Proposal of Replacing Classical Logic with Free Logic for Reasoning with Non-Referring Names in Ordinary Discourse

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Abstract. Reasoning carried out in ordinary language, can not avoid using non-referring names if occasion arises. Semantics of classical logic does not fit well for dealing with sentences with non-referring names of the language. The principle of bivalence does not allow any third truth-value, it does not allow truth-value gap also. The outcome is an ad hoc stipulation that no names should be referentless. The aim of this paper is to evaluate how far free logic with supervaluational semantics is appropriate for dealing with the problems of non-referring names used in sentences of ordinary language, at the cost of validity of some of the classical logical theses/ principles.

Keywords: Free Logic; presupposition language; Supervaluation

1 Introduction

In Ordinary language we often utter sentences without being concerned of whether or not the sentences refer to something existing. In our daily life, we use names / singular terms of persons, objects, animals that do not actually exist, and we also ascribe truth-values to sentences containing such non-referring names. For instance, in scientific discussions, scientists sometimes presume existence of unobserved entities to explain certain observed phenomena, otherwise unexplained. For repeated reference, scientists conveniently give names to such supposed entities. This is how the name ‘Vulcan’ came to be used in astronomical parlance. Besides this, we talk about persons who are no longer among us, or discuss about objects which are demolished. Some such sentences are; ‘Mother Teresa dedicated her life for serving people’, ‘Atlantis was a well-constructed city’. Moreover, we meaningfully talk about mythical characters, characters of fairy tales and fictions in propositions like, ‘Santa Claus gifts all children on the Christmas Eve’, ‘Rama killed Ravana’, where names like ‘Santa Claus’, ‘Rama’ do not refer to any actually existent entity. Thus, presence of non-referring names in ordinary language is unavoidable. Not only that, although one may not be bothered about the truth of sentences about mythical/ fictional characters, but truth values of sentences with non-referring names are matters of concern in scientific discourses and historical discussions, particularly, when such sentences occur in inferences.

An arbitrary assignment of truth-values to such sentences in the framework of classical first order predicate logic gives rise to several misconceptions and misinterpretations regarding historical and scientific discussions. For instance, on a bivalent interpretation of language, sentences like, ‘Mother Teresa is a wicked woman’ and ‘Mother Teresa dedicated her life for serving people’ may be true simultaneously, as the singular term ‘Mother Teresa’ has no reference. Similarly, ‘Mother Teresa dedicated her life for serving people’ and ‘Martians help people’ both may be true and thus a sentence with historical significance and an imaginary sentence both get the same stature. In the same way, scientific discussions about hypothetical objects (about which no experimental evidence is available) would become nonsensical in classical first order predicate logic.

In this paper free logic with supervaluational semantics is described from the perspective its treatment towards non-referring names. In the last section of this paper I demonstrate the viability of a variant of supervaluational semantics as a replacement of classical first order predicate logic for reasoning with ordinary and scientific discourses with non-referring names.

2. Concept of Free logic emerges as a solution of the problems of non-referring names

A number of logicians, namely, Karel Lambert [5], Henry Leonard [7], Hailperin and Leblanc [6] proposed to free classical logic from any existential assumption even with respect to names (singular terms). They were in favour of giving up the assumption that each singular term in the language must designate some object in the chosen domain of interpretation for that language. As a result, a host of logical system, named ‘free logic’ appeared in the logical scenario. Some important classical logical principles, namely the rule U.I. and the rule E.G., (or, their corresponding axioms) were not admitted in those systems.

Free logic offers a kind of first order system, which is completely free from existential assumption with respect to names, both general and singular, in the language of the system. Semantics for various systems of free logic differ from each other with respect to the nature of the domain of interpretation, the interpretation function from the vocabulary of the language to the domain of interpretation and the valuation function that assigns truth values to closed formulas, particularly to atomic sentences of the language. There are basically three approaches of free logic [8] – **negative semantics** assigns False to any atomic sentence with an empty term not of the form ‘E!t’ (t exists); **positive semantics** assigns True to some atomic sentence with empty singular term not of the form ‘E!t’ and **neutral semantics** stipulates that all atomic sentences with empty names not of the form ‘E!t’ are to be neither true nor false; i.e. truth-valueless. Assignment of truth or falsity to every sentence containing empty names as per positive and negative semantics is not always intuitive. For instance, sentences like “Pegasus flies over the clouds”, “2/0 < 2/0” being true as per positive free logic and sentence like "Santa Claus is an imaginary character" being false in negative free logic are counterintuitive and unacceptable. Therefore, they are clearly not suitable as logic for ordinary discourses.

3. Free logic under a chosen semantics; Supervaluational Semantics

In case of reference failure, Strawson [9] acknowledged truth-value gaps and thus stepped out of the realm of bivalence by admitting that some sentences with non-referring names can be neither true, nor false. The neutral semantics based on supervaluation goes appropriately with the true Strawsonian spirit regarding existential presupposition behind every true or false predication in ordinary language.

3.1. Presupposition as a Semantic Relation:

The concept of presupposition, originally proposed by P.F. Strawson [9], and formally defined by van Fraassen [3][4] as a semantic relation is as follows:

If A and B are two propositions, then a characterization of presupposition can be given in a language as,

A presupposes *B* iff *A* is neither true nor false unless *B* is true.

This is equivalent to,

If A is true, then B is true

and, If A is false, then B is true.

To understand, one might say, for example, that a proposition, 'Saina Nehwal is an athlete' is true when the presupposition 'Saina Nehwal exists' is true and Saina Nehwal is a member of the class associated with the predicate 'is an athlete'; similarly, the proposition, 'Saina Nehwal is an Australian' is false only if the presupposition, 'Saina Nehwal exists' is true, and provided, Saina Nehwal is not a member of the class associated with the predicate 'is an Australian'. So if a language contains only those singular terms that refer to existents, then each sentence of the language would have a truth-value.

Presupposition is different from other semantic relations, e.g., implication and necessitation. Implication is defined as the logical truth of $'A \supset B'(\sim A \vee B)$. For implication *modus tollens* is accepted as valid, whereas in case of presupposition it doesn't hold, since the analogue of *modus tollens* with respect to presupposition:

A presupposes B
(not B)
Therefore, (not A)

is not valid; if both the premises are true, the conclusion is not true (i.e. neither true nor false).

Another distinction is that the argument :

A presupposes B
(not A)
Therefore, B

is valid in case of presupposition, since if the premises are true, so is the conclusion; whereas, for implication this argument doesn't hold.

However, presupposition and implication have something in common, which is, if A either presupposes or implies B then the argument from A to B is valid.

3.2. Presupposition and Supervaluation semantics

van Fraassen [3] proposed supervaluation which is a function from the set of sentences in a given language to the set {T,F} of truth-values. Supervaluation is a super-structure built upon a set of classical valuations defined over a model $M = < D, f >$,where D is the domain of discourse and f is an interpretation function assigning some element from D to individual constants and a subset of D to each predicate symbol, which is the extension of the predicate in D. A *supervaluation s* over a model M is a function that assigns T(F) exactly to those statements assigned T(F) by all the classical valuations over the model M; otherwise, they are not defined. One difference with classical situation is that, with respect to supervaluation, even in a classical model the interpretation function f may not be defined for all individual constants (names) in the language under consideration (e.g., the language of ordinary discourse). The language for which supervaluation has been proposed as an admissible valuation is called a presuppositional language. The syntax of a presuppositional language say, L, i.e., the vocabulary and grammar of L are defined in the same way as in classical logic.

Suppose, a and b are two names in L (a presuppositional language) such that, $f(a) \in D$ and $f(b)$ that is not in D in the model M. Let us suppose that $f(a) \in f(P)$ for some predicate P. Moreover, there are exactly two classical valuations, v_1 and v_2, in M. Then the supervaluation V_s can be assigned as shown in **Table 1**:

Table 1. Supervaluation semantics based on two classical valuations, v_1 and v_2.

	v_1	v_2	V_s
Pa	T	T	T
$\sim Pa$	F	F	F
Pb	T	F	-
$\sim Pb$	F	T	-
$Pb \vee \sim Pb$	T	T	T
$(x)Px$	T	T	T
$(x)Px \supset Pb$	T	F	-

Here, dashes indicate truth-value gaps.

The sentence ‘$P(b) \vee \sim P(b)$’ is a tautology and gets T by the supervaluations, though ‘b’ is a non-referring name. A quantificational sentence, for instance, $(x)Px \supset Pb$ will be neither true nor false as the sentence gets T in v_1 and gets F for v_2.

The way supervaluational semantics deals with truth-value gaps in accordance with the concept of presupposition as mentioned earlier. The truth or falsity of all sentences, presupposes that the singular terms (names) occurring in the sentences refer to some existent entities i.e., some objects in the domain of interpretation. If this presupposition fails, the concerned sentences are assigned no truth-value, hence a truth-value gap occurs. For instance, in supervaluational semantics, the sentence, ‘Santa Claus gifts all children on the Christmas Eve’ is assigned neither ‘truth’, nor ‘falsity’, as the sentence gets T by the above classical valuation v_1 and is assigned F by some other classical valuation v_2 in the above model. Since, the name ‘Santa

Claus' does not denote any object, the existence-presupposition of the above sentence fails. Both v_1 and v_2 arbitrarily assigns truth-values, and there being no uniformity between them, none of them is admissible. Thus supervaluation defined over the set $\{v_1, v_2\}$ assigns no truth-value to the sentence.

3.3. Shortcomings of Supervaluation Semantics

The supervaluational semantics is not perfect for describing ordinary language, as well as the most obvious lacuna of supervaluational semantics is that it invalidates the principle of self-identity and the principle of substitutivity of identicals.

Self-identity sentences of the form '$a = a$', where 'a' is non-referring name, would be false in supervaluational semantics. But, intuitively the sentence is accepted to be logically true for whatever a is.

Under supervaluations the *salva veritate* substitutivity fails for propositional sentences. For a truth-valueless atomic formula A, $A \rightarrow A$ is logically true in supervaluational semantics. But if another truth-valueless formula B is substituted for the second occurrence of A, the resulting formula $A \rightarrow B$, is truth-valueless; not true.

Consider the case when 'a' is referring and 'b' is non-referring, i.e., $I(a) \in D$ and $I(b) \notin D$; moreover, consider that $a \in I(P)$. Now, consider two classical valuations v_1 and v_2, such that $v_1(Pa) = T$ and $v_2(Pa) = T$; and $v_1(Pb) = T$ and $v_2(Pb) = F$. Then '$(\forall x)Px \supset Pa$'is true, though '$(\forall x)Px \supset Pb$' is not. However, in standard first order predicate logic (FOP) both are true as endorsed by UI rule, known as the principle of Specification. This is however quite expected in a system of free logic.

4. Modified Supervaluational Semantics

To salvage these problems we need a modified supervaluational semantics for free logic, that allows 'incomplete' objects and truth-value gaps, which is proposed by Bencivenga [1][2].

This modified semantics is based on the concept of counterfactual theory of truth — the point of view where a sentence containing a non-referring name is True (False) if it **would** be True (False) **were** this term to denote something existent.. Let, ***U*** = <*D,I*> be a model- structure where *D*, the domain of interpretation of ***U***, is a set, possibly empty; and *I* is a unary interpretation function, total on the set of predicates and partial on the set of names, that assigns to every individual n-ary predicate a set of ordered n-tuples of members of *D*, and to every individual term for which it is defined a member of *D*.

Based on this model structure a partial valuation function υ_U from the set of wffs to {T,F} is defined, which admits truth-value gap. For a wff A , if all the singular terms in it have denotations in D then υ_U asserts truth values to A similar to classical logical valuation. If A is an atomic formula which is not of the form $E!t$ and contains a non-referring term $\upsilon_U(A)$ would be neither true nor false. If *A* is of the form *a=b* then $\upsilon_U(A) = F$ if exactly one of *I*(*a*) and *I*(*b*) is defined or both *I*(*a*) and *I*(*b*) are defined but $I(a) \neq I(b)$; $\upsilon_U(A) =$ undefined if neither *I*(*a*) and *I*(*b*) is defined and otherwise $\upsilon_U(A) = T$. This valuation function υ_U is called as 'factual valuation'.

Let ***U′*** = <*D′,I′*> be a **completion**, i.e., an **extension** of ***U***. Let ***U′*** = <*D′,I′*> be a **completion**, i.e., an **extension** of ***U***. ***U′*** is a completion of ***U*** if and only if it fulfils the following conditions:

i) *D′* must be a non-empty superset of *D*;

ii) for every predicate *P*, *I′*(*P*) is a superset of *I(P)*,

iii) *I′*(*t*) is defined for every singular term *t,* and is identical with *I(t)*whenever *I(t)* is defined.

A valuation in the model ***U*** ′,which is an extension of the model ***U*** is a total function $\upsilon_{U'(U)}$ from the set of well-formed (wff) formulas/sentences to the set of truth-values {T,F}. The valuation $\upsilon_{U'(U)}$ for ***U′*** from the point of view of ***U*** is determined by ***U*** whenever a definite truth-value is assigned to a wff in ***U*** by υ_U , and is determined by ***U′*** elsewhere.

On the other hand, the valuation function $\upsilon_{U'(U)}$ in ***U′*** is called 'formal valuation' which is based on some "mental experiments". Mental experiments are carried out to consider what would have happened if some objects existed corresponding to terms that do not, in fact, denote. A preference is given to reality or fact in determining the valuation $\upsilon_{U'(U)}$. $\upsilon_{U'(U)}$. A formal valuation $\upsilon_{U'(U)}$ in ***U′*** is defined in the following way:

1. a) If *A* is an atomic formula and $\upsilon_U(A)$ is defined in ***U***, then $\upsilon_{U'(U)}(A) = \upsilon_U(A)$

b) If *A* is an atomic formula and $\upsilon_U(A)$ is not defined in ***U***, then $\upsilon_{U'(U)}(A) = \upsilon_{U'}(A)$

2. If *A* is of the form '$\sim B$', then $\upsilon_{U'(U)}(A) = T$ if and only if $\upsilon_{U'(U)}(B) = F$.

3. If *A* has the form '$B \& C$', then $\upsilon_{U'(U)}(A) = T$ in ***U′*** if and only if $\upsilon_{U'(U)}(B) = \upsilon_{U'(U)}(C) = T$;

4. If *A* has the form '$\forall x B$', then $\upsilon_{U'(U)}(A) = T$ if and only if $\upsilon_{U'(U)}(B(t/x))$ = *T* in ***U′*** for every singular term *t* such that_$\upsilon_{U'(U)}(E!t) = T$. (It must be noted that $\upsilon_{U'(U)}(E!t) = \upsilon_U(E!t)$)

Now, considering the valuations $\upsilon_{U'(U)}$ for all the completions ***U′*** of ***U***, the supervaluation υ_U^s for the model ***U*** is defined. The valuation υ_U^s in ***U*** is a partial function from the set of wffs (sentences) in the chosen language L to the set of truth-values {T,F}, defined as follows:

1.a) If $\upsilon_{U'(U)}$ (*A*)= *T* for every valuations $\upsilon_{U'(U)}$ for all completion ***U′*** of ***U***, then υ_U^s (*A*)=*T*; in other words, *A* is super-true in ***U***.

b) If $\upsilon_{U'(U)}$ (*A*)= *F* for every valuations $\upsilon_{U'(U)}$ for all completion ***U′*** of ***U***, then υ_U^s (*A*)=*F*; in other words, *A* is super-false in ***U***.

c) υ_U^s (*A*) is not defined in ***U*** if not by virtue of (a)-(b).

As a consequence, a wff (sentence) is **logically true** in the above supervaluational model, i.e., SL-true if and only if υ_U^s (*A*) = *T* for every model ***U***.

Here as a corollary it can be noted that in the modified supervaluational semantics, classical principles are not always vacuously invalid because of containing non-referring names. Let's consider an instance of the principle of Specification, '$(\forall x)Px \rightarrow Pb$', where '*b*' is a non-referring name in the language, i.e., *I(b)* is not defined in ***U***. So, the whole sentence becomes truth-valueless in a model (supervaluational model) ***U***. Now an extension of ***U***, ***U′,*** is considered which assigns a denotation (counter-factual) to *'b'* and it is assumed that '*Pb'* is False in the extended model, ***U***, ***U′***. In the extension ***U′*** of ***U***, '*b*' is taken to be as if denoting but its ***U′*** is not taken to belong to the extension *I′(P)* in ***U′***. So, '*Pb*' is assigned False in ***U′***. Therefore, the sentence $'(\forall x)Px \rightarrow Pb'$ becomes False in ***U′***, as its antecedent is True and consequent is False. But depending on the predicate '*P*' and the term '*b*' it may be the case that '*Pb*' is False(True) in all extensions of ***U***; accordingly '$(\forall x)Px \rightarrow Pb$' may be super-true or super-false or even truth-valueless, unlike van Fraassen's approach where the principle was assigned truth value gap always.

5. Modified Supervaluation for reasoning with Ordinary Discourses with non-referring names

In this section I will attempt to justify the aptness of the modified supervaluational semantics as a logic of ordinary discourse. The mental experiment in evaluating the formal valuations in the modified supervaluational semantics makes it suitable for reasoning in ordinary discourses since while determining the formal valuations we can appropriately design the extended models as per the context of discussion. For instance, in the modified structure, sentence like 'Mother Teresa dedicated her life for serving people' would be super- true inspite of the term 'Mother Teresa' being non-referring, as the sentence would be true in all the formal (counterfactual) valuations if the extended models, based on which the valuations are given, are based on the historical facts known about Mother Teresa. In the similar way 'Mother Teresa is wicked' would become super-false. The term 'Mother Teresa' being an empty singular term no factual truth-value can be assigned to both of the sentences in the model- structure ***U***, i.e., the valuation υ_U for the two sentences are not defined and hence we must proceed to determining the counter-factual valuation for the sentences some completion ***U′*** of ***U***. While considering the completion of the model structure ***U***, truth-values of the two sentences are determined under the condition 'if Mother Teresa *were* alive' and all the known facts are used to define ***I'***.

Now based on the information that we have about Mother Teresa, the sentence 'Mother Teresa dedicated her life for serving people' would be true in all the completions ***U′*** of ***U,*** had Mother Teresa been alive. Hence the sentence would be super-true, i.e., any valuation function υ_U^s in ***U*** would assign *T* to the sentence. On the other hand, if Mother Teresa *were* alive, the sentence 'Mother Teresa is wicked' would be false in all of the completions and hence would be super-false. 'Pegasus has white hind legs' would be truth-valueless, since even if 'Pegasus' were existing there is no information regarding the colour of its hind leg, and hence, the sentence would be true in some completions and false in some other completions; finally, the sentence becomes truth-valueless in the valuation υ_U^s, which takes the logical product of the counter-factual valuations assigned in different completions.

It is noticeable that all of the above-mentioned three sentences would be true in positive free semantics, false in negative free semantics, truth-valueless in van Fraassen's supervaluational semantics and would get arbitrary truth-values in classical first order predicate logic, provided the names 'Mother Teresa' and 'Pegasus' lack references.

Most importantly, reasoning about scientific discourses is also possible using the modified supervaluation, because such reasoning involves hypothetical objects whose existence is not yet proved. For instance, the existence of 'Dark matter' has not been proved experimentally and scientists have not yet succeeded to detect any 'Dark matter' particle, but various astronomical observations strongly recommend the existence of such particles. Hence the term 'Dark matter' can be considered to be an empty term. In such a scenario, sentences 'Dark matter contributes to 70% of the mass of the universe' and 'Dark matter is blue particle', the modified supervaluation semantics would appropriately ascribe the correct truth-values to the two sentences because if Dark matter *were* existent, the first sentence would be true in all completions and the second one would be false in all completions; thus the first one would be super-true and the second one would be super-false. Thus, this example demonstrates the aptness of the modified supervaluational semantics for reasoning with scientific discourses.

Here, there is a difference with classical logic and the modified supervaluation semantics. As in modified supervaluational semantics, any completion of the structure ***U*** based on some possible, i.e., counter-factual world. So, the notion of super-truth in such a model is a stronger notion than the classical one, comparable to some extent with the modal notion of necessary truth. Though objectivity of the classical truth, which is the essence of the classical notion of truth, is retained in the notion of 'super-truth'; 'super-truth' avoids arbitrariness in a classical valuation (particularly with regard to the failure of presupposition due to lack of denotation of a name involved in the sentence under consideration) by selecting only what is common in all the valuations in a given situation

So, we can say that the non-classical modified supervaluational semantics offers an alternative for a classical logic, that is to be seriously taken at least with respect to ordinary discourses. The semantics is sound but is not strongly complete as the semantics deal with partial interpretation. Incompleteness in the strong sense of such a system is not a defect that is peculiar to such a system only, since, consistency or soundness is achieved. Gödel's incompleteness thesis, which says that in any consistent formal system there are sentences of the language of the system which can neither be proved nor can be disproved in the system justifies the workability of a consistent system. For example, Peano Arithmetic is not complete as some statements in Peano arithmetic, which are true, are neither provable nor disprovable. So, weak completeness must not persuade us to overlook the ability of supervaluational semantics to provide intuitively acceptable interpretation of ordinary discourses.

So, it can be said that free logic (the system of supervaluational semantics) is appropriate as the logic of ordinary discourses.

Acknowledgments: I am extremely grateful to Prof. Sanjukta Basu and Prof. Mihir Kumar Chakraborty for their guidance and technical suggestions.

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