The anomalous magnetic moment: classical calculations

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11 June 2019

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Abstract: Critics of the *Zitterbewegung* interpretation of quantum mechanics often ask what *predictions* come out of the model. The answer to this question must be modest: the model does not yield anything new or explain something that had hitherto remained unexplained. The *Zitterbewegung* interpretation just comes across as being more logical and consistent because it provides a *realist* interpretation of what the wavefunction actually *is*. However, in order to gain some basic credibility, *zbw* theorists should do more of an effort to show how the model explains standard results obtained by mainstream research. It should, for example, explain the anomalous magnetic moment as measured in single-electron cyclotron experiments. If it could do so in a convincing way, then it should be recognized as a valid and alternative interpretation of quantum mechanics.

This paper explores the geometry of the *zbw* model in very much detail and argues it can be done. In this paper, we do the calculations assuming the naked *zbw* charge has *zero* rest mass, and we find an anomalous magnetic moment that's off by a factor equal to $4/\pi \approx 1.27$ only. This is quite encouraging because we've used approximative formulas to calculate volumes. In addition, the model has a flexible assumption (the rest mass of the naked zbw charge may have some value *close* to zero rather than exactly zero) which we haven't exploited yet.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations, realist interpretation of quantum mechanics, anomalous magnetic moment, classical explanation of anomalous magnetic moment.

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The *zbw* oscillator model of an electron

There are several varieties of the *Zitterbewegung* model (Hestenes, 1990, 2008 and Burinskii, 2008, 2016, 2017). In our previous papers[1], we presented a very simple model – but it is one that is consistent with the Dirac's original interpretation.[2] It is probably useful to repeat the basics. We took Einstein's mass-energy equivalence relation (E = m·c^2) and, interpreting c as the tangential velocity of the naked charge (or the toroidal photon, as Burinskii refers to it[3]), substituted c for a·ω: the tangential velocity equals the radius times the angular frequency. We then can then use the Planck-Einstein relation (E = ħ·ω) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – packs (i) the electron's energy (E = E = m·c^2) and (ii) one unit of physical action (ΔS = h). The idea of an oscillation packing some amount of physical action may not be very familiar but it is quite simple: just re-write the Planck-Einstein relation as h = E·f = E/T. The cycle time T = h/E is equal to:

$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

Hence, this cycle time T is the time it takes for the *zbw* charge (or the naked charge, if you prefer that term) to go around the loop (λ_C) at the extreme velocity we assume it has ($v = c$):

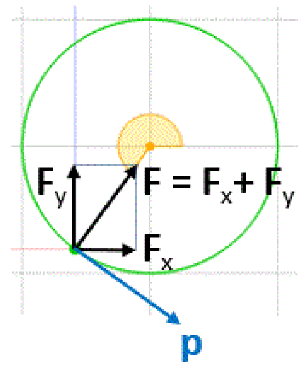
$$\mathrm{T} = \lambda_C/c = (h/\mathrm{m}c)\cdot(1/c) = h/\mathrm{E}$$

Physical action is the product of force, distance and time and it is, therefore, easy to associate it with the idea of a cycle—and elementary cycle, in this particular case. It is just an interpretation of the general quantum-mechanical rule that angular momentum comes in units of ħ: the ΔS = n·h and ΔL = n·ħ (n = 1, 2,…) are, therefore, equivalent.[4]

[1] See: Jean Louis Van Belle, *The electron as a harmonic electromagnetic oscillator*, 1 June 2019, http://vixra.org/abs/1905.0521.
[2] It was Erwin Schrödinger who stumbled upon the idea of the *Zitterbewegung* when he was exploring solutions to Dirac's wave equation for free electrons. It's worth quoting Dirac's summary of it: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)
[3] See Alexander Burinskii's 2008, 2016 and 2017 publications.
[4] We write ΔS and ΔL (instead of S and L) to show we are actually talking about the *difference* in action/momentum between two states with different energy levels. The associated difference in energy is ΔE = $\mathrm{E}_2 - \mathrm{E}_1 = (n_2 - n_1)\cdot\hbar\cdot\omega = (n_2 - n_1)\cdot h\cdot f$.

Figure 1 illustrates the model. We have a centripetal force (F) holding our *zbw* charge (the naked charge, which has zero rest mass) in its circular orbit around some center.

Figure 1: The *Zitterbewegung* model of an electron



Because the naked charge goes around at the speed of light (or *almost* the speed of light, as we will argue later), it acquires some mass which we'll denote as m_γ. We use the γ subscript here because the *zbw* charge does behave like a photon here: it acquires relativistic mass because of its extreme velocity. The only thing is that our *zbw* charge also has electric charge (all of the charge of the electron, in fact), which a photon doesn't have, of course! The point is: the zbw charge will also have some non-zero momentum $p = m_\gamma v = \gamma m_0 v = \gamma m_0 c$, even if m_0 (the rest mass of the naked charge) is zero.

Now, the angular momentum of the electron is equal to ħ/2 or some value very close to it.[5] We also know that angular momentum should be equal to the length of the lever arm (a) and the momentum $p = m_\gamma c$, so p is equal to $p = L/a$. It is useful to note that this formula – just like the others – is relativistically correct, so one should not cry wolf here. Hence, we get the following result:

1. $L = \hbar/2 \Leftrightarrow p = L/a = (\hbar/2)/a = (\hbar/2)\cdot mc/\hbar = mc/2$
2. $p = m_\gamma c$

$\Rightarrow m_\gamma c = mc/2 \Leftrightarrow m_\gamma = m/2$

This is the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is half of the (rest) mass of the electron. We interpreted this result in terms of a mathematical *equivalence* between the rotational motion and a two-dimensional oscillation—one perpendicular to (and, therefore, independent from) the other, each packing half of the total energy of the electron:

$$E_x = E_y = m_\gamma \cdot a^2 \cdot \omega^2 = m_\gamma \cdot c^2 = m \cdot a^2 \cdot \omega^2/2 = m \cdot c^2/2$$

Notation can be confusing here. E_x and E_y are often used to refer to the x- and y-component of the electric field vector (***E***), but that is not the case here: E_x and E_y is the *energy* (E) associated with the oscillation in the x- and y-direction respectively.

You may wonder if the math holds for relativistic speeds. Indeed, if the velocity of a mass on a spring – on *two* springs, in this case – becomes a sizable fraction of the speed of light, then we can no longer

[5] The anomalous magnetic moment or – to be precise – the anomalous g-ratio suggest angular momentum or magnetic moment, or both, are slightly off.

treat the mass as a constant factor: it will vary with velocity, and its variation is given by the Lorentz factor (γ). We will come back to this, but we quickly want to show our reasoning is consistent. The velocity in the *x* and *y* direction goes from *c* to –*c*, and is zero when the pointlike charge reverses direction, which is at distance *a* and –*a* from the center. Hence, at these two points, the rest mass of our *zbw* charge is, effectively, equal to $m_0 = 0$. The charge only acquires some non-zero mass because of its velocity. We write:

$$m_v = \gamma m_0 \text{ for } 0 \leq v \leq c$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

The relativistically correct force equation for *one* oscillator is:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

We do not need to solve this equation. We only want to get an energy conservation equation from it. We do so by multiplying both sides with $v = dx/dt$ and then some re-arranging yields the following:

$$v\frac{d(\gamma m_0 v)}{dt} = -kxv \Leftrightarrow \frac{d(m_v c^2)}{dt} = -\frac{d}{dt}\left[\frac{1}{2}kx^2\right] \Leftrightarrow \frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}kx^2 + m_v c^2\right] = 0$$

So what is the energy concept here? We recognize the potential energy: it is the same $kx^2/2$ formula we get for the non-relativistic oscillator. No surprises: potential energy depends on position only, not on *velocity*. However, the $(½)m_0v^2$ term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the $m_vc^2 = \gamma m_0c^2$ term. Both energies vary – with position and with velocity respectively – but the equation above tells us their *sum* is some constant. In fact, that's we refer to it as an energy conservation equation. So we can imagine the same game: two oscillators working in tandem. The reader should remind him- or herself that m_v is *not* the mass of the electron: it is the mass of the oscillating *zbw* charge. Hence, we can calculate the total energy by equating *x* to 0 (the charge passes the center of its oscillation along the horizontal or vertical axis): $kx^2/2$ will then be equal to zero, and m_vc^2 will be equal to $m_cc^2 = m_cc^2 = \gamma m_0c^2 = mc^2/2$. The m in the latter expression is, effectively, the rest mass of the electron.

The *zbw* electron as a perpetual current

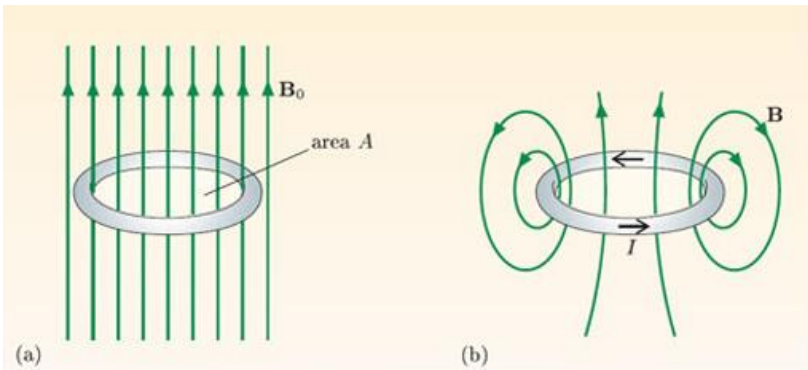
The zbw model of an electron can also be described in terms of a perpetual current in some sort of superconducting ring, as illustrated in Figure 2. As such, we think of its energy in terms of an electromagnetic field. To be precise, we will think of a magnetic field keeping the *zbw* charge in its orbit. This explains Hestenes' interpretation of the *zbw* model of an electron, which is equivalent to the oscillator model, and which he summarizes as follows:

> "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field."[6]

[6] Email from Dr. David Hestenes to the author dated 17 March 2019.

You may think this interpretation has a problem because we do not have any *real* material ring or wire in free space to hold and our guide our charge. However, the more advanced calculations of Hestenes (1990, 2008) and Burinskii (2008, 2016 and 2017) show that the scale and the magnitude of the force and the other variables don't require any wiring: Nature has tuned this LC circuit perfectly well.

Figure 2: A perpetual current in a superconducting ring[7]



We explored many interesting properties and implications of this model in the mentioned paper (we mentioned, most notably, that this allows for a realist interpretation of the wavefunction) but we won't repeat these here. We would just like to make one small point on the energy density inside the loop. We do so because we said little or nothing about that in our previous analysis. Let us use the *metaphor* of that superconducting ring again to analyze this. Figure 2(a) above shows a uniform magnetic field going through that ring made of superconducting material. The idea then is that we would cool the ring below the critical temperature and switch off the field. Lenz's law – Faraday's law of induction, really – then tells us the *change* in the magnetic field (so that's us flipping the *kill* switch, basically) will induce an electromotive force. Hence, we get an *induced* electric current, and its direction and magnitude will be such that the magnetic flux it generates will compensate for the flux change: the induced current in the superconducting circuit will just maintain the flux through the ring at the same value. However, while the flux will the same, you should note that the field looks different now: in Figure 2(a), we have a *uniform* magnetic field within the ring – the field in our apparatus, really – while in Figure 2 (b) we have a field that's produced by the current flowing in the ring now. The new field gives us the same flux, but the field density is now much larger close to the ring, and the field density at the center is rather weak, even if the total flux has the same value.

Why is this important? It is important because we will probably want to know, at some point in the analysis, where the (field) energy is actually located. Why? Our $m_\gamma = m/2$ formula establishes an equivalence between:

1. The moment of inertia of a point mass m_γ at a distance $r = a$ from the axis of rotation: $I = m_\gamma \cdot a^2$.
2. The moment of inertia of a *disk* with radius r and mass m: $I = m \cdot a^2/2$.

[7] Source: Open University, Superconductivity, https://www.open.edu/openlearn/science-maths-technology/engineering-and-technology/engineering/superconductivity/content-section-2.2#.

Hence, we must show that – somehow – the energy (or *mass*[8]) effective mass of the electron will be spread over the disk. *If* we assume its energy – and, therefore, its mass – is spread uniformly over the whole disk[9], *then* we can use the 1/2 form factor for the moment of inertia (I). Hence, we conceptually distinguish the moment of inertia of the pointlike charge (I_γ) and the moment of inertia of our electron (I_γ), and we write:

$$(1)\ \mathrm{L} = I_\gamma \cdot \omega = \mathrm{m}_\gamma a^2 \cdot \frac{c}{a} = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\mathrm{m}c^2}{\hbar} = \frac{\hbar}{2}$$

$$(2)\ \mathrm{L} = I_\mathrm{e} \cdot \omega = \frac{\mathrm{m}a^2}{2} \cdot \frac{c}{a} = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\mathrm{m}c^2}{\hbar} = \frac{\hbar}{2}$$

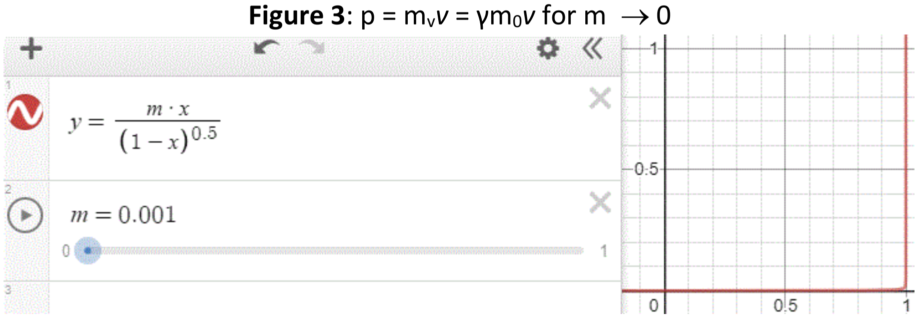
You may think this is rather obvious, but it isn't. It is a very deep and philosophical point. The energy is in the motion, but there is also energy in the magnetic field and we should, therefore, show how the magnetic energy is spread uniformly over the whole disk to validate the second of the two equations above. We haven't had the time to delve in this matter. The magnetic field becomes *weaker* as r goes to 0, and we know the energy *density* is proportional to the *square* of the magnetic field. Hence, if the magnetic field drops off as we move from the current ring to the center, we'd expect energy and, therefore, mass densities to *decrease* exponentially. This is a paradox which, hopefully, will not be too difficult to solve. We hope it's not a spoiler!

Let's move to the main topic of this paper.

The rest mass of the *zbw* charge

The *Zitterbewegung* model of an electron – or most *flavors* of that model, at least – assume the rest mass of the pointlike charge is zero. The idea here is Wheeler's idea of mass without mass: the naked charge has no mechanical mass. Its motion is what gives it mass. That mass is nothing but the equivalent mass of the energy in its motion. However, it is actually tempting to entertain the hypothesis that our *zbw* charge would have some *very tiny* mass. The reason is the following: the $p = m_\gamma v = \gamma m_0 v = \gamma m_0 c$ involves the product of zero (m_0) and infinity (γ for $v = c$), and such product does not seem to make any sense mathematically. To illustrate the issue, we used an online graphing tool (desmos.com) to illustrate what happens with the $p = m_v v = \gamma m_0 v$ function for m = 0.001 and v/c ranging between 0 and 1.

Figure 3: $p = m_v v = \gamma m_0 v$ for m $\to 0$



[8] Einstein's mass-energy equivalence relation – written as $E/m = c^2$ here – tells us that energy and mass are linearly proportional, and that the constant of proportionality is equal to c^2.
[9] This is a very essential point. It is also very deep and philosophical. We say the energy is in the motion, but it's also in the oscillation. It is difficult to capture this in a mathematical formula. In fact, we think this is the key paradox in the model.

It is quite enlightening: p is (very close to) zero for v/c going from 0 to 1 but then becomes infinity at v/c = 1 itself. This is, obviously, not a regular function: we don't have a unique value for it at v/c = 1. What can we say about this? We think a particle that has some momentum should have some non-zero rest mass. Let us go through the math.

At first sight, the $m_\gamma = \gamma m_0 = m/2$ is just like an $x \cdot y = k$ relation: we have two variables (γ and m_0), and their product is some constant (m/2), so they are inversely proportional to each other. However, the relationship is, obviously, much more complicated. To be precise, the variables are not m_0 and γ but γ and v, or v/c. In fact, if we think of $\beta = v/c$ as the variable, we may want to think of the other variable as some ratio between 0 and 1 too, so we can write it as m_0/m and re-write the equation accordingly. However, that doesn't help all that much. Let us try something else: if v is not equal to c, then it's actually the *radius* of that circular orbit that's going to change: $v = r \cdot \omega = r \cdot E/\hbar = r \cdot E/\hbar = r \cdot m \cdot c^2/\hbar$. Hence, we can write the relation as:

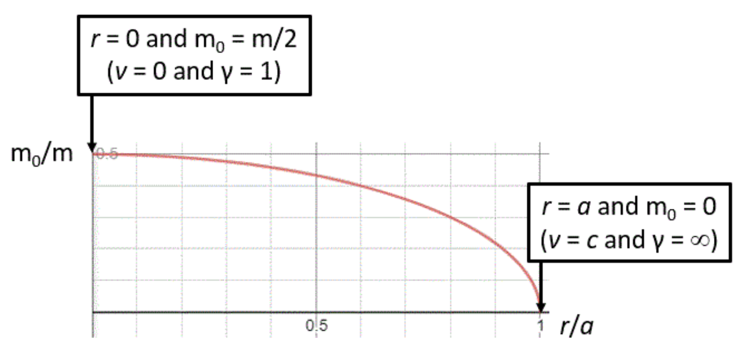
$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{r^2 \cdot m^2 \cdot c^4}{c^2 \cdot \hbar^2}}} = \frac{m_0}{\sqrt{1-\frac{r^2}{a^2}}} = \frac{m}{2}$$

That's interesting because we can rewrite this as:

$$\frac{m_0}{\sqrt{1-\frac{r^2}{a^2}}} = \frac{m}{2} \Leftrightarrow 2 \cdot \left(\frac{m_0}{m}\right) = \sqrt{1-\left(\frac{r}{a}\right)^2}$$

This is a function that makes us think of the $y^2 = 1 - x^2$ relation for a circle except for the 1/2 factor, but then we should note that the m_0/m ratio will effectively vary between 0 and 1/2, as opposed to r/a, which will – just like the x in the $x^2 + y^2 = 1$ relation – vary between 0 and 1. We get the following graph:

Figure 4: The m_0/m ratio as a function of the r/a ratio



However, this nice graph still doesn't give us a good *second* fundamental relation that would solve the problem: what's the *actual* m_0/m ratio? Is it zero ($m_0 = 0$), 1/2 (($m_0 = m/2$) or some value in-between?

We will let this matter rest for a while (this sounds a bit funny in this context) and first explore why it would depend on the r/a ratio.

The dependence of the anomalous magnetic moment on the *zbw* radius

It is easy to show why the anomalous magnetic moment would depend on the *Zitterbewegung* radius. If we denote this radius by *r* (which may or may not be equal to *a* = ħ/m*c*), then the formula for the angular momentum becomes:

$$\mathrm{L} = I_{\mathrm{e}} \cdot \omega = \frac{\mathrm{m}r^2}{2} \cdot \frac{v}{r} = I_{\gamma} \cdot \omega = \mathrm{m}_{\gamma} r^2 \cdot \frac{v}{r} = \mathrm{m}_{\gamma} \cdot r \cdot v = \frac{\mathrm{m} \cdot r \cdot v}{2}$$

The m is, once again, the *rest* mass of the electron[10], so the formula is just the one we mentioned already. However, we substituted *c* for *v* and *a* for *r*. The idea here is that the angular frequency ω remains the same (ω = E/ħ = *v*/*r*) because the rest mass (or rest energy) of the electron is what it is and, therefore, the radius *r* and *v* may be different from *a* and *c* but they are still related through the tangential velocity formula: $v = r \cdot \omega = r \cdot \mathrm{E}/\hbar = r \cdot \mathrm{m} \cdot c^2/\hbar$. Note that I_{e} and I_{γ} denote the moment of inertia of the electron and the *zbw* charge respectively.

To calculate the anomalous magnetic moment – which is actually an anomalous g-ratio[11] - we need the electric current $\mathrm{I} = \mathrm{q_e} \cdot \omega$. The current does *not* depend on *v* or *r*: qe is just the (naked) charge, and ω is the same angular frequency ω = E/ħ = *v*/*r*. As mentioned, we assume *v* and *r* may vary but their *ratio* remains the same. The magnetic moment is equal to the current times the area of the loop and is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi r^2 = \mathrm{q_e} \frac{\mathrm{m}c^2}{h} \cdot \pi r^2 = \mathrm{q_e} c \frac{\pi r^2}{2\pi a} = \mathrm{q_e} c \frac{r^2}{2a}$$

We substituted m*c*/h for $\lambda_c = 2\pi \cdot a$ in the formula above. For *a* = *r*, the formula simplifies to the one you know:

$$\mu = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2} \frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar$$

However, we don't simplify here. Let us have a look at the formula for the g-ratio:

$$g_r = \frac{\mu_r}{\mathrm{L}_r} = \frac{\mathrm{I} \cdot \pi r^2}{\mathrm{m}_{\gamma} \cdot r \cdot v} = \frac{\mathrm{I} \cdot \pi \cdot \mathrm{r}}{\mathrm{m}_{\gamma} \cdot v}$$

What can we do with this? Nothing much. However, note that we introduced a subscript (g_r) to distinguish the *actual* value for *g* from its theoretical value, which we get from equating *r* to *a* and *v* to *c*:

$$g = \frac{\mu}{\mathrm{L}} = \frac{\mathrm{I} \cdot \pi a^2}{\mathrm{m}_{\gamma} \cdot a \cdot c} = \frac{\mathrm{q_e} \cdot c \cdot \frac{a^2}{2a}}{\mathrm{m} \cdot a \cdot c/2} = \frac{\mathrm{q_e}}{\mathrm{m}}$$

You will say this doesn't look like the *g*-factor for the pure spin moment, and you are right. The convention is to write the g-factor as a multiple of $\mathrm{q_e}/2\mathrm{m}$, so it is a pure number:

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[10] We could write it with a subscript ($\mathrm{m} = \mathrm{m_e}$) but, for the sake of keeping the notation as simple as possible, we refrained from that.

[11] The gyromagnetic ratio is the ratio of the magnetic moment and the angular momentum. As mentioned, the anomalous magnetic moment is actually a misnomer. First, it is *not* a magnetic moment: it is the g-ratio. Second, as we try to show here, it may actually not be anomalous at all!

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We think this convention obscures the matter[12], so we'll just stick with our ratio – which is a real gyromagnetic ratio instead of some number – and let's see what happens. The anomaly is usually defined as the *difference* between real gyromagnetic ratio and the theoretical value ($g_r - g$) but we'll also write it as a ratio:

$$\frac{g_r}{g} = \frac{\dfrac{\mathrm{q_e}c\dfrac{r^2}{2a}}{\mathrm{m}\cdot r\cdot v/2}}{\dfrac{\mathrm{q_e}c\dfrac{a^2}{2a}}{\mathrm{m}\cdot a\cdot c/2}} = \frac{r^2}{a^2}\frac{a}{r} = \frac{r}{a}$$

This is a wonderful result: the anomaly is just the ratio between the *actual* or *effective* Zitterbewegung radius and its theoretical value. We can write it very simply:

$$g_r = (r/a)\cdot g$$

We know Schwinger's first-order value for the anomaly is $\alpha/2\pi \approx 0.00116141$. We also know and we know – *from experiments that measure this g-ratio* – that this first-order correction explains 99.85% of the anomaly. The second-, third-, or n^{th}-order corrections that one gets only need to explain 0.15%.

The α in the formula is the fine-structure constant ($\alpha \approx 1/137$), and it also relates the Compton radius to the Thomson radius. The Thomson radius is the classical electron radius: $r_e = \alpha\cdot a \approx a/137 \approx 2.818 \times 10^{-15}$ m. We get this radius from *elastic* scattering experiments. They are referred to as elastic because the photon seems to bounce off some hard *core*: there is no interference. In contrast, Compton scattering is usually explained by some electron-photon interference. It involves high-energy photons (the light is X- or gamma-rays) whose energy will be briefly absorbed before the electron comes back to its equilibrium situation by emitting another photon. The wavelength of the emitted photon will be longer. The photon has, therefore, less energy, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum.

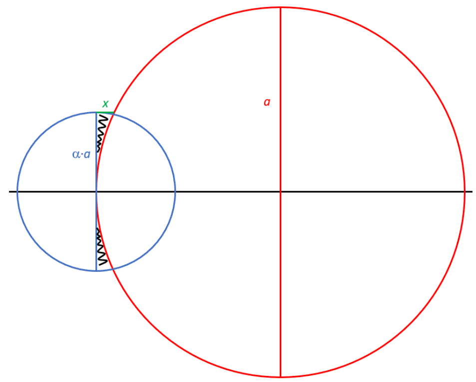
This picture is fully consistent with the *Zitterbewegung* model of an electron: the hard core is just the pointlike charge itself. It is, effectively, pointlike (10^{-15} m is the *femto*meter scale) but, as we can see, pointlike does not mean dimensionless. So what is going on here, and can we explain Schwinger's $\alpha/2\pi$ factor for the anomaly?

A classical explanation for the anomaly

Figure 5 is not to scale but illustrates the geometry of the situation. We think of the naked charge as a charged sphere with radius $\alpha\cdot a$ moving in a circular orbit with radius $r = a$.

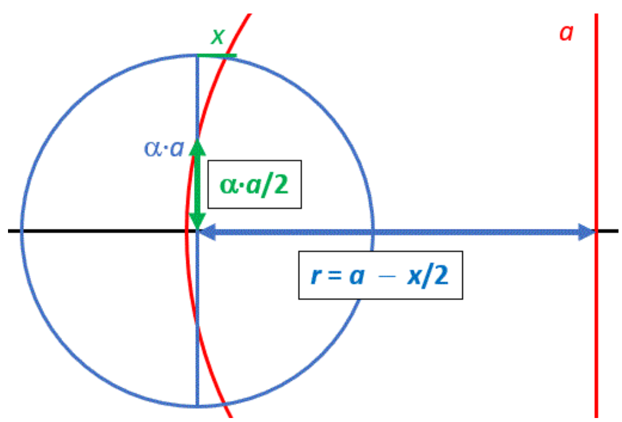
[12] See: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018 (http://vixra.org/abs/1812.0233).

Figure 5: Geometry of *zbw* charge and electron (1)



The points in the two triangular areas will move at a velocity *v* which is slightly higher than *c* = *a*·ω. Hence, *the **effective center of charge** is slightly changed here*. We could provide a formal definition here but we don't think it's necessary: it is obvious that we can define the concept of the *effective center of charge* in pretty much the same way as we would define the effective center of mass: we think of the *zbw* charge as consisting of a potentially infinite number of infinitesimally small charged points, which rotate about the center at a velocity that is defined by their distance from the center. The point is: if we want the charged sphere – on average and as a whole – to move around the center at the speed of light (*c*), then we have to reduce the effective *zbw* radius somewhat. This correction is approximated by the distance *x*/2 in Figure 6.

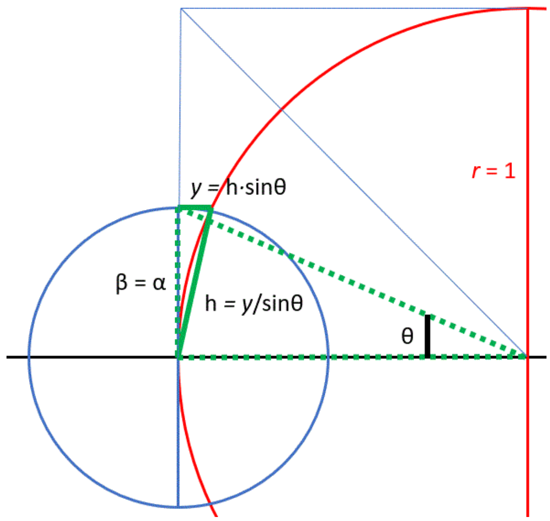
Figure 6: Geometry of *zbw* charge and electron (2)



All that remains to be done is to prove that the correction is equal (or not) to α/2π. Let's explore the geometry somewhat further. To facilitate calculations, we scaled everything by dividing the radii of the two circles (α·*a* and *a*) by a, so the large circle is the unit circle, and the radius of the sphere of charge is β = α.[13] Also, our length *x* now becomes *y* = *x*/*a*.

[13] We insert a new symbol (β) so that you would think of it as a variable rather than as a constant (the fine-structure constant). It is, strictly speaking, not necessary, but it helped my thinking so I hope it helps yours too.

Figure 7: Geometry of *zbw* charge and electron (3)



It now becomes obvious that we have two *similar* triangles here.[14] The first triangle is the triangle in the large circle, which represents the orbit of our *zbw* charge. The height of the first triangle – whose base is now equal to $r = 1$ (its base was a before re-scaling) – is equal to the base of the second triangle, which is the triangle in the small circle, which represents the circumference of the *zbw* charge, which was equal to $2\pi\cdot\alpha\cdot a$ before re-scaling. Hence, the height of the large triangle ($\beta = \alpha$) is the base of the small triangle.

The length of the hypothenuse (which we will denote by h as per the usual convention[15]) is equal to the ratio of y and $\sin\theta$. Conversely, the length we want to calculate (y, which we can then scale back to find x and $x/2$), will be equal to $y = h\cdot\sin\theta$. Now, because the large circle is the unit circle, we know that $\sin\theta$ will be equal to $\beta = \alpha$. Hence, we can write:

$$y = h\cdot\alpha = h\cdot\alpha$$

But what is the length of that hypothenuse? Not sure, but it is easy to see that it depends on θ, or on β. But *how* exactly? If $\beta = 1$, the θ angle will be equal to $\pi/4$, so that's one eighth of the circumference of the circle: $\beta = 1 \Leftrightarrow \theta = 2\pi/8 = \pi/4$. We can now take smaller values of β to approximate the actual $\beta = \alpha$, and it is easy to see we have a proportional relation here, as shown in Table 1.

Table 1: The proportionality between β and θ

β	**Θ**
$\beta = 1$	$\theta = 2\pi/8 = \pi/4$
$\beta = 1/2$	$\theta = (2\pi/8)\cdot(1/2) = (2\pi/8)/2 = \pi/8$
$\beta = 1/4$	$\theta = (\pi/4)\cdot(1/4) = \pi/16$
…	
$\boldsymbol{\beta = \alpha}$	$\boldsymbol{\theta = (\pi/4)\cdot\alpha}$

[14] Two triangles are similar – i.e. they have the same *shape* – if every angle of one triangle has the same measure as the corresponding angle in the other triangle. The corresponding sides of similar triangles have lengths that are in the same proportion, and this property is also sufficient to establish similarity.
[15] There is not much scope for confusing h with Planck's constant here, so we should not invent some other symbol here.

The y = h·sinθ = h·α and h = y/sinθ = y/α relations are nice but give us a tautology: y = y. We need to try something else. Let us try the small-angle approximation. Indeed, our ratio α is very small and, hence, we can write: sinθ ≈ θ = (π/4)·α (the latter identity is given in the table above). In addition, we can also use the small-angle approximation to write h as h ≈ α, so we get:

$$y = \text{h}\cdot\sin\theta \approx \alpha\cdot\theta = \alpha\cdot(\pi/4)\cdot\alpha = (\pi/4)\cdot\alpha^2$$

Let's scale everything back to the actual size by multiplying all lengths by a. We get:

$$x = a\cdot y = (\pi/4)\cdot\alpha^2\cdot a \Leftrightarrow x/2 = a\cdot y/2 = (\pi/8)\cdot\alpha^2\cdot a$$

Now, we are interested in the anomaly, which we can now write as:

$$\frac{g-g_r}{g} = \frac{a-r}{a} = \frac{x/2}{a} = \frac{(\pi/8)\cdot\alpha^2\cdot a}{a} = \frac{\pi\cdot\alpha^2}{8}$$

This result is good, but it is not good enough. It is good because it is a result that is expressed in terms of α and π – and, importantly, *nothing else* – but the formula differs from Schwinger's α/2π ≈ 0.00116141 factor. It's not even close numerically: $\pi\cdot\alpha^2/8 \approx 0.000021$. The result is off by a factor that's equal to $4/\pi^2\alpha \approx 55.5$, so that's a factor of the order of 1/α ≈ 137. That's not a disaster for a first attempt at calculation but it's, obviously, not good enough.

Is there any way out? We may think it's got something to do with the fact that we imagine the *zbw* charge to be some *sphere* of charge. Indeed, unlike our electron, we do *not* picture it as some disk. Our little triangle is a little *cone* of charge, so perhaps we should calculate the *ratio* between the volume of our cone of charge and the total volume of charge. We can do that. The formula for the volume of a cone is equal to $V = \pi\cdot b^2\cdot h/3$. The b and b here are the base (b) and the height (h) of the triangle that defines the volume of rotation here. Hence, b and h are equal to x and α·a here.

$$\frac{\mathrm{V}_r}{\mathrm{V}} = \frac{\frac{\pi\cdot x^2\cdot\alpha\cdot a}{3}}{\frac{4\pi\cdot(\alpha\cdot a)^3}{3}} = \frac{\pi\cdot(\pi\cdot\alpha^2\cdot a/4)^2\cdot\alpha\cdot a}{4\pi\cdot(\alpha\cdot a)^3} = \frac{\pi^2\cdot\alpha^2}{4^3}$$

As expected, this line of reasoning just confirms we get an anomaly that is of the same order:

$$\frac{g-g_r}{g} \propto \alpha^2$$

In contrast, Schwinger's factor is of the order α: no square. But… *Wait a minute!* We should actually look at the *volumes of rotation* here:

1. We have (the surface of) a little triangle going around a loop whose circumference is approximately 2π·a. The volume of rotation of this little triangle will approximate V = (h·b/2)·(2πR) = [(α·a)·(πα²a/8)/2)]·(2π·a) = $\pi^2\alpha^3a^3/8$. In fact, we have two little triangles and the relevant volume may, therefore, be equal to (h·b)·(2πR) = $\pi^2\alpha^3a^3/4$
2. The volume of rotation for our *zbw* charge will approximate V = (πr^2)·(2πR) = (πr^2)·(2πR) = (πα² a^2)·(2πa) = $2\pi^2\alpha^2a^3$.

The ratio of these two volumes is equal to:

$$\frac{V_x}{V} = \frac{\pi^2\alpha^3 a^3}{4} \cdot \frac{1}{2\pi^2\alpha^2 a^3} = \frac{\alpha}{8}$$

This is the right order of magnitude but our calculation is still off by a factor that's equal to 4/π, so that's almost 30%.[16] In light of the primitive analysis, this α/8 factor looks like a great first-order start, but we should improve on it. How could we do that?

Our formulas for the volume of rotation are, obviously, approximative only: we should calculate an integral to get the *exact* toroidal volumes. At the same time, the product of a surface with some *effective* radius R should give us the volume and, while we have an effective radius that is not *exactly* equal to *a*, the effective radius for the two volumes will be the same, so the error is (probably) not in the value that we are using for the radius R. The correction(s) that is (are) to be made need to take into account that the small triangles are *not* exact triangles, and that their surface is, therefore, *smaller* than h·b. Also, the ½ factor in the $\alpha \cdot a/2$ and $x/2$ lengths of the sides of these triangles is not correct. We can, of course, just plug in the mentioned 4/π ≈ 1.273 factor to get the right result – as we do below – but that doesn't explain much.

$$\frac{g - g_r}{g} = \frac{4}{\pi} \cdot \frac{V_x}{V} = \frac{\frac{4}{\pi} \cdot (h \cdot b) \cdot (2\pi R)}{(\pi r^2) \cdot (2\pi R)} = \frac{\frac{4}{\pi} \cdot (\alpha \cdot a) \cdot \frac{\pi\alpha^2 a}{8}}{\pi \cdot \alpha^2 a^2} = \frac{\alpha}{2\pi}$$

We conclude that, while the first stab at it yields encouraging results, we obviously still have some work to do.

Some additional classical considerations

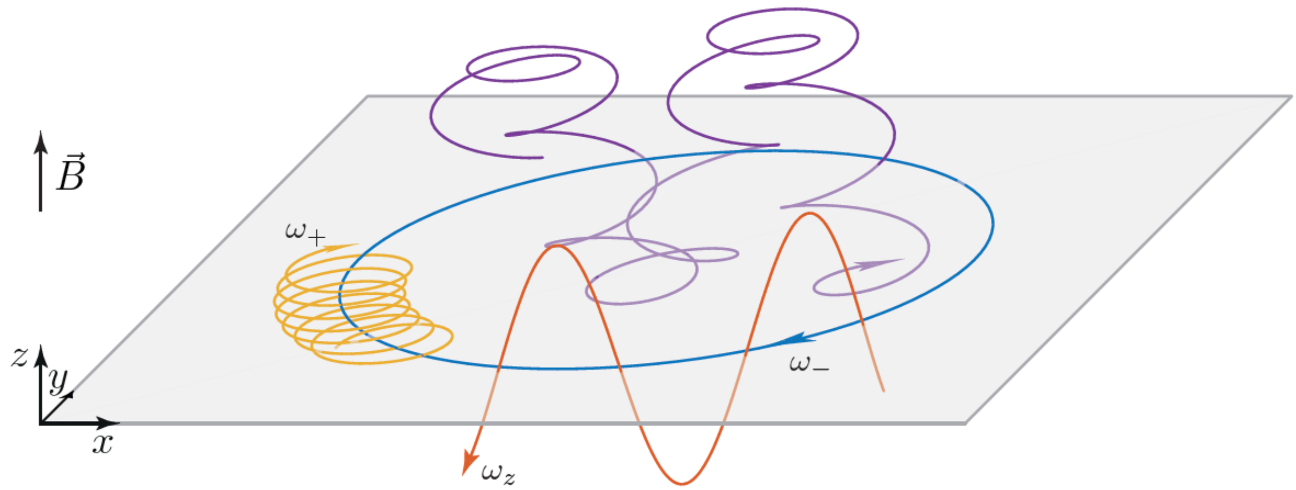
As mentioned, there is still some work ahead but, at the same time, it is quite remarkable that we are so close to the mark. There are, indeed, many other factors to consider. One important factor, for example, is the classical coupling between the spin and orbital angular momentum which one would expect to see in a one-electron cyclotron. The electron in the Penning trap that is used in these experiments is, effectively, not a spin-only electron. It follows an orbital motion – that is one of the three or four layers in its motion, at least – and, hence, if some theoretical value for the g-factor has to be used here, then one should also consider the g-factor that is associated with the orbital motion of an electron, which is that of the Bohr orbitals (g = 1).

Hence, one would expect to see a classical coupling between (1) the precession, (2) the orbital angular momentum and (3) the spin angular momentum, and the situation is further complicated because of the electric fields in the Penning trap, which add another layer of motion. The complexity of the situation is illustrated below.[17] Hence, we are hopefully that some more research will be able to narrow the gap between the *zbw* explanation and QFT calculations.

[16] The percentage depends on whether one prefers to calculate 1 – 4/π or, alternatively, 1 – π/4.

[17] The source of the illustration is the following course material, which we found particularly enlightening: *Cylotron frequency in a Penning trap*, Heidelberg University, Blaum Group, 28 September 2015. The motions are complicated because the Penning trap traps the electron using both electric as well as magnetic fields.

Figure 8: The three principal motions and frequencies in a Penning trap



One other serious consideration that may affect the result is the assumption that the rest mass of the naked (*zbw*) charge is equal to zero. As we mentioned, perhaps it is *close* to zero, but not exactly zero. This would be an easy way to make the calculations come out alright, but the associated calculations will be more complicated, and critics would be right in saying that we're making the theory fit experiment. At the same time, I'd say the following to these critics: the model on which these calculations are based would seem to have more appeal than the hocus-pocus on which current QFT calculations are based.

Finally, we would like to point out that, even if we would be left with a difference of almost 30%, such difference is still smaller than what famous theorists like Richard Feynman find acceptable for first approximations. For example, Richard Feynman's calculation of the size of an atom – which is widely quoted still – starts by saying that "we need not trust our answer to within factors like 2, π, etcetera" when doing this kind of "dimensional analysis." (Feynman's Lectures, Vol. III, Chapter 2, Section 4).

Conclusions

This paper explored the geometry of the *zbw* model in very much detail and calculated the order of magnitude of the anomalous magnetic moment assuming the naked *zbw* charge has *zero* rest mass. We found an anomalous magnetic moment that is off by about 25-30% only (as compared to the experimentally established value in, for example, the Harvard experiments[18]). This is encouraging because the calculations show the result will be some function of α and 2π, and of those two factors only. More research is needed to reduce the error because of (i) the approximative formulas we used and (ii) the need to include an analysis of other factors, which include the classical coupling between spin-only and orbital moment which one would expect to see in a one-electron cyclotron.

Jean Louis Van Belle, 11 June 2019

[18] See: Hanneke, Fogwell Hoogerheide, and Gabrielse (2011), *Cavity Control of a Single-Electron Quantum Cyclotron: Measuring the Electron Magnetic Moment*, *Physical Review A*. **83** (5):052122.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.
5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

We would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

We should, perhaps, also mention the following critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Finally, I would like to thank Prof. Dr. Alex Burinskii for taking me seriously. He is in a different realm – and he has made it clear that my writings are, perhaps, somewhat simple and may, therefore, serve pedagogic purposes only. However, his confirmation that I am not making any *fundamental* mistakes while trying to understand the fundamentals, have kept me going on this.