Riemann Hypothesis Yielding to Minor Effort

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ABSTRACT

This is one other ‘trilinear’ instance featuring a set of three elegant yet elementary, one-line demonstrations which point to largely the same solution-patterns as obtained from a variety of angles. While this core does lend unequivocal support to the Riemann Hypothesis, the latter may have to be qualified with respect to the Euler-Riemann Equivalence (identity) as its sole grounds.

[1] In the name of Jesus Christ. Unto Weierstrass and Schelling, Assange, and whosoever emerging victorious as brethren rather than teachers, and for that matter as soul-and-spirit-mates within one’s Inner Southeast. Unto those transcending the ridiculous evils and preposterous vanity of the wicked yoke that’s about as long-standing as it is wearing out.

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Central Result: A Layered yet Simple Nucleus

The present paper will shun [ultimate] *prior* generalizations of the Riemann Hypothesis (RH henceforth) per se[3]. That said, it will be shown how RH amounts to but one [core] solution-branch, with some latent realizations allowing a better glimpse of the solution structure or Re-Im linkage. Suffice it to first list the set of [open-form] reductions with the closed-form solutions readily implied, followed by an exposition of the underlying rationales demonstrating how *posterior* solution-extensions may enrich as well as qualify RH.

The entire line of reasoning embarks on the Euler-Riemann Equivalence (ERE) or identity depicting the zeta as a sum *and* a product, to be defined over natural versus prime numbers respectively:

$$\sum_{n=1}^{\infty} n^{-s} = \prod_{prime}^{\infty} \frac{p^s}{p^s - 1} \equiv \vartheta(s)$$

With the summation or left-hand side series terms referred to somewhat loosely as [point] 'zeta density,' the present paper will demonstrate that the [open-form] solutions can be inferred as:

$$(1.1)\ \ n^{-s} = \frac{log^n \varphi(0)}{n!}, \qquad \varphi(0) = \begin{cases} 0^{\varphi} \\ \varphi * 0 \\ \varepsilon, \quad |\varepsilon| \epsilon (0..1) \end{cases}$$

$$(1.2)\ \ n^{-s} = \varphi(0)^n, \qquad \varphi(0) = \begin{cases} 0^{\varphi} \\ \varphi log 0 \\ \varepsilon, \quad |\varepsilon| \epsilon (0..1) \\ n!^{-\frac{1}{n}} * 0^{\varphi} \end{cases}$$

$$(2)\ \ 1^{\pm s} = \varphi(0), \qquad \varphi(0) = \begin{cases} 0^{\varphi} \\ 0^{-1} \\ 1^{\pm T} \sim 0^{\pm \frac{1}{T} * T} \end{cases}$$

$$(3)\ s = \begin{cases} k_1 + k_2 * T \\ iT \end{cases}$$

It is straightforward to see that the general solution takes on the following form:

$$(A)\ \ s = -n * \frac{log\varphi(0)}{logn}$$

[3] Subsequent papers will feature the zeta as one special case of the CES or Lame structures subject to a family of rho-calculus, which in turn are inherently entwined with the previously exposited formalizations of *orduality* (Shevenyonov, 2016a; 2016b). All of these, however, can safely be assumed away for now without loss of continuity.

This obviously pertains to (1.2) as a general structure subsuming, *inter alia*, (1.1) with the phi assuming a logarithmic stretching and (2) with n=1. It also agrees with (3), perhaps seen as one other corner case[4]. This could in fact be an early hint at how the *s* structure may be defined across layers (far beyond *s=s(n)* as if to violate the Re(s)=1/2 conjecture positing invariance of sorts) as well as complex-*solution branches*[5]. On the other hand, the multiple overlaps across the solution concepts can be approached by first trying out the third solution, e.g. $s=T^6$, directly on the ERE. The resultant zeta will be finite, with some of the realizations likely amounting to zero. Please observe that (3) can otherwise be discerned as a particular restriction of (2) (or the third materialization of the phi), which in turn is a special branch of (1.2) under n=1. At the same time, (1.2) has (1.1) as one of its branches.

One way of showing the prior validity of (1.1) with an eye on a zero-valued zeta would be to directly substitute it into ERE, e.g. sum both sides *ad infinitum*[7], thus embarking on the *exponent* series.

$$\vartheta(s) = \sum_{n=1}^{\infty} \frac{log^n 0}{n!} = e^{log0} = 0, \ \ QED$$

Incidentally, the validity or feasibility of (1.2) can likewise be tested by tapping into a *power*-series summation (as opposed to exponentiation), with phi being a unity stretching (fixed point implied):

$$\vartheta(s) = \sum_{n=1}^{\infty} \varphi^n(0) = \frac{\varphi^{\infty}(0)-1}{\varphi(0)-1} - 1 = 0, \ \ \varphi(x) = x, \qquad x \equiv 0, \qquad QED$$

Subtraction of a unity value is to act on the missing n=0 in the original zeta series and the respective [zero] power in the summation domain.

At this point, two of the aforementioned solution-structures have been tested while showing clear-cut resemblance to the rest. These will now be obtained more rigorously from the ERE by deploying rather distinct and non-overlapping methodologies. In the meantime, please

[4] Some *infinity*-reaching solutions may go undetected numerically or experimentally, as they will be shown to be less than distinguishable technically from finite ones, RH or Re=1/2 in particular.

[5] These possibilities could tentatively be rethought along the *levels-of-variableness* lines as in Shevenyonov (2016c). However, this strands applicability could best be appreciated in the forthcoming paper's setup deploying an altogether different formal perspective to be tested on a variety of hypotheses.

[6] By which number I ofttimes mean a large value potentially tending to infinity

[7] While complex sets can *not* be *ordered* routinely—with the respective series terms seen as implied elements of the [open-form] solution set on top of making up point densities,--still it is presumed that solution-*branches* can uniquely be *compared* as belonging to respective classes. This assumption will be drawn upon as part of arriving at (1.2) via Taylor series expansion later in text. More broadly, this appears right up my *orduality* alley with some, otherwise ill-defined, objects being comparable or related in the more general or complete as well as simple sense of the term. Again, this latter perspective can be set aside as one not underpinning the exposition for most practical purposes.

note it remains to be shown later on that, whilst *Re(s)=1/2* does capture a dominant solution invariably, some of the hidden and possibly missing options may have strong bearing on it. In other words, any of these (RH included) could be seen as *sufficient* candidacy yet far from necessary (or exclusively as well as exhaustively nontrivial). Still, one has good reasons to see that (1.2), in being representative of the rest, captures the broad branch of solutions (including RH) meeting all of the above cut-offs.

Deliberation Cost: A Self-Rewarding Pathway

Branch (1) will be retrieved as an *exponent series*, whereas Branch (2) as a solution to a *functional equation*—both building on the right-hand side of the ERE.

The big idea behind the former possibility pertains to representing the infinite *product* as an infinite *sum*, which would in turn be juxtaposed against the original left-hand side sum of ERE:

$$\Sigma = \Pi \equiv \Sigma'$$

$$\exists X_n : \Pi \equiv \prod_{prime}^{\infty} (1-p^{-s})^{-1} = \prod_{n=1}^{\infty} e^{X_n} = e^{\sum X_n} = \sum_{n=1}^{\infty} \frac{(\sum X_n)^n}{n!}$$

$$\sum X_n = -\sum \log(1-p^{-s}) \equiv log\, \Pi$$

Now, with the latter product being fixed (tantamount to the right-hand side representation of the zeta as in the ERE), it follows that,

$$\vartheta(s) = \Pi = e^{log\Pi} = \sum \frac{log^n \Pi}{n!} = \sum n^{-s} \equiv 0$$

By again matching the comparable branches (or the respective series terms), (1.1) readily obtains while implying, among other things,

$$(n!\, n^{-s})^{\frac{1}{n}} = log 0$$

The above may either hold as *n*-invariance or otherwise with respect to particular *n* values (or patterns). The above structure could first be approached qualitatively, by suggesting that

$$(n!)^{\frac{1}{n}} < n = (n^n)^{\frac{1}{n}} < T, \qquad n^s \sim (-\frac{1}{T})^n \sim 0^n$$

The latter resembles (1.2) and (2) alike. Interestingly enough, the *n*th root of the *n*-factorial would prove *finite* even under *n* being arbitrarily large (tending to *T,* or infinity), which is why it has no countervailing impact on the magnitude of the solution. Technically, this factor can be assessed at[8],

$$(T!)^{\frac{1}{T}} < T = (T^T)^{\frac{1}{T}}$$

$$\exists X: X^{-n} \equiv \frac{1}{n!} \leftrightarrow \sum_{n=1}^{\infty}(\frac{1}{X})^n = \sum_{n=1}^{\infty}\frac{1}{n!} \leftrightarrow \frac{X^{-\infty}-1}{X^{-1}-1} - 1 = e \leftrightarrow X \equiv (n!)^{\frac{1}{n}} = \frac{e}{e-1}$$

One may decide at leisure whether this finite value is *n*-invariant or [quasi] asymptotic, even as it has nothing to do with implicit-functional (open-form) solutions or conventional asymptotic approximations per se. Somehow it could be upheld tentatively that, whatever the nature of the [finite] X factor,

$$(n!\,n^{-s})^{\frac{1}{n}} \sim \begin{cases} \varphi^{-1}(0) \\ log\varphi(0) \end{cases}$$

The two alternate materializations would seem to converge around phi being unitary (i.e. fixed point implied around zero) while agreeing with (1.1) and (1.2) alike.

Now, in order to fully capture the closed-form solution, consider a Stirling equivalence holding for *n*=N large:

$$N! \sim \sqrt{2\pi N} * (\frac{N}{e})^N, N \to T$$

$$log\vartheta(s) = (N!\,N^{-s})^{\frac{1}{N}} \sim (2\pi N)^{\frac{1}{2N}} * \frac{N}{e} * N^{-\frac{s}{N}} \equiv log0$$

Now consider the following plausible assumptions as per N large and *m* odd:

$$(2\pi)^{\frac{1}{2N}} \sim 1, \qquad -1 = e^{i\pi m}, \;\; N \sim T \sim log0$$

It then follows that,

$$N^{\frac{\frac{1}{2}-s}{N}} \sim e^{1+i\pi m} \leftrightarrow s \sim \frac{\log[\sqrt{N} * e^{-(1+i\pi m)}]}{\log N} = \frac{1}{2} - N(1+i\pi m)$$

It should therefore come as no surprise that the Re=1/2 core may have an infinite term added on. On second thought, one way around the issue could be to make use of the [quasi] asymptotic facility by referring to the exponent term in the Stirling denominator as not much different from unity:

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[8] The careful reader may, independently, long have come to appreciate that, while some of the techniques being deployed would appear to be [quasi] asymptotic, the zeta's nature is far from it—the same going for its *s* zeros which are maintained throughout as well as computed in precise and explicit terms.

$$e=(1+\frac{1}{N})^N \leftrightarrow \frac{N}{e}\sim(\frac{N^N}{(1+\frac{1}{T})^{N^2}})^{1/N}\sim\frac{N}{(1+0)^N}\sim N$$

At this rate, the effective solution can be put as,

$$s\sim\frac{1}{2}+it, \qquad t\equiv-\pi mT\sim-\infty$$

While this sheds some extra light on the *substantive structure* of the Im(s) in suggesting it is [infinitely] odd about half of the time, this might imply that RH features but a rather *special* (albeit highly concentrated) *sub*set of the broader effective feasibility domain. Moreover, the critical band may prove relevant with *s* taking on values as diverse as 0 or 1 in addition to *trivial* zeros, by embarking on the following trick[9]:

$$2i\pi k\sim 0\sim\frac{1}{T} \leftrightarrow s=\frac{1}{2}-i(\pi mT)=\frac{1}{2}*\left[1-\frac{1}{T}*m*T\right]=\begin{cases}\frac{1}{2}-0*\infty*\frac{m}{2}\\ \frac{1}{2}-\frac{m}{2}\end{cases}$$

Incidentally, the residual will likely prove finite (all the time under *m* sub infinity), yet will hardly ever make ½ sharp because of *m* [absolute value] being odd and hence non-zero.

Now, keeping a similar stance in mind, attempt a case of (1.1.1) or (1.2.2),

$$\varphi(0)=0^{\varphi} \leftrightarrow log\varphi(0)=\varphi log0$$

$$s=\frac{log\frac{n!}{\varphi^n}-nloglog0}{logn}$$

$$\forall k\in R: log0=\log(2i\pi k)=\log(2\pi k)+logi, \qquad logi=loge^{i(\frac{\pi}{2}\pm 2\pi k)}=i(\frac{\pi}{2}\pm 2\pi k)$$

$$log0=\log(2\pi k)*e^{iATAN}=\log(2\pi k)*[1+iTAN], \qquad TAN\equiv\frac{(\frac{\pi}{2}\pm 2\pi k)}{\log(2\pi k)}$$

$$loglog0=loglog(2\pi k)+iATAN$$

By embarking on Stirling's asymptotic one more time, one obtains that, for N large:

$$Re(s)=\frac{logN!}{logN}-\frac{N}{logN}*log\varphi-\frac{loglog(2\pi k)}{logN}\sim\frac{\frac{1}{2}*\log(2\pi N)+N*(logN-1)}{logN}-\Delta$$

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[9] This may not hold *outside* exponentiation, though, where *s* does belong. The forthcoming paper dwells on a broader, indeed generalized perspective for distinguishing as well as reconciling across such 'levels of variableness' setups.

$$\Delta \equiv \frac{N}{logN} * log\varphi + \frac{loglog(2\pi k)}{logN}$$

Under phi distributed as or comparable to N, the Re(s) part collapses to:

$$Re(s) \sim \frac{1}{2} * \left(1 + \frac{log2\pi}{logN}\right) + N - N - \frac{loglog(2\pi k)}{logN} = \frac{1}{2}, QED$$

The imaginary part could be gauged at anywhere near,

$$Im(s) \equiv t = -\frac{N}{logN} * ATAN\left(\frac{\frac{\pi}{2} \pm 2\pi k}{log2\pi k}\right)$$

For instance, with *k* hitting zero, the *t* imaginary part could be structured as follows:

$$t = t_{k=0,\ N=T} - \frac{T}{logT} * \pi l \ \forall l \in \boldsymbol{N}$$

Not only does it follow the *prime-number distribution* frequency (as one natural outcome of the ERE design), it does so as per each particular (large enough) value of *n*. And, for *k* very large, the PND pattern appears twice:

$$t = t_{k=T} - \pi(T) * ATAN(\pi(T))$$

Put simply, so much as just a small subset of potential solutions for *s* (which is what RH apparently amounts to) still features rich structural implications and linkages.

We now are in a position to turn to *Branch 2* for some extra insights along the lines of discerning the *s* patterns. The righthand side of the ERE can be rethought as a functional or operator equation as follows:

$$\vartheta(s) = \frac{\prod p^s}{\prod p^s - 1} \equiv \frac{\varphi(t)}{\varphi(t-1)} \leftrightarrow \varphi(t) = \vartheta(s)\varphi(t-1)$$

Assuming fixed point at unity, one readily infers:

$$\varphi(1) = 1 = p_1, \qquad \varphi(t) = \vartheta^t(s) * \varphi(1) = \vartheta(s)^{p_\infty^s - 1} \equiv \prod_{prime}^{\infty} p^s$$

If one now ventures an *[interim] generalization* of the zeta as aimed at capturing partial products for series running through M, it may follow that:

$$\vartheta_M(s) \equiv \prod_{prime}^{M<\infty} (1-p^{-s})^{-1}, \qquad \varphi_M(t) \equiv \prod_{prime}^{M<\infty} p^s = \vartheta_M{}^{p_M^s - 1}$$

In particular, for M=1, a structure is cleared appearing as,

$$1^s = (1^{-s})^{1^s - 1} \;\leftrightarrow\; 1^s = \begin{cases} 0^{\varphi} \\ \pm\infty \sim \pm T = \begin{cases} T \\ log0 \end{cases} \end{cases}$$

The above solution closely follows (2) as well as (1.1) and (1.2) at n=1. Alternatively, an explicit or closed-form solution would suggest:

$$s = \varphi \frac{\log(2i\pi)}{2i\pi} \sim \varphi * \frac{log0}{0}$$

While this may resemble a PND counterpart for complex numbers (or possibly around zero), this moreover appears to follow the general (A) solution structure. For *n* large, (A) could be rewritten as,

$$s \sim \frac{log0^{\varphi}}{\frac{1}{N} * \log(\frac{1}{N})} \sim \varphi * \frac{log0}{0}$$

In any event, the above structure (based on a partial or indeed singular primes product), can further be reduced to,

$$s = \frac{log2\pi + i(\frac{\pi}{2} \pm 2\pi k)}{2i\pi} = \varphi * [\left(\frac{1}{4} \pm k\right) - i\frac{log2\pi}{2\pi}]$$

While phi=2 does yield the RH result around k=0, this only really features a partial solution, with the *Im=t* structure still remaining of interest in just how it follows the PND pattern as previously spotted[10].

Rehashing on the RH Solution Tapestry

To draw a tentative bottomline, one should be able to appreciate a rich and well-structured variety of RH solutions which all—much to one's surprise!—point to largely the same patterns, with the Re=1/2 suggesting but a special yet dominant case. Incidentally, the overlap takes far wider swathes than that, all of the solutions featuring remarkable simplicity at a high level of completion[11]. In other words, not only do solution-branches prove comparable *across* themselves as shown throughout, they reveal recurring patterns *within* them as well. To illustrate this,

[10] Needless to say, the levels of variableness are: *k* and phi (in-branch alone).
[11] Which would seem a characteristically *ordual* outcome in its own right.

consider a bottom-up comparison [decision-procedure], starting with (3). Can the two solutions possibly be reconciled beyond assuming k2=*i* and k1=0? Suppose, in contrast, k2=0 for simplicity's sake. Now embark on the *log0* trick to see how the line may prove fuzzy between an all-imaginary versus an all-real solution:

$$s = k_1 = \frac{1}{2} \sim \frac{i\pi k}{2i\pi k} = i\pi k * \frac{1}{0} = i\pi k T \sim \begin{cases} \frac{i\pi 1}{T} * T = i\pi, & k = 0 \\ \pm i * \infty, & k \neq 0 \end{cases}$$

In fact, this is how RH could either disguise some infinity-reaching, purely imaginary solutions or, by contrast, end up appearing as one of these going undetected experimentally (observationally).

The rest of the in-branch bridges appear even more straightforward than that. For instance, per (2), 2.2 and 2.3 feature restrictions of phi to +1 or -1 values. For that matter, as regards 2.2 and 2.1:

$$n!^{-\frac{1}{n}} \sim \varepsilon, \qquad \varepsilon^N \sim 0, \qquad 0^{-1} \sim log0$$

Not least, whilst it has been deemed as outside this paper's scope (let alone a daring and risky enterprise), deployment of an [interim] generalization of zeta spanning a sub-infinity interval has paid off just handsomely, in that the implied solution structure fits squarely into the grand pattern as well as its inter- and intra-branch 'levels-of-narrowness.'

References

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