On the Origin of 1/f Noise Due to Entropy Damage

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Abstract

In this paper we provide two thermodynamic models to describe 1/f noise due to microscopic entropy damage fluctuations; that is, minor fluctuations of degradation occurring in system-environment interaction. As such, we find that flicker noise is a sensitive measure of entropy damage. The concepts provided are consistent with the literature on 1/f noise measurement observations in materials and helps provide a uniform understanding of 1/f phenomena.

1. Introduction:

A concise expression for the second law of thermodynamics in terms of entropy damage change (Δs_{damage}) [1] is

$$\Delta s_{damage} \geq 0 \tag{1}$$

Damage entropy (often termed generated entropy) change is a measurable quantity. In this paper we discuss this measure in terms of 1/f noise. Typically damage is readily observed at the macroscopic level. However, if we recognize that 1/f noise is perhaps one of the best tools in measuring this quantity at the microscopic level, it could prove to have practical importance in damage prognostics. Therefore, this suggests that if microscopic entropy damage occurring in the system-environmental interaction could be stopped, so too would flicker noise cease. In a simpler view, if there was no internal friction during the system-environment interaction, we would have a reversible process and an absence of flicker noise. In a sense, this conclusion is trivial since all thermodynamic processes generate entropy. However, it becomes non-trivial when one asserts that flicker noise is a sensitive measure of irreversibilities occurring. It is non-trivial when an entropy view provides consistency with 1/f noise observations in the literature and modeling helps to provide a universal understanding of the phenomena. Noise in operating systems is starting to be recognized as important for prognostics of failure [1, 2]. One of the more telling signs of the association of noise with failure is related to the human heart degradation where congestive heart patients compared with healthy hearts had a distinctly different noise spectrum [3].

A review on some key 1/f noise observation reported in the literature is provided here along with a discussion on how these are consistent with an entropy damage approach. Here we define microscopic "entropy damage" (generation entropy), as permanent entropy damage increase (positive Eq. 1) as compared with "entropy flow" which can increase or decrease (for example material heating or cooling). That is, entropy flow is not a source of flicker noise. However, it would be consistent to suggest that entropy flow (adding heat) for example, can cause degradation in the material, leading to entropy damage; thereby contributing to flicker noise indirectly and perhaps adding some confusion to the origin of flicker noise observations.

Consider a thermodynamic process of a current flowing through a resistor in a common type of 1/f noise measurement. We can view entropy damage occurring due to the act of the current flow that is associated with thermodynamic stress in the material (we denote W_i as work typically done by the current or possibly any other neighboring thermodynamic process – described later) generating entropy damage in the resistor $\Delta s_{resistor}$ as well entropy change associated to the current flow $\Delta s_{current}$ as it is being impeded by some sort of internal friction. The resulting *1/f* measurement is observed in the form of voltage fluctuations across the resistor. The higher the internal resistance to current flow and its susceptibility to damage in the resistor, the larger is the resistor's entropy $s_{resistor}$. In this view the entropy generated in the measurement process suggests the following dependence

$$\Delta s_{Damage} = \Delta s_{resistor}(W_i) + \Delta s_{current}(s_{resistor}) \geq 0 \tag{2}$$

The inequality equals zero for a theoretical reversible process. The entropy change is to the environment and the system. It might be helpful to think of the resistor as the environment and the current as the system or visa versa. Note with this model, if we had no current, therefore no current change, that it is still possible to get entropy damage and thus 1/f noise by the way W_i is defined (see discussion below).

1.1 How well is 1/f Noise Understood?

In a pedagogical review, Milotti [4] conclusion on 1/f noise was summed as: "Do we have by now an "explanation" of the apparent universality of flicker noises? Do we understand *1/f* noise? My impression is that there is no real mystery behind *1/f* noise, that there is no real universality and that in most cases the observed *1/f* noises have been explained by beautiful and mostly *ad hoc* models". In this publication we realize that there are numerous thermodynamic processes that have been shown to produce 1/f noise and well modeled for the processes of interest. However, here we are providing commonality with entropy generation solution, perhaps a better universal understanding of 1/f noise and how it is extremely important in using it as a tool to measure a material susceptibility to damage.

1.2 The Measurement Process Creates Irreversibilities

Although no process is truly reversible, a common thermodynamic argument, it is worthy of comment. We could state that if a system process is in thermal equilibrium, then the process is reversible. However in thermal equilibrium there is no measurement process!

Clarke and Voss [5] found that 1/f noise was present if there was no driving current at equilibrium but they could not guarantee true thermal equilibrium. In this view there could not be a measurement process, if thermodynamic equilibrium had been reached. Therefore, thermal equilibrium could not have been achieved during their measurement process. This goes to the point of Eq. 2 if $\Delta s_{current}=0$ that we can still have an entropy change. Recall that W_i is defined to be due to the measurement current or any other neighboring thermodynamic work process. This is discussed a bit more below.

2.0 Overview of Entropy Models

There has obviously been a large body of research on flicker noise over the years. One cannot begin to cite all the important contributions. In this paper, what we will do is connect the key dots that lead to our conclusion that 1/f noise is a sensitive measure of entropy damage by using two models. The first is a free energy flicker model that is compared to Schottky's original model. The second is a time domain entropy model that is transformed to the frequency domain, leading to 1/f noise. These models are also discussed as consistent with the literature finding for noise as a function of volume, material homogeneity, current, internal friction, frequency etc.

2.2 Time Domain Entropy Model

The definitions of entropy, s, for discrete and continuous variable X are,

Discrete *X, p(x):*

$$s(X) = -\sum p(x)\log_2 P(x) \tag{3}$$

and Continuous *X, f(x), Differential Entropy [6,7]:*

$$s(X) = -\int f(x) \log\,(f(x))\,dx \tag{4}$$

Here we are concerned with the continuous variations in time t distributed by *f(t)*. Consider a Gaussian spectral density due to a distribution of damage entropy processes and in turn current fluctuations amplitudes with pdf

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(t-\mu)^2}{2\sigma^2}\right) \tag{5}$$

Gaussian spectra density for 1/f noise time domain processes is often described as logical in the literature [such as 8,9]. Milotti [4] summarized the question noting:

> Voss [8] produced experimental plots of the quantity <*V*(*t*)|*V*0>/ *V*0 in several conductors and was able to show that the noise processes observed were reasonably Gaussian. Further, it was noted that the superposition of many non Gaussian microscopic processes can results as Gaussian at the macroscopic level (demonstrated via the central limit theorem). J. B. Johnson in his 1925 experiment [9] in vacuum tubes asserted that the spectral density characterizes a noise process completely only if the process is stationary, ergodic and Gaussian: does the observed *1/f* noise satisfy all these constraints.

When Eq. 5 is inserted into the differential entropy Eq. 4, the result for a temporal process is [6].

$$s(t) = \frac{1}{2}\log(2\pi e\,\sigma(t)^2) \tag{6}$$

The rate changes to the entropy is then

$$\frac{ds(t)}{dt} = \frac{1}{2}\frac{1}{\sigma(t)}\frac{\sigma(t)}{dt} \tag{7}$$

We write this as

$$\sigma^2(t) = \frac{1}{4}\left(\frac{\sigma(t)}{dt} \Big/ \frac{ds(t)}{dt}\right)^2 = k\,t^u \tag{8}$$

Here we assume some possible time dependence power with constant k for the moment. We write Eq. 8 in this form as we are interested in the Allan Variance

$$\text{Allan Variance}: \sigma^2(\tau) = \frac{1}{2(n-1)}\sum_i (\bar{y}(\tau)_{i+1} - \bar{y}(\tau)_i)^2 \tag{9}$$

In this model we assume the variance rate of change likely goes as the entropy rate in Eq. 8, so

$$u=0 \tag{10}$$

When this is the case, we get a *stationary process* and the equivalent frequency domain PSD spectrum *S* is transformed for $\sigma^2(t) \propto t^0$ to [7,10] to the frequency domain where

$$S(f) \propto 1/f \tag{11}$$

Note if we had some temporal dependence (non stochastic process) in the ratio $\sigma^2(t) \propto t$ in Eq. 8 where

$$u=1 \tag{12}$$

we would get Brownian noise as it transform to [7, 10]

$$S(f) \propto 1/f^2 \tag{13}$$

We note that the temporal model, u=0, indicates that the variance and entropy rates change together. Therefore, we anticipate 1/f noise provides more fundamental significance to entropy damage sensitivity then say brown noise (u=1).

2.1 Free Energy Flicker Model

It can prove helpful to have a second supportive entropy model to have a clearer understanding of flicker noise. As a measurable quantity, entropy damage, prior to macroscopic observations, can likely be observed with flicker noise measurements. The interaction of the system (current typically) with the environment (material, or visa versa), causes entropy damage as described in Eq 2. In the case of a 1/f noise measurement, the current and the material could both be degraded. We note an increase in entropy damage, Eq. 2, corresponds to a decrease in the free energy ϕ, i.e.

$$\frac{ds_{Damage}}{dt} > 0 \rightarrow \frac{d\phi}{dt} < 0 \tag{14}$$

Given a system interacting with the environment at temperature *T*, the thermodynamic probability P of a microstate L is

$$P_L \ \alpha \ Exp(-\phi_L / K_B T) \tag{15}$$

with free energy ϕ_L . For low frequencies fluctuations from a thermodynamic process, the microstate probability *P* is time dependent function of the free energy change

$$P_L(t) = \frac{1}{Z}\exp\left(-\frac{\Delta\phi_L(t)}{K_B T}\right) \text{ and Z is } \sum_i P_i = \frac{1}{Z}\sum_i \exp\left(-\frac{\phi_i}{K_B T}\right) = 1 \tag{16}$$

That is, Z is the normalized partition function. We then model the free energy ϕ_L via a Taylor expansion as

$$\phi(t) = \phi(0) + ty_1 + \frac{t^2}{2}y_2 + ... \tag{17}$$

where y_1 and y_2 are given by

$$y_1 = \frac{\partial\phi(0)}{\partial t} \ and \ y_2 = \frac{\partial^2\phi(0)}{\partial t^2} \tag{18}$$

Microscopic damage corresponds to free energy change which is assumed small, then taking the first term in the expansion

$$\Delta\phi_L = \phi(t) - \phi(0) \cong ty_L \tag{19}$$

Therefore the damage probability is

$$P_L(t) = \frac{1}{Z}\exp(-\lambda_L t) \tag{20}$$

where $\lambda = \frac{y_1}{K_\mathbf{B} T}$. Note that $\Delta\phi_L$ itself is not temperature dependent and the systems free energy goes as the thermodynamic work of damage, for an isothermal process

$$\delta W = \sum_a Y_a dX_a \propto \Delta\phi \tag{21}$$

where Y and dX are conjugate work variable like stress and strain, voltage and charge, etc. [1].

We are now in a position to compare Equations 20 with 22 below, which relates to Schottky's (1926) [11] original model. In his model, contribution to the vacuum tube current from cathode surface trapping sites, released electrons according to a simple exponential relaxation

$$N(t) = N_o \exp(-\lambda t) \tag{22}$$

In the entropy viewpoint, the origin would be due to damage fluctuations in the free energy observed from the cathode current and related to the surface trapping. The results lead to the Schottky's [11,4] spectrum model

$$S(\omega) = \frac{N_0^2 n}{(\lambda^2 + f^2)} \tag{23}$$

Bernamont [12] later pointed out that only a superposition of processes with a variety of relaxation rates, λ would yield 1/f noise for a reasonable range of frequencies. He showed that if λ is uniformly distributed between λ_1 and λ_2, and the amplitudes remain constant, the spectrum can be interpreted in the pink 1/f noise region

$$S(\omega) = \frac{N_0^2 n\pi}{2\omega(\lambda_2 - \lambda_1)}, \qquad \lambda_1 << \omega << \lambda_2 \tag{24}$$

and Brown noise for example for

$$S(\omega) = \frac{N_0^2 n}{\omega^2}, \qquad \lambda_1 << \lambda_2 << \omega \tag{25}$$

2.3 Particle Analogy "PhoDons" and Measurement Uncertainty

We briefly can be a bit creative and assign a word "Pho*D*on" for a fundamental *D*amage particle created with energy change $\Delta\phi_i$. The "phodon" associated energy of creation would then be somewhat analogous to phonon energy, but unlike phonons with modes of vibration in say a crystal structure, phodon wave nature becomes associated with random current fluctuation in the measurement process.

Degradation can then be thought of as creating phodon damage particles created with energies that depend on the material properties and interaction with the neighboring environment so that according to the second law and Eq. 2

$$Tds = \delta Q + Tds_{Damage} = \delta Q - d\phi + \delta W \tag{26}$$

Here we see the phodon creation causes a change to the free energy via the current work (consistent with Eq 2).

In Eq. 2, when the current flow is zero (i.e. $\Delta s_{current}=0$), it is apparent that phodons can be created. This is because we described W_i typically due to measurement current of any other thermodynamic process. This is possibly due to thermal fluctuations. As well, we might wonder if phodon creation can also be due to zero point fluctuation. When we talk about the difficulty of obtaining thermodynamic equilibrium, we cannot be sure how microscopic irreversibilities occur in materials.

Finally, we should be mindful of the difficulty of taking very low frequencies measurement (near 0), the observation time must be long enough to be certain of the frequency value $\Delta t \geq 1/\ \Delta\nu$ due to the uncertainty principle for measurement accuracy.

3.0 The Entropy Argument consistency with the 1/f literature

Modeling can only go so far, the physics needs to match up with the numerous experimental observations in the literature in order to be consistent with the concept of 1/f noise measurements being a sensitive measure of entropy damage. In this section we discuss some key common 1/f observations and how these are consistent with the damage entropy view.

Many of the features of flicker noise are illustrated by the phenomenological equation due to Hooge [13] in the form

$$S(f) = \gamma \frac{V_{DC}^{2+\beta} n\pi}{N_C f^{\alpha}}, \tag{27}$$

Here α, β, and γ are constants, V_{DC} is the applied voltage and N_C is the total number of charge carriers in the sample. Here we see that noise power $S(f) \sim \langle V^2 \rangle = (IR)^2$, where I is the driving current and R is the sample resistance which is direct internal friction (entropy) of the material. In this view of 1/f noise, in the thermodynamic process, the entropy damage occurring, in say the resistor-current interaction, corresponds to a change in the free energy from the initial to final state $\Delta W=\phi_i-\phi_f$ where $\phi_i>\phi_f$ for an isothermal measurement process. Damage entropy goes as the interaction which goes in turn by the intrinsic friction of resistance in the material to current flow. It has been found that wire-wound resistors have less flicker noise than carbon. Given the same *R* value, the wire wound resistor is apparently more stable and less susceptible to damage in thermodynamic processes [14]. Wire wound vs. carbon resistors is a great example of how materials can be characterized in terms of damage susceptibility to the current stress for which 1/f noise provides a sensitive measure of the materials stability showing the stronger stability in wire wound resistors.

Note the spectral density in (26) is independent of temperature which reinforces the fact of entropy generation compared with entropy flow (heat added) at the origin of 1/f noise. However, 1/f noise shows some atypical temperature dependent characteristic (Eberhard and Horn, 1978 [15]) where they noted an adhoc function of $\gamma(T)$. We would view this due to damage created by heat as part of the thermodynamic entropy damage process.

The flicker-noise voltage power in MOSFET origin has been associated with traps in the gate dielectric. Traps can be associated with entropy damage. Flicker Noise is often modeled as $K/(C_{ox}\ WL\ f)$, where *K* is the process-dependent constant, C_{ox} is the oxide capacitance in MOSFET devices, *W* and *L* are channel width and length respectively [16].The volume effect is perhaps a bit difficult to explain but we note that the impedance is complex Z=R+iX with oxide reactance Xc=1/wC. To show some consistency with Hooge, entropy is likely generated by the complex impedance where the reactance increases with oxide thickness, but should increase as resistance does decreases with width. Therefore in the case there is complexity with entropy generation with respect to the dimensions and how entropy is distributed in the material.

The phase noise of an oscillator is perhaps one of the most important parameters. Here flicker noise is known to dominate. Phase noise is important as it affects the purity of the carrier frequency in transmission. It is known that the unloaded Q in flicker noise goes as the inverse of Q to the forth power observed [17,18,19] in the low flicker frequency area (i.e. near the carrier frequency) as noted in oscillator power noise spectral density. Here again, damping (a inverse function of Q), a characteristic of another form of internal friction can be strongly associated with entropy generation. A higher Q also indicates a more stable material and less susceptible to entropy damage.

3.1 The Need for Noise Level Standards

Lastly we would like to suggest that 1/f measurement consistency in terms of noise level (i.e. PSD amplitude) can only be accurate if procedural standards are developed with calibration standards. That is, there has been a lot of work since Johnson's original observation of 1/f noise [9]. How well can we actually correlate one research effort to another in terms of noise level? One would anticipate a researcher noise level value would be self consistent. However, without measurement standards and consistent procedures, comparing noise levels of spectra from different researchers would likely yield inconsistencies. In our view, we anticipate that 1/f measurements could provide more contributions in characterizing materials as to their relative damage tolerance, if calibration methods were developed.

4.0 Conclusion

In this paper we provided a free energy and an entropy approach to demonstrate that 1/f noise measurements are important to observing fundamental degradation processes. This was supported with 1/f noise observation in the literature that indicated that such measurements were related to entropy damage concepts presented like internal friction of the material. We also created a phodon particle description of damage events to help break down damage fundamentals. In the time domain model, we found that the variance and entropy rates changed together compared to other noise processes. Therefore, we concluded that 1/f noise provides more fundamental significance to entropy damage sensitivity then other noise measurement. In general, the entropy approach is helpful as it provides a good broad understanding of 1/f noise and its importance to damage measurements in materials.

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