Stochastic, Granular, Space-time, and a new interpretation of (complex) time: a root model for both relativity and quantum mechanics

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A stochastic model is presented for the Planck-scale nature of space-time. From it, many features of quantum mechanics and relativity are derived. As mathematical points have no extent, the stochastic manifold cannot be tessellated with points and so a granular model is required. A constant grain size is not Lorentz invariant but, since volume *is* a Lorentz invariant, we posit grains with constant volumes. We treat both space and time stochastically and thus require a new interpretation of time to prevent an object being in multiple places at the same time. As the grains do have a definite volume, a mechanism is required to create and annihilate grains (without leaving gaps in space-time) as the universe, or parts thereof, expands or contracts. Making the time coordinate complex provides a mechanism. As this is a 'root' model, it attempts to explicate phenomena usually taken for granted, such as gravity and the nature of time. Both the General Relativity field equations (the master equations of Relativity) and the Schrödinger equation (the master equation of quantum mechanics) are produced.

INTRODUCTION

The precursor of this paper appeared years ago in Phys. Rev.[Ap.O]. The paper was highly cited and well regarded, e.g. Siser Roy called it 'profound'[1], Steven Miller 'Remarkable'[2], Luis de la Peña 'Pioneering work' [3]. Since then, there has been much activity in the stochastic approach, some of it spawned by the precursor paper.

However, that earlier paper, being behind a pay-wall, is not easily accessible. And also parts of it needed revisions. Hence the revised and slightly abridged version is included as the appendix to this paper.

Much of quantum mechanics may be derived if one adopts a very strong form of Mach's Principle, requiring that in the absence of mass, space-time becomes not flat but stochastic, a state of maximum entropy, and thus arguably providing no arrow of time. If one wanted to be philosophical, the idea of no arrow of time might suggest that in the absence of mass (and of photons), space-time should have no properties at all. And that might include the metric signature. In that case, the idea of dimension would seem to have no meaning.

The stochasticity is manifested in the metric tensor which is considered to be a collection of stochastic variables. The stochastic metric assumption is sufficient to generate the spread of the wave packet in empty space. The idea is that vacuum energy fluctuations imply mass fluctuations which imply curvature fluctuations which then imply fluctuations of the metric tensor. The metric fluctuations are then taken as fundamental and a stochastic space-time is theorized. A number of results from quantum mechanics are derived.

If one further notes that all observations of dynamical variables in the laboratory frame are contravariant components of tensors, and if one assumes that, locally, a Lagrangian can be constructed, then one can derive the uncertainty principle. Finally, the superposition of stochastic metrics and the identification of the negative of the determinant of the metric tensor as the indicator of relative probability yields the phenomenon of interference, as will be described for the two-slit experiment. (The above is from the precursor paper.)

Addressing some of the difficulties of the precurser paper, required an extension of the model: In-so-far as the fluctuations are not *in* space-time but *of* space-time, and points have no extent, a granular model was deemed necessary. For Lorentz invariance, the grains have constant 4-volumes. Further, as we wish to treat time and space similarly, we propose fluctuations in time. In order that a particle not appear at different points in space at the same time, we found it necessary to introduce a new model for time where time as we know it is emergent from an analogous coordinate, tau-time, τ, where 'τ-Time Leaves No Tracks' (that is to say, in the sub-quantum domain, there is no 'history'). The model provides a 'meaning' of curvature as well as a (loose) derivation of the Schwarzschild metric without need for the General Relativity field equations. In order to tessellate the space-time manifold, it was necessary to introduce a complex time with the imaginary component 'rolled-up' at the Planck scale. The imaginary component will be seen to be associated with mass and energy.

The purpose of the Stochastic, Granular, Complex Time model, 'SGCT', is to both fold the seemingly incomprehensible behaviors of quantum mechanics into the (one hopes) less incomprehensible properties of space-time, and also to generate many of the phenomena of relativity, as well as of quantum mechanics. We do this by working with space-time at the Planck scale.

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I. OVERVIEW

Although it is a remarkably reliable schema for describing phenomena in the small, quantum mechanics has conceptual problems; e.g. How can entanglement send information faster than light (without violating relativity)? What is happening in the two-slit experiment? How can it be that the wave function can instantaneously collapse? In what medium does the Ψ wave travel? Is the E=hf wave (the Compton wave) the same as the Ψ wave? What is the wave function? What explains superposition? Can the two-slit experiment (at least in theory) be performed with macroscopic masses? Is 'the Cat' alive or dead? (One should say at the outset that this stochastic space-time theory is a DeBroglie-Bohm rather than a Copenhagen model so Schrödinger's cat is not an issue; Waves interfere. Particles do not.) And finally, how can quantum mechanics peacefully coexist with relativity.

The mathematics of quantum mechanics works exceedingly well. What we attempt first in this paper is to provide a conceptual framework for the quantum phenomena described by the mathematical formalism.

Granular space-time theories suffer from the problem that if the grains have a specific size, then the theory cannot be Lorentz invariant. Accordingly, we'll model grains (which we call 'venues' to distinguish them from point-like 'events'), as having constant volumes (rather than constant dimensions) and volumes *are* Lorentz invariant.

But there is an issue with General Relativity as well as with our quantum mechanics model:

In differential geometry, Loveridge[4] has pointed out that the Ricci tensor governs the evolution of a small volume element (i.e. $\sqrt{-g}$) as it travels along a geodesic.

Following Loveridge, assume a very small spherical volume of dust o centered on point $x^\mu(0)$ moving along a direction T^μ. ($T^\mu \equiv \frac{dx^\mu}{d\tau}$). One has that $\frac{D^2}{d\tau^2}o - \frac{D^2_{flat}}{d\tau^2}o = -oR_{\mu\nu}T^\mu T^\nu$, where D is the covariant derivative along the path. The equation applies for both three and four dimensional volumes. The reason for subtracting the second term is that the choice of coordinates could give an apparent (not intrinsic) change of volume.

In Special Relativity, the Ricci tensor is zero. Which means that the volume element, $\sqrt{-g}$, is invariant. (For Special Relativity, this is easy to see: In a Lorentz transformation, as the length shrinks, time expands to leave the volume unchanged.) In General Relativity, in empty space-time, while the Riemann tensor is not zero, the Ricci tensor is. So, in empty space (i.e. exterior to a mass), the volume element is also invariant, i.e. $R_{\mu\nu} = 0$. In a mass though, the Ricci tensor is not zero so the volume element is not constant. Furthermore, our stochastic space-time quantum mechanics model postulates that the volume element, $\sqrt{-g}$, is not constant and is proportional to $\Psi^*\Psi$.

To address these issues, we *could* propose that there is a fifth dimension and that the five-dimensional volume element is *everywhere* constant. We could postulate that the fifth dimension coordinate (or the change of that coordinate with respect to the forth dimension time), is zero except when in a mass or where the wave function Ψ is non-zero (or where an electromagnetic field is present).

The square of the wave function itself would be proportional to the 4-dimensional volume element (see the Appendix). (Further on in this paper, we'll argue instead, that it is actually proportional to the *square* of the volume element.) External to a mass, the General Relativity field equations, i.e. $R_{\mu\nu} = 0$, would still hold (for both 4 and 5 dimensions, i.e. μ and ν range from 0 to 3, or from 0 to 4).

In order that the 5-dimensional line element not be observably different from the 4-dimensional line element, we could adopt the Kaluza-Klein[5] idea that the fifth dimension is 'rolled-up'.

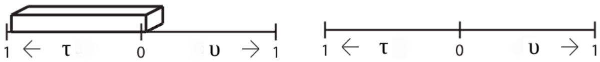
An alternative approach, (which we adopt instead of a fifth dimension), proposes that the forth dimension, time, is complex. And the imaginary component fulfills the same function as the aforementioned fifth-dimension. In particular, the volume element is the 3-volume element times the real time component times the magnitude of the imaginary component. The imaginary component is (as previously) rolled-up. We have taken the complex time approach (as opposed to the fifth-dimension approach) because, as will be seen, it gives better results, and also because the previous forth and fifth dimensions seemed to be very tightly connected, suggesting that they were aspects of the same quantity. Note: 'complex time' is not an entirely new idea, e.g. S. Hawking[6].

Another problem with a space-time of granules (with constant volumes) rather than of points is how to handle an expanding or contracting space-time or region of space-time. We need a mechanism to create and annihilate empty venues (venues not containing mass) without leaving gaps in the space-time manifold. Either the constants c, G, and/or h depend on the size of the universe and so change the Planck units in such a way as to preserve the number of venues, or the Susskind landscape model[7] is applicable and in addition the overall volume of the multiverse is constant and venues can migrate between universes, or there is a mechanism for the creation and annihilation of (mass-less) venues. We suggest the following mechanism:

As a part of the universe contracts, a venue's 4-volume must also contract. Since the contraction is a 'time' process, we suggest that the venue contraction is in the (real) time component. To keep the volume element constant, as the real time component contracts, the imaginary time component expands. We will (later in this paper) associate the imaginary time component with a venue's mass or energy (via the Compton frequency).

The following diagrams show coordinates τ and υ (the imaginary time component), and a rectangular solid representing a venue. (Note: τ-time, a modification of the usual t-time will be explained below.)

Again, as the τ coordinate of the venue contracts, to preserve the 5-volume, υ must expand. At some point, the contraction coordinate, τ, approaches zero while υ, approaches its circumference.



At the point where the contraction reaches zero, the υ component 'rolls over' to zero. The 5-volume is then zero and the venue blinks out of existence.

Creation of venues is similar: When a mass-less venue expands, it increases the real time coordinate. The imaginary time component decreases to compensate. The imaginary time component rolls over to give a high value to the component. The venue volume is then is far too high. The venue then splits, giving each new venue half the original real time component and half the original imaginary time component. As the expansion increases, the imaginary time components decrease as the venues' real time component increase, moving the two new venues towards equilibrium.

At no point then, is the space-time manifold not fully tessellated.

As we are treating space stochastically, for covariance we would like to treat both space and time similarly. To do that, we then let the stochasticity apply to time as well as space. This leads to an obvious problem: If a venue contains mass, then migrations can position the mass so it appears at multiple positions in space at the same time. E.g. A venue containing mass could migrate one unit backward in time, then one unit forward in, say, x, then one unit forward in time, resulting in the mass being at both (x,y,z,t) and (x+1,y,z,t). Preventing this necessitated a change in how we view time.

First, let's consider the idea of the 'world-line'. Moving forward from the present, we are predicting the future. And with quantum uncertainties (as well as with the intervention of outside forces) that future cannot be certain. And if there is no completely deterministic trajectory going forward, then arguably neither is there one going backward in time. The world-line then, seems to have limited utility in quantum mechanics. Instead of a world-line, we consider a 'world-double-cone', with its apex at 'now' that widens as one moves forward or backward in time.

We suggest that for the quantum world, t is not the (real component of the) fourth dimension, and that t is an emergent quantity, if not merely a human construct based on memory. The time coordinate, t, is a defined quantity in the laboratory frame whereas we suggest (below) another quantity, τ (tau-time) is appropriate in the quantum domain.

We'd like to treat the time dimension, t, in the same way as we treat spacial dimensions. But there is a big difference between a space and time coordinate: Consider the graphic below:

A particle (the black disk) starts at x=0, then moves to x=1, then 2, then 3. (We are considering space-time to be granular, hence the coordinate boxes.) There is a single instance of the particle.

But time is different:

A particle at rest is at t=0, then moves to t=1, etc. But when it goes from t=0 to t=1, it also remains at t=0. There are now two instances of the particle, etc. In other words, a particle at a particular time is still there as time advances, and the particle is at the advanced time as well.

We define then, a new quantity, τ (tau-time), that acts much like the usual time, but in accord with the first graphic, above, i.e. when the particle advances in time, it erases the previous instance. That is to say, 'τ-Time Leaves No Tracks'. Aside from fixing the problem of the same mass appearing at an enormous number of different locations at the same time, in the section on 'Migrations in Space and Time', τ will be seen to provide a solution to the collapse of the wave-function problem.

II. COMPLEX TIME AND ITS RELATION TO MASS

We define Total (complex) time T.

T=τ+iυ.

τ is the 'time leaves no tracks' version of t.

υ is the imaginary component of time. It is rolled-up at the Planck scale so in the macroworld T is indistinguishable from τ.

Letting τ and υ be represented by a real and imaginary coordinate axis, we define Time-length (duration) , $\mathrm{T_d}=\sqrt{\tau^2+(i\upsilon)^2}$. (We will use this in Section VIII.)

A property of time is that it (usually) advances. As υ is a component of time, we assume it advances as well. But υ is rolled-up, so, as it continuously advances, it continuously reaches a maximum and rolls over to zero. We represent this as a frequency.

Masreliez[8] and Mukhopadhyay[9] among others have suggested that a mass oscillates at its Compton frequency, (and without such oscillation, there would be no DeBroglie wave, or indeed a Ψ). We accept that suggestion. The Compton frequency ,f_c, is defined as $f_c = \frac{mc^2}{h}Hz$.

We first convert Hz to cycles/Planck time.

$$\frac{f_c}{\sqrt{\frac{hG}{c^5}}} = \frac{mc^2}{h}$$

Now we'll convert m from kilograms to Planck mass, m_p.

$\frac{f_c}{\sqrt{\frac{hG}{c^5}}} = \frac{mc^2}{h}\sqrt{\frac{hc}{G}}$

Simplifying, we have $f_c = m_p$.

This says that if the mass in a venue is zero, (from the viewpoint of the laboratory observer) the υ time does not advance (which allows the cretion/annihilation mechanism to work). The more mass in a venue, the more 'rapidly' υ advances until at a maximum venue mass of one Planck mass, the frequency has increased to one cycle per Planck time. And in that latter case, every Planck time, the resultant rotation advances υ to the same angular point, which is then indistinguishable from a frequency of zero. In short then, we associate mass with a frequency (the Compton frequency) of the imaginary time component.

Mass seems to come in chunks. In any case, mass does not seem to have a continuous range of values. And if mass is a discrete quantity, then so too is υ-time. And it is not unreasonable to assume τ-time is discrete as well. This time discreteness could be the reason an orbital electron can transition 'instantaneously' between orbits.

'Time' then can be considered made up of two characteristics: a coordinate (τ) going from minus to plus infinity, and υ, the imaginary time, representing an ordering schema as described by H. Reichenbach[10].

The imaginary component acts much like a separate (time-like) fifth dimension. This is vaguely similar to the idea that there is a fifth dimension which *is* mass, as proposed by Mashhoon & Wesson[11] and the Space-Time-Matter consortium[12].

III. WIENER (AND WIENER-LIKE) PROCESSES

Stochasticity is exhibited by venues migrating through the space-time manifold (without leaving gaps in the space-time). We required granularity since the (stochastic) space-time must tessellate the manifold. But point-like events have no volume which is to say that multiple events could migrate to the same 'point' in the manifold. (We will see though, that interior to a Schwarzschild singularity, gaps could occur if we accept an interpretation of the Kruskal metric saying that the 'interior' of a Kruskal wormhole is not in the space-time manifold [it multiply connects the manifold].) A 'point' then, is an idealization. So is a line. So the shortest possible length is also an idealization without any physical meaning. We maintain, on the other hand, that a volume is 'real'. So the idea of a smallest possible volume is likewise 'real'.

As mentioned above, there is another difference between time and space: Time (usually) moves forward. To reflect this, rather than a strictly Brownian motion approach where a particle can move in space left or right, up or down, in or out, in a reference frame where its velocity is zero, time for that particle can move forward (its usual behavior) or remain stationary. As a result, time can not go backwards in that reference frame.

The approach taken here considers a granular space-time undergoing Brownian Motion in both space and time. A (modified) Wiener Process is our starting point in modeling a granular, indeterminate space-time.

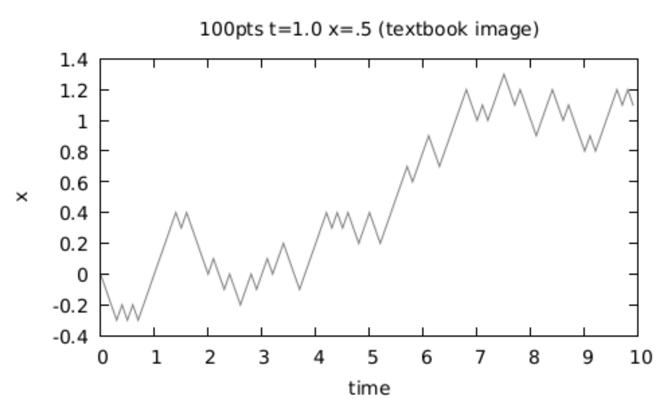
First, we consider Wiener migrations of venues in space.

A Wiener Process W is an idealization of Brownian motion. It is a random walk of n steps where n approaches infinity.

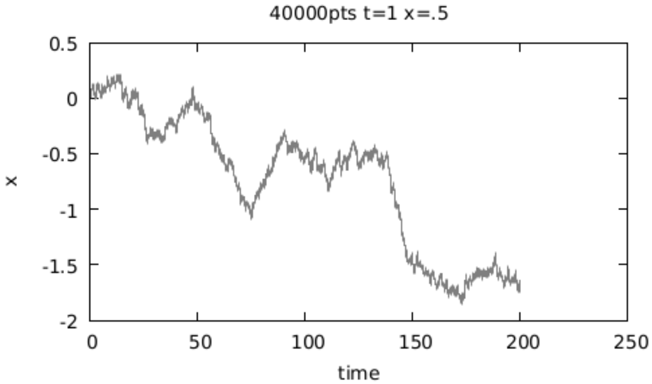
The ith step is defined as

$$W_i = W_i - 1 + \frac{X}{\sqrt{i}}$$

where X is a binary random variable (+ or - 1). As n gets large, the distribution of W_i tends towards the unit normal distribution. As can readily be seen, as i goes to infinity, the W graph is everywhere continuous but nowhere differentiable. The graph is fractal (in that it is scale independent). The graph is a 'space filling' curve with fractal dimension 1.5. Traversing between any two points along the curve requires covering an infinite distance. However, in any finite time interval, there are found all finite values of x. So, if we let i go to infinity, in the case where a venue can move, it can move to all values of x in an arbitrarily small time interval, e.g. faster than light, which would not be a problem for venues not carrying mass. In this granular model though, we do not let i go to infinity.

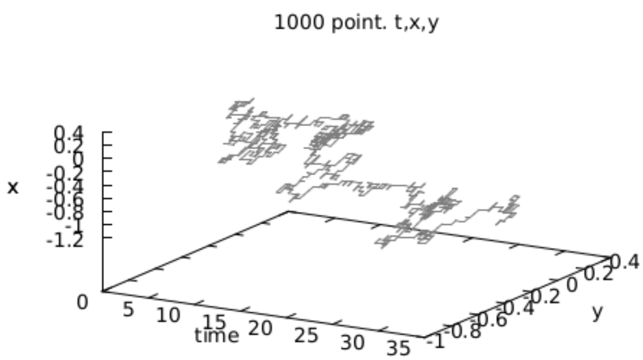


Here is something of a textbook example of a 100 point Wiener Process curve with measure=0.5. Note: 'measure' refers to the probability of a 'coin flip' being heads. E.g. a measure of 0.75 means there is a 75% probability of the coin being heads (or left vs. right, or up vs, down).



And above is a 40000 point example.

Extended to infinity, the variable i becomes a continuous variable, generally represented as t (time). In our granular model we do not extend to infinity. The above is for a 2-dimensional process (t vs x). To express t, x and y, two coins are flipped, one for x migration and the other for y.

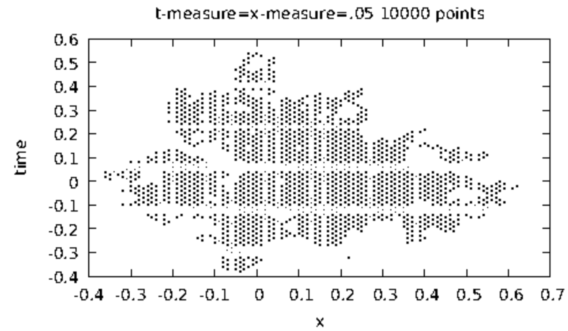


An x measure greater than .5 causes a tendency to drift up. Less than .5 tends downward

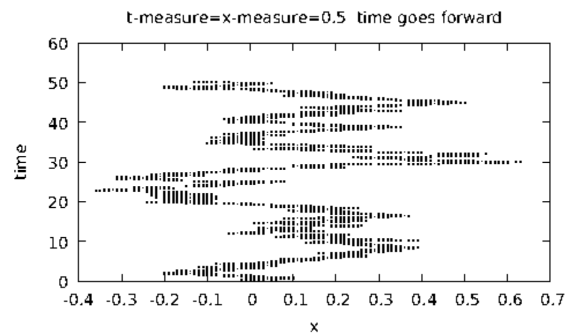
A. Migrations in both Space and Time; Time in Quantum Mechanics

We would like to treat time and space similarly. And so we will consider diffusion in space as well as in time.

Consider the graph (of 1000 points) below. (The vertical and horizontal lines are artifacts of the graphing software.) The graph represents the path of a a single venue migrating in x and also in t, both with a measure of 0.5, where the coordinate axes are laboratory x and laboratory t.



And here is the graph with the same data as above, but where, as described above in **I**, 'time moves forward' is taken into consideration. (However, from another reference frame, time can still go backwards.)

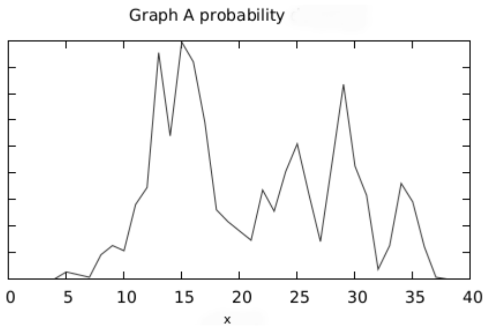


In either graph, there is an immediate problem:

Consider what these graphs signify: At any given laboratory-time t, the same venue will (simultaneously) be at a very large number of x coordinates. If there were mass/energy at the venue, this would be very problematic as causality and conservation of mass would be violated.

This problem has been addressed (in the introduction) by introducing τ (tau-time), and the 'τ-Time Leaves no Tracks' idea.

We can still consider the graphs, but we'll interpret them differently: If we take any (horizontal) time (τ) as a 'now', A venue (containing a mass) stochastically flits forward and back in space, and forward and being stationary in time. So that at 'now' there is one and only one particle. But where it is cannot be predicted. However, the likelihood of the particle being at a particular x (+/- dx) position is determined by the relative number of times the particle is at that position. In the case of the graphs, if we take as 'now' the τ-time slice at -0.2, for example, we find (by examining the data) the following probability curve (for the first graph):



This is analogous to $\Psi^*\Psi$. But the graph is a construct. It represents, but is not actually, the particle. When the particle is measured by, for example, being absorbed in a detector, it freezes (no longer moves stochastically). It no longer flits through time and space so the graph 'collapses' to the measured position. (that position is only determinable by the measurement.) This is analogous to the collapse of the wave function, but here (as the graph was merely a mathematical construct) there is no collapse problem.

There are a few points/speculations to be made about measurements. First, to be a true measurement, there must be a latch/flip-flop/memory so that the 'film' cannot be run backwards. As an example, consider the two slit experiment with electrons. If a measurement device is placed at a slit, there is no interference pattern. But when an electron goes through a slit, the orbital electrons in atoms of the wall of the slit will be distorted by the passage of the electron. This distortion is *almost* a measurement. But when the electron passes through the slit, the orbital electrons become un-distorted. The interference pattern is still produced because there is no latching of measurement information. A latch could be some mechanical contrivance, or even human (or non-human) memory. A fruit-fly observing at the slit will kill the interference pattern, but only for the fruit-fly. We think the process should be transitive; A human observing the fruit-fly's memory will cause the interference to be killed for the human as well. A measurement forges a connection between the thing being measured and the measurer–forcing them to have the same relative now. In the macro-world, virtually everything observes (via photons) everything else, forcing that macro-world (or a portion thereof) to have the same relative now. And measurements forces time to have tracks. Not that time is frozen, but looking back to a particular time will show uniquely what the world looked like at that time. E.g., if one were to do high-speed filming of particle 'tracks' in a cloud chamber, one would see the time-tracks.

Observation, a crucial part of a measurement, is conducted via photons. We speculate that *all* measurements are via photons (or, equivalently, by the electromagnetic field)?

The time leaves no tracks concept implies that there are multiple futures, and they all 'happen'. (This is somewhat redolent of the Everett many-world interpretation[13].) In SGCT, an observation from the laboratory will select a particular future (making a track).

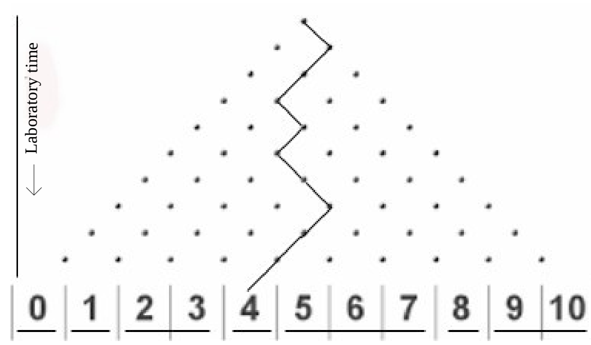
In the above, if the particle were in a potential well with perfectly reflecting walls, the above graph would (after a time) represent the probability density of finding the particle at a particular position in the well.

Again, the particle has always existed at only a single venue, but the venue migrations happen roughly at the rate of the Planck time, making the particle appear (in some sense) to be at multiple positions at a particular time. Further, (because of the properties of Wiener Processes) the particle appears to spread. If the particle were not constrained by the well, the graph would evolve (spread) arbitrarily rapidly. In that case the curve would represent the relative probability density of finding the particle at a particular position. The curve then would represent DeBroglie's 'ghost waves that guide the particle'[14].

(The jagged lines in the graph, as opposed to a smooth curve, is an artifact of the binning algorithm in the software.)

By Statement 1.4 of the precursor paper (the appendix of this paper), the particle location becomes less stochastic as mass increases (this will be demonstrated in the next subsection). There is a point where the stochasticity ceases. There one can use the usual t-time. So, we consider t-time (and also causality) to be an emergent quantity. In the rest of this paper, when we do not reference history, we will simply use t instead of τ. Also, as a result of measurement, when the above graph 'collapses', the time is fixed, so measurement causes time to 'leave tracks' causing τ to become t.

Now we can revisit Statement 3 of the precursor paper: The metric probability postulate, $P(x,t) = -kg$.



Consider the above diagram. (Note: time increases downward.)

A particle is placed at the apex (more accurately, we place a *venue* at the apex). It migrates stochastically until it ends up in one of the numbered bins. A typical migration path is shown in the diagram. If we repeat the process a large number of times, the number of particles in each bin (of the x coordinate) can be described by a binomial distribution centered on bin 5. (See Subsection B, below, for a graphic, and a more detailed description.) That binomial distribution can represent the probability density of finding a particle in a particular bin at laboratory time 10 (the bottom of the pegboard). As we increase the number of time steps, the distribution will flatten, until it is essentially flat. At that point the size of the invariant volume element $dV_I = \sqrt{-|g|}dx^1dx^2dx^3dx^4$ determines the probability density. And that is the original version of Statement 3.

In the diagram above, the bin underlines represent different volume elements at different regions of the space. A way of determining the probability densities is to count the number of possible migration paths to a particular region. (The space-time is discrete so the number is countable.) The probability densities are proportional to those numbers.

Consider the typical path shown above. We can represent it as follows:

t:	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
x:	+1	-1	-1	+1	-1	+1	+1	-1	-1	-1

Time increases by one unit at each transition.

At this point, the original version of Statement 3 still applies.

But now let there also be diffusion in time, as well as in space. We can repeat the pegboard analogy, but consider the venue time rather than venue mass. We will of course obtain the same probability distribution as previously. The distribution will represent the probability density of the quantum time having the value of the laboratory time at laboratory time equal to 10 (the bottom of the pegboard).

The peak of the distribution curve corresponds to the peak on the previous curve. Accordingly, the probability density for x to be at 5 (the middle of the pegboard) must be that previous distribution value at x=5 times the quantum time distribution at x=5. But the two distributions are the same, so the probability density is the square of the distribution. The same argument applies for any point along the base (x coordinate), i.e. the probability density for x is the square of the binomial distribution.

So, again considering the invariant volume element, this means that the probability densities are the square of the values of the original Series 3 values, i,e, $P(x,t) = -k|g|$.

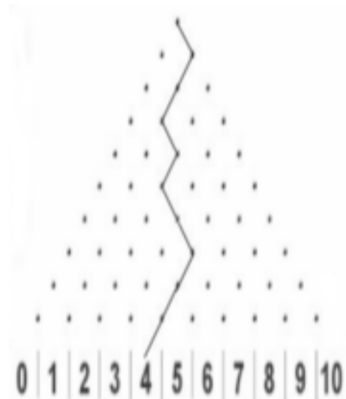
Note: As the probability density is not stochastic while the metric components are, that puts constraints on the metric tensor, i.e. the determinant of the metric tensor is constant while the metric components are not. So (stochastic) changes in one or more components are compensated by opposite changes in the others. This implies that while a venue is in constant flux, its dimensions continuously and unpredictably change while the venue maintains a constant volume. This also implies that the metric stochasticity is due to a single (and the same) random variable in each non-zero metric component (That variable will then drop out in the determinant.)

B. Generalization of the Pegboard

Expanding on the pegboard description: If a venue at the peak of the pegboard contains a Planck mass, the largest mass it can host, then there are no migrations (i.e. the mass is stationary) and the apex angle of the pegboard is zero which is to say the quantum time and laboratory time are the same. As one increases the angle, migrations and the speed of migrations increase. (Note that migrations are assumed to occur instantaneously, so 'speed' refers to the time interval between successive migrations.)

As the angle increases, one sweeps through particles of decreasing mass until at just under 90 degrees, one is at the mass(es) of neutrinos. The x coordinate migrations is near maximal and the speed is just under c (one Planck length per Planck time). At 90 degrees, the venue mass is zero and the pegboard represents the light cone. The apex angle can continue to increase to 180 degrees. At that point venues can migrate instantaneously.

Consider, for example, the following diagram representing a pegboard for a venue hosting a relatively massive particle:



We consider, as previously, the horizontal scale as bins of one Planck length. The vertical axis time interval then is greater than 1. It is easy to see that as the vertex angle goes to zero (representing a Planck mass which therefore is not migrating), the time interval goes to $\sqrt{2}$.

This presents a problem: Our Wiener-like migrations assumes that each migration is exactly one Planck length and one Planck time. And this (with a variable apex angle) is clearly not the case. This can be resolved by the use of complex time as follows:

As in Section II, we define Total (complex) time T. T=τ+iυ.

And again, letting τ and υ be represented by a real and imaginary coordinate axis, we define Time-length $T_d = \sqrt{\tau^2 + (i\upsilon)^2}$.

In accordance to the unity Planck length and Planck time migration model, instead of having τ always being unity (which it isn't), we'll have T_d always be unity, i.e, $1=\tau^2 - \upsilon^2$.

So as τ increases, υ increases. τ and υ change at the same rate so $1=\tau^2 - \upsilon^2$ is maintained.

As we have previously shown, the probability density goes as the volume element squared (and therefore as time squared). We assume it goes as for υ^2 as well as for τ^2.

An increase of the frequency of υ indicates an increase of mass, resulting in an increase of τ. And an increase of τ says that the migration of the venue hosting mass goes more slowly. So one could say that υ (rolling-over at the venue-mass Compton frequency) is a measure of the venue's mass while τ is a measure of the mass's inertia.

And because of the probability density going as time squared (as does τ and υ in the definition of $\mathrm{T_d}$), the pegboard time evolution is linear.

IV. VENUE MIGRATIONS IN EMPTY SPACE

Mach's Principle posits that the local properties of space-time depend on the mass distribution in the universe. We'll adapt the principle to the SGCT model. And we'll introduce another variable: 'Indeterminacy', the probability that a migration will actually happen.

As with 'Measure', Indeterminacy is implemented with a 'coin flip'. And we'll suggest that outside of a mass, the Indeterminacy decreases with decreasing distance from the mass/energy (i.e. space becomes more determinate as one approaches a mass). It will be seen that 'Measure' mainly influences quantum effects while Indeterminacy influences relativistic effects.

The space-time Indeterminacy decreases as one approaches a mass. But this is under-specified; masses can have different densities, so we wouldn't expect the Indeterminacy to necessarily vanish at the surface of a mass. We suggest however, that venues can migrate into a mass until, at some point the Indeterminacy vanishes. Yet we do not want masses to be pulled apart by the space-time so we'll posit that migrations of adjacent venues each containing mass must stay adjacent. And in that case, one could consider each of those venues having zero Indeterminancy compared to the others.

We'd expect that at some distance, Rs, from the center of the mass, the venues, would be trapped, i.e. unable to migrate away. This is highly suggestive of the event horizon of the Schwarzschild solution. We'll assume Rs (the Indeterminacy radius) and the Schwarzschild radius are the same.

The concept of Indeterminacy decreasing with closeness to mass has an interesting *possible* consequence relating to measurement: A measurement requires an exchange of energy between what is being measured and the measurer (an energy that can't be transformed away). But energy of this form (e.g. photons), being equivalent to mass, *possibly* forces determinacy of the photon relative to the object that absorbs and/or emits the photon.

So, for example, if one were to place a measuring apparatus at one slit in the two-slit experiment, activity at that slit (at the time it is measuring if a particle went through it) would be deterministic since the photon 'connects' the slit and the emitter (i.e. the laboratory). And therefore, since Indeterminancy=0 says that the system behaves classically (as opposed to quantum-mechanically), the interference pattern would seem not to happen. But once the particle passes through the slit, the connection is broken so the interference pattern *would* occur *unless* the measurement was 'recorded' (e.g. a flip-flop). See section X,B.

Insofar as measurements are accompanied by exchanges of photons, it's tempting to consider that photons are the carriers of causality.

Up to this point, we've considered the migration of just a single venue. The model though, assumes space-time is completely tessellated (tiled) by venues, i.e. there are no regions of space-time that are not fully covered by venues. While we can justify the migration of a single venue, migrations of venues in a completely tiled space-time is more problematic. One might even doubt that there can be any migrations at all in a fully-tiled space-time. We are modeling the stochasticity of space-time as a Wiener-like process on venues (grains). We assume that the space-time completely tessellates the space-time (i.e. there are no holes in the space-time). How then can migrations occur in a fully tiled space-time? We start by modeling any single venue as a Wiener-like process. Other venues must then also migrate to preserve the tiling.

The migration can proceed in one or two ways: The first is like the circulation in a perfect fluid. The 'diffusion' in that case, is via closed loops in the space-time. This can happen if the time of migration is not instantaneous. In that case the probability of a loop migrating synchronously is not zero, as all venues in the loop can migrate in the non-zero migration time.

The second way is the squishing-interchange of venues, as shown below: The diagrams represent an idealized pair of venues. The black and white venues continuously move to interchange their positions while keeping their volumes constant. If migrations happen instantaneously, then a loop migration has zero probability of occurring (since all venues in the loop would have to migrate at the exact same instant). In the instantaneous case, (the asynchronous case) when a venue migrates, the venues surrounding it do not *at that same instant* migrate. In that case the migrating venue selects some direction in which to migrate, and then interchanges (via squishing) with the (temporarily static) venue in that direction. As venues do not simultaneously migrate in 'lock step', asynchronous migrations are by far the most likely migration mechanism, and can be approximated as a Wiener process.

While the SGCT model is of a discrete, granular space-time, the discreetness is expressed in the venue volumes.

So local continuous processes (between adjacent venues) as the above are not disallowed. Indeed, the Diffusion equation, can apply. See Section VIII.

The migration problem persists though, as can be seen in Indeterminacy: Assume a spherical mass in an otherwise empty space. Indeterminacy is assumed to decrease as a venue migrates towards a mass. Even with Measure = 0.5, a venue will at some point approach arbitrarily close to the mass. But (letting r be the radial distance to the mass) as Indeterminacy is the probability that the venue will not migrate at the next coin flip, the venue will spend increasing amounts of 'time' as r decreases. In the case of multiple venues, there will be proportionally more of them in a volume element closer to the mass. This results in the 'piling-up' of venues as one gets closer to the mass. How can this be? We don't want to resort to venues 'pushing' against other venues since that would imply that the venues are overlaid onto space-time instead of them *being* space-time. Nor do we want to employ higher dimensions. An answer (perhaps the only answer) is curvature. But what is curvature? 't Hooft has theorized[15] that curvature is an artifact of the fact that we live in four dimensions but space-time is actually five dimensional (e.g. a two dimensional being on a sphere can measure curvature, but with the sphere embedded in a flat three dimensions, there is no real curvature.) We will take a different approach: Venues are assumed to have constant volume but not constant dimensions. Curvature will be described, below, as the thinning of space dimensions, (particularly the radial distance from a mass dimension) while the time dimension thickens.

Note that as for the motion of the particle the particle doesn't become 'fuzzy', but its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger grain size.

Two effects: like a smaller pollen grain in Brownian motion: the smaller the grain, the more it stochastically moves. But as the effective grain radius increases, the movement decreases as there is a larger circumference over which the movements can average.

Note then that the effective radius *rate* of increase decreases as the effective radius increases. To reiterate, this is because, as the particle grows in *effective* size the average effect of the venue migrations against the particle surface begin to average out (analogous to the case of Brownian motion where the jitter of a large pollen grain is less than that of a smaller grain).

We maintain that all physics that uses the radius should use the effective radius. radius= rest-radius + *Radius Quantum Correction:* $r = r_{c+}r_{qc}$. For an example of the effective radius, see the Schwarzschild metric derivation below.

One might consider the 'actual' radius as the covariant (and hence unobservable radius) whereas the effective radius is the contravariant (in principle, observable) radius.

We explore now whether the model might indeed reproduce the Schwarzschild metric.

A mass generates 'curvature', that is to say, a deformation of venues. While to a distant observer the venues are deformed to be spatially concentrated around the mass, to the venues near the mass there is no observable evidence of such concentration as the space-time itself is 'deformed' (by way of the venues) so any 'observer' in a venue would be unaware of the deformation.

Consider space-time with a single spherical mass m with an Indeterminacy radius Rs. The Wiener graphs are for some undefined unit of time. But as one increases the number of coin flips towards infinity, the time interval decreases to an infinitesimal, dt. For a granular space-time though, the number of coin flips isn't infinite and the time interval, though small, isn't infinitesimal. Once again, Indeterminacy is the probability of, given that the venue is at a position with that Indeterminacy, the venue migrates from that position at the next coin flip.

Since migrations slow as venues approach a mass, indeterminacy then, expresses the slowdown in time and the compression of space as the venue approaches Rs. [As we'll be frequently employing Indeterminacy, we'll represent it by the letter 'u' (from the German word for indeterminacy, Unbestimmtheit)].

As a venue migrates in towards R_s, u decreases. The probability density of the venue being at a particular radial distance, r, therefore, increases. This results in venues piling-up as they approach R_s. But as the venues 'tile' space-time, the only way they can pile up is by way of curvature (i.e. squishing in the radial dimension and compensating by lengthening in the real-time τ dimension): To a distant observer, the venues would decrease in size and migrate more slowly which is to say time would slow down.

Recalling (see Statement 2 of the precursor paper) that the contravariant distance to a black hole is $\int_0^{\bar{r}} dr = \bar{r}$, while the covariant distance is $\int_0^{\bar{r}} d(\frac{r}{1-2Gm/r}) = \infty$, we (in Cartesian coordinates) associate the contravariant distance with the number of *Planck lengths* from the observer to the point of observation and the covariant distance with the number of *venues* from the observer to the point of observation.

This implies that local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity, and because of that stochasticity, the measurements are also stochastic, and the measurements are contravariant, as can be seen by the raising of the covariant coordinates by the stochastic metric tensor).]

Now, near r = R_s, space-time becomes Q-classical (no quantum effects, as opposed here to R-classical: no general relativity effects) so a metric makes some sense. Since Measure (bias in the coin flips) is presumed not to be a function of location, we take the simplifying assumption that the metric tensor does not depend on Measure, but only on the Indeterminacy, u. And, for the moment, we'll ignore how a venue migrates *in* a mass (when R_s is less than the mass radius).

Since for a mass, we have spherical symmetry, we can let, $ds^2 = -f(u)dt^2 + g(u)dr^2 + r^2 d\Omega^2$where f and g are two (to be determined) functions of u, and $d\Omega^2 \equiv d\theta^2 + sin^2(\theta)d\varphi^2$is the metric of a 2-dimensional sphere. Consider f(u) and g(u). We wish dt to lengthen and dr to shorten as u decreases. ds can be thought of as the time element in the frame of the venue. So, for example, as u goes to zero, a big change in t will result in a small change of s, and a small change in r results in a large change in s. The *simplest* implementation of the above suggests that $f(u)$ is just u itself and $g(u)$ is u^{-1}i.e. $ds^2 = -udt^2 + u^{-1}dr^2 + r^2 d\Omega^2$.

Now, as to u, note that,

at r = infinity: u = 1,

at r = Rs : u =0, and

for r < Rs : u can become nonphysical (u<0).

The *simplest* expression for u satisfying the above is, $u = (1 - \frac{R_s}{r})$ which gives us

$$ds^2 = -(1 - \frac{R_s}{r})dt^2 + (1 - \frac{R_s}{r})^{-1}dr^2 + r^2 d\Omega^2$$

We have of course, as described earlier, equated the Schwarzschild radius with the Indeterminacy radius.

This is the result Karl Schwarzschild derived from the General Relativity field equations. One can easily go a bit further by noting that R_s can only be a function of the mass, and finding a product of mass with some physical constants to give a quantity with dimensions of length suggests $R_s = \frac{kGm}{c^2}$ where k is a constant. So we now have (setting units so that c=1),

$$ds^2 = -(1 - \frac{kGm_s}{r})dt^2 + (1 - \frac{kGm_s}{r})^{-1}dr^2 + r^2 d\Omega^2.$$

We still need to determine the value of the constant, k. But this is known territory. R_s was derived (by Karl Schwarzschild and others) by requiring the metric to reproduce the Newtonian result at large values of r and small values of mass, and we need not reproduce the derivation(s) here.

At first glance, there appears to be a problem with Schwarzschild metric and stochastic granular space-time theory in that masses can be arbitrarily smaller than the Planck mass. And that would allow the Schwarzschild radius to be vanishingly small, to the point of exposing the 'naked singularity' at r=0. And that is something we would like not to be possible.

But, as described earlier, any physical radius must be the *effective* radius (effective radius= rest-radius + Radius Quantum Correction). As a mass decreases to below the Planck mass, quantum effects occur which increase the effective radius. So a Schwarzschild radius of one Planck length is the minimum possible Schwarzschild radius. Masses less than one mass then increases the (effective) Schwarzschild radius (until the rate of increase decreases to zero). That the Schwarzschild radius of a Planck mass is the Planck length is then consistent with the granular hypothesis.

V. STOCHASTIC GRANULAR SPACE-TIME AND THE LORENTZ AETHER THEORY

We consider that our Stochastic Granular Space-time (SGCT) theory is (or can be made to be) a super-set of the Lorentz Aether Theory (LAT) where the aether is space-time itself (specifically, the 'grains'/venues making up the space-time). By doing so, we can appropriate the LAT derivation of the constancy of the speed of light. (We feel that any theory of space-time should contain an explanation of that constancy.)

As is widely known[16], the Michelson-Morley experiment failed to find the Lorentz aether, thus seemingly invalidating the Lorentz Theory[17]. Less widely known perhaps, is that the second version of Lorentz's theory (with H. Poincaré as second author) reproduced Einstein's Special Relativity (ESR) so well that there is no experimental way to decide between the two theories[17]. The second LAT theory differs from the first in that it posits that the aether is partially dragged along with a moving body in the aether. This is akin to frame dragging (e.g. the Lense-Thirring effect) in the Kerr Metric[18]. We will posit frame dragging in SGCT as well, i.e. the dragging along of venues by a moving object. (Note that the Kerr metric itself 'breaks' the continuity space-time. If it didn't, the frame dragging would 'wind-up' space-time, and it doesn't[19]. One might take this as an argument for a discrete space-time such as in SGCT.)

Although LAT derives the constancy of the speed of light whereas ESR takes it as a given, there are objections to LAT:

1. There is an 'aether', the makeup of which is not specified.
2. There is a privileged, albeit unobservable, reference frame where the aether is at rest (isotropic).
3. The (constant) velocity of light results from electromagnetic interactions with waves (and matter), and not from properties of space-time.

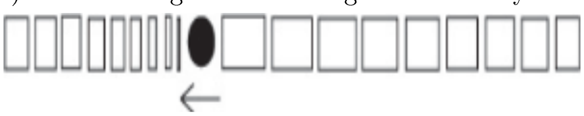
SGCT can address these issues: As for 1, the makeup of the aether, SGCT says the aether is the space-time itself. And in 1922, Einstein himself said essentially the same thing.

[Note: Einstein (translation): "Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it"]

2. A privileged reference frame, is also not an issue in SGCT. The stochastic nature of space-time makes it impossible to define a *global* rest frame. But we can consider a *local* privileged reference frame where the correlation region (the region where we can consider a background privileged frame) is large compared to the region where we are doing experiments. And the Unruh effect[20] implies that Lorentz frames are privileged.

3. The constancy of the speed of light not a result of the properties of space-time, can be addressed as well. While there is nothing wrong with the LAT derivation of the constancy, we can give a qualitative geometrical model as an alternate way of thinking about the constancy:

We suggest (and this is highly speculative) that frame-dragging occurs whenever a mass (non-zero rest mass) moves through space-time. Photons, as their rest mass is zero, moves without frame-dragging. This (as we will see) allows an argument showing the constancy of c.



Consider an object (here, the black circle) moving at high speed in the direction of the arrow. The object moves through the venues (here represented by the white rectangles). But due to venue frame dragging at high velocities, the venues are pushed ahead of the moving object. But venues are constant in volume, and the only way that they can 'pile-up' is by contracting in the direction of motion (and expanding in other dimensions). The object must move through these venues. As the object's speed increases, the contraction increases (rather in the way a 'curvature well' becomes ever deeper). To an external observer (making contravariant observations), the objects increase in velocity slows until it stops completely where the venue dimension in the direction of motion approaches zero. To that observer (as can be seen in the diagram above) the object is accelerating (which because of the Equivalence Principle, is under the influence of gravity). This establishes that a mass has a limiting velocity.

We have postulated then, that a particle with non-zero rest mass drags along (empty) venues as it moves, Photons, having zero rest mass, do not drag venues.

So, if a particle moving with respect to the local privileged reference frame emits a photon, the photon does initially travel with a velocity of c plus the velocity of the particle. But the particle is dragging venues. As the venue contracts in the direction of motion, since its volume is constant, it expands in the time dimension. And this makes the time a photon takes to pass through the venue constant. The photon has more venues to pass through than it would have if the particle were not moving. Because of the additional distance (i.e. number of venues) the photon needs to travel, its speed at the detector, would be a constant, which is to say c.

If the detector were extremely close to the emitter (on the order of Planck lengths) one would measure a value of the velocity greater than c. This length scale is too small to measure so the velocity greater than c is unobservable.

The SGCT model violates Galilean Relativity in that motion is not (in this model) relative. LAT violates it as well. This is allowed (in both cases) by having a privileged reference frame.

With SGCT then, there is a new phenomenon at play: 'Velocity Induced Frame-dragging. So, in addition to frame-dragging being generated by mass (or acceleration), it is also generated by an object's linear motion in the space-time aether. One way of perhaps justifying this is to consider the conservation of energy, as the sum of potential and kinetic energy. The former is gravity dependent while the other is motion dependent. Since gravity yields curvature, perhaps velocity does as well. Potential then, could be considered a result of Mach's Principle.

Frame-dragging has much in common with curvature, specifically Schwarzschild curvature. We might therefore expect the metric tensors to be similar. Indeed, without doing any calculations, we can guess at a metric for the moving object. Consider the g_{11}(the radial component of the Schwarzschild metric) $(1-\frac{2Gm}{rc^2})^{-1}$. The velocity induced model is not a function of mass, so m and G are unlikely to be in g_{11}. However, note that Gm/rc^2 has units of v^2/c^2, so we might expect g_{11} to be $(1-k\frac{v^2}{c^2})^{-1}$ where k is a constant. We would expect a (coordinate) singularity to occur when $v=c$, so that would make $k=1$. A similar argument can be made for g_{00}(the time component).

VI. BRIEF SUMMARY OF THE MODEL SO FAR

The aim of 'Stochastic space-time' is to introduce stochasticity into the structure of space-time itself, rather than into the properties of the particles in the space-time. This is a similar, geometrodynamic, approach to Nelson's groundbreaking model[21] that indeed has matter moving stochastically *in* the space-time.

Mass reduces stochasticity as one approaches the mass. And, insofar as stochasticity correlates to entropy which establishes the arrow of time, the 'length' of that arrow is not constant throughout space-time.

Because points have no extent, there seemed to be no way to prevent events (points) migrating to the same point. Therefore tessellating space-time would be problematic. So a granular model of space-time seemed necessary. Further, whereas the only geometrical property of an event is its coordinate location, grains, having extent, can have different values of $\mathbf{\Delta}$x, $\mathbf{\Delta}$y, $\mathbf{\Delta}$z, and $\mathbf{\Delta}$t (or rather $\mathbf{\Delta}\tau$ and $\mathbf{\Delta}\upsilon$). And that allows an explanation of curvature within four dimensions (as opposed to explaining it by embedding the four dimensional space-time manifold in a five dimensional Euclidean space). And as long as the volume of the grains (which we call 'venues') is constant, we do not violate Lorentz invariance.

In order that we treat time in the same way as we treat space (and not to have particles appear at different places at the same time), we needed a new version of time, τ-time. The implication is that our usual t-time is just a human construct, not actually intrinsic to space-time. The utility of having the volume element constant and the requirement of tessellating space-time in an expanding or contracting universe led us to considering complex-time.

VII. MASS AND GRAVITY

1. *Geometric Properties of Mass*

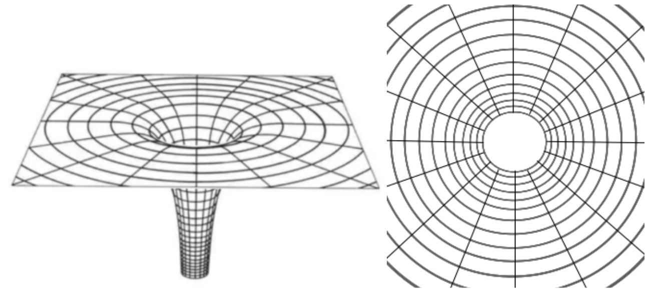
The SGCT model doesn't attempt to say what mass is, but instead examine the geometric properties pertaining to the mass.

The principal function of mass is (in the model) the stabilization of space-time, i.e, one would like the fluctuations in/of space-time not to rip apart masses. In particular, a mass causes adjacent mass-containing venues, because of stabilization, to act as a single larger venue (which is why E(mass)=hf works).

In empty space, in particular, the venues' dimension coordinates fluctuate (and this is required for the creation and annihilation of empty venues). The fluctuating mass can be associated with vacuum energy fluctuations and metric tensor fluctuations. The idea of metric tensor fluctuations was the initial idea behind our stochastic space-time theory (the precursor paper).

In (the usual interpretation of) General Relativity, mass causes 'curvature'. But what is curvature? Arguably, it is merely an artifact of describing space-time with one too few dimensions. For example, if a (two dimensional) ant were wandering on the surface of a sphere, he could measure curvature and determine that his environment was non-Euclidean. A three-dimensional (ignoring time) being would say the space(-time) *was* Euclidean and the ant was not able to see that third dimension. G. 't. Hooft has made a similar argument, as has J. Beichler. (And, of course, 'Campbell's Embedding Theorem'[22] states that any n-dimensional Riemannian manifold can be embedded locally in an n+1-dimensional Ricci-flat manifold.) But for us, rather than using a full extra dimension to explain curvature, we describe curvature as an artifact of the four-dimensional contraction or expansion of venue coordinate values. A feature of this granular model is that some phenomena attributed to the large-scale structure of space-time (e.g curvature) can be explained by extremely small scale phenomena (e.g. compression and expansion of venue dimensions).

Graphically, the left image below represents the traditional 'gravity well' curvature representation, and the right image is the left image but looking directly down from above.



The right image is still a three-dimensional representation. But, in the SGCT interpretation, it is a two-dimensional image, the 'depth' being due to the compression of venues. We have reduced space-time by one dimension (without losing information) and since there is no curvature, the space-time (arguably) is flat. (This is a kind of 'holographic principle'.)

Note: As previously noted, for the motion of a mass, the mass doesn't become 'fuzzy', but (because of migration external to the mass) its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger mass diameter.

A quantum particle apparently spreads. So, in some sense, the mass is effectively spread through the space-time. And the field equations act on the spread mass. And (since inside a mass, the Ricci tensor is not zero) the space-time near a *quantum* particle has a non-constant real 4-volume element.

In short then, there is relationship between a particle's mass and its radius; the higher the mass, the shorter the radius.

2. *Mass and Gravity*

Whilst Newton and Einstein *described* the action of gravity, a *mechanism* for gravity was not provided. SGCT, on the other hand, does suggest a mechanism: When venues are near a large mass (from outside the mass), their 3-volumes are compressed. The constancy of the volume is maintained by a corresponding expansion of the time coordinate. The space compression continues when a venue is interior to the mass. Here the imaginary time component is important and is non-zero. As one approaches The Schwarzschild radius, (at least one of) the venue's dimensions approaches zero while at the same time, the imaginary time component (which has expanded to maintain a constant volume element) rolls over to zero. The venue thus annihilates. A venue then comes in to take its place. So there is a continuous stream of venues approaching the Schwarzschild radius and annihilating. Venue creation in the space-time at large makes up for the loss of venues. If a venue holds a test mass, it will fall in toward the surface of the large mass. (This is largely because of the mass-free venues 'in front' of that venue.) The speed (as a function of the radial distance from the center from the large mass) can easily be calculated (and the result of the calculation compared with the Newtonian result):

Consider a spherical shell at some distance from a mass (M). As the venues at the shell migrate in toward the mass, the number of venues at the shell do not change, so as the shell's radius changes the venues must compress in the two dimensions perpendicular to the radial direction. To keep the volume constant, the real time component (the imaginary-time component is too small in scale to have any effect here) must expand as the square of a coordinate perpendicular to r (because of the *two* space coordinate compressions). Venues are being annihilated at a constant rate. So if one uses a stopwatch to monitor how fast a venue (containing a test particle) is falling toward the surface of the mass, it will appear to go faster as it approaches because of the slowing of the stopwatch. So the distance covered by the falling venue will go as the square of the rate of the slowing of time. This is to say that, $v^2 = \frac{constant}{r}$. There is also a contraction of the venue in the r direction, but that is a relativistic effect that we will ignore for the moment. The velocity equation from Newtonian physics is, $v^2 = \frac{2GM}{r}$. The *constant* is (related to) the rate at which venues are annihilated, so we can associate that rate with 2GM which gives a connection of SGCT to 'physics'. Further, since the Newtonian description of an object falling under the influence of gravity is a conversion of potential to kinetic energy, the SGCT derivation of the fall of gravity provides the link to kinetic and potential energy. More importantly though, it might explain the concept of energy in terms of SGCT.

To summarize: The volume, V, of a sphere is $\frac{4\pi r^3}{3}$. $\frac{dV}{dr} = 4\pi r^2$. But venue volume is invariant. So as the 3-volume, V, decreases proportionally to r^2, the real-time component must increase proportionally to r^2. However the r coordinate of a venue *decreases* as the venue approaches a mass, and the decrease is non-linear. So far from a mass, in the Newtonian domain, a venue's r coordinate is essentially constant. So that leaves t to increase proportionally to r^2.

Venue creation to replenish the venues lost to annihilation will happen over a large region of space-time. But one would expect that the closer one is to the mass responsible for the annihilation, the higher the rate of venue creation. The venues thus created would then cause a very small deviation from the inverse square law, and also from the General Relativity predictions. Because of the increase of venue creation as one approaches a mass, one would expect the deviation to be most evident with a test mass in a highly elliptical orbit around the mass responsible for the annihilations.

Note: Carlo Rovelli states that relativity's the slowing of time is the source of gravity[23]. SGCT says the annihilation of venues is the source of gravity and the rate of fall due to gravity is determined by the slowing of time.

VIII. THE DIFFUSION AND SCHRÖDINGER EQUATIONS

The SGCT model is essentially a description of diffusion of space-time. As such, one might think that the diffusion equation, $\frac{\partial \varphi}{\partial t} = D\frac{\partial^2 \varphi}{\partial x^2}$ would be part of that description. This, the one-dimensional diffusion equation, is easy to derive.

First consider the 'flux' ,j, (in the x direction) of a quantity through a section perpendicular to x (per unit area and per unit time). We ignore the bulk motion of the carrier (assume fluid). And let φ be the 'concentration' of the quantity.

We can see that $\frac{\partial \varphi}{\partial t} = -\frac{\partial j}{\partial x}$.

We can also see (Fick's first law [24]) that $j = -D\frac{\partial \varphi}{\partial x}$ where D is the Diffusivity coefficient.

(D is a proportionality factor between the diffusion flux and the gradient in the concentration of the diffusing substance. The higher the levels of diffusivity of a certain substance to another, the faster the diffusion rate by both of the substances. It is given the unit of length squared per unit time.)

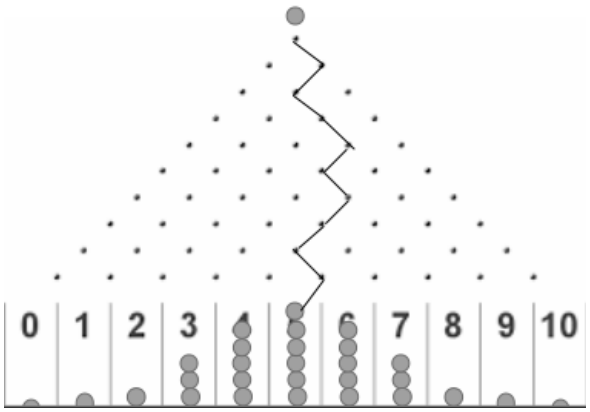
So $\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial \varphi}{\partial x}\right)$. And if D is constant, that yields the above diffusion equation,

$$\frac{\partial \varphi}{\partial t} = D\frac{\partial^2 \varphi}{\partial x^2} \ .$$

The equation is very similar to the (potential free) one dimensional Schrödinger equation, but with several significant differences: The 'diffusion constant' in the Schrödinger equation is complex, and the interpretations of the solutions differ. In the Diffusion equation, the solution, φ, (the concentration) can trivially be interpreted as a probability density (of a test particle having diffused to another position in space), whereas with the Schrödinger equation, it is the *square* of the solution that corresponds to the probability density. And significantly, the diffusion equation, while it describes diffusion in space, does not describe diffusion in time, nor does it consider the effect of the rolled-up imaginary time component.

We'll attempt now to include diffusion in time to see how that effects the solution of the diffusion equation.

We'll start by saying that the probability of a particle starting at $x_0 = t_0 = 0$ arriving at a point $x = x_1$ is proportional to the number of ways the particle in a fixed number of steps, n, (corresponding to n time increments in the laboratory frame) can arrive at point x_1. And we'll calculate it from the 'laboratory' frame where time is granular but not stochastic. As an example, consider the following diagram. (The following is similar to the Statement 3 analysis in Section IIIA.)



First we consider the cases where there is no time diffusion.

Examples:

The jagged line in the above diagram represents a typical path of the ball at the top of the diagram dropped down on the 'pegboard'. If many balls are dropped, the balls will fall into bins as above, and their numbers in each bin will result in a binomial distribution. (The Binomial distribution is equivalent to the Gaussian distribution when the number of axis points is large.)

In the diagram, we can consider the height the time axis (increasing downward) and the horizontal the x axis. The top ball then is initially at t=0, x=5. As it falls, at each time interval (when it encounters one of the pegs), it can move either one x unit to the left or right.

We can represent the typical path above as follows:

t:	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
x:	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1

Again, summing over all possible paths (with time increasing one unit per point of direction determination), the probability (here, the binomial) distribution results.

Consider the table now as a section of a much longer table. The sum of all paths gives a probability.

Now if we consider also diffusion in time, the top row of the table will no longer be all +1. The numbers in the top row will have the same number of variations as in the second row. And, as before, we us assume isotropy, i.e., the distributions are the same for x,y,z, and t. And so the number of paths will be the square of the number of paths of the non-time-diffusion paths. And so the probabilities of (the balls being in a particular bin) in the time diffusion paths will be the *square* of the number in the non-time-diffusion paths. In other words, the solution, φ, of the diffusion equation represents the square root of the probability density. (In our Brownian motion model where only one direction at a time migrates, the square property holds over three dimensions and not just over the one-dimension case above.)

Now, having included time-diffusion in the (interpretation of) the diffusion equation, we turn our attention to including the possible effects of complex time.

First regarding $\frac{\partial\varphi}{\partial t} = D\frac{\partial^2\varphi}{\partial x^2}$, Nettel in 'Wave Physics'[25] says: "If we are to have a solution to a first order differential equation, that solution will have to be an exponential function rather than a trigonometric one. Moreover, to avoid having the solution go to infinity or be exponentially damped as t goes to either plus or minus infinity but rather to get waves, the exponent in the solution will have to be imaginary. As the reader can easily check (if we include i in the equation), we get the solution $\psi(x,t) = e^{i(kx-\omega t)}$."

The diffusion equation is for diffusion in 3-space (although we've interpreted it as a diffusion also in time). But how do we include the rolled-up, imaginary time. Taking guidance from the above, we will again introduce another coordinate axis, an imaginary-time axis perpendicular to the real-time axis.

Imaginary time is rolled-up. It's coordinate then continues to increase, rolling around to zero, etc. This gives a complex frequency e^{iv}, where we can let υ be $kx - \omega\tau$.

So, now having an imaginary axis for υ-time, we'll again define a 'total time' T which will be the combination of τ-time and υ-time,

T = τ + iυ.

$\frac{\partial T}{\partial t} = 1$ and $\frac{\partial T}{\partial \upsilon} = i$

We have then, $\frac{\partial\Psi}{dT} = \frac{\partial\Psi}{\partial\tau} * \frac{\partial\tau}{\partial T} * \frac{\partial\Psi}{\partial\upsilon} * \frac{\partial\upsilon}{\partial T}$.

Substituting gives, $\frac{\partial\Psi}{\partial T} = \frac{\partial\Psi}{\partial\tau} * \frac{\partial\Psi}{\partial\upsilon} * (-i)$.

What can we say about $\frac{\partial\Psi}{\partial\upsilon}$?

As υ is periodic, then so to is $\frac{\partial\Psi}{\partial\upsilon}$,

and the period is at the Planck scale.

Now the Diffusion equation, though working in the macro (and to some extent) quantum domains, might not be expected to work at the Planck scale. We 'blur' the time in the Diffusion equation (i.e., take over a short (but not too short) time, then $\frac{\partial\Psi}{\partial\upsilon}$will average out to a constant. And as the solution depends on details of the physical situation, that constant, here called k, is (at least most all of the time) not zero.

We will use the 'total time' T, rather than τ in the equation. So now, we have,

$\frac{\partial\Psi}{\partial T} = ik\frac{\partial\Psi}{\partial\tau}$

and the Diffusion equation becomes,

$-ik^{-1}\frac{\partial\Psi}{\partial T} = D\frac{\partial^2\varphi}{\partial x^2}$

Of course, we could have just used Schrödinger's argument (about needing i for there to be waves) to arrive at his equation. but once we have included i, in $i\frac{\partial\varphi}{\partial t}$ the equation, though very useful, is nonphysical. The above argument is intended to provide a physical (i.e. geometric) description.

IX. ROTATIONS *IN* AND *OF* THE SPACE-TIME MANIFOLD

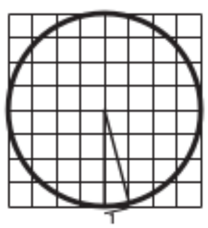
In the stochastic space-time model, analogously to the way a Brownian pollen grain moves under the collective collisions with water molecules, the more the object is at the quantum scale the more the *translatory* motion is due to migrations of venues. We expect that the *rotational* motion of an elementary particle is due to the rotations of venues in the space-time manifold.

Rotations in a discrete space-time has a possible problem: A 'point' near the axis of rotation could rotate to another point within the same venue. And that seems to conflict with the idea a venue cannot have any internal structure.

We'll consider then, that for an elementary particle, its rotations are entirely the result of venue rotation. This is consistent with the idea that mass is actually a venue property. And, if the particle were charged, it would not radiate since it is not the particle but the space-time that is rotating.

There is however a particular issue with (rigid) particle rotations in a granular manifold. We are mindful that in Special Relativity, there are no rigid objects. We are interested in whether granular space-time theory also does not allow rigid objects.

The diagram below shows a mass with the dark circle indicating its circumference. It is sitting on a background of venues (the squares). It also shows an arc length of one Planck length.



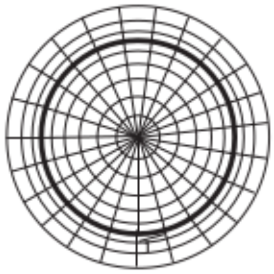
If the particle, rather than the space-time were rotating, then when the particle rotates through the one Planck length arc, a point close to the origin would rotate far less than a Planck length, leaving the point in the same venue. But, by hypothesis, a venue has no internal structure; e.g. there can't be two distinct points in a venue.

So if the above diagram applies, rigid rotations of the particle cannot happen.

We'll address this by assuming the particle is subject to General Relativity.

Earlier in this paper curvature was described as not a property of space-time, but merely an expression of the compression of venues. (E.g. a venue could compress in space dimensions while expanding in the conventional time dimension).

Consider the diagram below.



Here the particle (the dark circle represents its circumference) is set against a grid of venues distorted by the particle's mass.

We consider that the space-time exterior to the particle is described by the usual Schwarzschild metric and the interior by the Schwarzschild, perfect fluid interior solution:

$$c^2ds^2 = \frac{1}{4}\left(\left(1-\frac{2Gm}{c^2r_m}\right)^{\frac{1}{3}} - \left(1-\frac{r^2 2Gm}{c^2r_m^3}\right)\right)^2 c^2dt^2 - \left(1-\frac{2Gmr^2}{c^2r_m^3}\right)^{-1} dr^2 - r^2\left(d\Theta^2 + sin^2\Theta d\phi^2\right)$$

where r_m is the radius of the mass.

Notice that the 'curvature' increases as one approaches the surface from the exterior, and decreases as one proceeds from the radius towards the center. And, as long as the Schwarzschild radius is less than the particle radius, there is no Schwarzschild singularity. Further, because venues are not points, there is no singularity at the center either. The rotation now is like a 'pizza slice' or wedge, no point on any venue rotates into the same venue.

(Note that if we truly consider rotations, the Kerr metric would be more appropriate. But as we consider the rotations as going in all directions at the same time, Kerr would also not be the appropriate metric.)

There are however (at least) two reasons why the above schema doesn't work: First, while the argument seems reasonable in two dimensions, it does not work in three. For a spherical particle rotating as above, a point on a venue on the surface on the axis of rotation would rotate in the same venue. Second, if the Schwarzschild interior metric is roughly appropriate, then at the center of the particle, the 'curvature' would vanish. That is to say that the venues near the center would be minimally distorted. And, especially as in the schema, the number of venues circling the particle at any radius is the same, the venues near the center would be extremely distorted. We conclude then, that rotations are not rigid. Can we explain how even non-rigid rotations can be explained?

We can modify the above schema. First, we let the circling venues at any radius can circle the particle independently of the circling venues at any other radius. This, incidentally, would allow zero total angular momentum if the various rotating circles of venues were rotating in opposite directions. And second, we can allow rotations to occur simultaneously in any plane containing the center of the particle. This gives a geometric interpretation of particles rotating simultaneously in all directions and is suggestive of spin.

X. COMMENTS ON QUIDDITY, ENTANGLEMENT, AND THE TWO-SLIT EXPERIMENT

A. Information & Quiddity : Pilot-waves & Entanglement

There are two forms of information at play: one of which is restricted to travel at no greater than the speed of light and the other (e.g. collapse of the wave function, entanglement and the like) not so restricted. These are very different processes, and so using the word 'information' for the both of them is confusing. We'll reserve 'information' for the first case, and 'quiddity' for quantum information. (Quiddity means the inherent nature or essence of something. And the first three letters, qui, make it easy to remember QUantum Information.)

Information is carried by photons or mass (energy). Quiddity, as it travels faster than light (even infinitely fast), can not be carried by energy. In Stochastic Granular Space-time theory then, what can carry quiddity? The only thing left is empty venues.

While a venue has an invariant 4-dimensional volume with complex time, it can vary in its individual dimensions. As described earlier, the real-time 4-dimensional volume is related to the probability density, $\Psi^*\Psi$. So that probability density is a type of quiddity.

The wave function acts as a 'pilot wave' (as proposed by Louis de Broglie and David Bohm), moving in advance of a quantum particle. When the particle 'catches up' to a place where the pilot wave is, that wave then determines the particle's probability density.

Entanglement seems to work the same way: by the superluminal propagation of probability densities. Entanglement then is not an extremely strange peripheral property of quantum theory, but a necessary and central component of the theory. An entangled set of particles then could interact superluminaly, but an observer in the laboratory frame could not observe the result of the interaction until a time later, when a classical (subluminal) signal could have reached the interaction.

Arguably, entanglement is a process requiring superposition plus faster than light quiddity, and SGCT provides for both.

Our aim in the following is not to provide a theory/mechanism for entanglement, but to argue that Stochastic Granular space-time Theory allows for it, within the confines of four dimensions with complex time.

Bell's theorem[26] requires that to have entanglement, we must abandon 'objective reality' and/or 'locality'. Dropping locality means that things separated in space can influence each other instantaneously. Dropping objective reality means that a physical state isn't defined until it is measured (e.g. is the cat dead or alive?).

Weak measurement experiments[27, 28] building on the work of Yakir Aharonov and Lev Vaidman[29] imply that there *is* objective reality in quantum mechanics[30, 31] (in contradiction to the Copenhagen interpretation). By objective reality, we mean a particle does have a path (blurred somewhat by space-time fluctuations) regardless of whether it is being observed or not.

We're left then, with non-locality. SGCT is non-local. The issue, of course, is how to have non-locality whilst not violating Einstein's prohibition of information traveling faster than light. We slightly re-interpret that prohibition by positing that it is energy (as opposed to information) that can't travel faster than light.

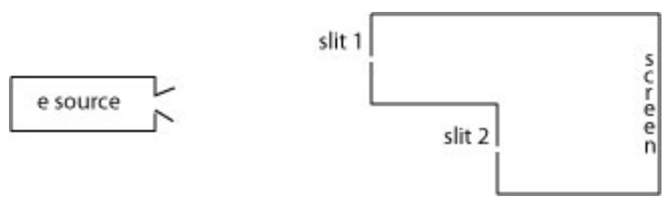
Empty venues carry no energy, and so (as we have seen) can migrate through space-time arbitrarily rapidly. The hope then is that we can find a way that empty venues can carry quiddity. (The Time Leaves No Tracks idea will help with that.)

A single empty venue seems not to fulfill that hope as a single empty venue's only quiddity is the fact of its existence, and since number of (empty) venues is not conserved, that fact doesn't seem to be able to explain entanglement.

We suggest though, that through some as yet *unknown* mechanism (which is why this is a suggestion and not a theory) that a number of empty venues can be bound together can migrate collectively through space-time (e.g. spiraling through space-time) they would then carry a more complex quiddity. So, for instance, two created entangled particle would carry this quiddity with them as they spread out (as a link between them). And through another unknown mechanism, a measurement of one particle forces the state of the other and then dissolves the link.

Again, this discussion is not an *explanation* of entanglement, but just an attempt to show that SGCT can contain such entangled states.

B. The Delayed-choice Two-slit Experiment



The diagram shows the 'delayed choice two-slit experiment': A low-intensity source directs electrons to a box containing two slits (slit 1 and slit 2). The beam intensity is such that there is only one electron traveling in the box at any time. As expected, an interference pattern is gradually produced on the screen at the back of the box. If a particle detector is introduced at slit 2 to determine which slit an electron passed through, then there will be no interference produced. One can arrange that the detector is optionally turned on only when the electron has passed by slit 1. If the detector is on at that point, then again, there will be no interference pattern produced. So it seems that when the electron gets to slit 2 and finds that the detector is on, it goes back in time to tell the electron to go through, or not go through slit-1.

How does the Granular Stochastic Space-time model explain this?

First, we (again) introduce the concept of an 'ephemeral' measurement: An electron has an associated electromagnetic field. As it goes through a slit, that field will interact with the electrons in the wall of the box at the slit. The box electrons then can tell if an electron has passed through a slit. And this could be considered a measurement; the box electrons could be considered a particle detector. But the interference pattern still occurs in this case. The difference is that the box electrons measurements are ephemeral; After the moving electron passes through the slit, the box electrons return to their undisturbed state, retaining no 'memory' of the measurement. The measurement is not preserved. The film can be run backward and it would be a valid physical situation. For there to be a true measurement then, there must be a mechanism to 'remember' the measurement – a latch or flip-flop of sorts. And that would mean the film could *not* be run backward. We regard measurement then, as a breaking of time-reversal symmetry. In the macro-world, everything is a measurement of sorts (viewing a scene gives an estimate of positions, etc.) and hence we can't run macro-world scenes backwards.

With quiddity (in this case, the pilot wave) able to move superluminaly, and indeed with no change in time (i.e. instantaneously in any reference frame), there isn't much to explain. The pilot wave precedes the electron going into the box. The pilot wave determines the probability of the electron being found at any point in the box at any time. If (at any time) the detector is switched on, that would change the geometry and hence the wave (at all points, future and past). The electron would continue its motion, catching up with the revised pilot wave and then moving accordingly. (This is much like the mechanism of entanglement).

XI. DISCUSSION

General relativity is a theory relating the large scale structure of space-time to the masses in it. Similarly, the stochastic space-time model relates the micro-structure of space-time to the behavior of masses at the quantum level. One says for general relativity, mass tells space how to bend, and space tells mass how to move. And in SGCT, we say mass tells space how to jell, and space tells mass how to jiggle. The model is neither exclusively in the domain of quantum mechanics nor General Relativity. It requires both theories in its development.

In the model, particles move (in an indeterminate manner) due to the space-time fluctuations exterior to the particle (similar to the way a Brownian Motion pollen grain moves). But unlike with Brownian motion, time (as well as space) fluctuates.

It is usual to think of mass as moving through space-time. And with a stochastic space-time, that implies two types of motion: motion *in* the space-time and motion *of* the space-time. But as we associate mass with a geometric object, the frequency of imaginary time. This suggests that there is only type of motion: motion *of* the space-time.

In free-space, there is no meaning in retracing a trajectory as, because of the space fluctuations, there is no well-defined 'place'.

In contrast to conventional QM where a non-massless particle is presumed to have a wave function (that is perhaps beyond the current measurement threshold), if Ψ is intimately related to the Compton wave, the SGCT model predicts that there isn't a wave function for a sufficiently large, spherical (non-interacting) mass. And that limiting mass is (very close to) the Planck mass. So, even in principle, the two slit experiment cannot be done with bee-bees, or marbles, or cannonballs.

This allows us to claim: In empty space, the Planck length is the smallest possible length, and the Planck time is the smallest possible time. In space-time holding mass we can say that the Planck time and lengths are dimensions of the smallest possible volume. And the Planck mass is the smallest possible purely classical (i.e. not subject to quantum mechanics) mass.

The above takes the Planck length and time as the smallest possible in free space, i.e. the quantum of space and time. But what is the smallest possible mass, the quantum of mass? The Planck mass is the upper bound for quantum masses. What is the lower bound? SGCT can't say. But that mass must be smaller than anything in nature. The mass of a neutrino isn't known, but it is in the order of 10^{-36}kg. The mass difference between the types of neutrinos will be smaller still. We can say then, that the quantum of mass is less than 10^{-28} Planck masses.

A previous paper[32] suggested that quantum oscillations of particles could be described as torsional vibrations occurring simultaneously in all directions It a model whereby such oscillating particles could pass through a polarizer admitting fifty percent throughput rather than just those particles aligned perfectly with the polarizer.

The object of the present model is to provide a conceptual basis for quantum mechanics–to show that the 'quantum weirdness' can be explained in terms of the behavior of space-time. And indeed, SGCT has managed to replicate some of the fundamental processes in conventional quantum theory.

The SGCT model embodies a very different than usual way of thinking about space-time. And, as such, it impacts almost all areas of 'modern physics', i.e. relativity and quantum mechanics.

Taking a single specific topic in modern physics and explicating it in terms of SGCT would require the reader to accept a large chunk of the model for perhaps just a little in the way of illumination. It was thought better then, to provide a qualitative description of the model's application to relativity and quantum mechanics as a whole, rather than a detailed quantitative description of a specific topic.

For the better part of a century, researchers have sought an understanding of the physical nature of quantum mechanics. Perhaps then, a mathematical solution to the problem is impossible and it is an example of the Gödel incompleteness theorem[33]. Does Gödel imply that there is no mathematical solution, or perhaps there is one but it can not be derived? Roger Penrose, Herman, Weyl, among others, felt that the theorem shows that we can never know many, if not most, of the physical laws of the universe.

The SGCT model is (at the moment) more phenomenological than mathematical; it is an assemblage of interconnected (hopefully self-consistent) phenomena–a scaffold onto which quantum phenomena can be attached. Each phenomenon attached to the scaffold might well be describable by mathematics, but the entire populated scaffold, because of Gödel, might not be. While the mathematical description of SGCT is thus far from complete, the model is specified sufficiently to allow (super)computer simulations. And perhaps, because of Gödel, computer simulations are the best we can do.

And finally, since venue migration is a diffusion process, if the universe is not infinite, or closed, the diffusion will continually increase the size of the universe.

ACKNOWLEDGMENTS

I gratefully acknowledge the helpful discussions with Dr. Norman Witriol. I especially thank and acknowledge Nicholas Taylor for discussions and ideas to the extent of almost being a co-author.

APPENDIX

THE ORIGINAL *STOCHASTIC SPACE-TIME AND QUANTUM THEORY* PAPER (REVISED)

INTRODUCTION

When considering the quantum and relativity theories, it is clear that only one of them, namely relativity, can be considered, in the strict sense, a theory. Quantum mechanics, eminently successful as it is, is an operational description of physical phenomena. It is composed of several principles, equations, and a set of interpretive postulates[Ap.1]. These elements of quantum mechanics are justifiable only in that they work. Attempts[Ap.2, Ap.3] to create a complete, self-contained theory for quantum mechanics are largely unconvincing. There are, in addition, a number of points where quantum mechanics yields troubling results. Problems arise when considering the collapse of the wave function, as in the Einstein-Podolsky-Rosen paradox[Ap.4]. Problems also arise when treating macroscopic systems, as in the Schrödinger cat paradox[Ap.5] and the Wigner paradox[Ap.6]. And quantum mechanics is not overly compatible with general relativity[Ap.6].

One way of imposing some quantum behavior on general relativity is the following: The uncertainty relation for time and energy implies that one can "borrow" any amount of energy from the vacuum if it is borrowed for a sufficiently short period of time. This energy fluctuation of the vacuum is equivalent to mass fluctuations which then gives rise to metric fluctuations via the general-relativity field equations.

An alternative approach is to impose, *ab initio,* an uncertainty on the metric tensor, and to see if by that, the results of quantum mechanics can be deduced. As this paper will shoe, with a few not particularly unreasonable assumptions, a large segment of the formalism of quantum theory can be derived and, more importantly, understood.
Mathematical spaces with stochastic metrics have been investigated earlier by Schweizer[Ap.7] for Euclidian spaces, and by March[Ap.9, Ap.10] for Minkowski space. In a paper by Blokhintsev[Ap.11], the effects on the physics of a space with a small stochastic component are considered. It is our goal, however, not to show the effects on physical laws of a stochastic space, but to show that the body of quantum mechanics can be deduced from simply imposing stochasticity on the space-time. Our method will be to write down (in Section II) a number of statements (theorems, postulates, etc.). We will then (in Section III) describe the statements and indicate proofs where the statements are theorems rather than postulates. Finally (in Section IV) we will derive some physical results, namely, the spread of the free particle (in empty space), the uncertainty principle, and the phenomenon of interference. The paper concludes (Sec. V) with a general discussion of the approach and a summary of results.

THE STATEMENTS

Statement 1. Mach's principle (Frederick's version).

1.1. In the absence of mass, space-time becomes not flat, but stochastic.

1.2. The stochasticity is manifested in a stochastic metric $g_{\mu\nu}$.

1.3. The mass distribution determines not only the space-time geometry, but also the space-time stochasticity.

1.4. The more mass in the space-time, the less stochastic the space-time becomes.

1.5 At the position of a mass "point", the space-time is not stochastic.

Statement 2, the contravariant observable theorem.

All measurements of dynamical variables correspond to contravariant components of tensors.

Statement 3. The metric probability postulate.

$P(x,t) = f\sqrt{-g},$ where for a one particle system $P(x,t)$ is the particle probability distribution. f is a real-valued function and g is the determinant of the metric. [But see ‡ in the description of the statements, below, for a revised interpretation.]

Statement 4. the metric superposition postulate.

If at the position of a particle the metric due to a specific physical situation is $g_{\mu\nu}(1)$ and the metric due to a different physical situation is $g_{\mu\nu}(2)$ then the metric at the position of the particle due to the presence of both of the physical situations is $g_{\mu\nu}(3)$,

$g_{\mu\nu}(3) = \frac{1}{2}[g_{\mu\nu}(1) + g_{\mu\nu}(2)].$

This is the case where the probabilities, $P_1$and P_2, of the two metrics are the same. In general though, Statement 4 becomes,

$g_{\mu\nu}(3) = P_1 g_{\mu\nu}(1) + P_2 g_{\mu\nu}(2)$.

Statement 5. The metric Ψ postulate.

There exists a local complex diagonal coordinate system in which a component of the metric at the location of the particle is the wave function Ψ.

DESCRIPTION OF THE STATEMENTS

Statement 1, Mach's principle, is the basic postulate of the model. It should be noted that requirement 1.5, that at the position of a mass point the space-time be not stochastic, is to insure that an elementary mass particle (proton, quark, etc.) is bound.

In our interpretation, a charged particle in stochastic motion does not radiate because it is the space–time rather than the particle which is stochastic. (This is in contrast to Nelson's formulation where the particle [when it's position is time-dependent] is simply posited not to radiate.)

Similarly, local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity (due to the stochasticity of the metric tensor), and because of that stochasticity, the measurements are also stochastic (and the measurements are contravariant [as can be seen by the raising of the covariant coordinates by the stochastic of the metric tensor]).

Statement 2, the contravariant observable theorem, is also basic. It is contended, and the contention will be weakly proved, that measurements of dynamical variables are contravariant components of tensors. By this we mean that whenever a measurement can be reduced to a displacement in a coordinate system, it can be related to contravariant components of the coordinate system. Of course, if the metric $g_{\mu\nu}$is well known, one can calculate both covariant and contravariant quantities. In our model however, the quantum uncertainties in the mass distribution imply that the metric cannot be accurately known, so that measurements can only be reduced to contravariant quantities. Also, in our picture, the metric is stochastic, so again we can only use contravariant quantities. We will verify the theorem for Minkowski space by considering an idealized measurement. Before we do, consider as an example the case of measuring the distance to a Schwarzschild singularity (a black hole) in the Galaxy. Let the astronomical distance to the object be $\bar{r}(\equiv \bar{\xi}^1)$. The covariant equivalent of the radial coordinate r is ξ_1, and

$\xi_1 = g_{1\nu}\xi^\nu = g_{11}\xi^1 = \frac{r}{1-2Gm/r}$,

so that the contravariant distance to the object is

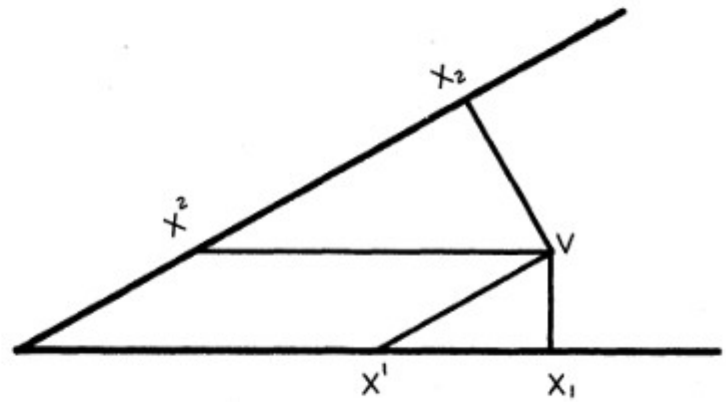


Figure 1. Covariant and contravariant components in oblique coordinates.

distance$= \int_0^{\bar{r}} dr = \bar{r}$,

while the covariant distance is

$\bar{\xi}_1 = \int_0^{\bar{r}} d(\frac{r}{1-2Gm/r}) = \infty$.

It is clear that only the contravariant distance is observable.

Returning to the theorem, note that when one makes an observation of a dynamical variable (e.g. position, momentum, etc.), the measurement is usually in the form of a reading of a meter (or meter-stick). It is only through a series of calculations that one can reduce the datum to, say, a displacement in a coordinate system. For this reduction to actually represent a measurement (in the sense of Margenau[Ap.12]) it must satisfy two requirements. It must be instantaneously repeatable with the same results, and it must be a quantity which can be used in expressions to derive physical results (i.e., it must be a physically "useful" quantity). It will be shown that for Minkowski space, the derived "useful" quantity is contravariant.

Note first (Fig. 1) that for an oblique coordinate system, the contravariant coordinates of a point V are given by the parallelogram law of vector addition, while the covariant components are obtained by orthogonal projection onto the axes[Ap.13].

We shall now consider an idealized measurement in special relativity, i.e., Minkowski space. Consider the space-time diagram of Fig. 2.

We are given that in the coordinate system $x^{'}$,$t^{'}$, an object (m, n) is at rest. If one considers the situation from a coordinate system x, t traveling with velocity v along the $x^{'}$axis, one has the usual Minkowski diagram[Ap.14] with coordinate axes Ox and Ot and velocity $v = tan(\alpha)$ (where the units are chosen such that the velocity of light is unity). OC is part of the light cone.

Noting that that the unprimed system is a suitable coordinate system in which to work, we now drop from consideration the original $x^{'}, t^{'}$coordinate system.

We wish to determine the "length" of the object in the x, t coordinate system. Let it be arranged that at

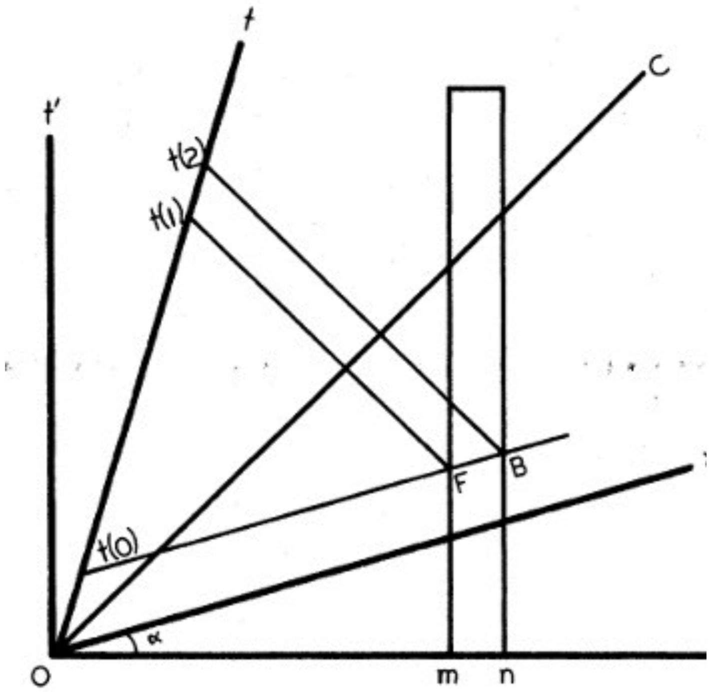


Figure 2. An idealized measurement.

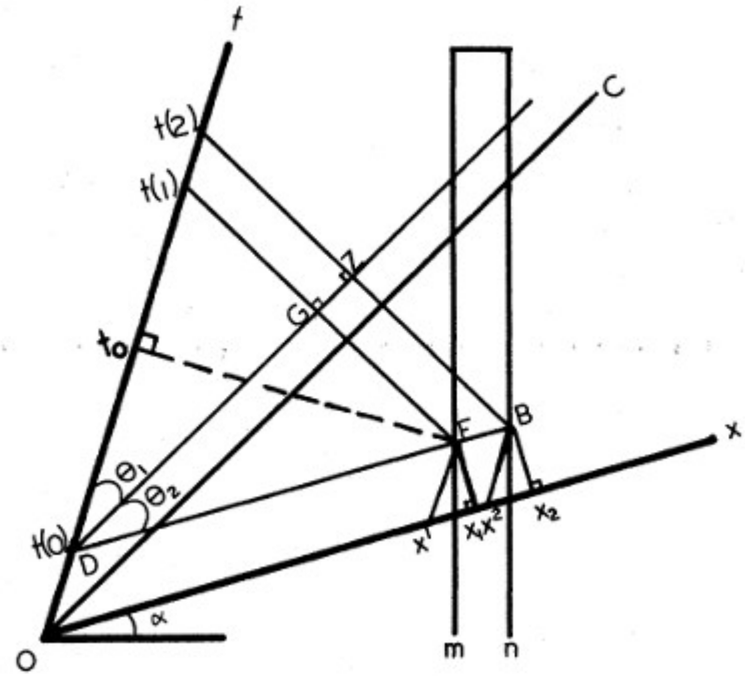


Figure 3. Analysis of the idealized measurement.

time $t(0)$ a photon shall be emitted from each end of the object (i.e., from points F and B). The emitted photons will intercept the t axis at times $t(1)$ and $t(2)$. The observer then deduces that the length of the object is $t(2) - t(1)$ (where $c = 1$). The question is: What increment on the x axis is represented by the time interval $t(2) - t(1)$? One should note that the arrangement that the photons be emitted at time $t(0)$ is nontrivial, but that it can be done in principle. For the present, let it simply be assumed that there is a person on the object who knows special relativity and who knows how fast the object is moving with respect to the coordinate system. This person then calculates when to emit the photons so that they will be emitted simultaneously with respect to the x, t coordinate system.

Consider now Fig. 3, which is an analysis of the measurement. Figure 3 is just figure 2 with a few additions: the contravariant coordinates of F and B, $x^1$and x^2 respectively. We assert, and it is easily shown, that $t(2) - t(1) = x^2 - x^1$. This is seen by noticing that $x^2 - x^1$= line segment B, F, and that triangle $t(2), t(0), Z$ is congruent to triangle $B, t(0), Z$. However, if we consider the covariant coordinates, we notice that $x_2 - x_1 = x^2 - x^1$. This is not surprising since coordinate differences (such as $x_2 - x_1$) are by definition (in flat space) contravariant quantities. To verify our hypothesis we must consider not coordinate differences which automatically satisfy the hypothesis, but the coordinates themselves. Consider a measurement not of the length of the object, but of the position (of the trailing edge m) of the object. Assume again that at time $t(0)$ a photon is emitted at F and is received at $t(1)$. The observer would then determine the position of m at $t(0)$ by simply measuring off the distance $t(1) - t(0)$ on the x axis. Notice that this would coincide with the contravariant quantity x^1. To determine the corresponding covariant quantity x_1, one would need to know the angle α (which is determined by the metric).

The metric $g_{\mu\nu}$ is defined as $\bar{e}_\mu \cdot \bar{e}_\nu$, where $\bar{e}_\mu$and $\bar{e}_\nu$ are the unit vectors in the directions of the coordinate axes x^μ and x^ν. Therefore, in order to consider an uncertain metric, we can simply consider that the angle α is uncertain. In this case measurement x^1 is is still well defined [$x^1 = t(1) - t(0)$], but now there is no way to determine x_1 because it is a function of the angle α. In this case then, only the contravariant components of position are measurable. [It is also easy to see from the geometry that if one were to use the covariant representation of $t(0)$, t_0, one could not obtain a metric-free position measure of m.]

Statement 3, the metric probability postulate, can be justified by the following: Consider that there is given a sandy beach with one black grain among the white grains on the beach. If a number of observers on the beach had buckets of various sizes, and each of the observers filled one bucket with sand, one could ask the following: What is the probability that a particular bucket contained the black grain? The probability would be proportional to the volume of the bucket.

Consider now the invariant volume element dV_I in Riemann geometry. One has that[Ap.15]

$dV_I = \sqrt{-|g|}dx^1dx^2dx^3dx^4$.

It is reasonable then, to take $\sqrt{-|g|}$ as proportional to the probability density $(\Psi^*\Psi)$ for free space.

See ‡ below for major revisions to Statement 3 (not in the original paper).

Note that the metric $g_{\mu\nu}$ is stochastic while the determinant of the metric is not. This implies that the metric components are not independent.

Consider again, the sandy beach. Let the black grain of sand be dropped onto the beach by an aircraft as it flies over the center of the beach. Now the location of the grain is not random. The probability of finding the grain increases as one proceeds toward the center, so that in addition to the volume of the bucket there is also a term in the probability function which depends on the distance to the beach center. In general then, we expect the probability function $P(x,t)$ to be $P(x,t) = A\sqrt{-|g|}$ where A is a function whose value is proportional to the distance from the center of the beach.

‡ Major changes from the original paper regarding Statement 3

Again, the metric probability postulate, can be justified by the buckets on a beach argument. And again, the probability that a particular bucket contained the black grain would be proportional to the volume of the bucket.

Consider the invariant volume element dV_I in Riemann geometry. One has that

$dV_I = \sqrt{-|g|}dx^1dx^2dx^3dx^4$.

(From here on, we'll represent the determinant of $g_{\mu\nu}$ by g rather then by $|g|$.)

At first sight then, it might seem reasonable to take $\sqrt{-|g|}$ as proportional to the probability density for free space.

The arguments above apply to the three-dimensional volume element. But we left out the other determinant of the probability density, the speed of the particle (the faster the particle moves in a venue, the less likely it is to be there.) And therefore, the larger the Δt the more likely the particle is to be found in the venue. So indeed (it seems as if) it is the four-dimensional volume element that should be used.

The metric probability statement above, as it stands, has problems:

First, if one considers the 'particle in a box' solution, one has places in the box where the particle has zero probability of being. And if $P(x,t) = k\sqrt{-g} = 0$, that means the determinant of the metric tensor is zero and there is a space-time singularity at that point. We address this problem by noting that the metric tensor is composed of the average, non-stochastic, background (Machian) metric $g^M_{\mu\nu}$ and the metric due to the Particle itself $g^P_{\mu\nu}$. We say then that the probability density is actually $P(x,t) = k(\sqrt{-g^T} - \sqrt{-g^M})$ where g^T is the determinant of the composite metric. In this case, $P(x,t)$ can be zero without either $g^T_{\mu\nu}$ or $g^P_{\mu\nu}$ being singular.

A second problem is that $P(x,t) = k\sqrt{-g}$ describes the probability density for a test particle placed in a space-time with a given (average) metric due to a mass, with determinant g. What we want, however, is the probability of the particle (not the test-particle) due to the metric contribution of the particle itself. Related to this is that $P(x,t) = k\sqrt{-g}$ doesn't seem to replicate the probability distributions in quantum mechanics in that the probability distribution, $\Psi^*\Psi$, is the square of a quantity (assuring that the distribution is always positive). But the differential volume element, $dV = \sqrt{-g}\ dxdydzdt$ is not the square of any obvious quantity. Further, $P(x,t) = k\sqrt{-g}$ is something of a dead end, as it gives $\Psi^*\Psi$ but no hint of what Ψ itself might represent. It would be nice if the probability density were proportional to the square of the volume element rather than to the volume element itself. With that in mind we'll again look at the probability density. (Multiple researchers[Ap.16, Ap.17] have agreed with Part A's $P(x,t) = k\sqrt{-g}$ and it is therefore with some trepidation that we consider that the probability density might be subject to revision.)

The initial idea was that, given a single particle, if space-time were filled with 3-dimensional boxes (venues), then the probability of finding a particle in a box would be proportional to the relative volume of the box. That was extended to consider the case where the particle was in motion. The probability density would then also depend on the relative speed of the particle. We will however, now argue that $P(x,t) \neq k\sqrt{-g}$, but instead $P(x,t) = -kg$ (essentially the square of the previous). But this will apply only when the quantum particle is measured (a contravariant measurement) in the laboratory frame. If however, one considers the situation co-temporally (i.e. covariantly) with the quantum particle, then $P(x,t)$ does equal $k\sqrt{-g}$, which is to say that the probability density is [co- or contra-variant] frame dependent.

There is another argument, but it assumes the main body of this *paper, relating to a time-like fifth dimension we call tau.*

Consider a quantum particle at a τ-time slice at, say, τ=now. And also consider a static quantum probability function (e.g. a particle in a well) at τ=now+1. (That function is a result of the quantum particle's migrations in time and space,) Then if we take a negligible mass test particle at τ=now, it will have a probability of being found at a particular location at τ=now+1 equal to that static probability function. And that function is proportional to the volume element (the square root of minus the determinant of the metric tensor). But what we're interested in is the probability function of the quantum particle as τ goes from now to now+1. We are considering the probability function at τ+1 as static. But it is the result of the migrations of the particle. At tau=now, it would then be the same probability function. So, as we go from now to now plus one, we would need to multiply the two (equal) probability functions. This results in the function being proportional to the determinant of the metric tensor (not its square root). This is rather nice as it allows us to suggest that the volume element is proportional to Ψ while the probability density is proportional to $\Psi^*\Psi$. Note that *this result is due to a mass interacting with the gravitational field it itself has generated. (This is analogous to the quantum field theory case of a charge interacting with the electromagnetic field it itself has created.)*

As yet another approach, consider the spread of probability due to the migration of venues. In the absence of a potential, the spread (due to Brownian-like motion) will be a binomial distribution in space (think of it at the moment, in a single dimension and time). But there is also the same binomial distribution in time. This, for example, expresses that the distant wings of the space distribution require a lot of time to get to them. The distribution then seems to require that we multiply the space distribution by the time distribution. The two distributions are the same so the result is the square of the binomial distribution. (The argument can be extended to the three spacial dimensions.) In the laboratory frame, time advances smoothly, which is to say that the time probability density distribution is a constant, so we do not get the square of the binomial distribution..

It seems then that there are both the distribution and its square in play. It might be that the covariant representation, i.e. the distribution 'at' the particle, is the binomial while a distant observer where time advances smoothly (not in the quantum system being observed) observes (i.e contravariant measurements) the square of the binomial distribution.

So now we have $P(x,t) = -kg$, which is to say that the probability density is proportional to the square of the volume element. This is rather nice as it allows us to suggest that the volume element is proportional to Ψ while the probability density is proportional to $\Psi^*\Psi$. (We will in a later paper suggest that the imaginary component of Ψ represents an oscillation of space-time.)

Statement 4, the metric superposition postulate, is adopted on the grounds of simplicity. Consider the metric (for a given set of coordinates) $g^{s1}_{\mu\nu}(x)$ due to a given physical situation $s1$ as a function of position x. Also let there be the metric $g^{s2}_{\mu\nu}(x)$ due to a different physical situation $s2$ (and let the probabilities of the two metrics be the same). What is the metric due to the simultaneous presence of situations $s1$ and $s2$? We are, of course, looking for a representation to correspond to quantum mechanical linear superposition. The most simple assumption is that

$g^{s3}_{\mu\nu}(x)= \frac{1}{2}[g^{s1}_{\mu\nu}(x) + g^{s2}_{\mu\nu}(x)]$.

However, this assumption is in contradiction with general relativity, a theory which is nonlinear in $g_{\mu\nu}$. The linearized theory is still applicable. Therefore, the metric superposition postulate is to be considered as an approximation to an as yet full theory, valid over small distances in empty or almost empty space. We expect, therefore, that the quantum-mechanical principle will break down at some range. (This may eventually be the solution to linear-superposition-type paradoxes in quantum mechanics.

Statement 5, the metric Ψ postulate, is not basic to the theory. It exists simply as an expression of the following: There are at present two separate concepts, the metric $g_{\mu\nu}$ and the wave function Ψ. It is the aim of this geometrical approach to be able to express one of these quantities in terms of the other. The statement that in some arbitrary coordinate transformation, the wave function is a component of the metric, is just a statement of this aim.

PHYSICAL RESULTS

We derive first the motion of a test particle in an otherwise empty space-time. The requirement that the space is empty implies that the points in this space are indistinguishable. Also, we expect that, on the average, the space (since it is mass-free) is (in the average) Minkowski space.

Consider the metric tensor at point Θ_1. Let the metric tensor at Θ_1 be $\tilde{g}_{\mu\nu}$ (a tilde over a symbol indicates that it is stochastic). Since $\tilde{g}_{\mu\nu}$ is stochastic, the metric components, do not have well-defined values. We cannot then know $\tilde{g}_{\mu\nu}$ but we can ask for $P\left(g_{\mu\nu}\right)$ which is the probability of a particular metric $g_{\mu\nu}$. Note then that for the case of empty space, we have $P_{\Theta 1}\left(g_{\mu\nu}\right)= P_{\Theta 2}\left(g_{\mu\nu}\right)$ where $P_{\Theta 1}\left(g_{\mu\nu}\right)$ is to be interpreted as the probability of metric $g_{\mu\nu}$ at point $\Theta 1$.

If one inserts a test particle into the space-time, with a definite position and (ignoring quantum mechanics for the moment) momentum, the particle motion is given by the Euler-Lagrange equations,

$$\ddot{x}^i + \left\{ {i \atop jk} \right\} \dot{x}^j \dot{x}^k = 0,$$

where $\left\{ {i \atop jk} \right\}$ are the Christoffel symbols of the second kind, and where $\dot{x}^j \equiv dx^j/ds$ where s can be either proper time or any single geodesic parameter. Since $\tilde{g}_{\mu\nu}$ is stochastic, these equations generate not a path, but an infinite collection of paths, each with a distinct probability of occurrence from $P(g_{\mu\nu})$. (That is to say that $\left\{ {i \atop jk} \right\}$ is stochastic; $\widetilde{\left\{ {i \atop jk} \right\}}$.)

In the absence of mass, the test particle motion is easily soluble. Let the particle initially be at (space) point Θ_0. After time dt, the Euler Lagrange equations yield some distribution of position $D_1(x)$. $[D_1(x)$ represents the probability of the particle being in the region bounded by x and $x + dx$.$]$ After another interval dt, the resulting distribution is $D_{1+2}(x)$. From probability theory[**?**], this is the convolution,

$$D_{1+2}(x) = \int_{-\infty}^{\infty} D_1(y) D_1(x-y) dy.$$

but in this case, $D_1(x) = D_2(x)$. This is so because the Euler-Lagrange equation will give the same distribution

$D_1(x)$ regardless of at which point one propagates the solution. This is because
$g_{\mu\nu}(x) \equiv \{g_{\mu\nu}(x_1),\, g_{\mu\nu}(x_2),\, g_{\mu\nu}(x_3)....\}$
are identically distributed random variables.
Thus,
$D_i(x) \equiv \{D_1(x),\, D_2(x), ...\}$
are also identically distributed random variables. The motion of the test particle(the free particle wave functions) is the repeated convolution $D_{1+2+....}(x)$, which by the central limit theorem is a normal distribution. Thus the position spread of the test particle at any time $t > 0$ is a Gaussian. The spreading velocity is found as follows: After N convolutions (N large), one obtains a normal distribution with variance σ^2 which, again by the central limit theorem, is N times the variance of $D_1(x)$. Call the variance of $D_1(x)$, a.
$Var(D_1) = a$.
The distribution $D_1$is obtained after time dt. After N convolutions then,

$$\Delta x = Var\left(D_{\sum_0^n i}\right) = Na.$$

This is obtained after N time intervals dt. One then has,

$$\frac{\Delta x}{\Delta t} = \frac{Na}{N},$$

which is to say that the initially localized test particle spreads with a constant velocity a. In order that the result be frame independent, $a = c$, and one has the results of quantum mechanics. At the beginning of this derivation it was given that the particle had an initial well-defined position and also momentum. If for the benefit of quantum mechanics we had specified a particle with a definite position, but with a momentum distribution, one would have obtained the same result but with the difference of having a different distribution D_1 due to the uncertainty of the direction of propagation of the particle.

In the preceding, we have made use of various equations. It is then appropriate to say a few words about what equations mean in a stochastic space-time.

Since in our model the actual points of the space-time are of a stochastic nature, these points cannot be used as a basis for a coordinate system, nor, for that reason, can derivatives be formed. However, the space-time of common experience (i.e., the laboratory frame) is non-stochastic in the large. It is only in the micro world that the stochasticity is manifested. One can then take this large-scale non-stochastic space-time and mathematically continue it into the micro region. This mathematical construct provides a non-stochastic space to which the stochastic physical space can be referred.

The (physical) stochastic coordinates $\tilde{x}^\mu$ then are stochastic only in that the equations transforming from the laboratory coordinates x^μ to the physical coordinates $\tilde{x}^\mu$ are stochastic.

For the derivation of the motion of a free particle we used Statement 1, Mach's principle. We will now use also Statement 2, the contravariant observable theorem, and derive the uncertainty principle for position and momentum. Similar arguments can be used to derive the uncertainty relations for other pairs of conjugate variables. It will also be shown that there is an isomorphism between a variable and its conjugate, and covariant and contravariant tensors.

We assume that we're able to define a Lagrangian, L. One defines a pair of conjugate variables as usual,

$$p_j = \frac{\partial L}{series\partial \tilde{q}^j}.$$

Note that this defines p_j a covariant quantity. So that a pair of conjugate variables so defined contains a covariant and a contravariant member (e.g. $p_j$and q^j). But since p_j is covariant, it is not observable in the laboratory frame. The observable quantity is just,

$$\tilde{p}^j = \tilde{g}^{j\nu} p_\nu,$$

but $\tilde{g}^{j\nu}$ is stochastic so that $\tilde{p}^j$ is a distribution. Thus if one member of an observable conjugate variable pair is well defined, the other member is stochastic. By observable conjugate variables we mean not, say, p_j, q^j derived from the Lagrangian, but the observable quantities $\tilde{p}^j, q^j$ where $\tilde{p}^j = \tilde{g}^{j\nu} p_\nu$; i.e. both members of the pair must be contravariant.

However, we can say more. Indeed, we can derive an uncertainty relation. Consider

$$\triangle q^1 \triangle p^1 = \triangle q^1 \triangle \left(p_\nu \tilde{g}^{\nu 1}\right).$$

What is the minimum value of this product, given an initial uncertainty $\triangle q^1$? Since p_j is an independent variable, we may take $\triangle p_j = 0$ so that

$$\triangle p^1 = \triangle \left(p_\nu \tilde{g}^{\nu 1}\right) = p_\nu \triangle \tilde{g}^{\nu 1}.$$

In order to determine $\triangle \tilde{g}^{\nu 1}$ we will argue that the variance of the distribution of the average of the metric over a region of space-time is inversely proportional to the volume,

$$Var\left(\frac{1}{V}\int_v \tilde{g}_{\mu\nu} dv\right) = \frac{k}{V}$$

In other words, we wish to show that if we are given a volume and if we consider the average values of the metric components over this volume, then these average values, which of course are stochastic, are less stochastic than the metric component values at any given point in the volume. Further, we wish to show that the stochasticity, which we can represent by the variances of the distributions of the metric components, is inversely proportional to the volume. This allows that over macroscopic volumes, the metric tensor behaves classically (i.e. according to general relativity).

For simplicity, let the distribution of each metric component at any point Θ be normal.

$$f_{\tilde{g}_{\mu\nu}}(g_{\mu\nu}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{g_{\mu\nu}}{\sigma})^2}.$$

Note also that if $f(y)$ is normal, the scale transformation $y \longrightarrow y/m$ results in $f(y/m)$ which is normal with

$$\sigma^2_{(y/m)} = \frac{\sigma^2_y}{m^2}$$

Also,for convenience, let
$f_{g_{\mu\nu}}$at $e_1(g_{\mu\nu}) \equiv f_{\Theta_1}(g_{\mu\nu})$.
We now require

$$Var(f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} \equiv \sigma^2_{((\Theta_1+\Theta_2+....+\Theta_m)/m)},$$

where $f_{(\Theta)}$ is normally distributed. Now again, the convolute $f_{(\Theta_1+\Theta_2)}(g_{\mu\nu})$ is the distribution of the sum of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2,

$$f_{(\Theta_1+\Theta_2)} = \int_{-\infty}^{\infty} f_{\Theta_1}(g^1_{\mu\nu}) f_{\Theta_2}(g^1_{\mu\nu} - g^2_{\mu\nu}) dg^2_{\mu\nu},$$

where $g^1_{\mu\nu}$ is defined to be $g_{\mu\nu}$ at Θ_1. Here, of course, $f_{\Theta_1} = f_{\Theta_2}$ as the space is empty so that,

$$f_{(\Theta_1+\Theta_2)/2} = f_{(g_{\mu\nu}/2at\Theta_1+g_{\mu\nu}/2at\Theta_2)}$$

is the distribution of the average of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2. $\sigma^2_{(\Theta_1+\Theta_2+....\Theta_m)}$ is easily shown from the theory of normal distributions to be,

$$\sigma^2_{(\Theta_1+\Theta_2+....+\Theta_m)} = m\sigma^2_\Theta.$$

Also, $f_{(\Theta_1+\Theta_2....+\Theta_m)}$ is normal. Hence,

$$\sigma^2_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} = \frac{m\sigma^2_\Theta}{m^2} = \frac{\sigma^2_\Theta}{m},$$

or the variance is inversely proportional to the number of elements in the average, which in our case is proportional to the volume. For the case where the distribution $f_{(g_{\mu\nu})}$ is not normal, but also not 'pathological', the central limit theorem gives the same result as those obtained for the case where $f_{(g_{\mu\nu})}$ is normal. Further, if the function $f_{(g_{\mu\nu})}$ is not normal, the distribution $f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)}$ in the limit of large m is normal,

$$f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} \longrightarrow f_{((\int_V g_{\mu\nu} d\tilde{V})/V)}.$$

In other words, over any finite (i.e. non-infinitesimal) region of space, the distribution of the average of the metric over the region is normal. Therefore, (anticipating Part B) in so far as we do not consider particles to be 'point' sources, we may take the metric fluctuations at the location of a particle as normally distributed for for each of the metric components $\tilde{g}_{\mu\nu}$. Note that this does not imply that the distributions for any of the metric tensor components are the same for there is no restriction on the value of the variances σ^2 (e.g., in general, $f_{(\tilde{g}_{11})} \neq f_{(\tilde{g}_{22})}$. Note also that the condition of normally distributed metric components does not restrict the possible particle probability distributions, save that they be single-valued and non-negative. This is equivalent to the easily proved statement that the functions

$$f_{(x,\alpha,\sigma)} = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2}(\frac{x-\alpha}{\sigma})^2\right)}$$

are complete for non-negative functions.

Having established that,

$$Var\left(\frac{\Theta_1+\Theta_2+....+\Theta_m}{m}\right) = \frac{\sigma^2_\Theta}{m},$$

consider again the uncertainty product,
$\triangle q^1 \triangle p^1 = p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}$.
$\triangle q^1$ goes as the volume [volume here is V^1 the one-dimensional volume]. $\triangle \tilde{g}^{\nu 1}$ goes inversely as the volume, so that $p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}$ is independent of the volume; i.e. as one takes q^1 to be more localized, p^1 becomes less localized by the same amount, so that for a given covariant momentum p_j (which we will call the proper momentum),$p_\nu \triangle q^1 \triangle g^{\nu 1}$ = a constant k. If also p_ν is also uncertain,
$p_\nu \triangle q^1 \triangle g^{\nu 1} \geq k$.
The fact that we have earlier shown that a free particle spreads indicates the presence of a minimum proper momentum. If the covariant momentum were zero, then the observable contravariant momentum $p^\nu = g^{\nu\mu} p_\mu$would also be zero and the particle would not spread. Hence,
$p_{\nu min} \triangle q^1 \triangle g^{1\nu} = k_{min}$.
or in general,
$\triangle q^1 \triangle (p_{\nu min} g^{\nu 1}) = \triangle q^1 \triangle p^1 > k_{min}$,
which is the uncertainty principle.

With the usual methods of quantum mechanics, one treats as fundamental, not the probability density $P(x,t)$, but the wave function Ψ [$\Psi^*\Psi = P(x,t)$ for the Schrödinger equation]. The utility of using Ψ is that Ψ contains phase information. Hence, by using Ψ the phenomenon of interference is possible. It might be thought that our stochastic space-time approach, as it works directly with $P(x,t)$, might have considerable difficulty in producing interference. In the following, it will be shown that Statements 3 and 4 can produce interference in a particularly simple way.

Consider again the free particle in empty space. By considering the metric only at the location of the particle, we can suppress the stochasticity by means of Statement 1.5. Let the metric at the location of the particle be $g_{\mu\nu}$.

We assume, at present, no localization, so that the probability distribution $P(x,t)$=constant. $P(x,t) = -Ag$ by Statement 3. Here A is just a normalization constant so that $-g = constant$. We can take the constant to be unity.

Once again, the condition of empty space implies that the average value of the metric over a region of space-time approaches the Minkowski metric as the volume of the region increases.

Now consider, for example, a two-slit experiment in this space-time. Let the situation $s1$ where only one slit is open result in a metric $g^{s1}_{\mu\nu}$. Let the situation $s2$ where only slit two is open result in a metric $g^{s2}_{\mu\nu}$. The case where both slits are open is then by statement 4,

$$g^{s3}_{\mu\nu} = \frac{1}{2}\left(g^{s1}_{\mu\nu} + g^{s2}_{\mu\nu}\right).$$

Let us also assume that the screen in the experiment is placed far from the slits so that the individual probabilities $-|g^{s1}|$ and $-|g^{s2}|$ can be taken as constant over the screen.

Finally, let us assume that the presence of the two-slit experiment in the space-time does not appreciably alter the situation that the metrics $g^{s1}_{\mu\nu}$ and $g^{s2}_{\mu\nu}$ are in the average $\eta_{\mu\nu}$ (that is to say that the insertion of the two-slit experiment does not appreciably change the geometry of space-time).

Now we will introduce an unphysical situation, a 'toy' model, the utility of which will be seen shortly

It is of interest to ask what one can say about the metric $g^{s1}_{\mu\nu}$. Around any small region of space-time, one can always diagonalize the metric, so we'll consider a diagonal metric. If the particle is propagated in, say, the x^3 direction and, of course, the x^4 (time) direction. We might expect the metric to be equal to the Minkowski metric, $\eta_{\mu\nu}$ save for g_{33} and g_{44}. (Here, we'll suppress the metric stochasticity for the moment, by, for example, averaging the metric components over a small region of space-time.) We will then, for the moment, take the following:

$$\tilde{g}^{s1}_{\mu\nu} = \left\| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & -t \end{matrix} \right\|, \text{ and so } |\tilde{g}^{s1}_{\mu\nu}| = -st,$$

where s and t are as yet undefined functions of position. In order that $|\tilde{g}^{s1}_{\mu\nu}|$ be constant, let $s = t^{-1}$ so that $\left|g^{s1}_{\mu\nu}\right| = \left|\eta_{\mu\nu}\right| = -1$.

Now we will introduce an nonphysical situation, the utility of which will be seen shortly.

Let $s = e^{i\alpha}$where α is some unspecified function of position. Consider the following metrics:

$$g^{s1}_{\mu\nu} = \left\| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{matrix} \right\|,$$

$$g^{s2}_{\mu\nu} = \left\| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\beta} & 0 \\ 0 & 0 & 0 & -e^{-i\beta} \end{matrix} \right\|,$$

where β is again some unspecified functions of position;

$$(-|\tilde{g}^{s1}_{\mu\nu}|)^{1/2} = (-|\tilde{g}^{s2}_{\mu\nu}|)^{1/2} = 1$$

(Note $|\frac{1}{2}A_{\mu\nu}| = \frac{1}{16}|A_{\mu\nu}|$),

$$(-|\tilde{g}^{s3}_{\mu\nu}|)^{1/2} = (-\tfrac{1}{16}|\tilde{g}^{s1}_{\mu\nu} + \tilde{g}^{s2}_{\mu\nu}|)^{1/2}$$
$$= [\tfrac{-1}{16}(2 + e^{i(\alpha-\beta)} + e^{-i(\alpha-\beta)})]^{1/2}$$
$$(-|\tilde{g}^{s3}_{\mu\nu}|)^{1/2} = \tfrac{1}{2}abs[cos\tfrac{1}{2}(\alpha-\beta)].$$

This is, of course, the phenomenon of interference. The metrics $g^{s1}_{\mu\nu}$, $g^{s2}_{\mu\nu}$, and $g^{s3}_{\mu\nu}$ describe, for example, the two-slit experiment described previously. The analogy of the function $e^{i\alpha}$ and $e^{-i\alpha}$ with Ψ and Ψ^* (the free particle wave functions) is obvious. The use of complex functions in the metric, however, is nonphysical. The resultant line element $ds^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu$ would be complex and hence nonphysical. The following question arises: Can we reproduce the previous scheme, but with real functions? The answer is yes, but first we must briefly discuss quadratic-form matrix transformations[Ap.18].

Let,

$$X = \left\| \begin{matrix} dx^1 \\ dx^2 \\ dx^3 \\ dx^4 \end{matrix} \right\|, \text{ and again let } G = |g_{\mu\nu}|.$$

Then $X^tGX = ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where X^t is the transpose of X. Consider transformations which leave the line element ds^2 invariant. Given a transformation matrix W,

$$X' = WX$$

and

$$X^tGX = X'^tG'X' = (X^t(W^t)^{-1})G'(W^{-1}X).$$

[Note: $(WX')^t = X'^tW'^t$.] However, $X^tGX \equiv (X^t(W^t)^{-1})(W^tGW)(W^{-1}X)$

so that $G' = W^tGW$.

In other words, the transformation W takes G into W^tGW. Now in the transformed coordinates, a metric $\tilde{g}^{s1}_{\mu\nu} \equiv G^{s1}$ goes to $W^tG^{s1}W$. Therefore,

$$\Psi_1^*\Psi_1 = -|W^tG^{s1}W|$$
$$= -|W^t|\ |g^{s1}_{\mu\nu}|\ |W|,$$
$$\Psi_3^*\Psi_3 = -\tfrac{1}{16}|W^tG^{s1}W + W^tG^{s21}W|.$$
$$= -\tfrac{1}{16}|W^t(G^{s1} + G^{s2})\ W|.$$
$$= -\tfrac{1}{16}|W^t|\ |G^{s1} + G^{s2}|\ |W|.$$

If we can find a transformation matrix W with the properties,

(i) $|W| = 1$,

(ii) W is not a function of α or β,

(iii) $W^tGW$is a matrix with only real components,

then we will again have the interference phenomenon with $g'_{\mu\nu}$ real, and again $\Psi_1^*\Psi_1 = \Psi_2^*\Psi_2 = 1$, and $\Psi_3^*\Psi_3 = \frac{1}{2}abs\left(cos\frac{\alpha-\beta}{2}\right)$. The appropriate matrix W is,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{Vmatrix} = W.$$

If, as previously,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{Vmatrix} \equiv \parallel \tilde{g}^{s1}_{\mu\nu} \parallel,$$

then,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -cos\alpha & sin\alpha \\ 0 & 0 & sin\alpha & cos\alpha \end{Vmatrix} = \parallel \tilde{g}^{'s1}_{\mu\nu} \parallel = W^t \tilde{g}^{s1}_{\mu\nu} W,$$

so that in order to reproduce the phenomenon of interference, the stochastic metric $\tilde{g}_{\mu\nu}$ will have off-diagonal terms. Incidentally, coordinates appropriate to $\tilde{g}^{s1}_{\mu\nu}$are,

$x^{1'} = x^1$,
$x^{2'} = x^2$,
$x^{3'} = \frac{-i}{\sqrt{2}}x^3 + \frac{1}{\sqrt{2}}x^4$,
$x^{4'} = \frac{1}{\sqrt{2}}x^3 - \frac{i}{\sqrt{2}}x^4$,

which is to say that with an appropriate coordinate transformation (which is complex), we can treat the free space probability distribution $\Psi^*\Psi$ in a particularly simple way. Since the components x^μ do not appear in predictions (such as $\Psi^*\Psi$), we may simply, as an operational convenience, take $\tilde{g}_{\mu\nu}$ to be diagonal, but with complex components.

DISCUSSION

Having recognized that quantum mechanics is merely an operational calculus, and also having observed that general relativity is a true theory of nature with both an operational calculus and a *Weltanschauung*, we have attempted to generate quantum mechanics from the structure of space-time. As a starting point we have used a version of Mach's principle where in the absence of mass, space-time is not flat, but undefined (or more exactly, not well defined) such that $P_\Theta(g_{\mu\nu}) = -k|g_{\mu\upsilon}|$ (where k is a constant) is, at a given point Θ, the probability distribution for $g_{\mu\nu}$ (in the Copenhagen sense[Ap.19]) .

From this, the motion of a free (test) particle was derived. This is a global approach to quantum theory. It should be noted that there are two logically distinct approaches to conventional quantum mechanics: a local, and a global formulation. The local formalism relies on the existence of a differential equation (such as the Schrödinger equation) describing the physical situation (e.g. the wave function of the particle) at each point in space-time. The existence of this equation is operationally very convenient. On the other hand, the global formulation (or path formulation, if you will) is rather like the Feynman path formalism for quantum mechanics[Ap.20], which requires the enumeration of the "action" over these paths. This formalism is logically very simple, but operationally it is exceptionally complex. Our approach is a local formalism. Statement 3, $P(x,t) = -Ag$, is local and provides the basis for the further development of stochastic space-time quantum theory. Statements 1 and 3 are then logically related. The remaining Statements 2, 4, and 5 are secondary in importance.

The conclusion is that with the acceptance of the statements, the following can be deduced:

(i) the motion of a free particle, and the spread of the wave packet,

(ii) the uncertainty principle,

(iii) the nature of conjugate variables,

(iv) interference phenomena,

(v) an indication of where conventional quantum mechanics might break down (i.e. the limited validity of linear superposition).

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