Quantization of Electromagnetic field in Rindler Space-time

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ABSTRACT

The article treats quantization of electromagnetic field that is defined in Rindler space-time. Likely the electromagnetic field, the potential did quantizated in inertial frame, the electromagnetic field, the potential can quantizate by the transformation of electromagnetic field or the transformation of the potential in the accelerated frame. We treats Lorentz gauge condition in quantization of electromagnetic potential..

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1. Introduction

The article treats quantization of electromagnetic field that is defined in Rindler space-time.

Rindler coordinate transformations are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0}{c}\xi^0), \quad x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \tag{1}$$

At first, in inertial frame, the quantization of electromagnetic potential is [2]

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_\mu^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

In this time, $\left|\vec{k}\right| = \frac{\omega}{c}$, $\vec{k} = (k_1, k_2, k_3)$, $\varepsilon^{(\lambda)} \cdot \varepsilon^{(\lambda')} = g^{\lambda\lambda'}$

$$i\vec{k}\cdot\vec{x} - i\omega t = ik_1 x + ik_2 y + ik_3 z - i\omega t$$

$$= ik_1[(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}] + ik_2\xi^2 + ik_3\xi^3 - i\frac{\omega}{c}[(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})] \tag{2}$$

According to [1], the quantization of Electromagnetic potential is in Rindler space-time,

$$\phi_\xi = \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}[\phi\cosh(\frac{a_0\xi^0}{c}) - A_x\sinh(\frac{a_0\xi^0}{c})]$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_{\xi^0}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\varepsilon_{\xi^0}^{(\lambda)}(k) = \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}[\varepsilon_0^{(\lambda)}(k)\cosh(\frac{a_0\xi^0}{c}) - \varepsilon_1^{(\lambda)}(k)\sinh(\frac{a_0\xi^0}{c})] \tag{3}$$

Hence, the quantization of Electromagnetic potential in X-axis

$$A_{\xi^1} = -\phi\sinh(\frac{a_0\xi^0}{c}) + A_x\cosh(\frac{a_0\xi^0}{c})$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_{\xi^1}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\varepsilon_{\xi^1}^{(\lambda)}(k) = -\varepsilon_0^{(\lambda)}(k)\sinh(\frac{a_0\xi^0}{c}) + \varepsilon_1^{(\lambda)}(k)\cosh(\frac{a_0\xi^0}{c}) \quad (4)$$

So, the quantization of Electromagnetic potential in Y-axis

$$A_{\xi^2} = A_y$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_2^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_{\xi^2}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\varepsilon_{\xi^2}^{(\lambda)}(k) = \varepsilon_2^{(\lambda)}(k) \quad (5)$$

Therefore, the quantization of Electromagnetic potential in Z-axis

$$A_{\xi^3} = A_z$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_3^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \varepsilon_{\xi^3}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) + a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\varepsilon_{\xi^3}^{(\lambda)}(k) = \varepsilon_3^{(\lambda)}(k) \quad (6)$$

Lorentz gauge condition is in inertial frame and accelerated frame in quantization of electromagnetic potential[1],

$$0 = \frac{1}{c}\frac{\partial\phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}$$

$$\varepsilon_0^{(\lambda)}(k)i\frac{\omega}{c} - ik_1\varepsilon_1^{(\lambda)}(k) - ik_2\varepsilon_2^{(\lambda)}(k) - ik_3\varepsilon_3^{(\lambda)}(k) = 0 \quad (7)$$

2. Quantization of electromagnetic field in inertial frame

Second, in inertial frame, the quantization of electromagnetic field is

$$E_x = -\frac{\partial\phi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}$$

$$=\int\frac{d^3k}{(2\pi)^3 2k_0}\sum_{\lambda=0}^{3}\bar{\varepsilon}_1^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t)-a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}_1^{(\lambda)}(k)=ik_1\varepsilon_0^{(\lambda)}(k)-\frac{i}{c}\,\omega\varepsilon_1^{(\lambda)}(k) \tag{8}$$

Hence, the quantization of magnetic field is in X-axis

$$B_x=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}$$

$$=\int\frac{d^3k}{(2\pi)^3 2k_0}\sum_{\lambda=0}^{3}\bar{\varepsilon}'^{(\lambda)}_1(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t)-a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}'^{(\lambda)}_1(k)=-ik_2\varepsilon_3^{(\lambda)}(k)+ik_3\varepsilon_2^{(\lambda)}(k) \tag{9}$$

So, the quantization of electric field is in Y-axis

$$E_y=-\frac{\partial\phi}{\partial y}-\frac{1}{c}\frac{\partial A_y}{\partial t}$$

$$=\int\frac{d^3k}{(2\pi)^3 2k_0}\sum_{\lambda=0}^{3}\bar{\varepsilon}_2^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t)-a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}_2^{(\lambda)}(k)=ik_2\varepsilon_0^{(\lambda)}(k)-\frac{i}{c}\,\omega\varepsilon_2^{(\lambda)}(k) \tag{10}$$

Therefore, the quantization of magnetic field is in Y-axis

$$B_y=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}$$

$$=\int\frac{d^3k}{(2\pi)^3 2k_0}\sum_{\lambda=0}^{3}\bar{\varepsilon}'^{(\lambda)}_2(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t)-a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}'^{(\lambda)}_2(k)=-ik_3\varepsilon_1^{(\lambda)}(k)+ik_1\varepsilon_3^{(\lambda)}(k) \tag{11}$$

Hence, the quantization of electric field is in Z-axis

$$E_z=-\frac{\partial\phi}{\partial z}-\frac{1}{c}\frac{\partial A_z}{\partial t}$$

$$=\int\frac{d^3k}{(2\pi)^3 2k_0}\sum_{\lambda=0}^{3}\bar{\varepsilon}_3^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t)-a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}_3^{(\lambda)}(k) = ik_3\varepsilon_0^{(\lambda)}(k) - \frac{i}{c}\omega\varepsilon_3^{(\lambda)}(k) \quad (12)$$

Therefore, the quantization of magnetic field is in Z-axis

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}'^{(\lambda)}_3(k)[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\bar{\varepsilon}'^{(\lambda)}_3(k) = -ik_1\varepsilon_2^{(\lambda)}(k) + ik_2\varepsilon_1^{(\lambda)}(k) \quad (13)$$

3. Quantization of electromagnetic field in Rindler space-time

Third, in the accelerated frame, the quantization of electromagnetic field is[1]

$$E_{\xi^1} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{(1+\frac{a_0\xi^1}{c^2})^2\phi_\xi\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial A_{\xi^1}}{\partial\xi^0} = E_x$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}^{(\lambda)}_{\xi^1}(k)[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\bar{\varepsilon}^{(\lambda)}_{\xi^1}(k) = \bar{\varepsilon}^{(\lambda)}_1(k) = ik_1\varepsilon_0^{(\lambda)}(k) - \frac{i}{c}\omega\varepsilon_1^{(\lambda)}(k) \quad (14)$$

Hence, the quantization of magnetic field is in X-axis in the accelerated frame,

$$B_{\xi^1} = \frac{\partial A_{\xi^3}}{\partial\xi^2} - \frac{\partial A_{\xi^2}}{\partial\xi^3} = B_x$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}'^{(\lambda)}_{\xi^1}(k)[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)]$$

$$\bar{\varepsilon}'^{(\lambda)}_{\xi^1}(k) = \bar{\varepsilon}'^{(\lambda)}_1(k) = -ik_2\varepsilon_3^{(\lambda)}(k) + ik_3\varepsilon_2^{(\lambda)}(k) \quad (15)$$

So, the quantization of electric field is in Y-axis in the accelerated frame.

$$E_{\xi^2} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^2}\{(1+\frac{a_0\xi^1}{c^2})^2\phi_\xi\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial A_{\xi^1}}{\partial\xi^0}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^2} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial A_{\xi^2}}{\partial\xi^0} = E_y\cosh(\frac{a_0\xi^0}{c}) - B_z\sinh(\frac{a_0\xi^0}{c})$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}_{\xi^2}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}_{\xi^2}^{(\lambda)}(k) = \cosh(\frac{a_0\xi^0}{c})\bar{\varepsilon}_2^{(\lambda)}(k) - \sinh(\frac{a_0\xi^0}{c})\bar{\varepsilon}'^{(\lambda)}_3(k)$$

$$= \cosh(\frac{a_0\xi^0}{c})(ik_2\varepsilon_0^{(\lambda)}(k) - \frac{i}{c}\omega\varepsilon_2^{(\lambda)}(k)) - \sinh(\frac{a_0\xi^0}{c})(-ik_1\varepsilon_2^{(\lambda)}(k) + ik_2\varepsilon_1^{(\lambda)}(k)) \quad (16)$$

Therefore, the quantization of magnetic field is in Y-axis in the accelerated frame,

$$B_{\xi^2} = \frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}'^{(\lambda)}_{\xi^2}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}'^{(\lambda)}_{\xi^2}(k) = \cosh(\frac{a_0\xi^0}{c})\bar{\varepsilon}'^{(\lambda)}_2(k) + \sinh(\frac{a_0\xi^0}{c})\bar{\varepsilon}_3^{(\lambda)}(k)$$

$$= \cosh(\frac{a_0\xi^0}{c})(-ik_3\varepsilon_1^{(\lambda)}(k) + ik_1\varepsilon_3^{(\lambda)}(k)) + \sinh(\frac{a_0\xi^0}{c})(ik_3\varepsilon_0^{(\lambda)}(k) - \frac{i}{c}\omega\varepsilon_3^{(\lambda)}(k)) \quad (17)$$

Hence, the quantization of electric field is in Z-axis in the accelerated frame,

$$E_{\xi^3} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^3}\{(1+\frac{a_0\xi^1}{c^2})^2\phi_\xi\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial A_{\xi^1}}{\partial\xi^0}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial A_{\xi^3}}{\partial\xi^0} = E_z\cosh(\frac{a_0\xi^0}{c}) + B_y\sinh(\frac{a_0\xi^0}{c})$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}_{\xi^3}^{(\lambda)}(k)\,[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x}+i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x}-i\omega t)]$$

$$\bar{\varepsilon}_{\xi^3}^{(\lambda)}(k) = \cosh(\frac{a_0\xi^0}{c})\bar{\varepsilon}_3^{(\lambda)}(k) + \sinh(\frac{a_0\xi^0}{c})\bar{\varepsilon}'^{(\lambda)}_2(k)$$

$$= \cosh(\frac{a_0\xi^0}{c})(ik_3\varepsilon_0^{(\lambda)}(k) - \frac{i}{c}\omega\varepsilon_3^{(\lambda)}(k)) + \sinh(\frac{a_0\xi^0}{c})(-ik_3\varepsilon_1^{(\lambda)}(k) + ik_1\varepsilon_3^{(\lambda)}(k)) \quad (18)$$

Therefore, the quantization of magnetic field is in Z-axis in the accelerated frame,

$$B_{\xi^3} = \frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c})$$

$$= \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0}^{3} \bar{\varepsilon}'^{(\lambda)}_{\xi^3}(k)\left[a^{(\lambda)}(k)\exp(-i\vec{k}\cdot\vec{x} + i\omega t) - a^{(\lambda)+}(k)\exp(i\vec{k}\cdot\vec{x} - i\omega t)\right]$$

$$\bar{\varepsilon}'^{(\lambda)}_{\xi^3}(k) = \cosh(\frac{a_0\xi^0}{c})\bar{\varepsilon}'^{(\lambda)}_3(k) - \sinh(\frac{a_0\xi^0}{c})\bar{\varepsilon}^{(\lambda)}_2(k)$$

$$= \cosh(\frac{a_0\xi^0}{c})(-ik_1\varepsilon^{(\lambda)}_2(k) + ik_2\varepsilon^{(\lambda)}_1(k)) - \sinh(\frac{a_0\xi^0}{c})(ik_2\varepsilon^{(\lambda)}_0(k) - \frac{i}{c}\omega\varepsilon^{(\lambda)}_2(k)) \quad (19)$$

4. Conclusion

Therefore, according to [1], likely the electromagnetic field, the potential did quantizated in inertial frame, the electromagnetic field, the potential can quantizate by the transformation of electromagnetic field or the transformation of the potential in the accelerated frame

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