Must a quantum mechanical particle sometimes be in two places at once?

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In this short paper, we point out that the interference of probabilities in the double slit experiment, or in a particle interferometer, should not necessarily lead us to think that a quantum mechanical particle's position is a meaningless concept or that continuous motion of a quantum mechanical particle is an impossibility. We do not need to conclude that a particle must sometimes be in two places at once, or that nature herself does not know exactly where a particle is etc. We show that the argument leading to that kind of conclusion, based on the interference of probabilities, is illogical when probability is viewed in a rational Bayesian fashion i.e. as accounting for rational degree of belief in an occurrence rather than the relative frequency of that occurrence in many trials. We lend support to the view that much progress may be made in the interpretation of the quantum formalism and in the formation of physical pictures of processes in quantum mechanics by viewing probability in a rational Bayesian manner.

Keywords Quantum Mechanics, uncertainty principle, Bayesian probability, Realism, QBism.

We start by sketching the typical argument leading to the conclusion that particles cannot be moving in definite orbits and can be in two places at once. We take the particle interferometer (Figure 1) as our experimental set up since the argument is clearer when the possible paths of the particle are well separated.

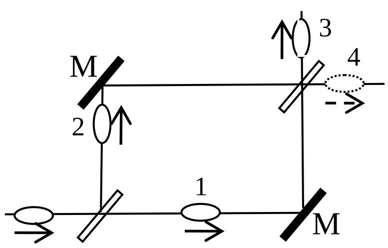

Figure 1. A simple particle interferometer. The incoming wave packet is split into packets 1 and 2 by the first beam splitter. These are reflected by mirrors M and are each partly transmitted and reflected at the final beam splitter to form out-going packets 3 and 4. Under fine-tuning packet 4 may be cancelled out.

With reference to Figure 1, the argument goes through 6 steps as follows. (i) The particle's initial motion has associated with it a certain time dependent normalised wave function ψ in fact spread all over space but highly concentrated in a small packet which after passage through the first beam splitter divides into two (half-normalised) wave functions, say ψ_1 and ψ_2, setting off in different ways (way 1 and way 2) from the first beam splitter, each wave function evolving according to the Schrödinger equation. (ii) In time, these waves interfere (like real waves would) so that the probability density for the particle being at some point (i.e. $|\psi_1+\psi_2|^2$) is not the same as $|\psi_1|^2+|\psi_2|^2$ in regions where ψ_1 and ψ_2 overlap (as in packets 3 and 4 in Figure 1). (iii) If the particle moves continuously through space it must set off from the first beam splitter either in way 1 or way 2. (iv) If it sets off in way 1 the probability density for the particle being at any point thereafter is $2|\psi_1|^2$ and if it sets off in way 2 it is $2|\psi_2|^2$ and (v) as the probabilities for the particle to set off one way or the other are each $\frac{1}{2}$, (vi) the net probability density should be $\frac{1}{2}\times 2|\psi_1|^2+\frac{1}{2}\times 2|\psi_2|^2=|\psi_1|^2+|\psi_2|^2$ not $|\psi_1+\psi_2|^2$. Hence we have a contradiction. The claim of continuous particle motion along one or other path must be false, and the particle must instead travel both ways at once around the interferometer.

Steps (i), (ii), (iii) and (v) of this argument are fine. Under the assumption of continuous motion the particle is expected, with equal probability, to set off one way or the other from the first beam splitter, and the probability density calculated by quantum mechanics is $|\psi_1+\psi_2|^2$ thereafter. The problem lies in step (iv). For when adopting the rational Bayesian approach to probability, we cannot speak of the probability of an event *supposing* the particle sets off in way 1 (or way 2) from the first beam splitter. We can only speak of the probability of an event supposing we *knew* the particle set off in way 1 (or way 2). This is because we are taking probabilities to be determined by knowledge not by physical conditions. But the acquisition of knowledge of which way the particle sets off is expected to change the particle motion thereafter on account of the uncertainty principle, so only when we actually acquire the necessary knowledge would it be rational to condition our probability distribution from $|\psi_1+\psi_2|^2$ to $2|\psi_1|^2$ (or from $|\psi_1+\psi_2|^2$ to $2|\psi_2|^2$). In the above argument, however, it is in no way supposed that we actually get to know which way the particle set out from the first beam splitter, so we have every reason to doubt the validity of step (iv) of the argument.

Inside the interferometer, where the wave packets are moving in separate paths, we are close to a classical limit, the de Broglie wavelength being very small compared to the packet dimensions. Under the assumption of continuous motion, we expect the particle to be moving inside one of the packets. We can find out which packet it chooses by placing a particle detector in one path. If this particle detector finds no particle, then we know (or fully expect that) the particle went the other way. We can then use our new knowledge to collapse our wave function from $\psi_1+\psi_2$ to $\sqrt{2}\psi_1$ (or from $\psi_1+\psi_2$ to $\sqrt{2}\psi_2$). There is no contradiction, and position measurements conducted in many trials following null-detection will confirm the correctness of our reasoning.

Note that collapse of the wave function may therefore be viewed as resulting from new knowledge, not from some unknown physical process as is commonly assumed. This is a great simplification in interpretation.

Now suppose the interferometer is fine-tuned so that (when no null-detection is performed inside the interferometer) the particle is found always to exit one way out of it. It might then seem strange that an observation, a null-detection (on one path inside the interferometer) which might be thought to cause virtually no disturbance, can result in the particle sometimes exiting the interferometer in a way it would seem never to do without that observation. This is evidently due to a change in the balance of the forces (whatever they are) governing the particle motion through the final beam splitter. After null detection and while the particle remains in the interferometer, the result of our acquisition of further knowledge results in our probability distribution over particle position sharpening (reducing to one wave packet rather than two), but becoming less sharp (two wave packets rather than one) after the particle has left the interferometer. Hence use of Bayesian reasoning to interpret the quantum mechanical formalism *can* lead to results of a kind not replicated in applications of Bayesian probability to classical mechanical processes, but that does not imply there is something wrong with Bayesian reasoning or with the assumption of continuous motion of a particle.

Having argued that a Bayesian *approach* to probability can help in the interpretation of quantum mechanics by removing the need for a particle to be in two places at once, we now note that the *rules* of Bayesian probability for use in pure state quantum theory must in some ways be different from the rules of Bayesian probability in ordinary life or in classical physics. The reason for this lies in the uncertainty principle, which greatly restricts the knowledge we can hold -the knowledge on which our probabilities are based.

Simultaneous knowledge of properties incompatible on account of the uncertainty principle is impossible. For example, we cannot know both the momentum and position of a particle to arbitrarily great precision. Neither can we know (or predict) in advance (to arbitrary precision) the position of a particle at two times during its motion. The position of a particle at one time and its position at a later time are incompatible properties. If we measure just the second we leave the first unknown, and if we measure the first we affect the second.

A consequence of these facts is that joint probabilities of incompatible properties are non-existent. Since we clearly cannot, for example, directly measure the z component *and* the x component of a particle's spin at one time in order to test (in repeated trials) any supposed joint probability distribution $p(\sigma_z, \sigma_x)$ over those variables, there is no need for that joint distribution. Use of the product rule $p(\sigma_z, \sigma_x) = p(\sigma_z) p(\sigma_x | \sigma_z)$ to *derive* $p(\sigma_z, \sigma_x)$ is not possible either, for, under our rational Bayesian interpretation, $p(\sigma_x | \sigma_z)$ is the probability of σ_x having acquired knowledge of the value of σ_z (rather than under the mere supposition that σ_z has a particular value). Acquisition of knowledge of the value of σ_z generally changes value of σ_x rendering the formula $p(\sigma_z, \sigma_x) = p(\sigma_z) p(\sigma_x | \sigma_z)$ inappropriate. So joint probabilities over incompatible variables are both untestable and incalculable, and are therefore rightly regarded as non-existent.

As a result, the usual sum and product rules of probability and hence the rule for conditional probabilities, or Bayes' rule, apply only in a sample space whose propositions refer to the possible values of a particular basic property. Suppose (only for simplicity of formulation) that our Hilbert space is of finite dimension N. Suppose, also, that x_i ($i = 1, \ldots N$) are the (mutually exclusive) propositions claiming the possible values x_i of the property x employed in a particular representation. Then, if $\Phi(x_i|Y)$ is our wave function under a pure-state of knowledge Y (acquired before the process in question starts), the probability of x_i is $P(x_i|Y) = |\Phi(x_i|Y)|^2$. The probability of the attribute claimed by the disjunction 'x_3 or x_5 …etc.' is

$$P(x_3 \vee x_5 \vee ...|Y) = P(x_3|Y) + P(x_5|Y) + ... \qquad (1)$$

Also, for any system attribute claimed by a disjunction A of the x_i ($i = 1,...N$), we have, for the probability of the conjunction of x_i and A the product rule

$$P(x_i \wedge A|Y) = P(A|Y)P(x_i|A \wedge Y) \qquad (2)$$

$A \wedge Y$ being the conjunction of propositions A and Y.

In the product rule (2) we take it that knowledge of the truth of A can be acquired in such a way that $A \wedge Y$ is, like Y, a pure-state of knowledge so that $P(x_i|A \wedge Y) =$ $|\Phi(x_i|A \wedge Y)|^2$. Only then can the product rule apply.

We can of course deduce from (1) and (2) the more general sum and product rules:

$$P(A \vee B|Y) = P(A|Y) + P(B|Y) - P(A \wedge B|Y),$$
$$P(B \wedge A|Y) = P(A|Y)P(B|A \wedge Y),$$

where A and B are any disjunctions of the x_i ($i = 1,...N$) and the same requirement regarding $A \wedge Y$ applies.

Hence the ordinary probability rules apply to the sample space whose atomistic propositions are the propositions x_i ($i = 1,...N$) of any one representation referring to any one particular time. The only difference lies in the limitations imposed on conditioning (from Y to $A \wedge Y$) as noted. An example of conditioning has been given above in connection with null detection in one branch of an interferometer, the x_i ($i = 1,...N$) there standing for the possible positions of the particle at the time the null measurement is performed. Dropping the wave function in one branch of the interferometer and renormalizing the wave function in the other is to follow a procedure consistent with the posterior probability at the time of the null measurement being given by

$$P(x_i|A \wedge Y) = \begin{cases} \dfrac{P(x_i|Y)}{P(A|Y)} & x_i \in \mathrm{A} \\ 0 & x_i \notin \mathrm{A} \end{cases}$$

as follows from (2), where, when $x_i \in \mathrm{A}$, we have $P(x_i \wedge A|Y) = P(x_i|Y)$. Here A stands for the set of propositions claiming the particle is at one or other point in the wave packet in the branch of the interferometer different from the branch where null-detection was performed, and A stands for the disjunction of the propositions in the set A.

What is the analytic reason for the 'interference of probabilities' characteristic of the double slit experiment and the particle interferometer? Well this is evidently to do with the relation the quantum formalism provides between the wave functions (or probability amplitudes) relating to *different* representations. If $x_i \ (i = 1,...N)$ and $x_i' \ (i = 1,...N)$ are the atomistic propositions in the sample spaces of different representations, then the formalism gives the relation

$$\Phi(x_i|Y) = \sum_{j=1}^{N} \Phi(x_i|x_j')\Phi(x_j'|Y) \tag{3}$$

between the wave functions $\Phi(x_i|Y)$ and $\Phi(x_j'|Y)$ under the same pure state of knowledge Y. The coefficients $\Phi(x_i|x_j')$ (themselves wave functions) are the 'transformation functions' from one representation to the other. We should call relation (3) 'Feynman's law' because Feynman seems to have been the first to see it as a law of probability rather than a law of physics. In particular, if the x_i ($i = 1,...N$) are standing for the possible positions of a particle at time t and $x_i' \ (i = 1,...N)$ are standing for the possible positions of the particle at the earlier time t', then the above relation gives us the wave function at one time in terms of the wave function at another. In the double slit experiment, for example, with t being a time the particle has arrived at the screen, and t' the time the particle passes through the slits, $\Phi(x_j'|Y)$ is non-zero only at the slits, and Feynman's law (3) reduces to

$$\Phi(x_i|Y) = \Phi(x_i|x_1')\Phi(x_1'|Y) + \Phi(x_i|x_2')\Phi(x_2'|Y)$$

for the wave function over the screen, x_1' and x_2' denoting the positions of the slits, while $\Phi(x_1'|Y)$ and $\Phi(x_2'|Y)$ are equal constants. The 'interference of probabilities' is here (and elsewhere) arising from Feynman's law understood as a law of rational Bayesian probability in pure-state quantum mechanics.

We are therefore arguing that by adopting a rational Bayesian approach to probability, and by interpreting the (non-relativistic) quantum formalism as a set of rules for calculating probabilities given our knowledge (always limited because of the uncertainty principle) we can avoid the need for a particle to be in two places at once and view it as being in continuous motion along a single path. Particle positions, and by extension, any basic property (such as a particle's momentum or a particle's spin component in a particular direction at any one time, etc.), can be taken to be *actually* possessed by a system, not just *potentially* possessed by it awaiting measurement.

Finally, attention is drawn to the connection between probability and frequency. This is vital because predicted probabilities are of course checked by observations in repeated trials. When a quantum mechanical process is repeated we assume the processes in each trial (or rather the propositions referring to them) are *logically independent*. This means knowledge of the outcome of one trial has no effect on the probabilities we should adopt for the outcomes of other trials. Under this condition a simple product rule correctly gives the joint probability distribution over the possible outcomes of all the trials. For example, in a single trial of the double slit experiment, the probability of arriving at point x_i on the screen is $P(x_i|Y) = |\Phi(x_i|Y)|^2$ and the joint probability of arriving at point $x_i^{(1)}$ in the first trial, at point $x_j^{(2)}$ in the second trial, … and at point $x_s^{(m)}$ in the final trial is $P(x_i^{(1)} \wedge x_j^{(2)} \wedge ... x_s^{(m)}|Y) = P(x_i^{(1)}|Y)P(x_j^{(2)}|Y)...P(x_s^{(m)}|Y)$. It then follows, in the usual way, that the expected number of cases in which the particle arrives at a particular place x_i on the screen is m times the probability $P(x_i|Y)$. That is, the fraction of times the particle arrives at x_i has a mean value $P(x_i|Y)$ and the standard deviation of this fraction from that mean value tends to zero as the number of trials tends to infinity. The algebraic proof of this is the same as in ordinary probability theory because the uncertainty principle does not restrict knowledge of the outcomes of different trials. The sample space in which the atomistic propositions are the conjunctions $x_i^{(1)} \wedge x_j^{(2)} \wedge ... x_s^{(m)}$ for all $i, j ... s$ is one in which the usual sum and product rules of probability apply. On the basis of our proposed Bayesian probability theory for quantum mechanics, there is, therefore, a way to check the probability distribution $P(x_i|Y)$, that quantum mechanics predicts, by conducting observations in many trials.

Note that, with the advent of quantum theory, the nature of science has changed. We no longer claim we can get to accurately measure all the dynamical properties of a system even in principle, nor get to know all the deterministic laws governing them. Instead, on the basis of the quantum formalism and any limited knowledge we may hold of a system's dynamical properties, we calculate the degrees of expectation we should rationally adopt for other dynamical properties. This includes the calculation of our expectations for the relative frequencies of observable outcomes in repeated trials of a process. We do not claim those frequencies are fixed by law. Violations of them might occur. They are not physically determined but only expected. Yet they can be relied upon, well enough, to confirm the correctness of the quantum formalism as a way of calculating degrees of expectation.

By restoring belief in the actual possession of properties by quantum mechanical systems, we open up the possibility of claiming *some* dynamical laws governing them; laws that can be safely claimed without violating the expectations calculated using the quantum formalism. For example, it seems possible to claim that the momentum of a free electron remains constant during its motion and that its spin component in any one direction remains constant so long as the electron experiences no magnetic field. In this way our understanding of the nature of quantum mechanical processes and the pictures we form of them might be improved.

We end by providing references to work developing ideas similar to those expressed here.

The idea that the wave function represents our *knowledge* of the dynamical properties of a quantum system is quite old. It was the view taken, for example, by Peierls [1].

The idea that Bayesian probability is better suited than conventional probability when it comes to understanding quantum theory has been argued by several authors. They include Appleby [2], Fuchs, Mermin and Schack [3], Benavoli, Facchini and Zaffalon [4], Marlow [5] and Jaynes [6]. In [7] Jaynes has made a convincing case for adopting a rational Bayesian approach to quantum statistical mechanics in which he sees the theory of mixed states (and density matrices) as providing new rules of Bayesian probability theory. It is true that these authors differ in their view as to what Bayesian probabilities refer to in quantum theory. Fuchs, Mermin and Schack, for example, follow the *subjective* Bayesian approach taking probabilities to refer to personal (measurement) experiences. They call their theory QBism. This contrasts with the view taken, for example, by Jaynes and the present author; that probabilities refer to physical attributes possessed by a quantum mechanical system and are rational degrees of expectation of those attributes given knowledge of certain dynamical properties of the system.

Some authors (including the author of this paper) are trying out new rules of Bayesian probability in which probabilities take complex values rather than real values, so wave functions themselves (rather than their squared moduli) are taken to be the probability distributions. See for example Youssef [8] and Hemp [9]; and in [10] Youssef provides a list of works by himself and by others taking the same path.

There are, of course, other arguments advanced (in addition to the one based on the ‘interference of probabilities’) for rejecting the reality of definite particle orbits or the possession of any quantum mechanical properties before their measurement. These include arguments leading to (violated) Bell type inequalities and to the Kochen Specker paradox. However, a Bayesian approach to probability leads one to doubt the validity of these arguments too. Jaynes [11] has raised questions regarding the validity of Bell type arguments from a Bayesian perspective, Youssef [12] has argued that Bell type arguments only confirm the need for a new approach to probability, and in [9] the present author has put forward reasons for thinking that Bayesian reasoning incorporating the uncertainty principle blocks the derivation of both Bell type inequalities and the Kochen Specker paradox.

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