# A New Hypothesis of the Relation between Momentum and Energy

Chen Zhipeng[1]

Beijing Relativity Theory Research Federation, Beijing

E-mail: 116422915@qq.com

**ABSTRACT**: The combination of Newton momentum and Einstein's mass-energy relation will cause many difficulties. A new hypothesis of the relation between energy and momentum is proposed to solve these difficulties. At the same time, the relation is validated.

**[keywords]** Momentum; Energy; Newton Momentum; Theory of Relativity; Quantum Mechanics

(In this formula, $c$ is the velocity of light, $\lambda$ is the wavelength, $u$ is the frequency, $v$ is the velocity of wave, $E$ is the energy, $h$ is Planck constant, $p$ is the momentum)

## 1. Current difficulties in Newton momentum

### 1.1 Substitution of matter waves into Newton momentum causes the difficulties of infinity

The difficulty of wavelength infinity caused by substituting De Broglie matter wave

into Newton momentum is: $\lambda = \frac{h}{p} = \frac{h}{mv}$ When $v$ approaches 0, (the wavelength of matter wave $\lambda \to \infty$ ) it does not conform to the facts.

### 1.2 Newton momentum does not conform to Einstein's relation between velocity and temperature

Planck -- Einstein temperature transformation formula is

$$T_0 = T / \sqrt{1-(v/c)^2} \qquad (1)$$

This formula shows that the temperature decreases as the velocity increases.

According to $\lambda = h/p$ and Wien's Displacement Law $\lambda T = b$, it can be obtained that: $T = \frac{bp}{h}$,

By substituting Newton momentum $p = mv$, it can be obtained that:

$$T = \frac{bmv}{h} \qquad (2)$$

Formula (2) shows that with the increase of velocity, the temperature of the object increases, which is inconsistent with Formula (1).

## 2. Establishment of a new relation between energy and momentum

In order to establish a new relation between energy and momentum, the following two assumptions must be made:

Assumption one: 1. Correspondingly, the velocity of light equals the product of wavelength and frequency, $c = \lambda u$.

2. The velocity of mechanical wave also satisfies the product of wavelength and frequency, $v = \lambda u$。

3. Then, analogous to material waves of matter particles (e.g. electrons, protons, etc.), it is concluded that material waves should also satisfy that the wave velocity is equal to the product of wavelength and frequency, $v = \lambda u$.

Assumption two. 1. Correspondingly, the photon has no rest mass and there is no rest photon.

2. Then by analogy, it can be obtained that quantum has no rest mass and there is no rest quantum. Since the photon satisfies $E = hu$, $p = \frac{h}{\lambda}$ ,then the quantum should also satisfy $E = hu$ , $p = \frac{h}{\lambda}$.

When the matter particle is quantum, then $E = hu$ and $E = mc^2$ are satisfied, then it can be obtained that $hu = mc^2$. And according to $v = \lambda u$, $p = \frac{h}{\lambda}$, it can be obtained that:

$$p = \frac{hu}{\lambda u} = \frac{mc^2}{v} = \frac{E}{v} \tag{3}$$

According to equation (3), it can be obtained that:

$E = mc^2 = hu = pv$ (hereinafter referred to as mass-energy momentum (4)

$E = pv$ shows that when the matter particle is quantum, it must be explained by Formula (4). Since the quantum has no rest mass, the velocity $v$ of the quantum must be greater than 0. As $v$ approaches $c$, $p$ approaches $mc$. And it can be obtained from $E = mc^2 = pv = hu$ that the faster the instantaneous velocity of the quantum motion is, the smaller the momentum of the quantum is.

## 3. Explanations of Ji Hao's Three Experiments

### 3.1 Ji Hao 's experiments do not conform to the results of substituting Newton momentum into the mass-velocity relation of special relativity.

Ji hao analyzed the experiments of mass-velocity relation validated by calorimetric method conducted by Bertozzi in the Nuclear Science Laboratory of the Physics Department of Massachusetts Institute of Technology. He found that the theoretical values of Bertozzi's experiments were electrons with five kinds of energy, 0.5MeV, 1.0MeV, 1.5MeV, 4.5MeV and 15MeV. However, only 1.5 MeV and 4.5 MeV electron energy measurements are given in the experimental report, and no 15 MeV electron energy measurements are given. Ji Hao questioned this and re-measured this experiment in the accelerator laboratory. The six nominal energies of the electron beams generated by the accelerator and the corresponding mass-velocity relation calculated by special relativity are shown in table 1. The electrons are injected vertically into a 0.1210T uniform magnetic field using a lead-iron collimator. According to the formula of substituting Newton momentum into the special relativity, the orbit radius of the electron circular motion should fall on the six points of 10.94 cm, 16.41 cm, 24.62 cm, 32.82 cm, 43.76 cm and 54.7 cm. However, Ji Hao's experiments showed that all six types of electrons fell within a small radius of about 18 cm on the photographic film -- the deflection radius of the electrons with different labeled energies did not conform to the prediction of substituting Newton momentum into special relativity (see table 1).

Table 1 Ji Han' s experiments, corresponding Newton' s theory prediction & relativistic prediction

| Labeled energy of electrons , MeV | 4 | 6 | 9 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|
| $v$ ( Relativistic prediction ) | 0.9908 c | 0.9969 c | 0.9986 c | 0.9992 c | 0.9995 c | 0,9997 c |
| $r$ ( Relativistic prediction ) | 10.94 | 16.41 | 24.62 | 32.82 | 43.76 | 54.7 |
| $r$ ( Newton's theory prediction ) | 1.40 | 1.404 | 1.40571 | 1.40755 | 1.40798 | 1.40868 |
| $r$ ( Ji Hao's experiments) | 17.8 | 17.9 | 18.0 | 18.1 | 18.2 | 18.3 |

Ji Hao' s experiments were carried out at the 2300C/D linear accelerator made by Varian. In the experiment, when the particle with a static mass of $m_0$ and a charge of $q$ is moving in the electromagnetic field with a velocity of $v$, the following equation can be obtained by substituting Newton Momentum and mass-velocity relation into Lorentz Force Equation:

$$\frac{d}{dt}\frac{m_0 \vec{v}}{\sqrt{1-(v/c)^2}} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad (5)$$

Using cylindrical coordinates, the electrons are assumed to move in the Z = 0 plane. In the uniform magnetic field, assume that P is the Newton momentum relation of the particle and R is the radius of the circular orbit of the particle. The basic formula of accelerator theory can be obtained as follows:

$$R = \frac{m_0 \vec{v}}{\sqrt{1-(v/c)^2}} = \frac{p}{qB} \qquad (6)$$

It can be known from table 1 that the Ji Hao' s experiments do not conform to the results of substituting Newton momentum into the mass-velocity relation of the relativity.

**3.2 Mass-energy momentum conforms to Ji Hao' s experiments**

Ji hao's experiments [4] re-conducted the experimental research on the Newton momentum and velocity relation, and found that after changing the magnetic field intensity, the results of the experiment did not conform to the Newton momentum predicted by relativity and the results of substituting the Newton momentum into the mass-velocity relation of special relativity. If the mass-energy momentum is used, it can be well consistent with Ji Hao's experiments.

When particles with a rest mass of $m_0$, and a charge of $q$, move at a velocity of $v$ in the electromagnetic field, the Lorentz Force Motion Equation satisfied by the mass-energy momentum can be obtained:

$$\frac{d}{dt}\frac{E}{\vec{v}} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad (7)$$

Using cylindrical coordinates, the electrons are assumed to move in the Z = 0 plane. In the uniform magnetic field, assume that R is the radius of the circular orbit of the particle and P is the momentum of the particle' s mass-energy momentum relation, then it can be obtained by substituting it into the commonly used formula in the accelerator theory that: $R = \frac{p}{qB} = \frac{E}{qBv}$ (8)

Ji hao's experiments on six kinds of energy electron beams showed that these six kinds of electrons all fell on the radius of about 18cm photographic film, indicating that the electrons with different energies almost all fell on the same circle. In other words, Ji Hao' s experiments do not conform to the theoretical value of substituting the Newton momentum into special relativity. If equation (8) is used, the orbital radius of electron' s circular motion is about 18cm, which perfectly explains Ji Hao's experiments.

**3.3 Calorimetric measurement experiments prove that the relativistic momentum does not increase with the increase of velocity after the velocity reaches a certain degree**

Ji Hao' s calorimetric measurement experiments were carried out in the Accelerator Laboratory of Fudan University and Shanghai Institute of Applied

Physics, Chinese Academy of Sciences, using the linear accelerator and five electron energies used in Bertozzi experiments. Ji Hao found that the experimental results were not consistent with the results of substituting Newton momentum into special relativity. For the electron beam with theoretical energy of 15MeV, the temperature measured by Ji Hao's experiment was an increment of 1.03 degrees Celsius. By substituting Newton momentum into special relativity, the temperature increment should be 6.29 degrees Celsius, which is a five-fold difference (see table 2).

Table 2. The relationship between the energy of the electron beam and the resulting temperature increment Δt

| The labeled kinetic energy of the electron MeV | 1.6 | 6 | 8 | 10 | 12 | 15 |
|---|---|---|---|---|---|---|
| $v/c$( Relativistic prediction ) | 0.97 | 0.9969 | 0.9981 | 0.9988 | 0.9992 | 0.9994 |
| Δt( Relativistic prediction °C) | 0.67 | 2.52 | 3.36 | 4.20 | 5.03 | 6.29 |
| Δt( Value measured by Ji Hao °C) | 0.97 | 1.00 | 1.03 | 1.03 | 1.03 | 1.03 |

According to the calculations in Table 2, if the lead platform is regarded as an isolated metal ball, the temperature increment caused by the electric field is calculated to be 0.89 degrees Celsius. The measured results show that the electron beams with theoretical energy values of 1.6 MeV, 6 MeV, 8 MeV, 10 MeV and 15 MeV can generate temperatures of 0.08, 0.11, 0.14, 0.14, 0.14, 0.14 and 0.14 degrees on lead targets, respectively.

According to Newton' s Mechanics formula, the energy is approximately 0.255MeV. The temperature rise of each beam on the lead target is about 0.12 degrees Celsius. Therefore, Ji Hao believes that the experimental results [5] are consistent with the results of Newtonian Mechanics.

When $v$ approaches $c$, it can be known that $p_N$ approaches $mc$ and mass-energy momentum $p_c = \frac{E}{v}$ also approaches $mc$. Then formula (4) also conforms to the measured value of Ji Hao.

### 3.4 Experiments on Electron Lorentz Force and Energy Measurement [6-7]

Ji Hao conducted his experiments [6] in the laboratory of Modern Physics of Fudan University. According to Ji Hao' s experiments of Electron Lorentz Force and Energy Measurement, when the motion direction of β-particle is perpendicular to the direction of static uniform magnetic field, it moves in a circle under the action of Lorentz Force of static magnetic field. It can be obtained by substituting formula (6) into Einstein's mass-energy relation that:

$$\frac{mc^2 \cdot v^2}{C^2 R} = evB \tag{9}$$

It can be obtained through experiments [6] that it does not accord with the value of substituting Newton's momentum into relativity. At the same time, it does not conform to the existing momentum-energy relation $E^2 - c^2 p^2 - m^2{}_0 c^4 = 0$, which indicates that the momentum-energy relation combined with Newton's momentum and special relativity has applicability.

Ji hao's experiments [2] were carried out on the femtosecond microwave synchrotron of Shanghai Institute of Applied Physics. According to Ji Hao's experiments, the actual effective force of the accelerator on the accelerated electrons depends on the electron velocity. The larger the electron velocity $v$ is, the smaller the actual force of acceleration will be. This indicates that the Ji Hao's experiments [2] do not conform to the Newton Momentum, and also do not conform to the momentum-energy relation of the combination of Newton momentum and special relativity.

According to equation (4) $p = E / v$, it can be known that the actual effective force of the accelerator on the accelerated electrons [2] depends on the velocity of the electrons. The larger the velocity of the electrons, the smaller the momentum of the energy added on the electrons, and the smaller the actual effective force of the accelerator.

At the same time, substituting equation (4) into Einstein's mass-energy relation can be well consistent with the experiment. The experiment also shows that in the same magnetic field, the greater the motion velocity of electrons, the smaller the Lorentz Force they are subjected to. The correctness of equation (4) is shown.

Ji Hao's three experiments do not conform to Newton momentum, nor do they conform to the results of substituting Newtonian momentum into special relativity. But they conform to mass-energy momentum $p = E / v$. It shows that Ji Hao's three experiments do not conform to the value of substituting Newtonian momentum into Einstein's special relativity, but they conform to the mass-velocity relation and mass-energy relation of special relativity.

## 4. One-dimensional square potential well for quantum equation of mass-energy momentum

The proof is as follows:

Set $E \to \frac{ih}{2\pi}\frac{\partial \psi}{\partial t}$, $P \to -\frac{ih}{2\pi}\frac{\partial \psi}{\partial x}$。

The CZP equation can be obtained from the mass-energy momentum, $E = pv$

Since $v = \lambda u$

$$\frac{ih}{2\pi}\frac{\partial \psi}{\partial t} = \left(-\frac{ihu\lambda}{2\pi}\frac{\partial \psi}{\partial x} + U\right)\psi \text{。}$$

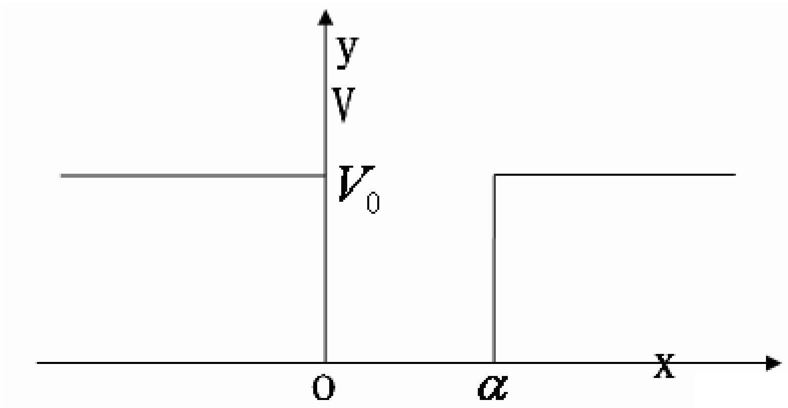
In order to simplify the calculation, the construction equation is $E = \frac{p^2(u\lambda)^2}{E}$, then it can be obtained that

$$\frac{ih}{2\pi}\frac{\partial \psi}{\partial t} = \left(-\frac{h^2(u\lambda)^2}{4E\pi^2}\frac{\partial^2 \psi}{\partial x^2} + U\right)\psi \qquad (10)$$

Considering a one-dimensional infinite potential well, the potential energy curve in the well can be expressed as follows:

(I) When $v(x) = 0$, then $0 < x < a$. (When $v(x) > 0$, then $x < 0, x > a$)

When only the energy of an infinite potential well is considered to be infinite, the potential energy curves inside and outside the well are shown as follows:




From (11), the quantum equations of steady state mass-energy and momentum inside and outside the well can be written as:

$$\frac{d^2\psi}{dx^2} + \frac{E^2}{(h/2\pi)^2(u\lambda)^2} = 0 \qquad (0 < x < a)$$

$$\frac{d^2\psi}{dx^2} + \frac{(E - V_0)^2}{(h/2\pi)^2(u\lambda)^2} = 0 \qquad (x < 0, x > a)$$

Where $E > 0$ represents the energy of particles. At $E < V_0$, set

$$k = \frac{E}{(h/2\pi)u\lambda}, \quad q = \frac{E - V_0}{(h/2\pi)u\lambda} \tag{11}$$

From (11) the two formulas above can be written as: $\frac{d^2\psi}{dx^2} + k^2 = 0$ $(0 < x < a)$

$$\frac{d^2\psi}{dx^2} + q^2 = 0 \qquad (x < 0, x > a)$$

These are two second order linear differential equations with constant coefficients. Set $(k > 0, q < 0)$, then the solutions are:

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx} \qquad (0 < x < a) \tag{12}$$

$$\psi(x) = B_1 e^{qx} + B_2 e^{-qx} \qquad (x < 0, x > a) \tag{13}$$

In the formula, $A_1$, $A_2$, $B_1$, $B_2$ are specific constants。The first term on the right of equation (10) $B_1 e^{qx}$ approaches ∞ when $x \to \infty$. The second term $B_2 e^{-qx}$ approaches ∞ when $x \to -\infty$. In order to guarantee the finiteness of the wave function, $B_1$, $B_2$ must be zero when $x < 0, x > a$. Therefore, (13) is written as: $\psi(x) = B_1 e^{qx}$ $(x < 0)$ (14)

$$\psi(x) = B_2 e^{-qx} \qquad (x > a) \tag{15}$$

It can be seen that the intensity of the wave functions in the region of $x < 0$ and $x > a$ will not be zero. This indicates that when the energy of the particle is less than the potential energy difference between inside and outside the well, the probability of the particle appearing outside the well is not zero. Constants $A_1$, $A_2$, $B_1$, $B_2$ can be determined by the standard conditions of wave function.

When we just consider the simple case where the well depth is infinite $V_0 \to \infty$.

Then, since when $V_0 \to \infty$, $q \to \infty$. According to (14) and (15), $\psi = 0$ at $x < 0, x > a$, which means that the probability of particle occurrence is zero.

And since the wave function must be continuous at $x=0$ and $x=a$, that is $\psi(0)=\psi(a)=0$. Substitute into (12) and obtain: $A_1+A_2=0$ and

$A_1e^{ika}+A_2e^{-ika}=0$.

It can be obtained through simultaneous equations that:

$e^{ika}-e^{-ika}=2i\sin ka=0$

In order to make this formula valid, the constant $k$ can not take any value, but can only take some discontinuous values that satisfy the following formula: $ka=\pm n\pi \qquad n=1,2,,3\ldots$

(16)

By substituting (15) into (10), the possible values of particle energy in an infinite deep well can be obtained as follows:

$E=\pm\frac{nhu\lambda}{2a}$, then it can be obtained that $E=\pm\frac{nhu}{2a/\lambda}$ (17)

When $2a=\lambda$, then it can be obtained that $E=\pm nhu$ (18)

For equation(17), it is shown that the one-dimensional square potential well of quantum has three forms, positive, negative, or positive and negative solutions. And the single solution of equation (18) is an integer multiple of $n$. Then in the equation (18), $\frac{n}{2a/\lambda}$ and energy $nhu$ can satisfy the condition only if they have quantization at the same time, where the quantization condition is an integer multiple of $n$, indicating that the particle has de Broglie volatility in motion. (19)

Since equation (17) must satisfy equation (16), then the value of $a$ in formula (17) can only be $\frac{1}{2}\lambda$ or an even multiple of $\frac{1}{2}\lambda$.

For equation (17), it is known that angular momentum $L=R\times P$ satisfies commutation relation $\left[L_i,L_j\right]=i\hbar\sum\varepsilon_{ijk}L_k$.

And it also satisfies the spin relation $\left[S_i,S_j\right]=i\hbar\sum\varepsilon_{ijk}S_k$ and so on.

Then the wave-particle duality statement can be obtained: the momentum and angular momentum of each quantum can correspond to the momentum and angular momentum of a quantum with the same frequency respectively.

For equation (17), it can be extended to matter waves.

That is, the energy, momentum and angular momentum of each particle in a de Broglie matter wave can correspond to the energy, momentum and angular momentum of a particle with the same frequency. (20)

## 5. Proof of mass-energy momentum by difference of anomalous magnetic momentum of electrons

I. The mass-energy momentum equation in the electromagnetic field is:

$$i\hbar\frac{\partial}{\partial t}=H\psi$$

$$H=\left(-\frac{ihu\lambda}{2\pi}\frac{\partial\psi}{\partial x}+U\right)\psi$$

(21)

The CZP equation of electrons (charge $-e$) in electromagnetic potential ($A,\varphi$) can be expressed as:

$$\left[i\hbar\frac{\partial}{\partial t}+e\varphi-u\lambda a(p+\frac{e}{c}A)\right]\psi=0 \qquad (22)$$

Under the non-relativistic limit, the formula can be obtained.

$$i\hbar\frac{\partial}{\partial t}\chi=ca.(P+\frac{e}{c}A)\varphi-e\phi\chi-2mc^2\chi \qquad (23)$$

Under the non-relativistic limit, it can be obtained from (21)

$$\chi\approx\frac{1}{2mc}\sigma.\left(P+\frac{e}{c}A\right)\varphi \qquad (24)$$

Substitute (24) into (22), it can be obtained that

$$\mu_B=\mu=\frac{e\hbar}{2mu\lambda} \qquad (25)$$

Considering that the velocity of electron was accelerated to about 0.997$C$ (about$10^{-3}$) in the experiment, the value of magnetic moment of electron obtained by formula (25) is closer to the measured value.

The solution of the Dirac equation is $\mu_B=\frac{e\hbar}{2mc}$, the calculated value $\mu=1.00116\mu_B$ is slightly different from the measured value (about $10^{-3}$).

## 6. Proof of Mass-energy Momentum by Strict Solution of Hydrogen Atom

From $i\hbar\frac{\partial}{\partial t}=H\psi$

$H=\left(-\frac{ihu\lambda}{2\pi}\frac{\partial\psi}{\partial x}+\frac{e_1^2}{r}\right)\psi$ (In the formula, $e_1=e/\sqrt{4\pi\varepsilon_0}$, $\varepsilon_0$ is dielectric constant of vacuum) (26)

The eigenvalue $E$ can be obtained from (26),

$$E=\frac{mc^3\varepsilon}{v}=\frac{mc^3}{v\sqrt{1+\frac{a^2}{\left(n-|k|+\sqrt{k^2-a^2}\right)^2}}} \qquad (27)$$

Formula (27) shows that the accurate energy value of hydrogen atom is related to the velocity, and it completely conforms to the experimental value. For Dirac equation, there are slight differences between $2s_{1/2}$ and $2p_{1/2}$, between $3s_{1/2}$ and $3p_{1/2}$, which are the values of hydrogen atom. These differences [7] happen to be the ratio to the Dirac equation$(v/c)$.


[1] Zhang Wenmo, Chen Zhipeng. Reflections on the contradiction between L.de Broglie matter waves and relativity. *Research on Contemporary Education,* 2008,25 (1)

[2]Ji Hao. *Frontier Science.* Three experiments that question Einstein's mechanics and traditional accelerator theory. 2010, 04 (3) :76-80

[3] Ji Hao. *Frontier Science(Quarterly) 2010.3. Volume 4. No.15.* Three experiments that question Einstein's mechanics and traditional accelerator theory

[4] Ji Hao. *Engineering Sciences.* Experiments on Electron Lorentz Force and ene rgy measurements. 2006.8(10):60-65

[5] Ji Hao. *China achievement of science and technology.* Experiments of measuri ng electronic energy by calorimetry. 2009. (1):34-35

[6] Ji Hao. *China Science & Technology.* Experiments on the motion of electrons with different energies in a uniform magnetic field. 2009 (6)

[7] Ka XingLin. Advanced Quantum Mechanics. Second edition. *Higher Education Press,* 2004:227-237.

[8] I Sick On the rms-radius of the proton. *Physics Letters B.* 2003 , 576 (1-2) :62-67