Riemann Hypothesis Yielding to Minor Effort--Part II:

A [Generalizing] One-Line Demonstration

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ABSTRACT[1]

The present attempt is by far the most parsimonious demonstration of a [controversial] rationale behind the RH, with a generalized solution likewise readily implied. It earmarks an analogy that may shine in many a forthcoming elaboration as will be proposed.

Keywords: Focal distributions, generalized Riemann Hypothesis (gRH), non-associativity in operational sequencing (or quasi-commutativity of the operators), fuzzy (alternating) set cardinality

Accidental Trivialization

The RH could be seen as 'trivial' from the outset. After all, any power series (over a natural field) taken to a zero value would somehow imply a complex [exponentiation] domain. And if it doesn't—as is evident in the 'trivial' zeros case with negative even values implied from the inverse gamma factor of the recursive representation[2]—the implied result would appear no more intuitive acceptable than does -1/2 as the infinite summation over unity[3].

It happens, the *non*-trivial zeros as postulated by the RH are far more 'trivial' than that. Please make sure that a one-line demonstration should suffice. Whether or not one goes with,

$$e^{2i\pi k} = 1 \leftrightarrow 2i\pi = 0 \; \forall k \in \boldsymbol{N}$$

[1] WP201955-512 *in memoriam* SU1492 jet crash victims.

[2] The Re(s)=1/2 is likewise hinted at insofar as (1-s) proves a conjugate in this case, with the essential structure of the solution being effectively intact, or sign-alternating in the imaginary part only.

[3] Be it inferred from a geometric progression or the Euler identity equivalent or taken as a convention, one will hardly fare fully content until after having grown accustomed to its pragmatic savings as in the string-theoretic applications.

or considers a qualification of the following sort,

$$\pm T \sim \frac{1}{\pm 2i\pi k}, \qquad T \to \infty$$

it is straightforward to see that (again, per the exponentiation domain only),

$$(1)\ Re(s) \equiv Re(\sigma + it) \equiv \sigma = Re\big([\sigma + it] * (\pm 2i\pi k) * (\pm T)\big) = Re(\pm i(2\sigma\pi kT) \mp 2\pi ktT)$$
$$= \mp 2\pi ktT \overset{\zeta(s)\neq 0}{\Longrightarrow} Re(s) = \frac{1}{2} * s$$

The entire proportion may turn an absolute value as per the special, zeta-at-zero case:

$$N^{-s} \equiv N^{-T\left(\frac{s}{T}\right)} = 0^{\frac{s}{T}} = 0 \ \leftrightarrow\ 0^{s} = 0^{T} = 0 = 0^{1} \overset{(1)}{\Rightarrow} Re(s) = \frac{1}{2}$$

This kind of absolute (on top of relative) symmetry may prove but a matter of mere *representation* though, as denied under any other zeta value—in which light the RH prediction could be somewhat reserved in just how fuzzy the line is between delusion and elusion. On the other hand, one would have yet to motivate the actual critical strip of (0..1), which can hardly fare on the strength of an addition versus subtraction of similar ½ values, even though this may provisionally lend itself to a somewhat generalized and differentiated nature of interrelationship between the real versus imaginary parts:

$$s = 2\pi tTk \pm 2\pi tTk' = Re(s) * \left(1 \pm \frac{k'}{k}\right) = \frac{1}{2} * \left(1 \pm \frac{k'}{k}\right)$$

Not only can the above capture the said critical strip as per the [candidate] nontrivial zeros, it can likewise account for the trivial domain as when

$$\left(1 \pm \frac{k'}{k}\right) = -4m\ \forall m \in \boldsymbol{N}$$

For one thing, one can think of alternate ways of capturing the two solution branches on the exact same premises (see Appendix). On the other hand, this kind of '*focal split*' will be reconsidered in a more elegant as well as consistent fashion as part of rationalizing an *n*-dimensional or *(n+1)-ion* (hypercomplex generalization) setup[4].

For now, one should be careful to approach the endogenization of the critical strip by invoking a phi-relaxation:

$$(1')\ N^{-s} = 0^{\varphi}, \varphi \not\equiv 1$$

[4] $s_n = \sigma + \sum_{i=1}^{n} e_i * t_i$

Arguably, the lower versus upper bounds confine the closure of $\varphi \in [0..2]$, such that[5]

$$s = \begin{cases} 0 = 2 * Re(s)_- = 2 * 0 \\ 2 = 2 * Re(s)_+ = 2 * 1 \end{cases}$$

Now, the simpler way of motivating the phi closure (alongside the critical strip) would be to consider the lower versus upper bounds for zeta as summation over,

$$\zeta(s) \equiv 0 \equiv \sum_{N=1}^{T\sim\infty} N^{-s} = T * 0^{\varphi} = \begin{cases} 0^{-1} * 0^{1+\epsilon} \sim 0^{0+\epsilon} \\ 0^{1+1+\epsilon} \sim 0^{2+\epsilon} \end{cases}$$

The former possibility suggests a full-blown summation (with all of the comparably small zero terms summed up as zero-representations times the inverse of zero), with *s* in the power counting as lump. In contrast, the latter suggests a weaker condition, with the real and imaginary parts treated as separable amidst the summation being irrelevant for zero terms (deem 1 as a 'compactly-supported function' qualifying the setup for the variational lemma as analogy[6]).

Generalizing toward Less Winding a Road

One may at this rate expect that, whilst in the *general* case, the lower bound (of the critical strip) would still stay put around 0, the upper bound would make *n*. In fact, this is what obtains under generalizations of the 'arithmetic zeta' sorts, with the real-part critical lines running ½ through *n*-½. While the rather involved apparatus would extend far beyond the intended minimalist perspective, a relationship of interest could still be proposed.

On the one hand, the upper bound could be inferred—along the lines of zero power counting—at,

$$(2)\ \ n = \sum_{\xi=1}^{n} 1 \equiv CS_+$$

Among other possibilities, this may pertain to the count of hypercomplex dimensions (e.g. 3 and 7 per quaternion and octonion setups respectively). On the other hand, the candidate critical-line real parts,

$$\Re_x(s) = x - \frac{1}{2}\,, x = \overline{1,n}$$

[5] The safer bet would be to maintain $\varphi \in 0 * [0..2] = [0^2..2 * 0]$ albeit possibly appearing as [0..0] on the face of it with the critical line likewise making 1*0=0. It is no wonder at this rate that the solutions seem extremely densely packed anywhere outside infinite scaling.

[6] $\int_a^b [*]\eta(x)dx = 0 = \sum_1^T N^{-s} = \sum_1^T [0^{\varphi}] * 1 * \Delta N \ \ \leftrightarrow [*] = [0^{\varphi}] = 0$

could tentatively be inferred as,

$$(3)\ \ x-\frac{1}{2}=\Delta\sum_{\xi=0}^{x-1}\xi=\Delta\frac{(x-1)x}{2}$$

While $\Delta x = 1$ for any integer by definition (and has therefore been omitted in the ultimate differential structure), the non-commutativity in (3), capturing the candidate critical-line *Re(s)* solutions, pertains to the fact that:

$$\Re_x(s)=\Delta\sum_{\xi=0}^{x-1}\xi\not\equiv\sum_{\xi=0}^{x-1}\Delta\xi=\sum_{\xi=0}^{x-1}1=x$$

That said, the gap is constant at exactly ½, and could be assumed as a summation (integration) constant bridging the two[7]. *Interior structure* in (3) appears to be a clear-cut generalization of—let alone intuitive motivation for—[the upper bound of] the *critical strip* as in (2):

$$(4)\ \ \left\{CS_+=\Re_n(s)+\frac{1}{2}\right\}\equiv\{(\sum_{\xi=1}^{n}1)-\frac{1}{2}=\lim_{x\to n}\Delta\sum_{\xi=0}^{x-1}\xi=\lim_{x\to n}(\sum_{\xi=0}^{x-1}\Delta\xi)-\frac{1}{2}\}$$

Aftermath & Inter-Match

It is straightforward to see how *n=1* of gRH yields the regular RH case. Whilst the former could be seen as a focal distribution (*n*-hyperelliptic as a representation of what I have referred to as Ψ in *ordual* terms from day one) in contrast to the latter having its *foci* converge around the singular *center*, both these setups showcase the candidate RH Re(s) solutions as probably having to do with condensed solution distributions. Whereas gRH points to a core of Re(s) candidate foci, RH features a double-dense singularity. By contrast, although the critical strip *bounds* may have tended to be treated as empty sets, they could in actuality pertain to rarefied ones (or indeed posit sparse matrices) with expected or ex-ante frequency (or solution density) decaying anywhere around the bounds or farther off the [non-trivial] core. Alternatively, rather than treated as rare-to-find (much less as nonexistent), these could be conjectured to elude regular representation—e.g. if only insofar as corner cases may fail to distinguish between [pure] real versus [pure] imaginary equivalents. Not least, while the cardinalities of the $\Re_x(s)$ versus *CS* sets are comparable to *n* versus that of the continuum, the former may not capture the effective solution set as their overlap (as if to hint at fuzzy or alternating solution-set cardinality).

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[7] Nonassociative operation-sequencing appearing as quasi-commutativity in the inter-operator relation, with ½ being the effective commutator:

$$\left(\sum,\Delta\right)-\left(\Delta,\sum\ \ \right)=\frac{1}{2}$$

APPENDIX

For every[8] *N*,

$$(A)\ N^{-s} = N^{-\sigma} * N^{-it} = 1^{-\sigma*\frac{logN}{log1}} * 1^{-it*\frac{logN}{log1}} = e^{-\left(\sigma*\frac{logN}{0}+it*\frac{logN}{0}\right)2i\pi k} = N^{\mp T*2k\pi(\sigma i-t)}$$

$$case\ \sigma \equiv \frac{1}{2}\colon N^{-s} = 0^{\pm k\pi*(i-2t)}$$

Alternatively[9],

$$(B)\ \ N^{-s} = e^{\mp 2\sigma i\pi kT*logN} * e^{\pm 2t\pi kT*logN} = (-1)^{\mp 2\sigma T*logN} * N^{\pm T*2t\pi k}$$
$$= (-1)^{\mp 2\sigma T*\frac{logN}{\log(-1)}*\log(-1)} * N^{\pm T*2t\pi k} = N^{\mp\sigma T*log(-1)^2 \pm T*2t\pi k}$$
$$= \begin{cases} N^{\mp\sigma T*\frac{1}{T}\pm T*2t\pi k} = N^{\mp\sigma\pm T*2t\pi k}\ (B.1) \\ N^{\mp 2\sigma T*i\pi k \pm T*2t\pi k} = N^{-T\pi k*(2\sigma i\mp 2t)}\ (B.2) \end{cases}$$

Obviously, the latter denotes the exact same result as in (A), notably under the Re(s)=1/2 setup. Insofar as such terms prove N-invariant, zeta as summation running ad infinitum could be treated in terms of,

$$(B')\ \zeta \equiv T * 0^{\pm k\pi*(2\sigma i-2t)} = 0^{\pm k\pi*(2\sigma i-2t)-1} \equiv 0 = 0^1$$

Again, by loosely venturing to maintain $0^T \sim 0 \sim 0^1$, the relative ½ proportions likewise ‘prove’ absolute, as is seen from the powers comparison. Otherwise, *s* may well turn out to be an infinity (taking on either sign) rather than unity. On second thought, implicit symmetry or indiscriminate treatment of the *Re(s)* equivalents need not imply the imaginary part will equal the real one. On the contrary, it will remain generalized:

$$(C)\ \ s = \sigma - 2\pi ktT = \sigma - \frac{2\pi kt}{2i\pi k} = \sigma + it = \sigma * \left(1 - \frac{1}{\sigma} * \frac{t}{i}\right) = \sigma - 2t\frac{\pi k}{2i\pi k}$$
$$= \sigma - 2 * \frac{1}{2} * \frac{t}{i} = \begin{cases} \frac{1}{2} + it \\ -m + (-m) + \epsilon, \qquad \epsilon \sim \pm 1/k\pi \end{cases}$$

The solution for sigma at ½ is readily invoked as a sufficient Diophantine, even as it neither necessarily collapses the overall *s* to explicit symmetry (applicable to *-m*) nor

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[8] This paper being self-sustained, cross-references will become invaluable later on. In passing, as pointed out in the latest one, I oft-times deploy T in place of [potential] infinity which–as will become apparent in the forthcoming expositions—allows for finer distinction and utter precision otherwise overlooked or denied.

[9] It is straightforward to appreciate that, in general, $X = a^{\frac{logX}{loga}}$ for any (*a, X*) with $a = \pm 1$ being just one case in point of particular convenience ad hoc.

compromises the trivial zeros domain[10]. In fact, it can be shown how both are consistent with (or are inferred from) the ubiquitous structure:

$$(D)\ N^{-s} \sim \varphi * 0^{i\pi}$$

Assuming away the exposition in (C), it can—informally so yet invariably consistent with the Euler identity—be argued that, as long as[11] $\varphi = N^{-it} = N^{-2i\pi*(t/2\pi)} \sim 0^{t/2\pi}$,

$$0^{i\pi} \sim N^{-T*i\pi} \sim N^{-\frac{i\pi}{2i\pi}} = N^{-\frac{1}{2}} = N^{-\sigma}$$

Now, to arrive at $s = -2m$ or $s \equiv 0mod(-2)$ based on a similar distribution, suppose

$$\zeta = T * N^{-s} = 0 = T^{-1} \ \leftrightarrow\ N^{-s} = T^{-2} = N^{-2T\varphi} \leftrightarrow\ s = \frac{2\varphi}{2i\pi k} \equiv -2m \ \leftrightarrow\ \varphi = -2i\pi km$$

$$N^{-s/2} = N^{-T\varphi} = 0^{\varphi} = 0^{i\pi*(-2km)}$$

This appears consistent with (*1'*), (A), (B), and (D).

Finally, one could think of natural ways for straddling as well as endogenizing the critical strip and its candidate *Re(s) core*.

$$N^{-s} \equiv N^{-\sigma} * N^{-it}$$

$$N^{-\sigma} = 1^{-\sigma*log_1 N} = e^{-2\sigma*i\pi kT*logN} = \begin{cases} N^{-T*2\sigma*i\pi k} = \begin{cases} 0^{2\sigma*i\pi k} \\ 0^{\sigma} \end{cases} \\ (-1)^{T*\frac{logN}{\log(-1)}*2\sigma k*\log(-1)} = N^{\frac{T^2}{T}*i\pi k*2\sigma} = N^{T^2*4\sigma\pi^2*k^3} \end{cases}$$

The above suggests how a representation, otherwise fully consistent with the prior phi-power distributions, might still reveal (or conceal) a fairly rich structure. The latter applies to the imaginary part as follows:

$$N^{-it} = \begin{cases} 0^{2it*i\pi k} = 0^{-2\pi kt} \\ N^{T*i\pi k*2it} = N^{-T*2\pi kt} \end{cases}$$

The two latter alternatives are trivially convergent (save for the alternating sign), in which light, by invoking the zero resultant of zeta again, it obtains that

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[10] $s = \sigma\left(1 - \frac{1}{\sigma} * \frac{t}{i}\right) = 2\sigma = -2m$ with $\frac{t}{i} = \sigma$ implied in (B') under |k| large

[11] One alternate way of representing this, without necessarily holding t to be $0mod2\pi$, could be as follows: $\varphi = N^{-it} = 0^{\frac{-it*logN}{log0}} = 0^{2t\pi k*logN}$. It can further be shown that, $0^{logN} = 0\ \forall N \in \boldsymbol{R}$: $0 = N^{\frac{\log 0}{logN}} = N^{-T/logN} \to 0^{logN} = 0$, QED. To simplify things, $0^{2t\pi k*logN} = 0^{logN^{2t\pi k}} \equiv 0^{logN'} = 0$. This is at the very least to illustrate just how hastily the fine intricacies of candidate solutions can be subsumed under generic aggregates or seen as densely packed and hence allegedly nondistinct.

$$(E)\ T * N^{-s} \equiv 0 \ \leftrightarrow \ N^{-s} = 0^2 = \begin{cases} 0^{\sigma - 2\pi kt} \\ 0^{-T*4\sigma\pi^2*k^3+2\pi kt} \end{cases}$$

The first assessment can be inferred at $\sigma = 2 + 2\pi kt = 2 - it * 0$ even though, by relaxing the infinite summation under the variational principle, one obtains

$$(F)\ \sigma_+ = \begin{cases} 1, & t\ finite \\ 1 - i\varphi, \quad t \to T, & \varphi\ compactly\ supported \end{cases}$$

Otherwise, the real part can be distributed as,

$$(G)\ \sigma_- \sim \frac{\pi kt - 1}{2\pi^2 k^3 T} \sim \frac{t/T}{2\pi k^2} \sim \begin{cases} 0, & t\ finite \\ \dfrac{it}{k} \equiv it' & \end{cases}$$

$$(H)\ \sigma_{average}(1) = \frac{1}{2} * (1 + 0) = \frac{1}{2} \sim \frac{1}{2} * (1 - i\varphi + it') = \sigma_{average}(2)$$

Interestingly enough, not only does Re(s) return ½ on *average* in either of the representations, it likewise shows how the *(0..1)* critical strip can have been motivated, even as these bounds conceal in-depth structure as posited by the above. Needless to say, though, averaged values are no closer to the ultimate solution-distributive patterns than are the elusive or focal-dominant representations. This questions the ultimate nature of RH predictions time and again, even though all of the above seems to lend formal support to its plausibly unaided statements.