The transferable complex belief model

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***Abstract*—We describe the transferable complex belief model, a model for representing quantified beliefs based on a newly defined complex belief function. The relation between the complex belief function and the probability function is derived when decisions must be made.**

***Index Terms*—Complex belief function, Quantified complex beliefs, Complex number.**

I. The complex belief function

Let Ω be a set of mutually exclusive and collective non-empty events, defined by

$$\Omega = \{E_1, E_2, \dots, E_i, \dots, E_N\}, \tag{1}$$

where Ω represents a frame of discernment.

The power set of Ω is denoted by 2^Ω, in which

$$2^\Omega = \{\emptyset, \{E_1\}, \{E_2\}, \dots, \{E_N\}, \{E_1, E_2\}, \dots, \{E_1, E_2, \dots, E_i\}, \dots, \Omega\}, \tag{2}$$

and $\emptyset$ is an empty set.

Definition 1: (Complex mass function)

A complex mass function $\mathbb{M}$ in the frame of discernment Ω is modeled as a complex number, which is represented as a mapping from 2^Ω to $\mathbb{C}$, defined by

$$\mathbb{M}: \quad 2^\Omega \to \mathbb{C}, \tag{3}$$

satisfying the following conditions,

$$\begin{aligned} \mathbb{M}(\emptyset) &= 0, \\ \mathbb{M}(A) &= \mathbf{m}(A)e^{i\theta(A)}, \quad A \in 2^\Omega \\ \sum_{A \in 2^\Omega} \mathbb{M}(A) &= 1, \end{aligned} \tag{4}$$

where $i = \sqrt{-1}$; $\mathbf{m}(A) \in [0, 1]$ representing the magnitude of the complex mass function $\mathbb{M}(A)$; $\theta(A) \in [-\pi, \pi]$ denoting a phase term.

In Eq. (4), $\mathbb{M}(A)$ can also expressed in the "rectangular" form or "Cartesian" form, denoted by

$$\mathbb{M}(A) = x + yi, \quad A \in 2^\Omega \tag{5}$$

with

$$\sqrt{x^2 + y^2} \in [0, 1]. \tag{6}$$

By using the Euler's relation, the magnitude and phase of the complex mass function $\mathbb{M}(A)$ can be expressed as

$$\mathbf{m}(A) = \sqrt{x^2 + y^2}, \text{ and } \theta(A) = \arctan(\frac{y}{x}), \tag{7}$$

where $x = \mathbf{m}(A)\cos(\theta(A))$ and $y = \mathbf{m}(A)\sin(\theta(A))$.

The square of the absolute value for $\mathbb{M}(A)$ is defined by

$$|\mathbb{M}(A)|^2 = \mathbb{M}(A)\bar{\mathbb{M}}(A) = x^2 + y^2, \tag{8}$$

where $\bar{\mathbb{M}}(A)$ is the complex conjugate of $\mathbb{M}(A)$, such that $\bar{\mathbb{M}}(A) = x - yi$.

These relationships can be then obtained as

$$\mathbf{m}(A) = |\mathbb{M}(A)|, \text{ and } \theta(A) = \angle\mathbb{M}(A), \tag{9}$$

where if $\mathbb{M}(A)$ is a real number (i.e., $y = 0$), then $\mathbf{m}(A) = |x|$.

The complex mass function $\mathbb{M}$ modeled as a complex number in the generalized Dempster–Shafer (GDS) evidence theory can also be called a complex basic belief assignment (CBBA).

If $|\mathbb{M}(A)|$ is greater than zero, where $A \in 2^\Omega$, A is called a focal element of the complex mass function. The value of $|\mathbb{M}(A)|$ represents how strongly the evidence supports the proposition A.

Definition 2: (Complex belief function)

A complex belief function can be defined by a mapping, $\mathbb{M}$: $2^\Omega \to \mathbb{C}$, called a complex basic belief assignment, satisfying the following axioms:

- $\mathbb{M}(\emptyset) = 0$,
- $\sum_{A \in 2^\Omega} \mathbb{M}(A) = 1$.

With regards to the complex basic belief assignment, the complex belief function in a proposition $A \in 2^\Omega$ can be defined by

$$\mathbb{B}el(A) = \sum_{B \subseteq A} \mathbb{M}(B), \tag{10}$$

Definition 3: (Complex plausibility function)

The complex plausibility function of proposition A, denoted as $\mathbb{P}l(A)$ is defined by a mapping from 2^Ω to $\mathbb{C}$

$$\mathbb{P}l(A) = 1 - \mathbb{B}el(\bar{A}) = \sum_{B \cap A \neq \emptyset} \mathbb{M}(B). \tag{11}$$

where $\bar{A}$ is the complement of A, such that $\bar{A} = \Omega - A$.

II. The pignistic probability derived from a complex belief function

Definition 4: (Complex pignistic probability transformation)

Let $\mathbb{M}$ be a complex basic belief assignment on the frame of discernment Ω and A be a proposition where $A \subseteq \Omega$, the complex pignistic probability transformation function is defined by

$$\mathbb{B}et(B) = \sum_{A \in 2^\Omega} \frac{|\mathbb{M}(A)|}{\sum_{C \in 2^\Omega} |\mathbb{M}(C)|} \frac{|B \cap A|}{|A|}, \tag{12}$$

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where $|A|$ represents the cardinality of A, $|B \cap A|$ represents the cardinality of intersection of $|B \cap A|$, and $|\mathbb{M}(A)|$ represents the absolute value of $\mathbb{M}(A)$.