(1+\cosh(7\sqrt{3}\,\pi))}\right)\text{^}(1/7)$$

$$\left(\frac{46080}{e^{-(\sqrt{3}\,\pi)/2}+e^{(\sqrt{3}\,\pi)/2}}-\frac{2}{2187\left(e^{-(3\sqrt{3}\,\pi)/2}+e^{(3\sqrt{3}\,\pi)/2}\right)}+\frac{2}{78125\left(e^{-(5\sqrt{3}\,\pi)/2}+e^{(5\sqrt{3}\,\pi)/2}\right)}-\frac{2}{823543\left(e^{-(7\sqrt{3}\,\pi)/2}+e^{(7\sqrt{3}\,\pi)/2}\right)}\right)\text{^}(1/7)$$

All 7th roots of 23040 sech((sqrt(3) π)/2)-sech((3 sqrt(3) π)/2)/2187+sech((5 sqrt(3) π)/2)/78125-sech((7 sqrt(3) π)/2)/823543:

- Polar form

$$e^{0}\sqrt[7]{23040\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}\approx 3.1416$$

(real, principal root)

$$e^{(2i\pi)/7}\sqrt[7]{23040\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

$\approx 1.9588+2.4562\,i$

$$e^{(4i\pi)/7}\sqrt[7]{23040\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

$\approx -0.6991+3.0628\,i$

$$e^{(6i\pi)/7}\sqrt[7]{23040\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

$\approx -2.8305+1.3631\,i$

$$e^{-(6i\pi)/7}\sqrt[7]{23040\,\text{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\text{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\text{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\text{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

$\approx -2.8305-1.3631\,i$

Alternative representations:

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} = \sqrt[7]{\frac{23040}{\cos\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}}$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} = \sqrt[7]{\frac{23040}{\cos\left(-\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}}$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} = \sqrt[7]{\frac{23040}{\frac{1^7}{\sec\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)}}}$$

Series representations:

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} = \frac{1}{105}\sqrt[7]{\pi}\left(\sum_{k=0}^{\infty}\frac{1}{\pi^2}(-1)^k(1+2k)\left(\frac{3241959373800000000}{1+k+k^2}-\frac{64339296875}{7+k+k^2}+\frac{4374\left(14493358+372709k+372709k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)\right)^\wedge(1/7)$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} = \left(\sum_{k=0}^{\infty}\left(\frac{23040(-1)^k(1+2k)}{\left(1+k+k^2\right)\pi}-\frac{(-1)^k(1+2k)}{2187\left(7+k+k^2\right)\pi}+\frac{(-1)^k(1+2k)}{78125\left(19+k+k^2\right)\pi}-\frac{(-1)^k(1+2k)}{823543\left(37+k+k^2\right)\pi}\right)\right)^\wedge(1/7)$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} =$$

$$\left(\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823543}+\frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78125}-\frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187}+46080(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)\right)^\wedge(1/7)$$

Integral representation:

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}} =$$

$$\sqrt[7]{\int_0^\infty\left(\frac{46080\,t^{i\sqrt{3}}}{\pi(1+t^2)}-\frac{2\,t^{3i\sqrt{3}}}{2187\pi(1+t^2)}+\frac{2\,t^{5i\sqrt{3}}}{78125\pi(1+t^2)}-\frac{2\,t^{7i\sqrt{3}}}{823543\pi(1+t^2)}\right)dt}$$

This value is practically equal to π .

((((((((([(23040 * 1/(1^7cosh(((Pi*sqrt(3))/2)))-1/(3^7cosh(((3Pi*sqrt(3))/2)))
+1/(5^7cosh(((5Pi*sqrt(3))/2)))-1/(7^7cosh(((7Pi*sqrt(3))/2)]))))))))))))^1/7 * 2 *
275

Input:

$$\left(23040\times\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)}+\frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}\right)^\wedge(1/7)\times 2\times 275$$

- $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$550\sqrt[7]{23040\,\mathrm{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)-\frac{\mathrm{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187}+\frac{\mathrm{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78125}-\frac{\mathrm{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823543}}$$

- $\mathrm{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1727.876450612266891627435827982653477555713003154386572171...

Property:

$$550 \sqrt[7]{23\,040 \operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right) - \frac{\operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187} + \frac{\operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125} - \frac{\operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543}}$$

is a transcendental number

Alternate forms:

$$\frac{110}{21}\left(3\,241\,959\,373\,800\,000\,000 \operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right) - 64\,339\,296\,875 \operatorname{sech}\left(\frac{3\sqrt{3}\,\pi}{2}\right) + 1\,801\,088\,541 \operatorname{sech}\left(\frac{5\sqrt{3}\,\pi}{2}\right) - 170\,859\,375 \operatorname{sech}\left(\frac{7\sqrt{3}\,\pi}{2}\right)\right)\text{^}(1/7)$$

$$550\left(\frac{46\,080 \cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)}{1+\cosh(\sqrt{3}\,\pi)} - \frac{2\cosh\left(\frac{3\sqrt{3}\,\pi}{2}\right)}{2187\,(1+\cosh(3\sqrt{3}\,\pi))} + \frac{2\cosh\left(\frac{5\sqrt{3}\,\pi}{2}\right)}{78\,125\,(1+\cosh(5\sqrt{3}\,\pi))} - \frac{2\cosh\left(\frac{7\sqrt{3}\,\pi}{2}\right)}{823\,543\,(1+\cosh(7\sqrt{3}\,\pi))}\right)\text{^}(1/7)$$

$$550\left(\frac{46\,080}{e^{-(\sqrt{3}\,\pi)/2}+e^{(\sqrt{3}\,\pi)/2}} - \frac{2}{2187\left(e^{-(3\sqrt{3}\,\pi)/2}+e^{(3\sqrt{3}\,\pi)/2}\right)} + \frac{2}{78\,125\left(e^{-(5\sqrt{3}\,\pi)/2}+e^{(5\sqrt{3}\,\pi)/2}\right)} - \frac{2}{823\,543\left(e^{-(7\sqrt{3}\,\pi)/2}+e^{(7\sqrt{3}\,\pi)/2}\right)}\right)\text{^}(1/7)$$

Alternative representations:

$$\sqrt[7]{\frac{23\,040}{1^7 \cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7 \cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\; 2\times 275 = 550\sqrt[7]{\frac{23\,040}{\cos\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7} - \frac{1}{\cos\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7} + \frac{1}{\cos\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7} - \frac{1}{\cos\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}}$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\;2\times 275 =$$

$$550\left(\frac{23040}{\cos\left(-\frac{1}{2}\,i\,\pi\sqrt{3}\right)1^7}-\frac{1}{\cos\left(-\frac{3}{2}\,i\,\pi\sqrt{3}\right)3^7}+\frac{1}{\cos\left(-\frac{5}{2}\,i\,\pi\sqrt{3}\right)5^7}-\frac{1}{\cos\left(-\frac{7}{2}\,i\,\pi\sqrt{3}\right)7^7}\right)\hat{}\,(1/7)$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\;2\times 275 =$$

$$550\sqrt[7]{\frac{23040}{\frac{1^7}{\sec\left(\frac{1}{2}\,i\,\pi\sqrt{3}\right)}}-\frac{1}{\frac{3^7}{\sec\left(\frac{3}{2}\,i\,\pi\sqrt{3}\right)}}+\frac{1}{\frac{5^7}{\sec\left(\frac{5}{2}\,i\,\pi\sqrt{3}\right)}}-\frac{1}{\frac{7^7}{\sec\left(\frac{7}{2}\,i\,\pi\sqrt{3}\right)}}}$$

Series representations:

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\;2\times 275 =$$

$$\frac{110}{21}\sqrt[7]{\pi}\left(\sum_{k=0}^{\infty}\frac{1}{\pi^2}(-1)^k(1+2k)\left(\frac{3241959373800000000}{1+k+k^2}-\frac{64339296875}{7+k+k^2}+\frac{4374\left(14493358+372709k+372709k^2\right)}{\left(19+k+k^2\right)\left(37+k+k^2\right)}\right)\right)\hat{}\,(1/7)$$

$$\sqrt[7]{\frac{23040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)}-\frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)}+\frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)}-\frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\;2\times 275 =$$

$$550\left(\sum_{k=0}^{\infty}\left(\frac{23040(-1)^k(1+2k)}{\left(1+k+k^2\right)\pi}-\frac{(-1)^k(1+2k)}{2187\left(7+k+k^2\right)\pi}+\frac{(-1)^k(1+2k)}{78125\left(19+k+k^2\right)\pi}-\frac{(-1)^k(1+2k)}{823543\left(37+k+k^2\right)\pi}\right)\right)\hat{}\,(1/7)$$

$$\sqrt[7]{\frac{23\,040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\ 2\times 275 =$$

$$550\left(\sum_{k=0}^{\infty}\left(-\frac{2(-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}}{823\,543} + \frac{2(-1)^k e^{-5/2\sqrt{3}(1+2k)\pi}}{78\,125} - \frac{2(-1)^k e^{-3/2\sqrt{3}(1+2k)\pi}}{2187} + 46\,080(-1)^k e^{-1/2\sqrt{3}(1+2k)\pi}\right)\right)\wedge(1/7)$$

Integral representation:

$$\sqrt[7]{\frac{23\,040}{1^7\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3^7\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5^7\cosh\left(\frac{5}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)}}\ 2\times 275 =$$

$$550\sqrt[7]{\int_0^\infty\left(\frac{46\,080\,t^{i\sqrt{3}}}{\pi(1+t^2)} - \frac{2\,t^{3i\sqrt{3}}}{2187\,\pi(1+t^2)} + \frac{2\,t^{5i\sqrt{3}}}{78\,125\,\pi(1+t^2)} - \frac{2\,t^{7i\sqrt{3}}}{823\,543\,\pi(1+t^2)}\right)dt}$$

The result, 1727,87 is very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5). Furthermore, we note that:

$$\left(23\,040\times\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)} + \frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}\right)\wedge(1/7)\times 2\times 275$$

The value is thence, the length of a circle C = 2πr = 2π*275 = 1727,876... where π is equal to the precedent expression:

$$\left(23\,040\times\frac{1}{1^7\cosh\left(\frac{1}{2}\left(\pi\sqrt{3}\right)\right)} - \frac{1}{3^7\cosh\left(\frac{1}{2}\left(3\pi\sqrt{3}\right)\right)} + \frac{1}{5^7\cosh\left(\frac{1}{2}\left(5\pi\sqrt{3}\right)\right)} - \frac{1}{7^7\cosh\left(\frac{1}{2}\left(7\pi\sqrt{3}\right)\right)}\right)\wedge(1/7)$$

Now, we have from (5) that is equal to $\frac{19\pi^7}{55700} = 1{,}0302616$ or $\frac{19\pi^7}{56700} = 1{,}0120912\ldots$

[((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input:

$$\frac{\coth(\pi)}{1^7} + \frac{\coth(2)\pi}{2^7} + \frac{\coth(3)\pi}{3^7} + \frac{\coth(4)\pi}{4^7} + \frac{\coth(5)\pi}{5^7}$$

- $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$\frac{1}{128}\pi\coth(2) + \frac{\pi\coth(3)}{2187} + \frac{\pi\coth(4)}{16384} + \frac{\pi\coth(5)}{78125} + \coth(\pi)$$

Decimal approximation:

1.030877123223701704727529179789658441898399900664552699051...

that:

1/(3*1575) * (19Pi^7 / 1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input interpretation:

$$\frac{1}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7} + \frac{\coth(2)\pi}{2^7} + \frac{\coth(3)\pi}{3^7} + \frac{\coth(4)\pi}{4^7} + \frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

12.145094460901...

The result 12,145 is a very good approximation to the value of black hole entropy 12,19.

(142.28)/(3*1575) * (19Pi^7 /1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input interpretation:

$$\frac{142.28}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7} + \frac{\coth(2)\pi}{2^7} + \frac{\coth(3)\pi}{3^7} + \frac{\coth(4)\pi}{4^7} + \frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

1728.00...

or:

Input interpretation:

$$\frac{142.28}{4725}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

1728.00…

Alternative representations:

$$\frac{\left((19\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=\frac{2703.32\,\pi^7\left(\frac{\pi\left(1+\frac{2}{-1+e^4}\right)}{2^7}+\frac{\pi\left(1+\frac{2}{-1+e^6}\right)}{3^7}+\frac{\pi\left(1+\frac{2}{-1+e^8}\right)}{4^7}+\frac{\pi\left(1+\frac{2}{-1+e^{10}}\right)}{5^7}+\frac{1+\frac{2}{-1+e^{2\pi}}}{1^7}\right)}{1.03087712322370000\times 4725}$$

$$\frac{\left((19\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=\frac{2703.32\,\pi^7\left(\frac{i\cot(i\pi)}{1^7}+\frac{i\pi\cot(2i)}{2^7}+\frac{i\pi\cot(3i)}{3^7}+\frac{i\pi\cot(4i)}{4^7}+\frac{i\pi\cot(5i)}{5^7}\right)}{1.03087712322370000\times 4725}$$

$$\frac{\left((19\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=\frac{2703.32\,\pi^7\left(-\frac{i\cot(-i\pi)}{1^7}-\frac{i(\pi\cot(-2i))}{2^7}-\frac{i(\pi\cot(-3i))}{3^7}-\frac{i(\pi\cot(-4i))}{4^7}-\frac{i(\pi\cot(-5i))}{5^7}\right)}{1.03087712322370000\times 4725}$$

Series representations:

$$\frac{\left((19\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=0.554995\,\pi^6+0.00226243\,\pi^8+\sum_{k=1}^{\infty}\left(\frac{1.10999\,\pi^6}{1+k^2}+\pi^8\left(\frac{0.0173436}{4+k^2\pi^2}+\frac{0.00152262}{9+k^2\pi^2}+\frac{0.000270993}{16+k^2\pi^2}+\frac{0.0000710393}{25+k^2\pi^2}\right)\right)$$

$$\frac{\left((19\,\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\,\pi}{2^7}+\frac{\coth(3)\,\pi}{3^7}+\frac{\coth(4)\,\pi}{4^7}+\frac{\coth(5)\,\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=$$

$$\sum_{k=-\infty}^{\infty}\left(\frac{0.554995\,\pi^7}{\pi+k^2\,\pi}+\frac{0.00867179\,\pi^8}{4+k^2\,\pi^2}+\frac{0.000761309\,\pi^8}{9+k^2\,\pi^2}+\frac{0.000135497\,\pi^8}{16+k^2\,\pi^2}+\frac{0.0000355197\,\pi^8}{25+k^2\,\pi^2}\right)$$

Integral representation:

$$\frac{\left((19\pi^7)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)\right)142.28}{1.03087712322370000\times 4725}=$$

$$\int_{\frac{i\pi}{2}}^{2}\Bigg(-0.0043359\,\pi^8\,\text{csch}^2(t)-\frac{0.00025377\,\pi^8\left(3-\frac{i\pi}{2}\right)\text{csch}^2\left(\frac{\frac{i\pi}{2}-3t+\frac{i\pi t}{2}}{-2+\frac{i\pi}{2}}\right)}{2-\frac{i\pi}{2}}+$$

$$\frac{1}{2-\frac{i\pi}{2}}\left(\pi-\frac{i\pi}{2}\right)\Bigg(-0.554995\,\pi^7\,\text{csch}^2\left(\frac{-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}}{-2+\frac{i\pi}{2}}\right)+$$

$$\frac{1}{\pi-\frac{i\pi}{2}}\left(4-\frac{i\pi}{2}\right)\Bigg(-0.0000338742\,\pi^8$$

$$\text{csch}^2\left(\frac{2i\pi-\frac{i\pi^2}{2}-\frac{4\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{-2+\frac{i\pi}{2}}+\frac{i\pi\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{2\left(-2+\frac{i\pi}{2}\right)}}{-\pi+\frac{i\pi}{2}}\right)-$$

$$\frac{1}{4-\frac{i\pi}{2}}7.10393\times10^{-6}\,\pi^8\left(5-\frac{i\pi}{2}\right)\text{csch}^2\Bigg(\frac{1}{-4+\frac{i\pi}{2}}$$

$$\Bigg(\frac{i\pi}{2}-\frac{1}{-\pi+\frac{i\pi}{2}}5\left(2i\pi-\frac{i\pi^2}{2}-\frac{4\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{-2+\frac{i\pi}{2}}+\right.$$

$$\left.\frac{i\pi\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{2\left(-2+\frac{i\pi}{2}\right)}\right)+\frac{1}{2\left(-\pi+\frac{i\pi}{2}\right)}i$$

$$\pi\left(2i\pi-\frac{i\pi^2}{2}-\frac{4\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{-2+\frac{i\pi}{2}}+\right.$$

$$\left.\frac{i\pi\left(-i\pi+\frac{i\pi^2}{2}-\pi t+\frac{i\pi t}{2}\right)}{2\left(-2+\frac{i\pi}{2}\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)dt$$

The result, 1728 is very near to the value of the mass of the candidate “glueball” $f_0(1710)$ that is 1723 (+ 6 – 5).

1.976111/(3*1575) * (19Pi^7 /1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input interpretation:

$$\frac{1.976111}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

24.00005...

The result 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

0.988055/(3*1575) * (19Pi^7 /1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input interpretation:

$$\frac{0.988055}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

12.0000...

The result 12, is very near to 12,19 that is the value of the black hole entropy.

63.23554/(3*1575) * (19Pi^7 /1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]

Input interpretation:

$$\frac{63.23554}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\pi}{2^7}+\frac{\coth(3)\pi}{3^7}+\frac{\coth(4)\pi}{4^7}+\frac{\coth(5)\pi}{5^7}\right)$$

- $\coth(x)$ is the hyperbolic cotangent function

Result:

768.0016...

The result 768,0016 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for m_q = 4.7 MeV/c^2 = 0.0047 GeV/c^2 , that is the mass of quark down is 4.8±0.5±0.3 = 4.7 MeV/c^2.

[[[[(0.988055)/(3*1575) * (19Pi^7 /1.0308771232237) * [((cothPi)/1^7)+((coth2Pi)/2^7)+((coth3Pi)/3^7)+((coth4Pi)/4^7)+((coth5Pi)/5^7)]]]] ^1/5

Input interpretation:

$$\left(\frac{0.988055}{3\times 1575}\left(19\times\frac{\pi^7}{1.0308771232237}\right)\left(\frac{\coth(\pi)}{1^7}+\frac{\coth(2)\,\pi}{2^7}+\frac{\coth(3)\,\pi}{3^7}+\frac{\coth(4)\,\pi}{4^7}+\frac{\coth(5)\,\pi}{5^7}\right)\right)^\wedge(1/5)$$

- coth(x) is the hyperbolic cotangent function

Result:

1.643752...

The result is 1,643752 value near to the fourteenth root of Ramanujan's class invariant 1164.2696, (that is 1,65578), to the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$ 1,6557 for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5 and a good approximation to the mass of the proton.

Now, we have from (7):

[(1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]

Input:

$$\frac{1}{(1^2+2^2)(\sinh(3)\,\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\,\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\,\pi-\sinh(\pi))}$$

- sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}$$

Decimal approximation:

0.010409030328236639391838653533273903174095172586202059414...

Alternate forms:

$$\frac{1}{25\,\pi\sinh(7)-25\sinh(\pi)}+\frac{1}{13\,\pi\sinh(5)-13\sinh(\pi)}+\frac{1}{5\,\pi\sinh(3)-5\sinh(\pi)}$$

$$-\frac{1}{5\,(\sinh(\pi)-\pi\sinh(3))}-\frac{1}{13\,(\sinh(\pi)-\pi\sinh(5))}-\frac{1}{25\,(\sinh(\pi)-\pi\sinh(7))}$$

$$\frac{1}{5\left(-\frac{\pi}{2\,e^3}+\frac{e^3\,\pi}{2}-\sinh(\pi)\right)}+\frac{1}{13\left(-\frac{\pi}{2\,e^5}+\frac{e^5\,\pi}{2}-\sinh(\pi)\right)}+\frac{1}{25\left(-\frac{\pi}{2\,e^7}+\frac{e^7\,\pi}{2}-\sinh(\pi)\right)}$$

Alternative representations:

$$\frac{1}{(1^2+2^2)(\sinh(3)\,\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\,\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\,\pi-\sinh(\pi))}=\frac{1}{5\left(\frac{\pi}{\operatorname{csch}(3)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{13\left(\frac{\pi}{\operatorname{csch}(5)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{(9+4^2)\left(\frac{\pi}{\operatorname{csch}(7)}-\frac{1}{\operatorname{csch}(\pi)}\right)}$$

$$\frac{1}{(1^2+2^2)(\sinh(3)\,\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\,\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\,\pi-\sinh(\pi))}=\frac{1}{5\left(-\frac{i\,\pi}{\csc(3\,i)}+\frac{i}{\csc(i\,\pi)}\right)}+\frac{1}{13\left(-\frac{i\,\pi}{\csc(5\,i)}+\frac{i}{\csc(i\,\pi)}\right)}+\frac{1}{(9+4^2)\left(-\frac{i\,\pi}{\csc(7\,i)}+\frac{i}{\csc(i\,\pi)}\right)}$$

$$\frac{1}{(1^2+2^2)(\sinh(3)\,\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\,\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\,\pi-\sinh(\pi))}=\frac{1}{5\left(\frac{1}{2}\,\pi\left(-\frac{1}{e^3}+e^3\right)+\frac{1}{2}\,(e^{-\pi}-e^{\pi})\right)}+\frac{1}{13\left(\frac{1}{2}\,\pi\left(-\frac{1}{e^5}+e^5\right)+\frac{1}{2}\,(e^{-\pi}-e^{\pi})\right)}+\frac{1}{(9+4^2)\left(\frac{1}{2}\,\pi\left(-\frac{1}{e^7}+e^7\right)+\frac{1}{2}\,(e^{-\pi}-e^{\pi})\right)}$$

Series representations:

$$\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}=$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{5^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\right)\Big/$$

$$\left(325\left(\sum_{k=0}^{\infty}\frac{\pi\left(3^{1+2k}-\pi^{2k}\right)}{(1+2k)!}\right)\left(\sum_{k=0}^{\infty}\frac{\pi\left(5^{1+2k}-\pi^{2k}\right)}{(1+2k)!}\right)\sum_{k=0}^{\infty}\frac{\pi\left(7^{1+2k}-\pi^{2k}\right)}{(1+2k)!}\right)$$

$$\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}=$$

$$\left(\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(50\pi I_{1+2k_1}(7)-50 I_{1+2k_1}(\pi)\right)\left(26\pi I_{1+2k_2}(5)-26 I_{1+2k_2}(\pi)\right)+\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(50\pi I_{1+2k_1}(7)-50 I_{1+2k_1}(\pi)\right)\left(10\pi I_{1+2k_2}(3)-10 I_{1+2k_2}(\pi)\right)+\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(26\pi I_{1+2k_1}(5)-26 I_{1+2k_1}(\pi)\right)\left(10\pi I_{1+2k_2}(3)-10 I_{1+2k_2}(\pi)\right)\right)\Big/$$

$$\left(\left(\sum_{k=0}^{\infty}10\left(\pi I_{1+2k}(3)-I_{1+2k}(\pi)\right)\right)\left(\sum_{k=0}^{\infty}26\left(\pi I_{1+2k}(5)-I_{1+2k}(\pi)\right)\right)\sum_{k=0}^{\infty}50\left(\pi I_{1+2k}(7)-I_{1+2k}(\pi)\right)\right)$$

$$\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}=$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\right)\Bigg/$$

$$\left(325\sum_{k=0}^{\infty}-\frac{i\left(-\left(3-\frac{i\pi}{2}\right)^{2k}\pi+\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}\right)}{(2k)!}\sum_{k=0}^{\infty}-\frac{i\left(-\left(5-\frac{i\pi}{2}\right)^{2k}\pi+\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}\right)}{(2k)!}\sum_{k=0}^{\infty}-\frac{i\left(-\left(7-\frac{i\pi}{2}\right)^{2k}\pi+\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}\right)}{(2k)!}\right)$$

Integral representations:

$$\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}=$$

$$\left(65\pi^2\int_0^1\int_0^1(5\cosh(5t_1)-\cosh(\pi t_1))(3\cosh(3t_2)-\cosh(\pi t_2))\,dt_2\,dt_1+125\pi^2\int_0^1\int_0^1(7\cosh(7t_1)-\cosh(\pi t_1))(3\cosh(3t_2)-\cosh(\pi t_2))\,dt_2\,dt_1+325\pi^2\int_0^1\int_0^1(7\cosh(7t_1)-\cosh(\pi t_1))(5\cosh(5t_2)-\cosh(\pi t_2))\,dt_2\,dt_1\right)\Bigg/$$

$$\left(\left(\int_0^1 5\pi(3\cosh(3t)-\cosh(\pi t))\,dt\right)\left(\int_0^1 13\pi(5\cosh(5t)-\cosh(\pi t))\,dt\right)\int_0^1 25\pi(7\cosh(7t)-\cosh(\pi t))\,dt\right)$$

$$\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}=$$

$$\left(13\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+25\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+65\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\Big/\left(325\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\quad \text{for } \gamma>0$$

And:

18 * 1/[[1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]]

Input:

$$18\times\frac{1}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}$$

- $\sinh(x)$ is the hyperbolic sine function
- Units »

Exact result:

$$\frac{18}{\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}}$$

Decimal approximation:

1729.2677062503402806558424805676602676894476395119871994 53...

Alternate forms:

$$\frac{18}{\frac{1}{25\pi\sinh(7)-25\sinh(\pi)}+\frac{1}{13\pi\sinh(5)-13\sinh(\pi)}+\frac{1}{5\pi\sinh(3)-5\sinh(\pi)}}$$

$$\frac{18}{\frac{1}{5\left(-\frac{\pi}{2e^3}+\frac{e^3\pi}{2}-\sinh(\pi)\right)}+\frac{1}{13\left(-\frac{\pi}{2e^5}+\frac{e^5\pi}{2}-\sinh(\pi)\right)}+\frac{1}{25\left(-\frac{\pi}{2e^7}+\frac{e^7\pi}{2}-\sinh(\pi)\right)}}$$

$$(5850(\pi\sinh(3)-\sinh(\pi))(\pi\sinh(5)-\sinh(\pi))(\pi\sinh(7)-\sinh(\pi)))/ \\ \left(103\sinh^2(\pi)+13\pi^2\sinh(3)\sinh(5)+25\pi^2\sinh(3)\sinh(7)+65\pi^2\sinh(5)\sinh(7)-\right. \\ \left.38\pi\sinh(3)\sinh(\pi)-78\pi\sinh(5)\sinh(\pi)-90\pi\sinh(7)\sinh(\pi)\right)$$

Alternative representations:

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}= \\ \frac{18}{\frac{1}{5\left(\frac{\pi}{\operatorname{csch}(3)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{13\left(\frac{\pi}{\operatorname{csch}(5)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{(9+4^2)\left(\frac{\pi}{\operatorname{csch}(7)}-\frac{1}{\operatorname{csch}(\pi)}\right)}}$$

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}= \\ \frac{18}{\frac{1}{5\left(-\frac{i\pi}{\csc(3i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{13\left(-\frac{i\pi}{\csc(5i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{(9+4^2)\left(-\frac{i\pi}{\csc(7i)}+\frac{i}{\csc(i\pi)}\right)}}$$

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}= \\ \frac{18}{\frac{1}{5\left(\frac{1}{2}\pi\left(-\frac{1}{e^3}+e^3\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{13\left(\frac{1}{2}\pi\left(-\frac{1}{e^5}+e^5\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{(9+4^2)\left(\frac{1}{2}\pi\left(-\frac{1}{e^7}+e^7\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}}$$

Series representations:

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}=$$

$$\left(5850\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\left(\frac{7^{1+2k_3}\pi}{(1+2k_3)!}-\frac{\pi^{1+2k_3}}{(1+2k_3)!}\right)\right)/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+$$

$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{5^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\right)$$

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}=$$

$$\left(5850\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)\left(2\pi I_{1+2k_3}(7)-2I_{1+2k_3}(\pi)\right)\right)/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)+\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)+$$

$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(5)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)\right)$$

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(5850\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_3}\pi}{(2k_3)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_3}}{(2k_3)!}\right)\right)\Big/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+$$

$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\right)$$

Integral representations:

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\pi^3\int_0^1\int_0^1\int_0^1(3\cosh(3t_1)-\cosh(\pi t_1))(5\cosh(5t_2)-\cosh(\pi t_2))(7\cosh(7t_3)-\cosh(\pi t_3))\,dt_3\,dt_2\,dt_1$$

$$\frac{18}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(5850\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\Big/$$

$$\left(13\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds + 25\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds + 65\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right) \text{ for } \gamma > 0$$

The result 1729,267 is very near to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5).

1/4 * 1/[[1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]]

Input:

$$\frac{1}{4}\times\frac{1}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}$$

- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{1}{4\left(\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}\right)}$$

Decimal approximation:

24.017607031254726120220034452328614829020106104333155547 96...

Alternate forms:

$$\frac{1}{\frac{4}{5\pi\sinh(3)-5\sinh(\pi)}+\frac{4}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{4}{25(\pi\sinh(7)-\sinh(\pi))}}$$

$$\frac{1}{\frac{4}{5\left(-\frac{\pi}{2e^3}+\frac{e^3\pi}{2}-\sinh(\pi)\right)}+\frac{4}{13\left(-\frac{\pi}{2e^5}+\frac{e^5\pi}{2}-\sinh(\pi)\right)}+\frac{4}{25\left(-\frac{\pi}{2e^7}+\frac{e^7\pi}{2}-\sinh(\pi)\right)}}$$

$$(325(\pi\sinh(3)-\sinh(\pi))(\pi\sinh(5)-\sinh(\pi))(\pi\sinh(7)-\sinh(\pi)))/ \\ (4(103\sinh^2(\pi)+13\pi^2\sinh(3)\sinh(5)+25\pi^2\sinh(3)\sinh(7)+65\pi^2\sinh(5)\sinh(7)- \\ 38\pi\sinh(3)\sinh(\pi)-78\pi\sinh(5)\sinh(\pi)-90\pi\sinh(7)\sinh(\pi)))$$

Alternative representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4}= \\ \frac{1}{4\left(\frac{1}{5\left(\frac{\pi}{\operatorname{csch}(3)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{13\left(\frac{\pi}{\operatorname{csch}(5)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{(9+4^2)\left(\frac{\pi}{\operatorname{csch}(7)}-\frac{1}{\operatorname{csch}(\pi)}\right)}\right)}$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4}= \\ \frac{1}{4\left(\frac{1}{5\left(-\frac{i\pi}{\csc(3i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{13\left(-\frac{i\pi}{\csc(5i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{(9+4^2)\left(-\frac{i\pi}{\csc(7i)}+\frac{i}{\csc(i\pi)}\right)}\right)}$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4}= \\ \frac{1}{4\left(\frac{1}{5\left(\frac{1}{2}\pi\left(-\frac{1}{e^3}+e^3\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{13\left(\frac{1}{2}\pi\left(-\frac{1}{e^5}+e^5\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{(9+4^2)\left(\frac{1}{2}\pi\left(-\frac{1}{e^7}+e^7\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}\right)}$$

Series representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4}=$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\left(\frac{7^{1+2k_3}\pi}{(1+2k_3)!}-\frac{\pi^{1+2k_3}}{(1+2k_3)!}\right)\right)\Big/$$

$$\left(4\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+\right.\right.$$
$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+$$
$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{5^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\right)\right)$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4}=$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)\left(2\pi I_{1+2k_3}(7)-2I_{1+2k_3}(\pi)\right)\right)\Big/$$

$$\left(4\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)+\right.\right.$$
$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)+$$
$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(5)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)\right)\right)$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4} =$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_3}\pi}{(2k_3)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_3}}{(2k_3)!}\right)\right)\Big/$$

$$\left(4\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+\right.\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+$$

$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\right)\right)$$

Integral representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4} =$$

$$\pi^3\int_0^1\int_0^1\int_0^1(3\cosh(3t_1)-\cosh(\pi t_1))(5\cosh(5t_2)-\cosh(\pi t_2))(7\cosh(7t_3)-\cosh(\pi t_3))\,dt_3\,dt_2\,dt_1$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)4} =$$

$$\left(325\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right) \Bigg/$$
$$\left(4\left(13\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+25\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+65\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\right) \quad \text{for } \gamma > 0$$

The result 24,0176 ≈ 24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

8 * 1/[[1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]]

Input:

$$8\times\frac{1}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}$$

- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{8}{\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}}$$

Decimal approximation:

768.5634250001512358470411024745156745286433953386609775348...

Alternate forms:

$$\frac{8}{\frac{1}{25\pi\sinh(7)-25\sinh(\pi)}+\frac{1}{13\pi\sinh(5)-13\sinh(\pi)}+\frac{1}{5\pi\sinh(3)-5\sinh(\pi)}}$$

$$\frac{8}{\frac{1}{5\left(-\frac{\pi}{2e^3}+\frac{e^3\pi}{2}-\sinh(\pi)\right)}+\frac{1}{13\left(-\frac{\pi}{2e^5}+\frac{e^5\pi}{2}-\sinh(\pi)\right)}+\frac{1}{25\left(-\frac{\pi}{2e^7}+\frac{e^7\pi}{2}-\sinh(\pi)\right)}}$$

$$(2600(\pi\sinh(3)-\sinh(\pi))(\pi\sinh(5)-\sinh(\pi))(\pi\sinh(7)-\sinh(\pi)))/\left(103\sinh^2(\pi)+13\pi^2\sinh(3)\sinh(5)+25\pi^2\sinh(3)\sinh(7)+65\pi^2\sinh(5)\sinh(7)-38\pi\sinh(3)\sinh(\pi)-78\pi\sinh(5)\sinh(\pi)-90\pi\sinh(7)\sinh(\pi)\right)$$

Alternative representations:

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}=\frac{8}{\frac{1}{5\left(\frac{\pi}{\operatorname{csch}(3)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{13\left(\frac{\pi}{\operatorname{csch}(5)}-\frac{1}{\operatorname{csch}(\pi)}\right)}+\frac{1}{(9+4^2)\left(\frac{\pi}{\operatorname{csch}(7)}-\frac{1}{\operatorname{csch}(\pi)}\right)}}$$

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}=\frac{8}{\frac{1}{5\left(-\frac{i\pi}{\csc(3i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{13\left(-\frac{i\pi}{\csc(5i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{(9+4^2)\left(-\frac{i\pi}{\csc(7i)}+\frac{i}{\csc(i\pi)}\right)}}$$

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}=\frac{8}{\frac{1}{5\left(\frac{1}{2}\pi\left(-\frac{1}{e^3}+e^3\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{13\left(\frac{1}{2}\pi\left(-\frac{1}{e^5}+e^5\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{(9+4^2)\left(\frac{1}{2}\pi\left(-\frac{1}{e^7}+e^7\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}}$$

Series representations:

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(2600\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\left(\frac{7^{1+2k_3}\pi}{(1+2k_3)!}-\frac{\pi^{1+2k_3}}{(1+2k_3)!}\right)\right)\Big/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+\right.$$
$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+$$
$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{5^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\right)$$

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(2600\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi))(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi))(2\pi I_{1+2k_3}(7)-2I_{1+2k_3}(\pi))\right)\Big/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi))(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi))+\right.$$
$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi))(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi))+$$
$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(2\pi I_{1+2k_1}(5)-2I_{1+2k_1}(\pi))(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi))\right)$$

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(2600\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_3}\pi}{(2k_3)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_3}}{(2k_3)!}\right)\right) \Bigg/$$

$$\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+$$

$$\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\right)$$

Integral representations:

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\pi^3\int_0^1\int_0^1\int_0^1(3\cosh(3t_1)-\cosh(\pi t_1))(5\cosh(5t_2)-\cosh(\pi t_2))(7\cosh(7t_3)-\cosh(\pi t_3))\,dt_3\,dt_2\,dt_1$$

$$\frac{8}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}} =$$

$$\left(2600\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)/$$

$$\left(13\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+25\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+65\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4s)}-e^{\pi^2/(4s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right) \quad \text{for } \gamma > 0$$

The result 768,5634 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for $m_q = 4.7\ MeV/c^2 = 0.0047\ GeV/c^2$, that is the mass of quark down is 4.8±0.5±0.3 = 4.7 MeV/c^2.

1/8 * 1/[[1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]]

Input:

$$\frac{1}{8}\times\frac{1}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}$$

- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{1}{8\left(\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}\right)}$$

Decimal approximation:

12.008803515627363060110017226164307414510053052166577773 98...

Alternate forms:

$$\frac{1}{\frac{8}{5\pi\sinh(3)-5\sinh(\pi)}+\frac{8}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{8}{25(\pi\sinh(7)-\sinh(\pi))}}$$

$$\frac{1}{\frac{8}{5\left(-\frac{\pi}{2e^3}+\frac{e^3\pi}{2}-\sinh(\pi)\right)}+\frac{8}{13\left(-\frac{\pi}{2e^5}+\frac{e^5\pi}{2}-\sinh(\pi)\right)}+\frac{8}{25\left(-\frac{\pi}{2e^7}+\frac{e^7\pi}{2}-\sinh(\pi)\right)}}$$

$$(325(\pi\sinh(3)-\sinh(\pi))(\pi\sinh(5)-\sinh(\pi))(\pi\sinh(7)-\sinh(\pi)))/ \\ (8(103\sinh^2(\pi)+13\pi^2\sinh(3)\sinh(5)+25\pi^2\sinh(3)\sinh(7)+65\pi^2\sinh(5)\sinh(7)- \\ 38\pi\sinh(3)\sinh(\pi)-78\pi\sinh(5)\sinh(\pi)-90\pi\sinh(7)\sinh(\pi)))$$

Alternative representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8}= \\ \frac{1}{8\left(\frac{1}{5\left(\frac{\pi}{\text{csch}(3)}-\frac{1}{\text{csch}(\pi)}\right)}+\frac{1}{13\left(\frac{\pi}{\text{csch}(5)}-\frac{1}{\text{csch}(\pi)}\right)}+\frac{1}{(9+4^2)\left(\frac{\pi}{\text{csch}(7)}-\frac{1}{\text{csch}(\pi)}\right)}\right)}$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8}= \\ \frac{1}{8\left(\frac{1}{5\left(-\frac{i\pi}{\csc(3i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{13\left(-\frac{i\pi}{\csc(5i)}+\frac{i}{\csc(i\pi)}\right)}+\frac{1}{(9+4^2)\left(-\frac{i\pi}{\csc(7i)}+\frac{i}{\csc(i\pi)}\right)}\right)}$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8}= \\ \frac{1}{8\left(\frac{1}{5\left(\frac{1}{2}\pi\left(-\frac{1}{e^3}+e^3\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{13\left(\frac{1}{2}\pi\left(-\frac{1}{e^5}+e^5\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}+\frac{1}{(9+4^2)\left(\frac{1}{2}\pi\left(-\frac{1}{e^7}+e^7\right)+\frac{1}{2}\left(e^{-\pi}-e^{\pi}\right)\right)}\right)}$$

Series representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8} =$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\left(\frac{7^{1+2k_3}\pi}{(1+2k_3)!}-\frac{\pi^{1+2k_3}}{(1+2k_3)!}\right)\right)\Big/$$

$$\left(8\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{5^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+\right.\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{3^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)+$$

$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{5^{1+2k_1}\pi}{(1+2k_1)!}-\frac{\pi^{1+2k_1}}{(1+2k_1)!}\right)\left(\frac{7^{1+2k_2}\pi}{(1+2k_2)!}-\frac{\pi^{1+2k_2}}{(1+2k_2)!}\right)\right)\right)$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8} =$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)\left(2\pi I_{1+2k_3}(7)-2I_{1+2k_3}(\pi)\right)\right)\Big/$$

$$\left(8\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(5)-2I_{1+2k_2}(\pi)\right)+\right.\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(3)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)+$$

$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(2\pi I_{1+2k_1}(5)-2I_{1+2k_1}(\pi)\right)\left(2\pi I_{1+2k_2}(7)-2I_{1+2k_2}(\pi)\right)\right)\right)$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8} =$$

$$\left(325\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_3}\pi}{(2k_3)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_3}}{(2k_3)!}\right)\right) /$$

$$\left(8\left(13\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+\right.\right.$$

$$25\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(3-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)+$$

$$\left.\left.65\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{i\left(5-\frac{i\pi}{2}\right)^{2k_1}\pi}{(2k_1)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_1}}{(2k_1)!}\right)\left(\frac{i\left(7-\frac{i\pi}{2}\right)^{2k_2}\pi}{(2k_2)!}-\frac{i\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k_2}}{(2k_2)!}\right)\right)\right)$$

Integral representations:

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8} =$$

$$\pi^3\int_0^1\int_0^1\int_0^1(3\cosh(3t_1)-\cosh(\pi t_1))(5\cosh(5t_2)-\cosh(\pi t_2))(7\cosh(7t_3)-\cosh(\pi t_3))\,dt_3\,dt_2\,dt_1$$

$$\frac{1}{\left(\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}\right)8} =$$

$$\left(325\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\Big/$$

$$\left(8\left(13\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+\right.\right.$$

$$25\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(3 e^{9/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds+$$

$$\left.\left.65\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(5 e^{25/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^s\left(7 e^{49/(4 s)}-e^{\pi^2/(4 s)}\right)\sqrt{\pi}}{4 s^{3/2}}\,ds\right)\right) \quad \text{for } \gamma > 0$$

The result 12.0088 is a good approximation to the value of black hole entropy 12,19

[[[[[[[1/8 * 1/[[1/((1^2+2^2)(sinh3Pi-sinhPi))+1/((2^2+3^2)(sinh5Pi-sinhPi))+1/((3^2+4^2)(sinh7Pi-sinhPi))]]]]]]]^1/5

Input:

$$\sqrt[5]{\frac{1}{8}\times\frac{1}{\frac{1}{(1^2+2^2)(\sinh(3)\pi-\sinh(\pi))}+\frac{1}{(2^2+3^2)(\sinh(5)\pi-\sinh(\pi))}+\frac{1}{(3^2+4^2)(\sinh(7)\pi-\sinh(\pi))}}}$$

- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{1}{2^{3/5}\sqrt[5]{\frac{1}{5(\pi\sinh(3)-\sinh(\pi))}+\frac{1}{13(\pi\sinh(5)-\sinh(\pi))}+\frac{1}{25(\pi\sinh(7)-\sinh(\pi))}}}$$

Decimal approximation:

1.643992938689302181012517489850117366192411300677053042630...

Alternate forms:

$$\frac{1}{2^{3/5}\sqrt[5]{\frac{1}{25\pi\sinh(7)-25\sinh(\pi)}+\frac{1}{13\pi\sinh(5)-13\sinh(\pi)}+\frac{1}{5\pi\sinh(3)-5\sinh(\pi)}}}$$

$$\frac{1}{2^{3/5}} 5^{2/5} \left((13(\pi\sinh(3)-\sinh(\pi))(\pi\sinh(5)-\sinh(\pi))(\pi\sinh(7)-\sinh(\pi)))/(103\sinh^2(\pi)+13\pi^2\sinh(3)\sinh(5)+25\pi^2\sinh(3)\sinh(7)+65\pi^2\sinh(5)\sinh(7)-38\pi\sinh(3)\sinh(\pi)-78\pi\sinh(5)\sinh(\pi)-90\pi\sinh(7)\sinh(\pi))\right)\hat{}\,(1/5)$$

$$\frac{1}{2^{3/5}\sqrt[5]{\frac{1}{5\left(\frac{1}{2}(e^{-\pi}-e^{\pi})+\frac{1}{2}\left(e^3-\frac{1}{e^3}\right)\pi\right)}+\frac{1}{13\left(\frac{1}{2}(e^{-\pi}-e^{\pi})+\frac{1}{2}\left(e^5-\frac{1}{e^5}\right)\pi\right)}+\frac{1}{25\left(\frac{1}{2}(e^{-\pi}-e^{\pi})+\frac{1}{2}\left(e^7-\frac{1}{e^7}\right)\pi\right)}}}$$

Continued fraction:

[1; 1, 1, 1, 4, 4, 3, 1, 1, 1, 1, 133, 3, 4, 1, 10, 1, 3, 13, 1, 3, 1, 1, 4, 1, 5, 2, 4, 8, 2, ...]

Continued fraction:

[1; 1, 1, 1, 4, 4, 3, 1, 1, 1, 1, 133, 3, 4, 1, 10, 1, 3, 13, 1, 3, 1, 1, 4, 1, 5, 2, 4, 8, 2, ...]

The result is 1,64399 value near to the fourteenth root of Ramanujan's class invariant 1164.2696, (that is 1,65578), to the numerical result for $\theta_{(2)}$ as a function of $\theta_{(0)}$ 1,6557 for the D7-brane in $AdS_2 \times S^2$-sliced thermal AdS_5 and a good approximation to the mass of the proton.

With regard the right-hand side, we remember that (from https://www.oreilly.com/library/view/fundamentals-of-silicon/9781118313558/b02.xhtml):

$$\sinh(\theta) = \frac{\exp(\theta) - \exp(-\theta)}{2}$$

$$\cosh(\theta) = \frac{\exp(\theta) + \exp(-\theta)}{2}$$

$$\tanh(\theta) = \frac{\sinh(\theta)}{\cosh(\theta)} = \frac{\exp(\theta) - \exp(-\theta)}{\exp(\theta) + \exp(-\theta)}$$

$$\coth(\theta) = \frac{1}{\tanh(\theta)} = \frac{\cosh(\theta)}{\sinh(\theta)} = \frac{\exp(\theta) + \exp(-\theta)}{\exp(\theta) - \exp(-\theta)}$$

$$\operatorname{sech}(\theta) = \frac{1}{\cosh(\theta)} = \frac{2}{\exp(\theta) + \exp(-\theta)}$$

$$\operatorname{csch}(\theta) = \frac{1}{\sinh(\theta)} = \frac{2}{\exp(\theta) - \exp(-\theta)}$$

thence from:

$$\left(\frac{1}{\pi} + \coth\pi - \frac{\pi}{2} tanh^2 \frac{\pi}{2}\right) / 2\sinh\pi$$

we have that:

-(((((1/Pi+cothPi-(1/2(Pi tanh^2 Pi)))))/(2sinhPi)

Input:

$$-\frac{\frac{1}{\pi} + \coth(\pi) - \frac{1}{2}\left(\pi \tanh^2(\pi)\right)}{2\sinh(\pi)}$$

- $\coth(x)$ is the hyperbolic cotangent function
- $\tanh(x)$ is the hyperbolic tangent function
- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$-\frac{1}{2}\operatorname{csch}(\pi)\left(\frac{1}{\pi} - \frac{1}{2}\pi \tanh^2(\pi) + \coth(\pi)\right)$$

- $\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

0.010263231861593347945496638553746930019442470402465850950...

Alternate forms:

$$\frac{\tanh(\pi)\left(\pi^2 - 2\pi\coth^3(\pi) - 2\coth^2(\pi)\right)\operatorname{sech}(\pi)}{4\pi}$$

$$-\frac{\frac{1}{\pi} + \frac{\cosh(\pi)}{\sinh(\pi)} - \frac{\pi\sinh^2(\pi)}{2\cosh^2(\pi)}}{2\sinh(\pi)}$$

$$-\frac{\coth(\pi)\left(4\pi\operatorname{csch}(\pi) + \pi^2\operatorname{sech}^3(\pi) + 4\operatorname{sech}(\pi) - \pi^2\cosh(2\pi)\operatorname{sech}^3(\pi)\right)}{8\pi}$$

Alternative representations:

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = -\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{-e^{-\pi}+e^{\pi}}$$

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = -\frac{1+\frac{1}{\pi}-\frac{1}{2}\pi\left(i\cot\left(\frac{\pi}{2}+i\pi\right)\right)^2+\frac{2}{-1+e^{2\pi}}}{-e^{-\pi}+e^{\pi}}$$

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = -\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{2\,i\cos\left(\frac{\pi}{2}+i\pi\right)}$$

Series representations:

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = -\frac{\left(-1+\pi+32\left(\sum_{k=1}^{\infty}\frac{1}{5-4k+4k^2}\right)^2+2\pi\sum_{k=1}^{\infty}q^{2k}\right)\sum_{k=1}^{\infty}q^{-1+2k}}{\pi} \quad \text{for } q=e^{\pi}$$

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = \frac{\left(1-32\left(\sum_{k=1}^{\infty}\frac{1}{5-4k+4k^2}\right)^2+\pi\sum_{k=-\infty}^{\infty}\frac{1}{\pi+k^2\pi}\right)\sum_{k=1}^{\infty}q^{-1+2k}}{\pi} \quad \text{for } q=e^{\pi}$$

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = -\frac{\left(-4+\pi^2-4\sum_{k=1}^{\infty}\frac{1}{1+k^2}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)\sum_{k=1}^{\infty}q^{-1+2k}}{2\pi} \quad \text{for } q=e^{\pi}$$

Integral representations:

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = \frac{-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2}{4\pi^2\int_0^1\cosh(\pi t)\,dt}$$

$$-\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)} = \frac{i\left(-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2\right)}{\pi\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{s^{3/2}}\,ds} \quad \text{for } \gamma>0$$

We observe that the result 0,01026323 is near to the result of the expression of the right-hand side, i.e. 0,01040903 (0.01026 ≈ 0.01040), with a minimal difference of 0,0001458.

With the value, with minus sign, we have the following expressions:

18 * 1/ (((((1/Pi+cothPi-(1/2(Pi tanh^2 Pi)))))/(2sinhPi)

Input:

$$18\times\frac{1}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\left(\pi\tanh^2(\pi)\right)}{2\sinh(\pi)}}$$

- $\coth(x)$ is the hyperbolic cotangent function
- $\tanh(x)$ is the hyperbolic tangent function
- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{36\sinh(\pi)}{\frac{1}{\pi}-\frac{1}{2}\pi\tanh^2(\pi)+\coth(\pi)}$$

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Decimal approximation:

−1753.833513920588068430472033407649322493627749119087654 27...

Alternate forms:

$$-\frac{72\pi\cosh(\pi)\coth(\pi)}{\pi^2-2\pi\coth^3(\pi)-2\coth^2(\pi)}$$

$$\frac{36\sinh(\pi)}{\frac{1}{\pi}+\frac{\cosh(\pi)}{\sinh(\pi)}-\frac{\pi\sinh^2(\pi)}{2\cosh^2(\pi)}}$$

$$-\frac{144\pi\sinh(\pi)\tanh(\pi)}{-4\pi-4\tanh(\pi)-\pi^2\tanh(\pi)\operatorname{sech}^2(\pi)+\pi^2\cosh(2\pi)\tanh(\pi)\operatorname{sech}^2(\pi)}$$

Alternative representations:

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}}=\frac{18}{\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}}=\frac{18}{\frac{1+\frac{1}{\pi}-\frac{1}{2}\pi\left(i\cot\left(\frac{\pi}{2}+i\pi\right)\right)^2+\frac{2}{-1+e^{2\pi}}}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = \frac{18}{\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{2\,i\cos\left(\frac{\pi}{2}+i\pi\right)}}$$

Series representations:

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{72\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{72\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+\pi^2-2\pi\sum_{k=-\infty}^{\infty}\frac{1}{\pi+k^2\pi}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{72\,i\,\pi\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

Integral representation:

$$\frac{18}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = \frac{18\,i\,\pi^{3/2}}{-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{s^{3/2}}\,ds \text{ for } \gamma>0$$

The value -1753,8335 is a good approximation (with minus sign) to the value of the mass of the candidate "glueball" $f_0(1710)$ that is 1723 (+ 6 – 5), and practically equal to the values 1759± 6; 1750(+ 6− 7).

8 * 1/ (((((1/Pi+cothPi-(1/2(Pi tanh^2 Pi)))))/(2sinhPi)

Input:

$$8\times\frac{1}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\left(\pi\tanh^2(\pi)\right)}{2\sinh(\pi)}}$$

- $\coth(x)$ is the hyperbolic cotangent function
- $\tanh(x)$ is the hyperbolic tangent function
- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{16\sinh(\pi)}{\frac{1}{\pi}-\frac{1}{2}\pi\tanh^2(\pi)+\coth(\pi)}$$

Decimal approximation:

−779.481561742483585969098681514510809997167888497372290790...

Alternate forms:

$$-\frac{32\pi\cosh(\pi)\coth(\pi)}{\pi^2-2\pi\coth^3(\pi)-2\coth^2(\pi)}$$

$$\frac{16\sinh(\pi)}{\frac{1}{\pi}+\frac{\cosh(\pi)}{\sinh(\pi)}-\frac{\pi\sinh^2(\pi)}{2\cosh^2(\pi)}}$$

$$-\frac{64\pi\sinh(\pi)\tanh(\pi)}{-4\pi-4\tanh(\pi)-\pi^2\tanh(\pi)\operatorname{sech}^2(\pi)+\pi^2\cosh(2\pi)\tanh(\pi)\operatorname{sech}^2(\pi)}$$

Alternative representations:

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}}=\frac{8}{\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}}=\frac{8}{\frac{1+\frac{1}{\pi}-\frac{1}{2}\pi\left(i\cot\left(\frac{\pi}{2}+i\pi\right)\right)^2+\frac{2}{-1+e^{2\pi}}}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}}=\frac{8}{\frac{1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2}{2i\cos\left(\frac{\pi}{2}+i\pi\right)}}$$

Series representations:

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{32\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{32\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+\pi^2-2\pi\sum_{k=-\infty}^{\infty}\frac{1}{\pi+k^2\pi}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = -\frac{32 i\pi\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

Integral representation:

$$\frac{8}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)}{2\sinh(\pi)}} = \frac{8 i\pi^{3/2}}{-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{s^{3/2}}\,ds \text{ for } \gamma>0$$

We note that the value -779,4815 is very near to the value 775,023 of nonperturbative contribution to the mass of a 1S quarkonium (with minus sign) for $m_q = 4.68\ MeV/c^2 = 0.00468\ GeV/c^2$, that is the mass of quark down is 4.8±0.5±0.3 = 4.68 MeV/c^2.

Input:

$$\frac{\pi^2\times 0.00468\times 624\times 0.012}{425\left(\frac{4}{3}\times 5.13\times 0.00468\right)^4}$$

Result:

775.023...

1/4 * 1/ (((((1/Pi+cothPi-(1/2(Pi tanh^2 Pi)))))/(2sinhPi)

Input:

$$\frac{1}{4} \times \frac{1}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\left(\pi \tanh^2(\pi)\right)}{2\sinh(\pi)}}$$

- $\coth(x)$ is the hyperbolic cotangent function
- $\tanh(x)$ is the hyperbolic tangent function
- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{\sinh(\pi)}{2\left(\frac{1}{\pi}-\frac{1}{2}\pi \tanh^2(\pi)+\coth(\pi)\right)}$$

Decimal approximation:

−24.358798804452612061534333797328462812411496515542884087 2...

Alternate forms:

$$\frac{\sinh(\pi)}{\frac{2}{\pi}-\pi \tanh^2(\pi)+2\coth(\pi)}$$

$$-\frac{\pi \cosh(\pi)\coth(\pi)}{\pi^2-2\pi\coth^3(\pi)-2\coth^2(\pi)}$$

$$\frac{\sinh(\pi)}{2\left(\frac{1}{\pi}+\frac{\cosh(\pi)}{\sinh(\pi)}-\frac{\pi \sinh^2(\pi)}{2\cosh^2(\pi)}\right)}$$

Alternative representations:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi \tanh^2(\pi)\right)4}{2\sinh(\pi)}}=\frac{1}{\frac{4\left(1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2\right)}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi \tanh^2(\pi)\right)4}{2\sinh(\pi)}}=\frac{1}{\frac{4\left(1+\frac{1}{\pi}-\frac{1}{2}\pi\left(i\cot\left(\frac{\pi}{2}+i\pi\right)\right)^2+\frac{2}{-1+e^{2\pi}}\right)}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi \tanh^2(\pi)\right)4}{2\sinh(\pi)}}=\frac{1}{\frac{4\left(1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2\right)}{2i\cos\left(\frac{\pi}{2}+i\pi\right)}}$$

Series representations:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)4}{2\sinh(\pi)}} = -\frac{\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)4}{2\sinh(\pi)}} = -\frac{\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{-2+\pi^2-2\pi\sum_{k=-\infty}^{\infty}\frac{1}{\pi+k^2\pi}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)4}{2\sinh(\pi)}} = -\frac{i\pi\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}}{-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2} \text{ for } q=e^{\pi}$$

Integral representation:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)4}{2\sinh(\pi)}} = \frac{i\pi^{3/2}}{4\left(-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2\right)}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{s^{3/2}}\,ds \text{ for } \gamma>0$$

The result -24,35879 is very near to 24, that represent (with minus sign) the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

1/8 * 1/ (((((1/Pi+cothPi-(1/2(Pi tanh^2 Pi)))))/(2sinhPi)

Input:

$$\frac{1}{8}\times\frac{1}{\frac{\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\left(\pi\tanh^2(\pi)\right)}{2\sinh(\pi)}}$$

- $\coth(x)$ is the hyperbolic cotangent function
- $\tanh(x)$ is the hyperbolic tangent function
- $\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{\sinh(\pi)}{4\left(\frac{1}{\pi}-\frac{1}{2}\pi\tanh^2(\pi)+\coth(\pi)\right)}$$

Decimal approximation:

−12.179399402226306030767166898664231406205748257771442043 6...

Alternate forms:

$$\frac{\sinh(\pi)}{\frac{4}{\pi}-2\pi\tanh^2(\pi)+4\coth(\pi)}$$

$$-\frac{\pi\cosh(\pi)\coth(\pi)}{2\left(\pi^2-2\pi\coth^3(\pi)-2\coth^2(\pi)\right)}$$

$$\frac{\sinh(\pi)}{4\left(\frac{1}{\pi}+\frac{\cosh(\pi)}{\sinh(\pi)}-\frac{\pi\sinh^2(\pi)}{2\cosh^2(\pi)}\right)}$$

Alternative representations:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}}=\frac{1}{\frac{8\left(1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2\right)}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}}=\frac{1}{\frac{8\left(1+\frac{1}{\pi}-\frac{1}{2}\pi\left(i\cot\left(\frac{\pi}{2}+i\pi\right)\right)^2+\frac{2}{-1+e^{2\pi}}\right)}{-e^{-\pi}+e^{\pi}}}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}}=\frac{1}{\frac{8\left(1+\frac{1}{\pi}+\frac{2}{-1+e^{2\pi}}-\frac{1}{2}\pi\left(-1+\frac{2}{1+e^{-2\pi}}\right)^2\right)}{2i\cos\left(\frac{\pi}{2}+i\pi\right)}}$$

Series representations:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}} = -\frac{\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{2\left(-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)} \text{ for } q = e^{\pi}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}} = -\frac{\pi\sum_{k=0}^{\infty}\frac{\pi^{1+2k}}{(1+2k)!}}{2\left(-2+\pi^2-2\pi\sum_{k=-\infty}^{\infty}\frac{1}{\pi+k^2\pi}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)} \text{ for } q = e^{\pi}$$

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}} = -\frac{i\pi\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}}{2\left(-2+2\pi+\pi^2+4\pi\sum_{k=1}^{\infty}q^{2k}+4\pi^2\sum_{k=1}^{\infty}(-1)^k q^{2k}+4\pi^2\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)} \text{ for } q = e^{\pi}$$

Integral representation:

$$\frac{1}{\frac{\left(\frac{1}{\pi}+\coth(\pi)-\frac{1}{2}\pi\tanh^2(\pi)\right)8}{2\sinh(\pi)}} = \frac{i\pi^{3/2}}{8\left(-2+2\pi\int_{\frac{i\pi}{2}}^{\pi}\operatorname{csch}^2(t)\,dt+\pi^2\left(\int_0^{\pi}\operatorname{sech}^2(t)\,dt\right)^2\right)}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{s^{3/2}}\,ds \text{ for } \gamma > 0$$

The result -12,17939 is practically equal to the value of black hole entropy 12,19 with minus sign

Appendix A

On the number 24 in string theory and Meson f0(1710)

From:

http://math.ucr.edu/home/baez/numbers/24.pdf

So: our calculation only gives consistent answers if the partition function

$$Z(t) = e^{\frac{i}{24}t} \prod_{n=1}^{\infty} \frac{1}{1 - e^{-int}}$$

is unchanged when we add 2π to t. Alas, it *does* change:

$$Z(t + 2\pi) = e^{\frac{2\pi i}{24}} Z(t)$$

But $Z(t)^{24}$ does *not* change! This is the partition function of 24 strings with one direction to wiggle — which is just like one string with 24 directions to wiggle.

So, bosonic string theory works best
when spacetime has $24 + 2 = 26$ dimensions!

From:

Introduction to String Theory - Winter term 2011/12
Timo Weigand - Institut fur Theoretische Physik, Universitat Heidelberg

- We first **introduce a cutoff Λ to regularise the divergent expression** in such a way that the divergence appears as we remove the cutoff by sending $\Lambda \to \infty$. A convenient cutoff procedure here is e.g. to rewrite the vacuum energy as

$$\frac{\pi}{\ell}\frac{d-2}{2}\sum_{n=1}^{\infty} n \to \frac{\pi}{\ell}\frac{d-2}{2}\sum_{n=1}^{\infty} n e^{-\frac{\pi n}{\ell \Lambda}} = \frac{\pi}{\ell}\frac{d-2}{2}\sum_{n=1}^{\infty} n (e^{-\frac{\pi}{\ell\Lambda}})^n. \tag{3.102}$$

From $\sum_{n=1}^{\infty} n q^n = q\frac{d}{dq}\sum_{n=1}^{\infty} q^n = q\frac{d}{dq}\frac{1}{1-q} = \frac{q}{(1-q)^2}$ we find

$$\frac{\pi}{\ell}\frac{d-2}{2}\sum_{n=1}^{\infty} n = \frac{\pi}{\ell}\frac{d-2}{2}\lim_{\Lambda\to\infty}\frac{e^{-\frac{\pi}{\ell\Lambda}}}{\left(1-e^{-\frac{\pi}{\ell\Lambda}}\right)^2} \tag{3.103}$$

$$= \frac{d-2}{2}\lim_{\Lambda\to\infty}\Big(\frac{\ell}{\pi}\Lambda^2 - \frac{\pi}{\ell}\frac{1}{12} + \mathcal{O}(\frac{1}{\Lambda})\Big). \tag{3.104}$$

- The expression for the vacuum energy has two non-vanishing contributions: The term proportional to Λ^2 is the divergent piece. It is important that this term scales like ℓ. Therefore, this term can be absorbed by adding a cosmological constant term proportional to $\Lambda^2 \int d^2\sigma\sqrt{-h}$ to the bare Polyakov action via **renormalisation**. This **counterterm** in the bare action then cancels off the divergence arising in the quantum computation of the vacuum energy.

  In addition there is the finite term $-\frac{d-2}{2}\frac{\pi}{\ell}\frac{1}{12}$. This term is present only due to the finite size of the string because it disappears in the limit $\ell \to \infty$. Unlike for the divergent term, there exists no local counterterm that we could add to the action such as to absorb this term. This term is therefore physical and defines the **Casimir energy** of the string.

A priori one might wonder why we *have to* cancel the entire piece of the term scaling like ℓ by adding a suitable cosmological constant - can't we just keep a finite fraction of it and declare it as part of $-a$? The reason why this is not possible in string theory is conformal invariance: The cosmological constant term breaks conformal invariance explicitly in the action - unless the classical counterterm and the quantum term cancel exactly. Therefore $-a$ must be identified with the Casimir energy, i.e.

$$a = \frac{d-2}{24} \qquad \text{for (d} - \text{2) transverse (NN) or (DD) oscillators.} \tag{3.105}$$

For $d = 26$ this gives $a = 1$ as found by requiring Lorentz invariance for the open string with (NN) conditions and also for the closed string. It therefore holds also for the (DD) string.

Remarks:

- Note that $a \neq 0$ still breaks conformal invariance, but merely in form of an acceptable conformal anomaly, i.e. of a quantum anomaly of the conformal symmetry.

- Very soon we will get to know powerful CFT techniques yielding another derivation of the Casimir energy making the relation to the central term c in in the Virasoro algebra clear.

There is an amusingly quick and efficient manner to re-derive the Casimir energy, i.e. the finite piece of the vacuum energy, by means of **ζ-function regularisation**, which is a formal way to regularize the sum $\sum_{n=1}^{\infty} n$. It makes use of the ζ-function $\zeta(s)$, defined as

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \tag{3.106}$$

We need the following two properties of the ζ-function which are proven in standard textbooks on complex analysis:

- $\zeta(s)$ is convergent for $\mathrm{Re}(s) > 1$.
- $\zeta(s)$ allows for an unique analytic continuation to $s = -1$ with

$$\zeta(-1) = -\frac{1}{12}.$$

With two eyes wide shut - or alternatively with the above proper procedure of regularization and renormalisation in mind - we deduce that the contribution to "a" from 1 (NN) or (DD) direction (i.e. one integer moded boson), given precisely by $-\frac{1}{2}\zeta(-1)$, is

$$\boxed{+\frac{1}{24} \quad \text{per (NN) or (DD) direction.}} \tag{3.107}$$

This also gives a quick way to read off the result for a string with (DN) boundary conditions, for which the mode expansion is half-integer. To this end we use

$$\zeta(s,q) = \sum_{n=1}^{\infty} (n+q)^{-s} \tag{3.108}$$

with analytic continuation

$$\zeta(-1,q) = -\frac{1}{12}(6q^2 - 6q + 1).$$

The contribution from one (DN) direction (i.e. one half-integer moded boson) is therefore ($q = \frac{1}{2}$)

$$\boxed{-\frac{1}{48} \quad \text{per (DN) direction.}} \tag{3.109}$$

If you don't believe this result you can derive it in a similar manner as for the (NN)/(DD) string around (3.103).

This allows us to write down the normal ordering constant for the critical string with m (DN) directions and $d-m$ (DD) or (NN) as

$$a = \frac{\#(NN) + \#(DD)}{24} - \frac{\#(DN)}{48} = \frac{\#(NN) + \#(DD) + \#(DN)}{24} - \frac{\#(DN)}{16} = \frac{d-2}{24} - \frac{m}{16}.$$

The mass relation for open string with m (DN) directions is therefore

$$\alpha' M^2 = N + \alpha'(T\Delta x)^2 - a = N + \alpha'(T\Delta x)^2 - \frac{d-2}{24} + \frac{m}{16}. \tag{3.110}$$

Significance of the ghost Virasoro algebra and criticality

The algebra of the conformal transformations of the full action $S = S_X + S_g$ is generated by the combined Virasoro generators

$$L_m^{\text{tot}} = L_m^{(X)} + L_m^{(g)} - a^{\text{tot}}\,\delta_{m,0}, \tag{3.170}$$

where we conventionally include a total normal ordering constant a^{tot} into the definition of L_m^{tot}. It is the sum of the normal ordering constant for the X and for the ghost fields,

$$a^{\text{tot}} = a^{(X)} + a^{(g)}. \tag{3.171}$$

- The first piece is just given by $a^{(X)} = \frac{d-2}{24} + \frac{2}{24}$ corresponding to the $d-2$ transverse X-oscillations familiar from lightcone quantisation together with the contribution from the X^0 and X^{d-1} components. These are not absent here since we are in a covariant gauge.
- To compute $a^{(g)}$ we observe that the ghost system counts as one anti-commuting set of integer moded scalars. By a similar computation as performed in section 3.2.3 its contribution to the Casimir energy is

$$a^{(g)} = -\frac{1}{12}. \tag{3.172}$$

Since there is no factor of $\frac{1}{2}$ in the definition of $L_0^{(g)}$ this is just minus the contribution of a commuting set of integer-moded scalars.

- Therefore

$$a^{\text{tot}} = \frac{d-2}{24} + \frac{2}{24} - \frac{1}{12} = \frac{d-2}{24} \equiv a. \tag{3.173}$$

The ghost system cancels the contribution from the unphysical non-transverse polarisations, a feature that we will encounter again in the framework of BRST quantisation.

One may verify that the combined Virasoro generators satisfy the commutation relations

$$\boxed{[L_m^{\text{tot}}, L_n^{\text{tot}}] = (m-n)\, L_{m+n}^{\text{tot}} + \delta_{m+n,0}\Big(\frac{c^{\text{tot}}}{12}(m^3-m) + 2m(a-1)\Big)} \tag{3.174}$$

with the central extensions governed by the quantities

$$c^{\text{tot}} = c^{(X)} + c^{(g)}\,, \qquad c^{(X)} = d \quad (\text{for propagation in } \mathbb{R}^{1,d-1}), \qquad c^{(g)} = -26. \tag{3.175}$$

The presence of the central term c^{tot} is equivalent to a Weyl anomaly of the full action $S_X + S_g$, or equivalently to a Weyl anomaly in the path integral

$$\int \mathcal{D}X e^{i\,S_X}(\det P). \tag{3.176}$$

The central term and thus also the Weyl anomaly of the path integral is absent iff

$$d = 26\,, \quad a = 1. \tag{3.177}$$

Thus criticality arises as a self-consistency requirement of the Faddeev-Popov treatment of the path integral.

This gives us a final interpretation of the meaning of criticality: It is the requirement that the X-theory cancels the conformal anomaly of the ghost system,

$$\boxed{0 \stackrel{!}{=} c^{(X)} + c^{(g)} = c^{(X)} - 26} \tag{3.178}$$

so that the anomaly of the full quantum theory is absent. What is actually fixed is not the number of spacetime dimensions, but the central extension $c^{(X)}$.

Relation of c to vacuum energy

The above transformation rule provides a very simple and efficient derivation of the Casimir energy on the cylinder in terms of the central extension c of the Virasoro algebra. The starting point is the intuitive assertion that on the complex plane the Casimir energy vanishes. Thus, in view of the relation between the Hamiltonian and the energy-momentum tensor this implies that the one-point function on the plane is zero, $\langle T_{\text{plane}}\rangle = 0$. Therefore

$$\langle T_{\text{cyl.}}\rangle(w) = -\frac{c}{24}\left(\frac{2\pi}{\ell}\right)^2. \tag{4.78}$$

This beautifully matches with our earlier computation of the vacuum energy if we remember that $w = i\xi^-, \bar{w} = i\xi^+$ in terms of the *Minkowski signature* lightcone coordinates. This gives an extra factor of -1 in relating the one-point function of $T_{\text{cyl.}}(w)$ to the physical value of the vacuum energy on the Minkowski signature cylinder. Altogether one finds with $H = \frac{2\pi}{\ell}(L_0 + \tilde{L}_0)$ and $L_0 = \frac{\ell}{4\pi^2}\int_0^\ell d\sigma T_{--}(\xi^-)$, $\tilde{L}_0 = \frac{\ell}{4\pi^2}\int_0^\ell d\sigma T_{++}(\xi^+)$ that the vacuum energy associated with a single string field in Minkowski signature is

$$\langle H_{\text{cyl.}}\rangle = (-1)\left(\frac{2\pi}{\ell}\right)\left(-\frac{\ell}{4\pi^2}\right)\int_0^\ell d\sigma(\langle T_{\text{cyl.}}(w)\rangle + \langle T_{\text{cyl.}}(\bar{w})\rangle) = -\frac{2\pi}{\ell}\frac{c}{12}. \tag{4.79}$$

Taking into account that a single string field has a conformal anomaly $c = 1$, this agrees with the result $\langle H\rangle = \frac{2\pi}{\ell}(-a - \tilde{a})$ with $a = \frac{1}{24}$ for an integer-moded boson.

This is an example of a general principle in quantum field theory:

> The path integral in compactified Euclidean time yields the partition function at temperature $T = \frac{1}{2\pi\tau_2}$.

ii) To generalise this to a torus with modulus $\tau = \tau_1 + i\tau_2$ we must in addition translate the fields by $2\pi\tau_1$ in spatial direction,

$$Z(\tau) \;=\; \langle 1\rangle_{\tau,\otimes X^i} = \sum_n \langle n|\, e^{2\pi i\tau_1 P - 2\pi\tau_2 H}\,|n\rangle$$

or more compactly

$$\boxed{Z(\tau) = \text{Tr}e^{2\pi i\tau_1 P - 2\pi\tau_2 H}.} \tag{5.98}$$

With the relations

$$P = \frac{2\pi}{\ell}(L_0 - \bar{L}_0), \qquad H = \frac{2\pi}{\ell}(L_0 + \bar{L}_0 - \frac{c+\bar{c}}{24}) \qquad \text{with} \qquad \frac{2\pi}{\ell} \equiv 1 \qquad \text{from now on,}$$

this can written as

$$Z(\tau) = (q\,\bar{q})^{-\frac{d}{24}}\ \text{Tr}q^{L_0}\,\bar{q}^{\bar{L}_0}, \qquad q = e^{2\pi i\tau}. \tag{5.99}$$

We now express L_0 in terms of the momentum operator and the number operator,

$$L_0 = \frac{\alpha'}{4}k^2 + N, \qquad \bar{L}_0 = \frac{\alpha'}{4}k^2 + \bar{N}, \tag{5.100}$$

and express the trace as an integral over the momentum modes times the trace over the oscillator part of a string state,

$$\text{Tr}\,q^{L_0}\,\bar{q}^{\bar{L}_0} = \int\frac{d^dk}{(2\pi)^d}\underbrace{\text{Tr}'}_{\substack{\text{trace over}\\ \text{oscillators}}}\ \langle k; N, \bar{N}|\,(q\bar{q})^{\frac{\alpha'}{4}k^2}q^N\bar{q}^{\bar{N}}\,|k; N, \bar{N}\,\rangle. \tag{5.101}$$

The inner product over the momentum state gives a factor of the spacetime volme $\langle k|k\rangle = \delta^{(d)}(k-k) \equiv V_d$, and with $(q\,\bar{q})^{\frac{\alpha'}{4}k^2} = (\exp(4\pi\tau_2))^{\frac{\alpha'}{4}k^2}$ we find

$$Z(\tau) = V_d\,(q\,\bar{q})^{-\frac{d}{24}}\int\frac{d^dk}{(2\pi)^d}\,e^{-\pi\tau_2\alpha' k^2}\cdot\text{Tr}'q^N\,\bar{q}^{\bar{N}}. \tag{5.102}$$

The computation of the trace was performed in detail on Assignment 6, to which we refer for details. The result is

$$\mathrm{Tr}' q^N \bar{q}^N = \prod_{n=1}^{\infty}(1-q^n)^{-d}(1-\bar{q}^n)^{-d}. \tag{5.103}$$

We finally perform a Wick rotation $k^0 \to ik^0$ to render the integral $\int d^d k$ finite and perform the Gaussian integration. This yields the final expression for the partition function $Z(\tau)$,

$$Z(\tau) = iV_d \left(Z_X(\tau)\right)^d, \tag{5.104}$$

$$Z_X(\tau) = (4\pi^2\alpha'\tau_2)^{-\frac{1}{2}} \cdot |\eta(\tau)|^{-2} \qquad \text{with} \tag{5.105}$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1-q^n) \qquad \text{the Dedekind } \eta \text{ function.} \tag{5.106}$$

The next task is to compute the one-loop integral of the ghost insertions. For reasons of time we do not present this computation here. It can be found e.g. in [P], Chapter 7, p. 212. We merely quote the final result: The ghost sector yields a factor of $|\eta(\tau)^2|^2$. Setting now $d = 26$ the one-loop amplitude is

$$Z(\tau) = iV_{26} \int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2} (4\pi^2\alpha'\tau_2)^{-13} \cdot |\eta(\tau)|^{-48}. \tag{5.107}$$

The IR regime corresponds to the limit $\tau_2 \to \infty$, in which the torus (or cylinder with both ends identified) becomes very long. In this regime we can expand the inegral over the η-function as

$$\begin{aligned} \int^{\infty} d^2\tau |\eta(\tau)|^{-48} &= \int^{\infty} d^2\tau \, (q\bar{q})^{-1} | \prod_{n=1}^{\infty}(1-q^n)|^{-48} \\ &\cong \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int^{\infty} d\tau_2 \, (q\bar{q})^{-1}(1+24q+\ldots)(1+24\bar{q}+\ldots) \\ &\cong \int^{\infty} d\tau_2 \left((q\bar{q})^{-1} + 24^2 + \ldots\right), \end{aligned}$$

where terms of the type $q + \bar{q}$ vanish upon performing the integral over the τ_1-coordinate. Thus we deduce the following IR-behaviour

$$iV_{26} \int^{\infty} \frac{d\tau_2}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-13} \left[\underbrace{e^{4\pi\tau_2}}_{\text{tachyon}} + \underbrace{24^2}_{\substack{\text{massless}\\\text{modes}}} + \underbrace{\ldots}_{\substack{\text{massive}\\\text{modes}}} \right] \tag{5.115}$$

- The first term is divergent. This divergence, however, is an artefact due to the appearance of the tachyon. Since the tachyon will be removed in the final superstring theory we can safely ignore this nuisance.
- The next term is due to the massless states. The long-distance behaviour is therefore governed by massless states. Their contribution to Z_{T^2} is finite.

- As in the closed sector, the ghost contribution turns out to cancel the oscillator trace of precisely two non-transverse directions. With this in mind the amplitude is

$$Z_{C_2} = \int_0^\infty \frac{dt}{2t} \mathrm{Tr}\, e^{-2\pi t(L_0 - \frac{c}{24})} \tag{5.118}$$

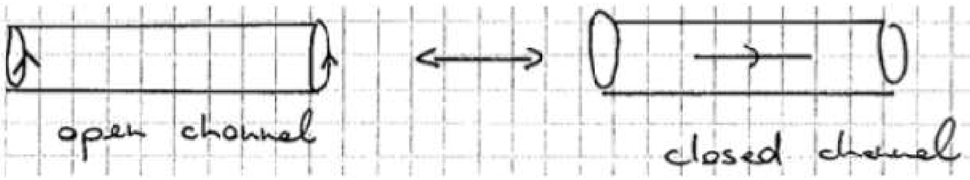
$$= iV_{26} \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-13} \mathrm{Tr}'_{\otimes_{i=1}^{24} X^i} q^{L_0 - \frac{1}{24}}. \tag{5.119}$$

For a stack of N concindent D-branes filling all of spacetime this is

$$Z_{C_2} = iV_{26} N^2 \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-13} \eta(it)^{-24}. \tag{5.120}$$

Open versus closed channel

- In the UV-divergent limit $t \to 0$, the cylinder is infinitely long.
- The remarkable insight is the following: We can either view the long cylinder as describing an open string stretching between the boundaries at $\sigma^1 = 0, \pi$ and running in the loop described by the Euclidean time σ^2. Or, alternatively, we may interpret the annulus as a closed string propagating at tree-level from the left to the right. The two interpretations of the cylinder are referred to as open and closed string channel.



- Technically, the two viewpoints are related by interchanging the role of the Euclidean time and the spatial coordinate on the worldsheet. From our analysis of $PSL(2,\mathbb{Z})$ transformations of the torus on Assignment 10 we recall that an S-duality transformation $\tau \to -\frac{1}{\tau}$ exchanges the coordinates σ^1 and σ^2. The same applies to the cylinder with it taking the role of τ. Including a conventional rescaling of the spatial coordinate the transition from the open to the closed string channel is accomplished by

$$t \longrightarrow s = \frac{\pi}{t}. \tag{5.121}$$

With the help of the transformation of the Dedekind function

$$\eta(it) = t^{-\frac{1}{2}}\eta\Big(\frac{i}{t}\Big) = \Big(\frac{s}{\pi}\Big)^{\frac{1}{2}}\eta\Big(\frac{is}{\pi}\Big) \tag{5.122}$$

the annulus amplitude in closed string channel is

$$Z_{C_2} = iV_{26}\, N^2 \frac{1}{2\pi(8\pi^2\alpha')^{\frac{1}{2}}}\int_0^\infty ds\, \eta\Big(\frac{is}{\pi}\Big)^{-24}. \tag{5.123}$$

- The UV limit $t \to 0$ in the open channel has translated in the IR limit $s \to \infty$ of the closed channel. This describes a closed string tree-level process with the string propagating over long Euclidean time. Thus we have reinterpreted the UV-divergence as an IR-divergence. This is in fact a general feature of string amplitudes:

All UV divergencies in string amplitudes can be reinterpreted as IR divergencies of dual diagrams.

- In fact, we can make the propagation of the closed strings visible in the limit $s \to \infty$ by expanding

$$\eta\Big(\frac{is}{\pi}\Big)^{-24} = \underbrace{e^{2s}}_{\text{tachyon}} + \underbrace{24}_{\text{massless}} + \mathcal{O}(e^{-2s}). \tag{5.124}$$

The tachyonic term is again an artifact of the bosonic theory. Of importance is the second term. It shows that **the IR divergence is due to the exchange of massless closed string states at zero momentum.**

The contribution of each field X^μ, ψ^μ to the normal ordering constants $a_{\rm NS}$, $a_{\rm R}$ follows e.g. by ζ-function regularisation.

- From the discussion of the bosonic string we recall the following reasoning that led to the **normal ordering constant of one periodic boson**: From

$$L_0^{(b)} = \frac{1}{2}\alpha_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n + \frac{1}{2}\sum_{n=-\infty}^{-1}\alpha_{-n}\alpha_n \tag{6.84}$$

with

$$\frac{1}{2}\sum_{n=-\infty}^{-1}\alpha_{-n}\alpha_n = \frac{1}{2}\sum_{n=1}^{\infty}\alpha_n\alpha_{-n} = \frac{1}{2}\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n + \underbrace{\frac{1}{2}\sum_{n=1}^{\infty}n}_{=-a} \tag{6.85}$$

we concluded that

$$a = -\frac{1}{2}\sum_{n=1}^{\infty} n = -\frac{1}{2}\zeta(-1) = \frac{1}{24}. \tag{6.86}$$

- Likewise for **periodic fermions** we compute

$$L_0^{(f)} = \frac{1}{2}\sum_{n=1}^{\infty} r\, b_{-r} b_r + \underbrace{\frac{1}{2}\sum_{n=1}^{\infty}(-r) b_r b_{-r}}_{\frac{1}{2}\sum_{n=1}^{\infty} r\, b_{-r} b_r - \frac{1}{2}\sum_{n=1}^{\infty} r} \tag{6.87}$$

and conclude

$$a = -\frac{1}{24}. \tag{6.88}$$

- For anti-periodic bosons (upper sign) and anti-periodic fermions fermions (lower sign), the normal ordering constant is

$$a = \mp\frac{1}{2}\zeta(-1,q)|_{q=\frac{1}{2}} \equiv \mp\frac{1}{2}\sum_{n=0}^{\infty}(n+q)^{-1}|_{q=\frac{1}{2}} = \mp\frac{1}{48}. \tag{6.89}$$

This is summarised in the following table.

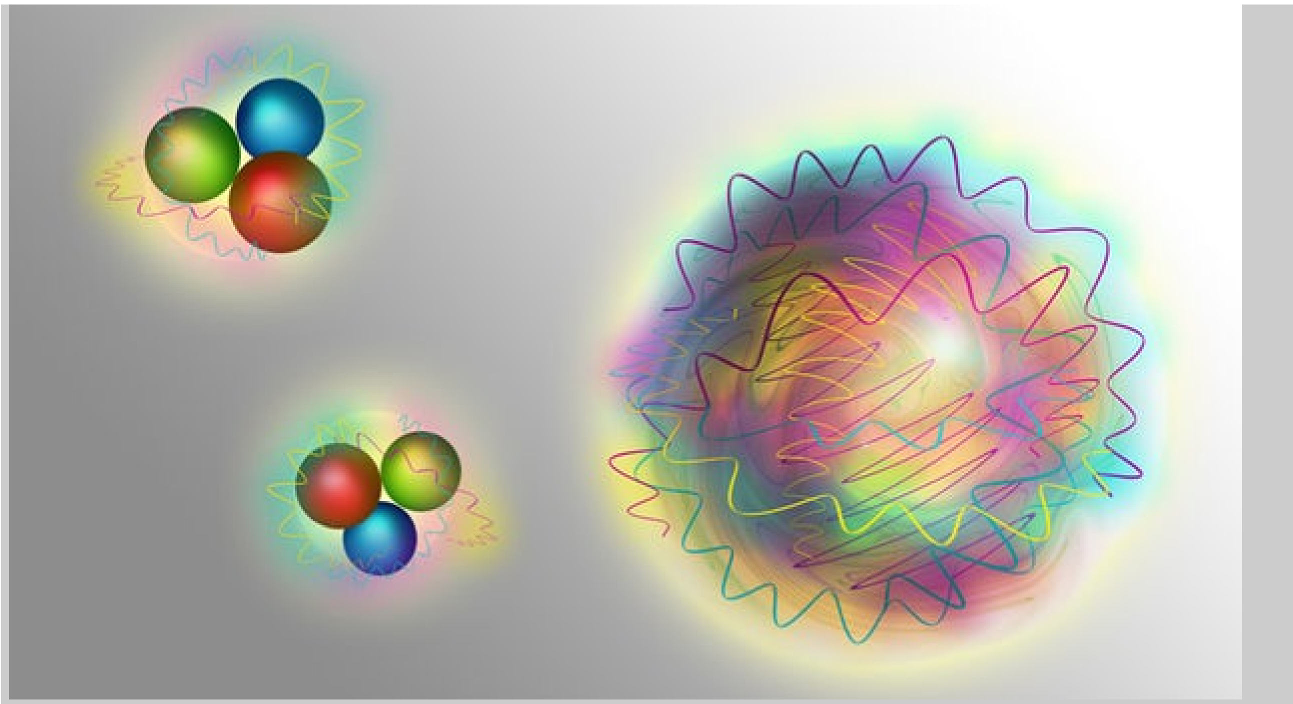
1 periodic boson	$a = +\frac{1}{24}$
1 anti-periodic boson	$a = -\frac{1}{48}$
1 periodic fermion	$a = -\frac{1}{24}$
1 anti-periodic fermion	$a = \frac{1}{48}$

(6.90)

We note as 12, 24 and 48 are fundamental numbers in the mathematics of string theory

Meson f0(1710) could be so-called "glueball" particle made purely of nuclear force

Colin Jeffrey
October 16th, 2015 (https://newatlas.com/meson-f01710-glueball-particle/39866/)



Researchers at TU Wien claim to have discovered the elusive "glueball" - a particle created from pure nuclear force(Credit: TU Wien)

Terms to describe the strange world of quantum physics have come to be quite common in our lexicon. Who, for instance, hasn't at least heard of a quark, or a gluon or even Schrodinger's cat? Now there's a new name to remember: "Glueball." A long sought-after exotic particle, and recently claimed to have been detected by researchers at TU Wien, the glueball's strangest characteristic is that it is composed entirely of gluons. In other words, it is a particle created from pure force.

First mooted as a particle in 1972 when physicists Murray Gell-Mann and Harald Fritsch wondered about possible bound states of recently-discovered gluons, scientists have sought the particle in the intervening decades. Originally dubbed "gluonium," but now called glueballs, these strange particles of pure force are exceptionally unstable and can only be indirectly detected by monitoring their decay as they disassemble into lesser particles.

More recently, physics Professor Anton Rebhan and his PhD student Frederic Brünner from TU Wien have theorized that a strong nuclear decay resonance, called f0(1710), observed in the data from a number of particle accelerator experiments is strong evidence for the elusive glueball particle.

Quarks are small elementary particles that make up such things as neutrons and protons. Binding these quarks together is the strong nuclear force which, in turn, couples the larger particles.

"In particle physics, every force is mediated by a special kind of force particle, and the force particle of the strong nuclear force is the gluon," said Professor Rebhan.

Elementary particles come in two kinds: those that carry force (bosons), such as photons, and those that make up matter (fermions), such as electrons. In this context, gluons may be viewed as more complex forms of the photon. However, as photons are the force carriers for electromagnetism, gluons exhibit a similar role for the strong nuclear force. The major difference between the two, however, is that gluons are able to be influenced by their own forces, whereas photons are not. As a result, photons cannot exist in force-bound states, though gluons, which are attracted by force to each other, make a particle of pure nuclear force possible.

In this way, many researchers believe that many of the unexplained particles discovered in particle accelerator experiments could indicate the presence of pure nuclear force particles, or glueballs. Contentiously, however, some scientists are of the opinion that the signals detected in the experiments may also just be some sort of conglomeration of quarks and antiquarks. This is particularly difficult to prove either way, though, as – whatever the mysterious particle is – it is too short-lived to be directly detectable.

Nevertheless, two mesons (a meson is a subatomic particle composed of one quark and one antiquark), entitled f0(1500) and f0(1710) have been determined via calculations to be the most likely candidates for the glueball particle. For some time, scientists believed that f0(1500) met many of the mathematical criteria for being the front-runner as the glueball particle, although much of this bias was also largely due the fact that many researchers believed that the production of heavy (strange) quarks in the decay of f0(1710) was implausible because gluon interactions do not normally distinguish between heavier and lighter quarks.

"Unfortunately, the decay pattern of glueballs cannot be calculated rigorously," said Professor Rebhan. "Our calculations show that it is indeed possible for glueballs to decay predominantly into strange quarks."

Despite the inconsistencies to accepted quark behavior, the decay pattern calculated by the two TU Wien researchers, which shows disassembly into two lighter particles, actually lines-up exceptionally well with the pattern measured for f0(1710). The researchers have shown that other decay patterns into two particles or more is possible, and have also calculated their decay rates.

Though these alternative glueball decays have yet to be measured, two experiments to be conducted at the Large Hadron Collider at CERN (TOTEM and LHCb) and one accelerator experiment in Beijing (BESIII) over the next few months are expected to produce data that will hopefully support the TU Wien researcher's hypothesis.

"These results will be crucial for our theory," said Professor Rebhan. "For these multi-particle processes, our theory predicts decay rates which are quite different from the predictions of other, simpler models. If the measurements agree with our calculations, this will be a remarkable success for our approach."

If the measurements and calculations do, in fact, agree, the evidence for f0(1710) being a glueball would be highly credible. Such a confirmation would also once again demonstrate that higher dimensional gravity research can be effectively utilized to solve particle physics problems. According to the researchers, this would be one more overarching support of Einstein's theory of general relativity, the centenary of which occurs next month.

The results of this research were recently published in the journal Physical Review Letters.

Appendix B

From: "SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS" MAXIE D. SCHMIDT - https://arxiv.org/abs/1609.02803v2

Corollary 4.7 (Special Values of Ramanujan's φ-Function). *For any $k \in \mathbb{R}^+$, the variant of the Ramanujan φ-function, $\varphi(e^{-k\pi}) \equiv \vartheta_3(e^{-k\pi})$, has the integral representation*

$$\varphi\left(e^{-k\pi}\right) = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{k\pi}\left(e^{2k\pi} - \cos\left(\sqrt{2\pi k}t\right)\right)}{e^{4k\pi} - 2e^{2k\pi}\cos\left(\sqrt{2\pi k}t\right) + 1}\right] dt. \tag{33}$$

Moreover, the special values of this function corresponding to the particular cases of $k \in \{1, 2, 3, 5\}$ in (33) have the respective integral representations

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}\left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi}\cos\left(\sqrt{2\pi}t\right) + 1}\right] dt \tag{34}$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{2}+2}}{2} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{2\pi}\left(e^{4\pi} - \cos\left(2\sqrt{\pi}t\right)\right)}{e^{8\pi} - 2e^{4\pi}\cos\left(2\sqrt{\pi}t\right) + 1}\right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{3}+1}}{2^{1/4}3^{3/8}} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{3\pi}\left(e^{6\pi} - \cos\left(\sqrt{6\pi}t\right)\right)}{e^{12\pi} - 2e^{6\pi}\cos\left(\sqrt{6\pi}t\right) + 1}\right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{5+2\sqrt{5}}}{5^{3/4}} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{5\pi}\left(e^{10\pi} - \cos\left(\sqrt{10\pi}t\right)\right)}{e^{20\pi} - 2e^{10\pi}\cos\left(\sqrt{10\pi}t\right) + 1}\right] dt.$$

From the first of (34):

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}\left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi}\cos\left(\sqrt{2\pi}t\right) + 1}\right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4{,}44288293815}{3{,}625609908} = 1{,}2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1{,}3313353638}{1{,}2254167025} = 1{,}08643481 \ldots$$

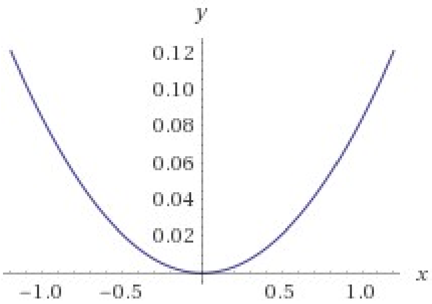
For the integral, we have calculate as follows:

integrate [(2.71828^0.89)/(sqrt6.283185307)][4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)1.33416))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)1.33416))+1]x

Indefinite integral:

$$\int \frac{2.71828^{0.89} \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 1.33416\right)\right)\right) x}{\sqrt{6.283185307} \left(e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798 x^2 + \text{constant}$$

Plot of the integral:



(x from −1.2 to 1.2)

Alternate form assuming x is real:

$$0.0837798 x^2 + 0 + \text{constant}$$

Thence: 1 + 0.0837798 = 1.0837798

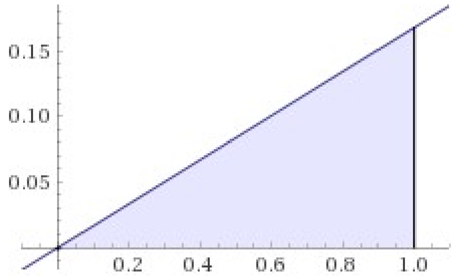
and:

integrate [(2.71828^0.89)/(sqrt6.283185307)][4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)1.33416))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)1.33416))+1] x, [0, 1]

Definite integral:

$$\int_0^1 \frac{2.71828^{0.89} \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 1.33416\right)\right)\right) x}{\sqrt{6.283185307} \left(e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798$$

Visual representation of the integral:



Open code

Riemann sums:

left sum	$0.0837798 - \frac{0.0837798}{n} = 0.0837798 - \frac{0.0837798}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$

(assuming subintervals of equal length)

Indefinite integral:

$$\int \frac{2.71828^{0.89}\left(4\, e^{3.14159265}\left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 1.33416\right)\right)\right) x}{\sqrt{6.283185307}\left(e^{12.56637} - \left(2\, e^{6.283185307}\right)\left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798\, x^2 + \text{constant}$$

Thence: 1 + 0.0837798 = 1.0837798

With regard the integral, from 0 to 0,58438 for t = 2, where (2.71828^2)/(sqrt6.283185307) = 2,94780 for t=2, we have:

integrate (2.94780)[4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)2))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)2))+1] x, [0,0.58438]

$$\int_0^{0.58438} \frac{2.94780\left(4\, e^{3.14159265}\left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 2\right)\right)\right) x}{e^{12.56637} - \left(2\, e^{6.283185307}\right)\left(\cos\left(\sqrt{6.283185307}\right) 2\right) + 1} dx = 0.0864364$$

Thence, 1 + 0,0864364 = 1,0864364; 1,08643481 ≅ 1,0864364.

In conclusion, the value of this, defined by us, "New Ramanujan's Constant" is 1.08643.

In this and others our papers, we have used 1,08643 as a new "Ramanujan's constant" and we can see as this constant is fundamental for some results that we have obtained in various equations analyzed and developed.

References

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S. Ramanujan "**Modular equations and approximations to π**" - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

S. Ramanujan "**Theorems on summation of series; e.g**." - page extracted from original manuscript