Developing a Lorentz Invariant form of the Schrodinger Equation

The Schrodinger Equation $i\hbar \frac{\partial \psi}{\partial t} = \widehat{H} \psi$, when applied to particle wave functions Ref [1], becomes (where the operator $\widehat{H} = -\frac{\hbar^2}{m} \nabla^2$):

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{m} \nabla^2 \psi \qquad (1)$$

However, the Schrodinger equation is not Lorentz Invariant, so it cannot be applied to the wave functions of moving particles.

The Classical Wave Equation *is* Lorentz Invariant and is also satisfied by particle wave functions, Ref [1]:

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi \qquad (2)$$

Rearranging (1) gives:

$$-i\hbar \frac{m}{\hbar^2} \frac{\partial \psi}{\partial t} = \nabla^2 \psi \qquad (3)$$

Then simplifying gives:

$$-i \frac{m}{\hbar} \frac{\partial \psi}{\partial t} = \nabla^2 \psi \qquad (4)$$

Therefore, substituting (4) into (2) gives:

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -i \frac{m}{\hbar} \frac{\partial \psi}{\partial t} \qquad (5)$$

So

$$\frac{i\hbar}{mc^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial \psi}{\partial t} \qquad (6)$$

Thus, for a moving particle's wave function, a Lorentz Invariant (Relativistic) form of the Schrodinger Equation $i\hbar \frac{\partial \psi}{\partial t} = \widehat{H} \psi$ can be written:

$$\frac{-\hbar^2}{mc^2} \frac{\partial^2 \psi}{\partial t^2} = \widehat{H} \psi \qquad (7)$$

This equation *can* be used with the wave functions of moving particles.

References:

[1] Traill. D. A. *"Wave functions for the electron and positron",* 2013-2019, http://vixra.org/pdf/1507.0054vH.pdf (Last accessed 30/5/2019).