Algorithm Capable of Proving Goldbach's Conjecture

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1. Abstract

I created an algorithm capable of proving Goldbach's Conjecture. This is not a claim to have proven the conjecture. The algorithm and all work contained in this document is original, so no outside sources have been used. This paper explains the algorithm then applies the algorithm with examples. The final section of the paper contains information to accompany my thoughts on why I believe Goldbach's Conjecture can be proven with the use of my algorithm.

Keywords— Goldbach; Algorithm; Number Theory; Mathematics

2. Introduction

Goldbach's Conjecture states that all even integers greater than 2 are the sum of two prime integers (hereafter, referred to as "prime" or "primes"). This is one of the oldest unproven conjectures because, although even integers have been tested to large values, it takes one example of an even integer that is not the sum of two primes to prove the conjecture false.

3. Algorithm Description

The purpose of the algorithm is to eliminate all even integers that are not the sum of one particular prime plus any other prime.

4. Definitions

Algorithm prime: The particular prime to which the algorithm is being applied.

Eliminated: Not the sum of the algorithm prime plus any other prime.

Strand: Having a starting point from which numbers are systematically eliminated. Strands are exclusive to the algorithm prime.

Starts with: The starting point of each strand. Do not eliminate this point.

Skips by: The increment by which even integers are eliminated. Strand 1 always skips by 6.

Significance: The first even integer that is not eliminated after applying previous strands. These points can be found by using:

$$8 * [NumberofCurrentStrand] + [PreviousSignificance] = [CurrentSignificance] \tag{1}$$

The "previous significance" for the first significance point is the (algorithm prime +1).
The number of significance points equals the number of strands needed to eliminate all sums that are not the sum of the algorithm prime plus any other prime. When applying the algorithm, one strand at a time, some even integers are eliminated more than once. Significance points can act as a shortcut for the algorithm (see "Note 2").

Note 1: 2 cannot be an algorithm prime; besides 2+2, 2+ any other prime has an odd sum and the focus of this work is on even sums.

Note 2: To use significance points as a shortcut, apply strand 1 and then simply eliminate even integers from each strand's significance point onward.

5. Algorithm

Strand 1 starts with an integer (determined by adding 3 to the chosen algorithm prime) and eliminates the even integers that skip by 6 from the starting point. The strands thereafter are the previous "starts with" +2 and the previous "skips by" +4.

6. Example

Our example desired maximum integer is 53 and the example algorithm prime is 3. First determine how many strands are needed:
Significance point 1: 8*1 (because this is the first significance point) + (algorithm prime +1)
In the example I chose 3 as the algorithm prime
The second significance point onward is 8* number of significance + previous significance point= new significance.

Significance point 1: (8*1) + (3+1) = 12
Now to find the second significance point, 12 is the "previous significant point".

Significance point 2: (8*2) + 12 = 28
Now to find the third significance point, 28 is the "previous significant point".

Significance point 3: (8*3) + 28 = 52
To ensure we do not need one additional strand to reach our desired max integer (53), one can see where the 4th strand would become significant:
To find the fourth significance point, 52 is the "previous significant point".

Significance point 4: (8*4) + 52 = 84
This means we only need three strands of the algorithm to find all the even integers that are not the sum of 3 + another prime between 0 and 53.
The first strand starts with 3+ the algorithm prime (3) and skips by 6. So starting with 6, but not including the starting point, we eliminate 12, 18, 24, 30, 36, 42, 48. Strand 2 starts with +2 and skips by +4 more than the last: starting on 8, skip by every 10th number, eliminating 18, 28, 38, 48. The third strand starts on 10 and skips by 14, eliminating 24, 38, 52. Some even integers are eliminated more than once, so look to "Note 2". The remaining even integers are the sums of the algorithm prime (3) + one other prime.

7. Discussion

As previously mentioned: for this example we chose the number 3 to be the algorithm prime. This means that after eliminating all even sums corresponding to the strands of the algorithm, the remaining even integers are the sums of 3+ one other prime. This does not account for the sums of 5+ any other prime or 7+ any other prime, etc.
To find the sums of alternate prime + prime combinations, choose a different algorithm prime. The remaining even integers will always be the sum of the algorithm prime plus other primes.
All evens ≥ 12 are the sum of prime + composite.
The largest number of consecutive primes is 3: 3,5,7. The largest gap in composites is 2 (where the twin primes occur). Using 3,5,7 alone, each composite can make three even sums per 1 prime + composite combination. When primes are added over the gap, there will be no gap in consecutive even sums.
There is one composite before the gap and one composite after. Over the course of 4 odd numbers with the gap on numbers 2 and 3, there are 6 consecutive sums.
Example: list of odd numbers with a twin prime gap: 15,17,19,21.
3+15=18, 5+15= 20, 7+15= 22
3+21= 24, 5+21= 26, 7+21= 28.
There is no break in the even sum between 7+15 and 3+21 because the number of consecutive primes is larger than the number in gap. This will always be true because every third odd number greater than 3 is divisible by 3. The gap will never exceed 2 and every composite starts with being added to 3,5, and 7 to account for all possible

prime+ composite combinations. With no gap in even sums of prime + composite addends *geq* 12 , all evens *geq* 12 are the sum of prime + composite.
Moving on to why this algorithm may be of use for proving Goldbach's Conjecture: In nature, this algorithm is realistically applied to all odd integers. When this is the case, we end up with something that looks like the figure below; where black represents prime + prime and purple is prime + composite and composite + composite.

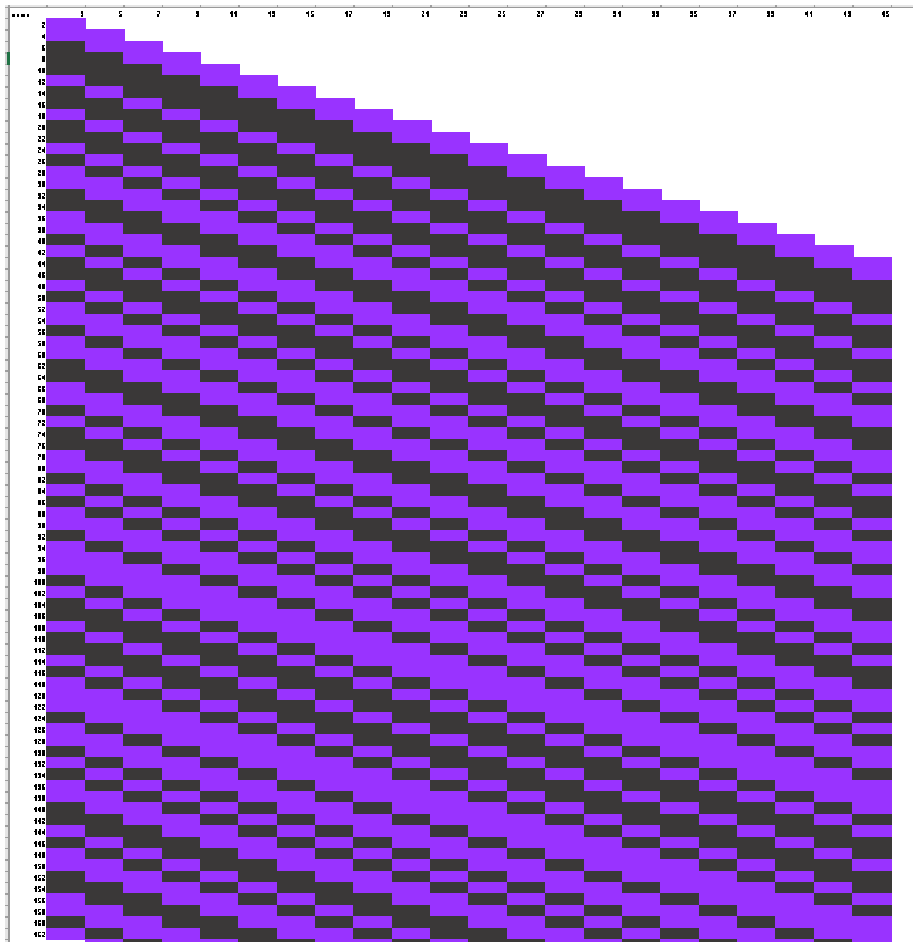

Figure 1: This is the algorithm applied to all odd integers

When the algorithm is applied to a prime integer, the algorithm eliminates prime + composite sums; the remaining even integers are the sum of prime + prime.
When the algorithm is applied to a composite integer, the algorithm eliminates composite + composite and the remaining integers are prime + composite.
The proof lies in being able to show that prime + prime and prime + composite follow the same "pattern". This is because it is easy to show that all evens are the sum of prime + composite. If prime + prime follows the same pattern, then naturally all even integers will also be the sum of prime + prime.
Considering the algorithm eliminates integers in a predictable manner, the the integers that remain standing are equally predictable.
Example: Every 6th number from any given starting point is eliminated. Say 'S' is all integers, 'M' is every 6th number, then the remaining integers will be 'S without M'.
The following image illustrates prime + prime and prime + composite following the same pattern. The difference between image 1 and image 2 is that all addend combinations are present in image 1, while image 2 has removed composite + composite.

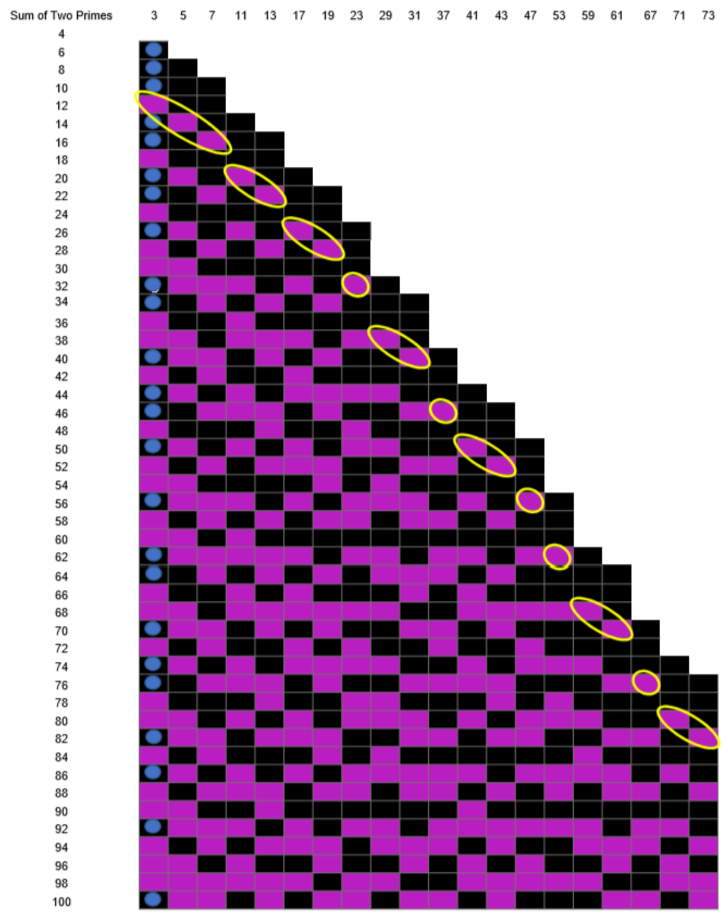

Figure 2: Eliminating Composite + Composite addends, we can see that Prime + Prime (black) and Prime + Composite (purple) follow the same pattern.

References/Bibliography

N/A- this is all original work.

Data Availability

N/A- Everything is provided in this paper. To figure out the algorithm, I did trial and error scratch work with pen and paper. From there I noticed patterns and made notes of them, which I put into this document.

Supplimentary Materials

N/A

Contact

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Conflict of Interest

N/A