Some Integrals

Edgar Valdebenito

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Abstract. This note presents some definite integrals.

Integrals

$$\int_0^{1/3}\left(\sqrt[4]{\frac{1-x+\sqrt{1-2x-3x^2}}{2x}}-\sqrt[4]{\frac{1-x-\sqrt{1-2x-3x^2}}{2x}}\right)dx=\frac{\pi}{2\sqrt{12+6\sqrt{3}}} \tag{1}$$

$$\int_0^{\infty}\left(\sqrt{\coth\frac{x}{2}}-\sqrt{\tanh\frac{x}{2}}\right)\frac{\sinh(2x)}{\left(1+3\cosh^2 x\right)^2}dx=\frac{\pi}{8\sqrt{12+6\sqrt{3}}} \tag{2}$$

$$\int_0^{\pi/2}\left(\sqrt{\cot\frac{x}{2}}-\sqrt{\tan\frac{x}{2}}\right)\frac{\sin(2x)}{\left(3+\cos^2 x\right)^2}dx=\frac{\pi}{8\sqrt{12+6\sqrt{3}}} \tag{3}$$

$$\int_1^{\infty}\left(\sqrt[4]{\frac{x+1}{x-1}}-\sqrt[4]{\frac{x-1}{x+1}}\right)\frac{x}{\left(1+3x^2\right)^2}dx=\frac{\pi}{16\sqrt{12+6\sqrt{3}}} \tag{4}$$

$$\int_1^{\infty}\left(\sqrt[4]{x}-\sqrt[4]{x^{-1}}\right)\frac{x^2-1}{\left(1+x+x^2\right)^2}dx=\frac{\pi}{2\sqrt{12+6\sqrt{3}}} \tag{5}$$

$$\int_0^{\ln\sqrt{3}}\left(\sqrt[4]{\frac{1+\sqrt{3}\sinh x}{1-\sqrt{3}\sinh x}}-\sqrt[4]{\frac{1-\sqrt{3}\sinh x}{1+\sqrt{3}\sinh x}}\right)\frac{\sinh x}{\cosh^3 x}dx=\frac{3\pi}{16\sqrt{12+6\sqrt{3}}} \tag{6}$$

$$\int_1^{\sqrt{3}}\left(\sqrt[4]{\frac{2x+\sqrt{3}\left(x^2-1\right)}{2x-\sqrt{3}\left(x^2-1\right)}}-\sqrt[4]{\frac{2x-\sqrt{3}\left(x^2-1\right)}{2x+\sqrt{3}\left(x^2-1\right)}}\right)\frac{x\left(x^2-1\right)}{\left(1+x^2\right)^3}dx=\frac{3\pi}{64\sqrt{12+6\sqrt{3}}} \tag{7}$$

$$\int_0^1 \left(\sqrt[4]{\frac{1+x}{1-x}} - \sqrt[4]{\frac{1-x}{1+x}} \right) \frac{x}{\left(3+x^2\right)^2}\, dx = \frac{\pi}{16\sqrt{12+6\sqrt{3}}} \tag{8}$$

$$\int_0^{1/2} \left(\sqrt[4]{\frac{\sqrt{1-x^2}+\sqrt{3}x}{\sqrt{1-x^2}-\sqrt{3}x}} - \sqrt[4]{\frac{\sqrt{1-x^2}-\sqrt{3}x}{\sqrt{1-x^2}+\sqrt{3}x}} \right) x\, dx = \frac{3\pi}{16\sqrt{12+6\sqrt{3}}} \tag{9}$$

$$\int_{\sqrt{3}/2}^1 \left(\sqrt[4]{\frac{x+\sqrt{3}\sqrt{1-x^2}}{x-\sqrt{3}\sqrt{1-x^2}}} - \sqrt[4]{\frac{x-\sqrt{3}\sqrt{1-x^2}}{x+\sqrt{3}\sqrt{1-x^2}}} \right) x\, dx = \frac{3\pi}{16\sqrt{12+6\sqrt{3}}} \tag{10}$$

References

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