Riemann Hypothesis Yielding to Minor Effort—Part III:

Ubiquitous Matching Paradox Irrelevant if Reduced to Extensions

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ABSTRACT[1]

This paper suggests one way around the issue of *complex matching* (ordering or comparison) which may give rise to [excessive] multiplicity of RH solutions beyond branching. More importantly, the RH appears to posit some counter-intuitive analogies in areas as diverse as Mikusinski operators and Euler equation (EE), which analytical or variational extensions could be seen as one way in which the general (or complete) bears on simplicity (in line with the *orduale programme*). While both could be seen as precursors or special approaches to functional analysis, these representational parallels have been arrived at from very distinct standpoints without any prior regard for each other. Arcane realizations such as the Veblen ordinals could further result from mere juxtaposition of the two.

Keywords: Euler equation (variations), Mikusinski operators (generalized functions or distributions), Veblen ordinals

Paths Most Traveled By: Lessons [Almost] Learned

If the jaded reader has by any chance noticed my first paper in the series (*viXra:1904.0235*), they may definitely have had their grudge and second thoughts on how the N-multiplicity of solutions should be construed. Invoke the complex non-orderability of sets (or awkward matching of series terms within the ERE), and this becomes a [lame] excuse and a source of mounting paradoxes or indeterminacy in its own right (dub it another continuum hypothesis or its variety if you like). The silvery lining, though, would be that one can still think of remedies so amazingly sparing and parsimonious, this low a deliberation cost alone could put them into question (unless one has lost the nerve to cope with more paradoxes). Indeed, the

[1] WP20190421-0506 *In memoriam* the kids of the Zimnyaya Vishnya mall fire incident, the Japanese school children that were stabbed by a knife attacker, and all those innocent martyrs that have fallen prey to accidental maniacs' mental (and spiritual) disorder while praying their way or to geopolitical fanatics' insane ambition without having any discretion or say over.

averages (as appropriately chosen) of any equivalence (be it ERE or otherwise) could be seen as *[self] identity* while positing one way around the bijective mapping or matching chasm.

Now revert to the exponential representation of the Riemann zeta (not necessarily considered at zero or any particular value) to see that:

$$(1)\ \ \widehat{N^{-s}} \equiv (\widehat{\frac{log^N\zeta}{N!}}) \leftrightarrow \widehat{N}^{-s} = \frac{log^{\widehat{N}}\zeta}{\widehat{N}!}$$

The N-hjatt could be seen as non-linear (CES or Lame type) averaging, and might not exactly seem to allow for unique match beyond 'reasonable doubt.' That said, considering a summation over the respective left- and right-hand side terms, one arrives at zeta identically, in which light the interpretation holds (at least weakly) for all practical purposes for lack of superior alternatives. Moreover, by assuming away the somewhat Diophantine nature of discerning the zeta (or *s*) from the above without any prior knowledge of what it is the N average amounts to, one can still think of a potential shortcut (otherwise confined to an open-form, implicit-functional reduction) by invoking:

$$(1.1)\ \ \widehat{N} \equiv (\frac{\zeta}{T})^{-1/s}, T \to \infty$$

So Much for Minkowski *Raum*: Making Room for Mikusinski Space

Multiply, for simplicity, both sides of (1) times N-hat, arriving at:

$$(2)\ \ \widehat{N}^{1-s} = \frac{log^{\widehat{N}}\zeta}{\Gamma(\widehat{N})}$$

Somehow, this immediately led my imagination all the way back to the *Mikusinski operators* as a kind of [dual] analogy, as in[2]:

$$(A)\ \ (\underline{s} - \alpha)^{-\lambda} = \left\{\frac{t^{\lambda-1}}{\Gamma(\lambda)} e^{\alpha t}\right\}$$

Now, by assessing the above at alpha equal 0 versus 1 and *(t, lambda-1)* identically matched to *(log zeta, N-hat)*, it follows that:

$$(2.1)\ \ (\underline{s} - \alpha)^{-(\widehat{N}+1)} = \left\{\frac{log^{\widehat{N}}\zeta}{\Gamma(\widehat{N}+1)}\zeta^{\alpha}\right\} = \left\{\widehat{N}^{-s} * \zeta^{\alpha}\right\} = \left\{\frac{\zeta^{1+\alpha}}{T}\right\}$$

[2] In the original (Mikusinski, 1953), he deploys the definition: $s^{-1} \equiv l$. However, in order to avoid notational confusion with an eye on the similarly looking zeta and differentiation operator, I opted to proceed with, $\underline{s} \equiv L^{-1}$ likely to come in handy throughout.

$$(2.1A)\quad (\underline{s}-1)^{-(\widehat{N}+1)} = \{\widehat{N}^{-s} * \zeta\} = \{\zeta^2/T\}$$

$$(2.1B)\quad \underline{s}^{-(\widehat{N}+1)} = \{\widehat{N}^{-s} * 1\} = \{\zeta/T\}$$

In order to reconcile (2.1) and (2), one may want to do away with the braces, which could be straightforward by invoking their nature as stemming from that of the integration (or inverse *s*) operator. The two latter equivalences can thus be rewritten to arrive at[3],

$$(2.1')\quad (\underline{s}-\alpha)^{-(\widehat{N}+1)} * \underline{s} = \frac{\zeta^{1+\alpha}}{T}$$

$$(2.1A')\quad (\underline{s}-1)^{-(\widehat{N}+1)} * \underline{s} = \frac{\zeta^2}{T} = \widehat{N}^{-s} * \zeta$$

$$(2.1B')\quad \underline{s}^{-(\widehat{N}+1)} * \underline{s} = \underline{s}^{-\widehat{N}} = \frac{\zeta}{T} = \widehat{N}^{-s}$$

The latter suggests the initially conjectured duality:

$$(2.2)\ \underline{s}^{-\widehat{N}} = \widehat{N}^{-s}$$

Final reconciliation yields:

$$\widehat{N}^{-s} = \frac{log^{\widehat{N}}\zeta}{\Gamma(\widehat{N}+1)} = \frac{(\underline{s}-\alpha)^{-(\widehat{N}+1)} * \underline{s}}{\zeta^{\alpha}}$$

[3] Strictly speaking, the braces may have to be treated along the lines of: $s\{a(t)\} = \{a'(t)\} + a(0)$. However, for special cases like $s^{-1} = \{1\}$, clearly $1 = s^{-1}s = s\{1\} = 1$. This need not hold for constants only: By multiplying the general definition as in (A) at alpha=0, one obtains that $\underline{s}^{-\lambda}\underline{s} = \underline{s}\left\{\frac{t^{\lambda-1}}{\Gamma(\lambda)}\right\} = \underline{s}\underline{s}^{-1}\frac{t^{\lambda-1}}{\Gamma(\lambda)}$, or $\underline{s}^{1-\lambda} = \frac{t^{\lambda-1}}{\Gamma(\lambda)}$. In fact, this does hold for lambda anywhere near 1, or for $a(t) \equiv \frac{t^{\lambda-1}}{\Gamma(\lambda)}$, such that $a(0) = 0$ outside that neighborhood or 1 within it. Not least, bearing in mind that $e^{-\lambda s}\{f(t)\} = f(t-\lambda)$ for $\lambda < t$ and 0 otherwise, by considering the overlap or convergence at $\lambda \to t \leftarrow -\frac{logs}{s}$, it follows that $s\{f(0)\} \to f(0)$ as long as $s^{\pm 1/s}$ tends to 1 yet not in general. Incidentally, the same could be attained insofar as $\lambda \to t \leftarrow 0^{-}$could suggest that $e^{-\lambda} \sim loge = 1$—which may, at the same time, provide one alternate solution for (2.1B), namely (perhaps valid for large N-average only)

$$(2.1B'')\quad \underline{s}^{-(\widehat{N}+1)} = \left\{\frac{log^{\widehat{N}}\zeta}{\Gamma(\widehat{N}+1)}log\zeta\right\} = \left\{\frac{log^{\widehat{N}+1}\zeta}{\Gamma(\widehat{N}+1)}\right\}$$

In actuality though, $\underline{s} = loge^{\underline{s}} = -\frac{1}{\lambda}\lim_{\lambda\underline{s}\to 0^{-}}\frac{e^{-\lambda\underline{s}}-1}{-\lambda\underline{s}}$, which implies $e^{-\lambda\underline{s}} \sim 1 + (\lambda\underline{s})^2 = 1 + 0^2 = 1$. This reworks the initial relationship as, $1\{f(t)\} = f\left(t+\frac{0}{\underline{s}}\right)$, i.e. either $s^{\pm\lambda}\{f(t)\} = f(t)$ or $\underline{s}^h\{f(t)\} = f(\underline{s}t) = \begin{cases} f(1) \\ f(0) \end{cases}$, where the *h* power points to homogeneity degree and the argument-differentiation outcomes to the corner values of the convolution while depending on whether *t* pertains to a *line* (variable) or a specific *point* (value). Alternatively, $\underline{s}^{h-1}f(t) = \underline{s}^h\{f(t)\} = f(\underline{s}t) = \underline{s}^h f(t)$ iff the function is either homogeneous of degree $\pm T$ (infinite) or is represented by its specific value (varying around a constant). Incidentally, this may befit either a zero function (such as the zeta at its specific 0 value or e^{log0}representation) or the respective reduction of the $\widehat{N}^{-S}$ terms to characteristic powers of zero (both proposed in the previous two papers on the RH).

Summation over (2.2) running *ad infinitum* returns the zeta:

$$(2.2')\ T\underline{s}^{-\widehat{N}} = \zeta$$

Based on this alone, the nature or estimate of N-averaged can be inferred as,

$$(2.2A')\ \ \widehat{N} = \frac{logT - log\zeta}{log\underline{s}}$$

Alternatively, the solution per a particular zeta amounts to,

$$(2.2B')\ \ s = \frac{\widehat{N}log\underline{s}}{log\widehat{N}} = \frac{logT - log\zeta}{log\widehat{N}}$$

Further reconciliation obtains by dividing, respectively, *(2.1')* over *(2.1B')* and *(2.1A')* over *(2.1B')*:

$$(2.1'')\ \ (\frac{\underline{s} - \alpha}{\underline{s}})^{-(\widehat{N}+1)} = \zeta^{\alpha}$$

$$(1 - \frac{1}{\underline{s}})^{-((\widehat{N}+1))} = \zeta$$

Herr Euler & Monsieur Veblen Shall Make 'Em Equal (or Reconciled at Any Rate)

Now reconsider (1) in terms of:

$$\widehat{N}! = \left[(\frac{log^{\widehat{N}}\zeta}{\widehat{N}!})^{-1/s}\right]!$$

More generally, for any [interim or interior] X:

$$X = \left[(\frac{log^{X}\zeta}{X!})^{-1/s}\right]$$

$$\prod_{X}^{\widehat{N}} X \equiv \widehat{N}! = \left[(\frac{log^{\Sigma_X^{\widehat{N}} X}\zeta}{\prod_X^{\widehat{N}} X\,!})^{-1/s}\right]$$

$$(3)\ \ \widehat{N}! = (\widehat{N}!\,log^{-\widehat{N}}\zeta)^{1/s}! = (\Pi X!\,log^{-\Sigma X}\zeta)^{1/s}$$

One is then moved to generalize (3) along the following lines (while setting zeta at zero):

$$(3')\ \ \widehat{N} = h \circ \left[\varphi\big(\widehat{N}\big) * 0^{\widehat{N}}\right] = \varphi^{-1} \circ h \circ [\Pi\varphi(\mathrm{X}) * 0^{\Sigma X}]$$

It is straightforward to observe that the phi stretching generalizes the factorial or gamma, while *h(.)* pertains to taking the argument to the power of 1/s, such that:

$$(3.1)\quad \gamma(X) \equiv \varphi(X) * 0^X$$

$$(3.A)\; X = h \circ \gamma(X) = \varphi^{-1} \circ h \circ [\gamma(X) * \prod_{\xi}^{X-1} \gamma(\xi)] = \varphi^{-1} \circ ([h \circ \gamma(X)] * \left[h \circ \prod_{\xi}^{X-1} \gamma(\xi)\right])$$

Not only is *h(.)* homogeneous[4], its associativity with respect to [lowercase] gamma over products (as the flipside of homogeneity) is coupled with their being mutually inverse:

$$h \circ \prod_{\xi}^{X-1} \gamma(\xi) = \prod_{\xi}^{X-1} h \circ \gamma(\xi) = \prod_{\xi}^{X-1} \xi = \frac{X!}{X}$$

In particular, for $X \equiv \widehat{N} = (\frac{\zeta}{T})^{-1/s}$,

$$\frac{T}{\zeta} = \gamma \circ (\frac{T}{\zeta})^{1/s} \equiv \gamma \circ h \circ \frac{T}{\zeta}$$

$$(3.B)\;\; X = h \circ \gamma(X) = \gamma \circ h(X)$$

This could be one other instance showcasing how the line could be fuzzy between associativity versus commutativity (*viXra:1905.0210*), while tracing up to a shared source or pattern.

Incidentally, the above could serve as basis for inferring a peculiar yet instrumental calculus based on a particular zeta value, e.g.:

$$0^s = 1 = 1^{s^2},\; 1^s = 0,\; 2^{s-1} = 0^2 = 1^{2s} = (\pm T)^{-2},\; 1^{s^3} = 0^{s^2}$$

The inference and implications will for now be assumed away alongside the *non*-zero zeta extensions for (*3'*). Suffice it at this stage to invoke the *Euler equation* so as to possibly detect some common structural representations[5]. The square bracket part (taken at zero in line with the variational lemma) looks (as per the special monodimensional case) as follows:

$$[.] \equiv \left[\frac{\partial F}{\partial f} - \frac{d}{dx}\frac{\partial F}{\partial f'}\right] = 0$$

[4] Any restriction or 'narrowing' of the form, $h(ab) = h(a)h(b)$, should do the candidate generalizing job.

[5] The careful reader will have observed the previous paper did propose that the RH bears some striking resemblance to variational foundations. Whilst it may, in hindsight, seem like $\eta(x) = 1$ was an unfortunate choice for a candidate 'compactly supported function' if only because it never equals zero at the corners, still it could be seen as a corner case of a tossup representation as its power share tends to a very small value: $\exists\alpha$: $0 = \Sigma\widehat{N}^{-s} \equiv \int_a^b [.]\,\eta(\xi)d\xi = T * \frac{\zeta}{T} = T * 0^2 = T * 0^{2(1-\alpha)} * 0^{2\alpha}$, such that $\eta(T) = \eta(1) = 0^{2\alpha} = \begin{cases} 0, & \alpha > 0 \\ 1, & \alpha \to 0 \end{cases}$.

Now rework the above as follows, by applying the proposed convention:

$$(4)\ \forall a:\ \frac{\partial}{\partial a} \equiv L_a^{-1} \equiv s_a \equiv \tilde{a}$$

In this light, the EE takes on the following form:

$$(4.A)\ L_f^{-1} = L_x^{-1} \circ L_{L_x^{-1} \circ f}^{-1}$$

$$(4.A')\ \ s_f = s_x \circ s_{s_x \circ f}$$

$$(4.B)\ \tilde{f} = \tilde{x} \circ \widetilde{\tilde{x} \circ f}$$

For it to resemble (3.A) or (3.B), it would have to look as follows:

$$\tilde{f} \equiv \tilde{x} \circ \tilde{x}^{-1} \circ \tilde{f} = \begin{cases} \tilde{h} \circ \tilde{\gamma}(\tilde{f}) \\ \tilde{\gamma} \circ \tilde{h}(\tilde{f}) \end{cases}$$

From comparing with (4.B), this necessitates

$$(5)\ \ \widetilde{\tilde{x} \circ f} = \tilde{x}^{-1} \circ \tilde{f}$$

In particular,

$$(5.1)\ \ \widetilde{\tilde{h} \circ f} = \tilde{\gamma} \circ \tilde{f}$$

$$(5.2)\ \ \widetilde{\tilde{\gamma} \circ f} = \tilde{h} \circ \tilde{f}$$

One straightforward way of generalizing beyond—indeed supplying a reconciliatory extension of both—the Mikusinski and Euler equivalences could be to violate (5) on margin or in major ways.

Even in its present form, though, (5) can be reduced to a fairly involved:

$$(5.1')\ f^{\widetilde{\frac{\underline{s}_1}{s}}} = \begin{cases} 0^{\tilde{f}} * \underline{s}_\Gamma \circ (\tilde{f} + 1) \\ \tilde{f}^{\underline{s}_s} \end{cases}$$

$$(5.2')\ \tilde{f}^{\frac{\underline{s}_1}{s}} = \begin{cases} \underline{s}_{0^f * \underline{s}_\Gamma \circ (f+1)} \\ \widetilde{f^{\underline{s}_s}} \end{cases}$$

In passing, it remains to be seen whether or not this resembles *Veblen ordinals*.