Addendum

Find a normal subgroup in A_4 of order 4.
Consider the subgroup in A_4

$$H = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}.$$

Let $N(H) = \{a \in A_4 | aHa^{-1} = H\}$. Since $H \subset N(H)$ and $(1\ 2\ 3) \in N(H)$, $|N(H)| > |H|$. Thus $|N(H)| > 4$. By Lagrange's theorem, $|H|$ divides $|N(H)|$ and $|N(H)|$ divides $|A_4|$. Thus 4 divides $|N(H)|$ and $|N(H)|$ divides 12. To conclude $|N(H)| = 12$ and hence $N(H) = A_4$.