The Possibility of Physical Waves as the Basis of Wave Mechanics

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Abstract: A new possibility is presented for the development of an alternative picture of wave mechanics, based on physical waves. In this approach, it is postulated that particles emit physical waves that play a role in the mediation of interactions with other particles. Doppler-shifted echoes of these postulated waves are shown to give a new explanation for Bragg scattering with the apparent wavelength ***h/p***, the de Broglie wavelength. The issue of conservation of mass-energy is discussed. Experimental tests of this hypothesis are proposed.

Keywords: quantum mechanics, relativistic, physical waves, rest mass

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1 Introduction

This paper presents a possibility for the development of an alternative picture of wave mechanics, based on the emission of physical waves by all particles with rest mass. These physical waves are presumed to play a role in the mediation of interactions with other particles. These waves will be called m_0*-waves* to distinguish them from the purely mathematical waves of the Schrödinger equation. It is postulated that the m_0-waves radiate away from every particle in all directions at speed $\boldsymbol{c}$, with frequency $\boldsymbol{\nu_0 = m_0c^2/2h}$ (for a free particle), in the rest frame of the particle; $\boldsymbol{m_0}$ being the rest mass of the particle, $\boldsymbol{c}$ the speed of light, and $\boldsymbol{h}$ the Planck constant. This postulated internal oscillation frequency of the particle differs by a factor of two from the internal oscillation frequency asserted by de Broglie [1].

In the rest frame of the particle, the wavelength of the postulated waves is thus

$$\lambda_0 = \frac{c}{\nu_0} = \frac{2h}{m_0 c} \tag{1}$$

which is twice the Compton wavelength of the particle. This factor of two will permit a new explanation of wave-like behavior with the apparent wavelength, ***h/p***, the de Broglie wavelength. The form of wave-like behavior considered in this paper is Bragg scattering; and in certain unusual cases of Bragg scattering of electrons, experimental tests can be specified that would either support the current proposal or falsify it.

Although the m_0-waves are described as radiating from the particle in all directions at speed $\boldsymbol{c}$, spherical symmetry for the emission of these waves in the particle's rest frame is not assumed, and no specific form

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of the waves other than periodicity is assumed. For example, there could be some sort of spiral symmetry associated with spin.

It is assumed that all interactions mediated by the postulated m_0-waves would obey conservation of mass-energy, linear momentum, and angular momentum, within the limits of the uncertainty principle; then there would be no net energy transfer to or from the m_0-waves. It is unknown whether any energy is associated with the m_0-waves themselves; certainly, no ordinary form of energy could be associated with them, because a continuous emission of any conventional form of energy would cause a gradual depletion in the particle's mass-energy, which is known not to occur.

This paper considers only particles that are essentially free, that is, particles that are free from the influence of any potential that is more than a negligible fraction of their rest-mass-energy.

2 Bragg Scattering

To begin with, the Bragg scattering of the m_0-waves emitted by a particle will be considered, and then the influence of the Bragg-scattered m_0-wave on the motion of the particle itself will be taken up. The scattered m_0-waves that return to the emitting particle could have particularly strong influence on the motion of the particle, because these reflected waves have frequencies close to the internal oscillation frequency of the particle, shifted only by the Doppler effect.

2.1 Normal Incidence (1-D Case)

Consider first the scattering of a particle's own m_0-waves from a set of crystal planes. The m_0-waves are assumed to scatter according to Huygens' Principle. The waves scattered by different points in any single plane of the crystal interfere constructively only if the scattering is at equal angles of incidence and reflection relative to that plane; and the scatterings from sequential planes interfere constructively only if the difference in path lengths from scattering by the different planes happens to be an integer number of wavelengths. Although the particle is pictured as radiating these m_0-waves in all directions, only echoes that return to the particle could affect its motion. For a particle moving in a direction normal to a set of crystal planes, one then needs to consider only m_0-waves that are reflected parallel to the axis of motion, i.e. normal to the crystal planes, because only these constructively interfering specular reflections will return to the emitting particle. See Figure 1. Let $\boldsymbol{\lambda_f}$ denote the wavelength of the Doppler-shifted m_0-waves that are emitted in the direction parallel to the particle's forward motion , and $\boldsymbol{\lambda_b}$ denote the corresponding wavelength for the backward-emitted waves, as seen in the frame of the crystal lattice. Consider the case in which the particle speed is such that the Doppler-shifted waves in either the forward and backward directions (or both) are resonant with the crystal spacing, $\boldsymbol{d}$. That is

$$2d = n_f \lambda_f, \tag{2}$$

and

$$2d = n_b \lambda_b, \tag{3}$$

where n_f and n_b are integers,.   Equations 2 and 3 are just the 1-D Bragg equations for the Doppler-shifted m_0-waves. At these Bragg resonances for the m_0-waves, the reflections from a large number of planes add constructively and could be expected to have a much larger effect on the particle than would be the case when the particle is not at a speed corresponding to one of these resonances.

For special values of ***d***, it is possible for the Bragg conditions (Eq.s 2 and 3) to be satisfied for both the forward and backward emitted waves. These special spacing values may come about through thermal motion of the lattice, as will be discussed in more detail later. It seems plausible that a maximal interaction of the particle with the lattice would occur under these double resonance conditions.

The wavelengths of the Doppler-shifted m_0-waves in the 1-D geometry are given by

$$\lambda_f = \lambda_0\gamma(1-\beta), \tag{4}$$

and

$$\lambda_b = \lambda_0\gamma(1+\beta) \tag{5}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, and v is the particle speed in the frame of the crystal lattice [2].

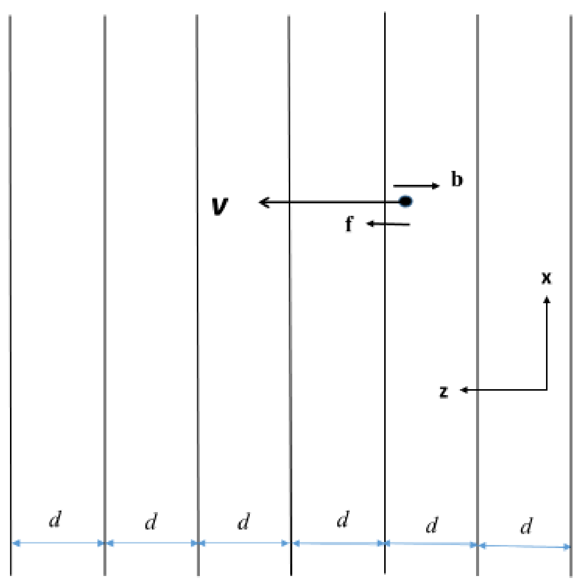

Figure 1. A particle moves at speed v through a lattice with regular plane spacing d at normal incidence to the crystal planes. The forward direction (f) and the backward direction (b) are indicated as parallell to the particle motion or in the opposite direction.

Substituting the expressions in Eq.s 4 and 5 into Eq.s 2 and 3, respectively, and with slight re-arrangement, one obtains

$$n_f = \frac{2d}{\lambda_0 \gamma (1-\beta)} \ , \tag{6}$$

and

$$n_b = \frac{2d}{\lambda_0 \gamma (1+\beta)} \ . \tag{7}$$

Subtracting Eq. 7 from Eq. 6 and simplifying the resulting expression, one obtains

$$2d = \frac{n\lambda_0}{2\beta\gamma} \ , \tag{8}$$

or

$$2d = n\frac{h}{\gamma m_0 v} \ , \tag{9}$$

or

$$2d = n\frac{h}{p} \ , \tag{10}$$

where $\boldsymbol{n = n_f - n_b}$ is also an integer, the quantity ($\boldsymbol{\gamma m_0 v}$) is the particle momentum ($\boldsymbol{p}$), and Eq. 10 is the 1-D Bragg relationship for the particle with the apparent wavelength, $\boldsymbol{h/p}$. So it has been seen that simultaneous satisfaction of the two Bragg relationships for the physical m_0-waves would lead to the Bragg relationship for the particle.

There generally will not be exact, simultaneous integer solutions of Eq.s 6 and 7 for any arbitrary choices of $\boldsymbol{d}$ and $\boldsymbol{\beta}$, but for heavy particles like neutrons, simultaneous integer solutions of Eq.s 6 and 7 may be found for spacing very close to any value of $\boldsymbol{d}$, and well within the compressive modes of lattice oscillations about $\boldsymbol{d}$. For lighter particles such as electrons, the coarser granularity of the solutions of these two equations may provide the possibility of experimental tests of the present hypotheses. More details of the simultaneous solutions of these equations and experimental tests will be discussed in later sections.

2.2 Oblique Incidence (3-D -Case)

The three dimensional case can be analyzed in just two dimensions if the $\boldsymbol{x}$ and $\boldsymbol{y}$ axes are chosen such that motion is entirely in the $\boldsymbol{x}$-$\boldsymbol{z}$ plane, as shown in Figure 2.

The simplified three-dimensional case (really just two dimensional) is treated by a coordinate transformation [2] as indicated in Figure 2. In the new laboratory frame, the particle is seen moving with speed $\boldsymbol{v}$ at angle $\boldsymbol{\theta}$ with respect to the $\boldsymbol{x}$ axis. One may consider an $\boldsymbol{x'}$-$\boldsymbol{z'}$ frame where the particle appears to move normal to the crystal planes, as in the one-dimensional case already analyzed. The origin of the $\boldsymbol{x'}$-$\boldsymbol{z'}$ frame moves at speed $\boldsymbol{v \cos \theta}$ in the $\boldsymbol{x}$ direction, parallel to the crystal planes. Eq.8 becomes

$$2d = \frac{n\lambda_0}{2\beta'\gamma'} \ , \tag{11}$$

where

$$\beta' = \frac{\beta sin\theta}{\sqrt{1-\beta^2 cos^2\theta}} \quad , \tag{12}$$

and

$$\gamma' = \frac{1}{\sqrt{1-(\beta')^2}} \qquad , \tag{13}$$

so that it may be shown that

$$\beta'\gamma' = \beta\gamma sin\theta. \tag{14}$$

Thus

$$2d = \frac{n\lambda_0}{2\beta\gamma sin\theta} \quad , \tag{15}$$

or

$$2dsin\theta = n\frac{h}{p} \quad ; \tag{16}$$

and the usual three dimensional form of the Bragg relation is seen to be obtained. This simplified 3-D case is just the Lorentz transformation of the 1-D case from a frame moving parallel to the crystal planes of interest.

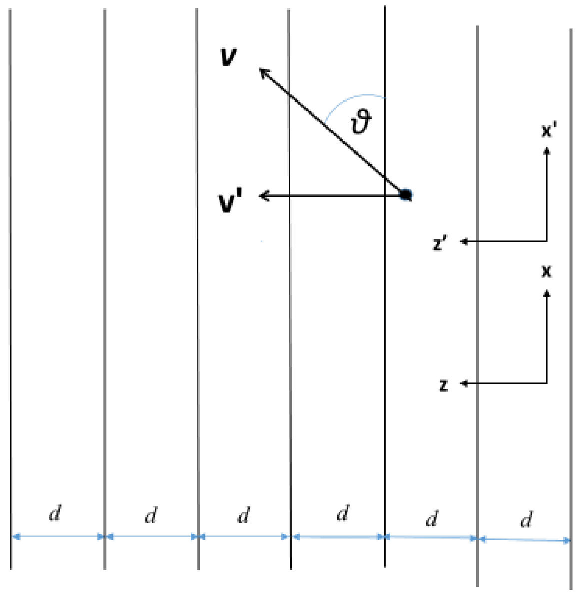

Figure 2. The particle moves with speed ***v*** at glancing angle ***θ***, with respect to the crystal planes. In the ***x***'-***z***' frame the motion of the particle is normal to the planes and has speed ***v'***. The origin of the ***x'-z'*** frame moves at speed ***v cos θ*** in the ***x*** direction. The plane spacing ***d*** is exactly the same in both frames.

3 Granularity and the Solutions of Equations 6 and 7

3.1 The 1-D Case

The 1-D Bragg condition (Equation 8) may be written as

$$2d = \frac{n\,\lambda_{Comp}}{\beta\gamma} \quad , \tag{17}$$

where $\lambda_{Comp} = \lambda_0/2$ is the Compton wavelength of the particle.

As previously mentioned, simultaneous solutions of Equations 6 and 7 are not possible for arbitrary values of ***d*** and ***β***. Equations 6 and 7 can be rewritten as

$$n_f = \frac{d}{\lambda_{Comp}}\sqrt{\frac{1+\beta}{1-\beta}} \quad , \tag{18}$$

and

$$n_b = \frac{d}{\lambda_{Comp}}\sqrt{\frac{1-\beta}{1+\beta}} \quad . \tag{19}$$

Simultaneous solution of these two equations in ***d*** and ***β*** gives

$$d = \lambda_{Comp}\sqrt{n_f n_b} \quad , \tag{20}$$

and

$$\beta = \frac{n_f - n_b}{n_f + n_b} \quad . \tag{21}$$

For first-order Bragg scattering, these become

$$d_I = \lambda_{Comp}\sqrt{n_f(n_f - 1)} \quad , \tag{22}$$

$$\beta_I = \frac{1}{2n_f - 1} \quad . \tag{23}$$

The subscript, Roman numeral ***I***, denotes first-order.

For a nominal crystal plane spacing ***d***, the nearest spacing value satisfying Eq. 22 can be found by inspecting the integers near the solution of Eq. 24:

$$\eta_{f,I} = \frac{1 + \sqrt{1 + 4\frac{d^2}{\lambda_{Comp}^2}}}{2} \quad , \tag{24}$$

where $\eta_{f,I}$ is a real number, usually not an integer. For nominal values of the crystal spacing ***d*** in the range 0.05 nm to 0.5 nm, $\eta_{f,I}$ values would range from about 21 to 207 for electrons, and about 37891 to 378906 for neutrons, based on λ_{Comp} data from CODATA 2010 [3]. Letting n_p denote some particular integer near $\eta_{f,I}$, then $(n_p + 1 , n_p)$ and $(n_p, n_p - 1)$ would be adjacent (n_f, n_b) pairs for first order Bragg scattering, and the spacing between these adjacent d_I values of Eq. 22 would become Δd_I as in Eq. 25 below.

$$\Delta d_I = \lambda_{Comp} n_p \left(\sqrt{1 + \frac{1}{n_p}} - \sqrt{1 - \frac{1}{n_p}}\right) = \lambda_{Comp}\left(1 + \left(\frac{1}{8}\right)\left(\frac{1}{n_p^2}\right) + \cdots\right) , \tag{25}$$

or about one Compton wavelength. In the case of electrons, this spacing can be several percent of the lattice spacing, a rather coarse granularity.

3.2 Experimental Test in the 1-D Case

For the postulated mechanism of Bragg scattering to occur, the lattice planes must either be spaced fortuitously at one of the simultaneously resonant values (Eq. 20) or they must sweep through the resonant spacings because of thermal oscillations. Since the lattice planes move much more slowly than the m_0-waves, momentary resonances could easily be established within the slower motions of the lattice, provided that the special $\boldsymbol{d}$ values were within the range of the thermal motions. In the case of electron scattering at low temperatures, the root-mean-squared (RMS) zero-point thermal motions of the planes [4] of some rigid lattices are slightly less than $\Delta \boldsymbol{d_I}$ (approximately the Compton wavelength). Therefore, for some materials (e.g. tungsten and osmium) at liquid nitrogen temperatures and for electron beams with very precisely defined values of $\boldsymbol{\beta}$, the special resonant spacing may be nearly out of the reach of thermal oscillations, and if the proposed explanation for Bragg scattering is true, then the predicted granularity in Bragg scattering may be observable. In general, for odd orders of Bragg scattering, it is easily seen numerically from Eq. 20 that resonant $\boldsymbol{d}$ values would be approximately half-(odd)integer multiples of $\boldsymbol{\lambda_{Comp}}$; while for even orders, the resonant $\boldsymbol{d}$ values would be approximately integer multiples of $\boldsymbol{\lambda_{Comp}}$. Since these spacings are interspersed, a change in the ratio of first order and second order scattering probabilities might be observable as the temperature of a tungsten crystal in reduced to the point that only the small zero-point motion occurs and fewer of the resonant spacings are accessible by thermal motion.

Although tests with osmium may be somewhat more sensitive than with tungsten, it should be noted that the high toxicity of oxides of osmium could require prohibitively elaborate safety measures.

It will be shown below that Equations 20 and 25 for 1-D Bragg scattering hold also in the 3-D case, and additional flexibility in observing the predicted granularity will be seen to pertain in the 3-D case.

3.3 The 3-D Case

In the 3-D case, Equations 6 and 7 apply to the $\boldsymbol{x' - z'}$ frame, and may be written with $\boldsymbol{\beta'}$ and $\boldsymbol{\gamma'}$ replacing $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. Then Equations 12, 13, and 14 may be employed to find expressions for $\boldsymbol{n_f}$ and $\boldsymbol{n_b}$ in terms of $\boldsymbol{d}$, $\boldsymbol{\beta}$, and $\boldsymbol{\theta}$.

$$n_f = \frac{d/\lambda_{Comp}}{\sqrt{1+\frac{\beta^2 sin^2\theta}{1-\beta^2}} - \frac{\beta sin\theta}{\sqrt{1-\beta^2}}} \quad , \tag{26}$$

and

$$n_b = \frac{d/\lambda_{Comp}}{\sqrt{1+\frac{\beta^2 sin^2\theta}{1-\beta^2}} + \frac{\beta sin\theta}{\sqrt{1-\beta^2}}} \quad . \tag{27}$$

Multiplying equations 26 and 27 together, one gets

$$n_f n_b = \frac{(d/\lambda_{Comp})^2}{\left[1+\frac{\beta^2 sin^2\theta}{1-\beta^2}\right] - \frac{\beta^2 sin^2\theta}{1-\beta^2}} \quad ,$$

or

$$d = \lambda_{Comp}\sqrt{n_f n_b} \quad , \qquad (28)$$

once again, as in the 1-D case (Eq. 20). Then the values of $\boldsymbol{d_I}$ and the granularity of the spacings are all exactly the same as in the 1-D case above.

Having $\boldsymbol{d}$ from Eq. 28 and recalling that $\boldsymbol{n = n_f - n_b}$, one may solve Eq. 15 for $\boldsymbol{\beta}$:

$$\beta = \frac{n_f - n_b}{\sqrt{4 n_f n_b sin^2\theta + (n_n - n_b)^2}} \quad , \qquad (29)$$

which reduces to Eq. 21 for $\boldsymbol{\theta} = \pi / 2$.

3.4 Experimental Tests in the 3-D Case

Returning to the possibility of an experimental test, it may be seen that for Bragg scattering of monoenergetic electrons from the 110 planes of a tungsten crystal, Eq. 29 would lead to a fine periodicity in fixed-energy rocking curves near the nominal Bragg angle, as shown in Table 1. Similarly, for Bragg scattering of electrons at a fixed angle, there would be a fine periodicity in scattering as a function of the electron's kinetic energy near the nominal Bragg energy, as shown in Table 2. The fine spacing of these Bragg scattering peaks is not predicted by conventional wave mechanics.

The values in Table 1 and Table 2 apply to scattering within the crystal. Refractive corrections may have to be applied when an electron enters or leaves the crystal at a vacuum boundary. These corrections would be less significant at higher electron energies.

3.5 Existing Experimental Data

So far, no reports of low-temperature data for electron scattering on tungsten have been found, but some room-temperature data have been reported. Stern and Gervais [5] have reported the Bethe "inner potential" for tungsten 110 to be 20 ± 1 eV. This finding should be useful to other experimenters in identifying the Bragg peaks. In a second paper Stern and Friedman [6] report examination of rocking curves and "pseudo-rocking curves" for tungsten 110. The observed laboratory voltages for the Bragg peaks in the pseudo-rocking curves in [Fig. 3, Ref. 6] are consistent with the 20 volt displacement from [5]; these are 7th, 8th, and 9th order Bragg peaks. Stern and Friedman report unexpected dips in these Bragg peaks that are somewhat suggestive of the fine spacing from Eq.29 as discussed above. They say that dynamical scattering theory has provided "no really complete explanation of these effects." However, the quantitative agreement of the spacing of the dips is not very good. Pairs of (***n_f***, ***nb***) values such as (97, 88) and (96, 87) for the 9th order case would give a fine spacing of 14.3 eV (at 15° from the normal), while the width of the dip in [Fig. 3, Ref. 6] is about 22 eV. The dips observed for 7th and 8th order peaks are similarly wider than Equation 29 would predict. Nevertheless, the dips observed by Stern and Friedman may have given a glimpse of the granularity predicted in this paper. Data from scattering at lower temperatures would provide a much more definitive test.

Table 1 Fine Angular Spacing for Electrons Scattering from Tungsten 110 Planes.

Fixed Kinetic Energy	θ	Δθ (average)	n_f , n_b
(Electron Volts)	(degrees)	(degrees)	
253	9.89		93, 92
253	10	0.11	92, 91
253	10.11		91, 90
54.4	21.75		93, 92
54.4	22	0.25	92, 91
54.4	22.26		91, 90
15.3	44.38		93, 92
15.3	45	0.63	92, 91
15.3	45.64		91, 90
9.29	63.70		93, 92
9.29	65	1.35	92, 91
9.29	66.39		91, 90
7.87	76.95		93, 92
7.87	80	3.87	92, 91
7.87	84.68		91, 90

Table 2 Fine Kinetic Energy Spacing for Electrons Scattering from Tungsten 110 Planes.

Fixed Scattering Angle, θ	Kinetic Energy	ΔK.E. (average)	n_f , n_b
(degrees)	(electron volts)	(electron volts)	
10	247.5		93, 92
10	253.0	5.53	92, 91
10	258.6		91, 90
22	53.2		93, 92
22	54.4	1.19	92, 91
22	55.6		91, 90
45	14.9		93, 92
45	15.3	0.33	92, 91
45	15.6		91, 90
65	9.1		93, 92
65	9.3	0.20	92, 91
65	9.5		91, 90
80	7.7		93, 92
80	7.9	0.17	92, 91
80	8.0		91, 90

4 Discussion

Some beginning steps have been proposed in the direction of a new theory of wave mechanics based on physical waves. In making a complete theory beyond the present small steps, very many additional factors would have to be understood, including the detailed structure of the m_0-waves and the mechanisms by which the various interactions are mediated, as well as the phenomena of particle creation and annihilation. If the proposed experimental tests verify the predicted departure from current quantum theory, then it would be reasonable to pursue a fuller theory of m_0-waves.

5 Conclusions

Beginning steps toward a possible new theory of wave mechanics have been proposed. General suggestions for experimental tests have been given, based on the prediction of a granularity in electron diffraction that is not predicted by current quantum theory.

6 References

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