From Pythagorean theorem to cosine theorem.

MSC 51M04

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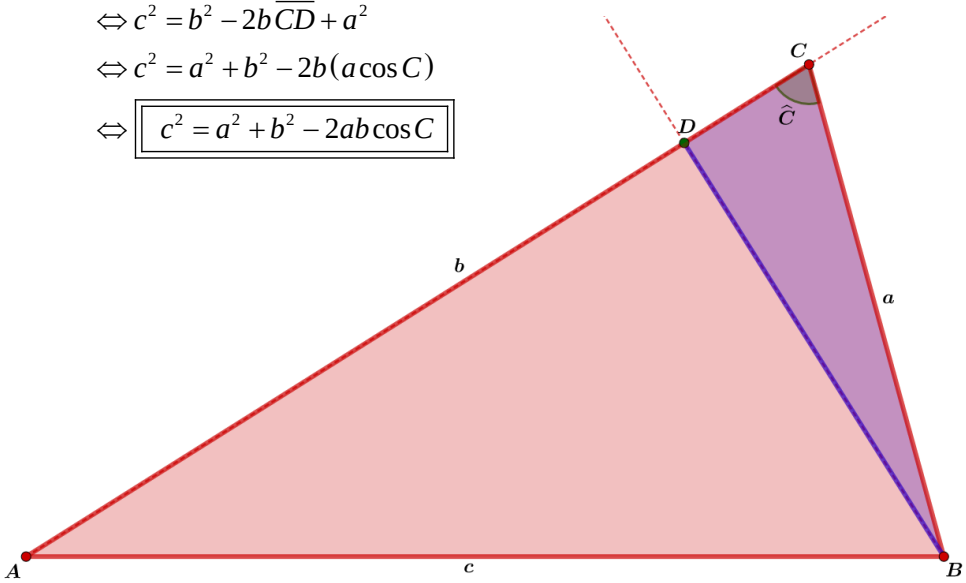
Abstract

Easy and natural demonstration of the *cosine theorem*, based on the extension of the *Pythagorean theorem*.

Applying **Pythagoras´ theorem** to $\overset{\triangle}{ABD}$:

- $\boxed{C < \frac{\pi}{2}}$

$$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 \Leftrightarrow c^2 = \left(b - \overline{CD}\right)^2 + \overline{BD}^2$$
$$\Leftrightarrow c^2 = b^2 - 2b\overline{CD} + \left(\overline{CD}^2 + \overline{BD}^2\right)$$
$$\Leftrightarrow c^2 = b^2 - 2b\overline{CD} + a^2$$
$$\Leftrightarrow c^2 = a^2 + b^2 - 2b(a\cos C)$$
$$\Leftrightarrow \boxed{\boxed{c^2 = a^2 + b^2 - 2ab\cos C}}$$



- $\boxed{C > \frac{\pi}{2}}$

$$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 \Leftrightarrow c^2 = \left(b + \overline{CD}\right)^2 + \overline{BD}^2$$
$$\Leftrightarrow c^2 = b^2 + 2b\overline{CD} + \left(\overline{CD}^2 + \overline{BD}^2\right)$$
$$\Leftrightarrow c^2 = b^2 + 2b\overline{CD} + a^2$$
$$\Leftrightarrow c^2 = a^2 + b^2 + 2b[a\cos(\pi - C)]$$
$$\Leftrightarrow \boxed{\boxed{c^2 = a^2 + b^2 - 2ab\cos C}}$$