The Concept of Basic Clocks and Simultaneity in TSR (v2, 2019-05-22)

Per Hokstad

phokstad@gmail.com

Abstract. The concept of 'basic clocks' was introduced in Hokstad (2018b). These are moving clocks to be applied in the theory of special relativity (TSR). They experience an initial synchronization at a common location, and provide the basis for defining time and simultaneity. Here we further motivate and elaborate the concept, *e.g.* by utilizing symmetry. We provide a general criterion of simultaneity, for any event on any reference frame of arbitrary velocity. Further we allow a chain of events with successive 'initial synchronizations'. Such a feature is required in the 'travelling twin' example, in order to handle the turning of the twin. Finally, the approach is used to discuss the interpretation of the case of the μ-mesons.

Key words: Simultaneity, Lorentz transformation, symmetry, time vector, travelling twin, μ-mesons.

1 Introduction

Today one establishes simultaneity within a single inertial reference frame (RF) by synchronized clocks, *e.g.* see standard textbooks like Giulini (2005) and Mermin (2005). We say that events with the same clock reading, 'time', t, on a specific RF, are *simultaneous in the perspective* of this frame. However, from a holistic viewpoint this is not a very satisfactory definition, as the various RFs will disagree with respect to simultaneity; also see further discussions in Debs and Redhead (1996).

The present work elaborates and combines ideas presented in Hokstad (2018a) and Hokstad (2018b). We still restrict to consider one space coordinate, x, and assume the existence of an infinite set of (imagined) RFs at various velocities. All clocks on each RF are synchronized in the standard way. Further, each RF has located a clock at its origin, and at an initial time, 0, the origins of these RFs are at the same location. At this instant, we synchronize all these so-called *basic clocks* (BCs), allocating the value 0. So, this synchronization is carried out when the clocks are at *the same location at the same time*, and we may refer to this as a 'point of initiation'.

We now claim that each of these BCs *remain* synchronized (in some sense). Any two of them move away from each other at a constant speed, but there is a symmetric situation; so there is no way to claim that one of the two clocks will go faster than the other.

So when the BCs at the origins of two specific RFs show the same time, this corresponds (in some sense) to simultaneous events 'at a distance'. Actually, we could consider this to be a consequence of the standard assumption of symmetry between the RFs. This leads to a rather strong form of simultaneity, as all observers will agree on this. The argument does not restrict to consider just two RFs.

Ch. 3 includes a discussion of the Lorentz Transformation (LT), and how the BC can be used to formulate a symmetric version of this; *cf.* Hokstad (2018a). We see this as a motivation for the definition of the suggested time vector, but it may not be essential for the following text. In an appendix we elaborate on alternative versions of the LT, providing some background for Ch. 4.

In Ch. 4 we give our definition of the time vector of an event, *cf.* Hokstad (2018b); specified as a complex variable. The real part equals the clock reading of the BC present at the event, and its absolute value equals the clock reading. Ch. 5 elaborates on the measure of simultaneity. Previous results are extended, as we provide an explicit simultaneity criterion depending also on the RF's velocity. Further, the analysis allows a sequence of 'points of initiation', a feature that is needed when we consider the travelling twin paradox; as the turning of the twin exemplifies the need of such a new 'point of initiation'.

We suggest that the given approach might affect the way we interpret time and simultaneity in TSR, and in Ch. 6 include a discussion of a couple of standard examples; the travelling twin paradox and the case of the μ-mesons.

2 Basic clocks (BCs) and notation

Consider an arbitrary RF, *K*. There are synchronized clocks at virtually any position. When there is a clock reading, *t* at a position, *x,* this specifies an event, (*t*, *x*) relative to *K*. Further, we introduce

$$w=x/t$$

This equals the velocity of an object that has moved from the origin of *K* and has arrived to the position *x* at 'time', *t*.

If a RF moves relative to *K* with velocity *w* we will denote it K_w. Further, (t_w, x_w) will specify an event on K_w. Now assume that we have an infinite set of auxiliary K_w's, ($-c < w < c$), where *c* is the speed of light. Initially, at 'time' 0, the origins of all K_w's are located at the same position, and we refer to this event as the 'point of initiation'. We refer to the clocks located at the origins of the RFs as basic clocks (BCs), and these are all set to 0 by this 'point of initiation'. Further, on each RF all clocks are synchronized in a standard way with its own BC.

We note that at any later event there will now be present a BC. In particular, the BC at the origin of K_w, ($w = x/t$), will be present at the event (*t, x*) on *K,* see Fig. 1. We refer to this BC reading as t_w^{BC}. Thus, t_w^{BC} equals the clock reading, t_w on K_w at the position, x_w=0. So, at each event there will be exactly one such BC reading. If we do not specify the RF where the BC is located, we simply write t^{BC}. Now it is a well-known consequence of the Lorentz Transformation (LT) that

$$t_w^{BC} = \sqrt{t^2 - (x/c)^2}, \ (w = x/t) \tag{1a}$$

Alternatively

$$t_w^{BC} = t\sqrt{1 - (w/c)^2}, \ (w = x/t) \tag{1b}$$

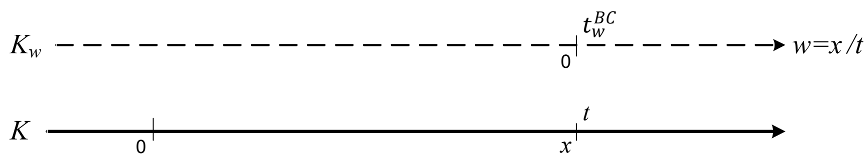

Figure 1 An event (*x, t*) on *K*, and the corresponding basic clock reading, t_w^{BC} of this event. Thus, K_w carries the BC for this event, as it moves relative to *K* at a velocity *w = x/t*.

3 The Lorentz Transformation (LT) and symmetry

As a further introduction, we consider some aspects of the LT, being related to BCs and to the symmetry of RFs. So now we consider *two* specific RFs, *K* and K_v.

3.1 The Lorentz Transformation (LT)

The LT states that when the event (t_v, x_v) on K_v is 'equivalent to' the event (*t, x*) on *K*, then

$$t_v = \frac{t-(v/c^2)x}{\sqrt{1-(v/c)^2}} \tag{2}$$

$$x_v = \frac{x-vt}{\sqrt{1-(v/c)^2}} \tag{3}$$

Fig. 2 gives an illustration of the relation, (2) for events where the clock readings equal *t* all over *K*. The corresponding clock readings on K_v equal $t_v = t_v(x)$. Thus, we may replace t_v in (2) by $t_v(x)$, to stress its dependence of the position, *x*.

The BC readings on these two RFs are found at *x* = 0 and *x*= *vt*, respectively, (marked as 'o' in the figure). Further, at position, x^*, we have $t_v = t$, (*cf.* Hokstad 2018a). Finally, we also indicate the clock readings, $t_{-v}(x)$ of the RF, K_{-v} by a red, stippled line. This, of course, is symmetric to $t_v(x)$ around the vertical axis.

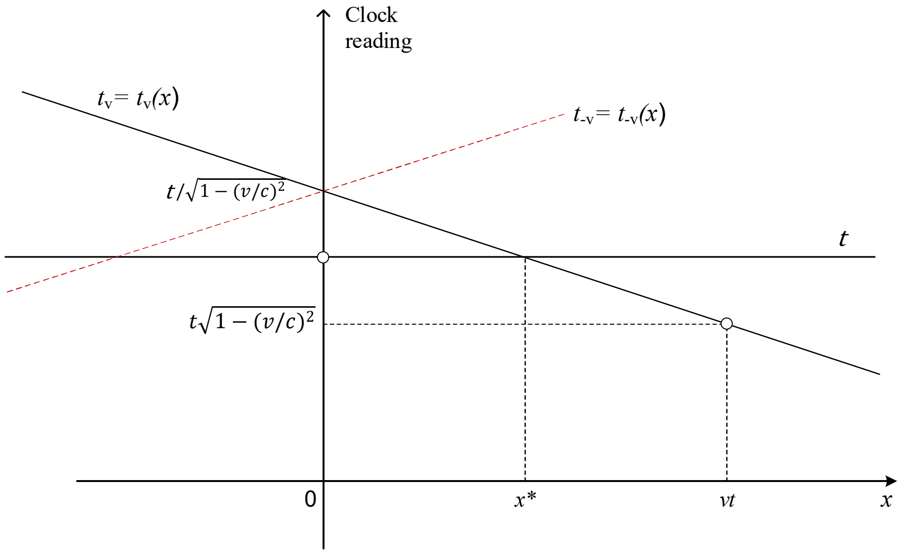

Figure 2 Illustration of the LT. Perspective of the RF, *K*: Clock readings equal *t* all over *K*. The clock readings on K_v equals t_v, as a function of the position, *x* on *K*. The Basic Clocks at the origins of *K* and K_v are marked as small circles, 'o'.

3.2 The symmetry of the two RFs

Fig. 2 gives an asymmetric illustration of the LT. The clock readings equal t all over K, which means that we have chosen the perspective of K. In Fig. 3 we give a symmetric presentation of the same relation. We may achieve this by introducing an auxiliary RF with its origin always located at the midpoint between the origins of K and K_v, and then choose the perspective of this auxiliary RF; *cf.* (Hokstad 2018a). Fig. 3 presents the clock readings of these three RFs as a function of x_{Aux}. By increasing 'time', the clock readings of all these RFs moves upwards along the vertical axis.

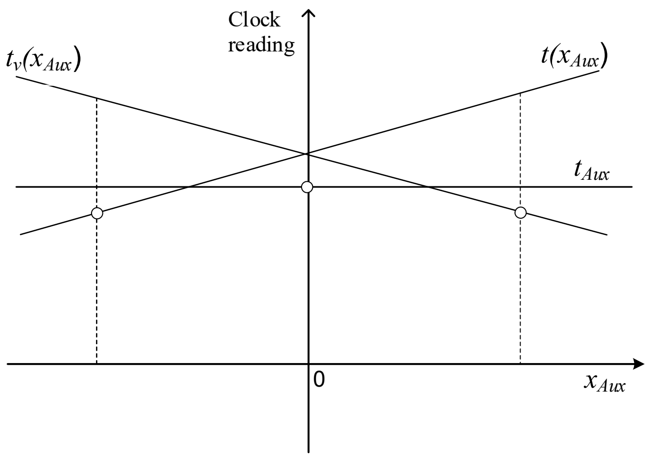

Figure 3 Alternative illustration of the LT. Perspective of an auxiliary, RF, K_{Aux}: with its origin always located at the midpoint between the origins of *K* and K_v. The origins of the three RFs (with a BC) are marked as 'o'.

Fig. 4, gives an alternative symmetric illustration of the clock readings on K and K_v, focusing on the observations at the origins of these RFs, (with the BCs again marked as 'o'). At the 'point of initiation' these BCs are at the same location, and by increasing 'time', they will in the figure move apart along a fixed horizontal line. Further, the figure has two lines, representing K and K_v. They have fixed slopes, given by the angle θ_v, where

$$\sin\theta_v = v/c \tag{4}$$

Now consider the triangle *DEF*. First, we choose the distance, $DE = t^{BC}$. It follows that $DF = t$, and finally $EF = \sqrt{t^2 - (t^{BC})^2} = x/c$. Thus, all distances in the figure have a simple interpretation, measured in time units; (x/c equals the time required for a light flash to go the distance, x).

Here *F* and *E* represent the same event described by *K* and K_v, respectively. The point *F* represents the event (t, x) on *K*, and point *E* the identical event $(t_v, 0)$ on K_v. Thus, t_v equals the t^{BC} at this position *E/F*. Similarly, point *D* corresponds to the event $(t^{BC}, 0)$ on *K*.

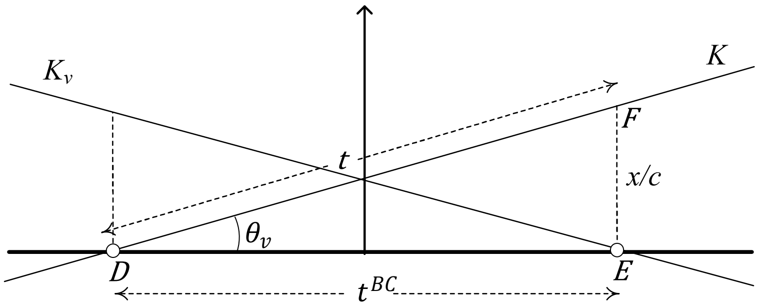

Figure 4 A fully symmetric illustration of the LT between RFs, *K* and K_v. focusing on the location of the two BCs.

So, by this illustration we split up the clock reading, t, in two orthogonal components, x/c and $t^{BC} = \sqrt{t^2 - (x/c)^2}$. Further, contrary to Fig. 2, we see that Fig. 4 is symmetric with respect to the two BCs (the two RFs).

4. The time vector and Basic Clocks (BCs)

Here we introduce and present basic features of the time vector, suggested in Hokstad (2018b). Also see Appendix A for some details related to the following results.

4.1 A single RF

We return to the situation described in Ch. 2, focusing on a specific RF, *K,* and consider an event (t, x) with $w = x/t$. Thus, the relevant BC of the event is located at the origin of the RF, K_w; *cf.* Fig. 1. By including a full set of RFs, $\{K_w\}_{-c<w<c}$, there will be a BC present at any event.

For an event (t, x) on *K* we now introduce a two-dimensional time vector, $\vec{t}$. We present it as a complex variable, and let the real part be the clock reading of the BC of the event (*cf.* (1)):

$$Re\,\vec{t} = t^{BC} = t_w^{BC} = \begin{cases} \sqrt{t^2 - (x/c)^2} = t\sqrt{1-(w/c)^2}, & t > |\,x\,|/c \\ 0, & t \le |\,x\,|/c \end{cases} \tag{5}$$

The imaginary part equals

$$Im\,\vec{t} = x/c$$

Thus, we define this time vector as[1]

$$\vec{t} = t^{BC} + i \cdot x/c \tag{6}$$

Now, $w = x/t$ equals the velocity (relative to *K*) of the RF where the relevant BC is located. If we want to specify this w, we write t_w^{BC} rather than t^{BC}.

We note that t^{BC} is a generic feature of the event; being independent of the RF, *K,* which we choose for specifying it. The imaginary part, x/c, however, depends on the chosen RF, *K*. It equals the time required

[1] In the following we will often implicitly assume $t > |\,x\,|/c$, thus, writing $t^{BC} = t\sqrt{1-(w/c)^2}$. The general expression is given in (5).

for a flash of light to go from the origin of *K* to the position of the event (x). This represents a distance in time, from the 'point of initiation', as specified on this *K*.

The absolute value of the time vector becomes:

$$\left|\vec{t}\right| = \begin{cases} \left|x\right|/c\,, & t < \mid x \mid/c \\ t, & t \geq \mid x \mid/c \end{cases} \tag{7}$$

Thus, it is only when $t > |x|/c$ that $\left|\vec{t}\right|$ equals the clock reading, t of the event. The reason is that we only observe $t^{BC} > 0$ for $t > |x|/c$; that is when $|v|<c$. Actually, at a fixed position, x, there will be no BC present until $t > |x|/c$. Further, the first BC that arrives to a position will (in the limiting case, $v=c$) read $t^{BC} = 0$. This seems a weakness of the time vector, with respect to providing a full description of the event (t, x). Alternative definitions of the time vector for $t < |x|/c$ is possible; or we could also restrict the definition of the time vector to $t > |x|/c$.

There are various alternative ways to formulations the time vector. First, we can write it in polar form. Defining $\varphi \in (-\pi/2,\ \pi/2)$ by

$$\sin\varphi = w/c \; (= x/ct) \tag{8}$$

we have ($|w| < c$)

$$\vec{t} = te^{i\varphi} = t(\cos\varphi + i\sin\varphi) = t(\sqrt{1-(w/c)^2} + i(w/c)) \tag{9}$$

When $\varphi = 0$, we have $w = x = 0$. Then the event in question occurs at the origin of *K,* and the relevant BC is the one located on *K* itself. In this case, only, the time vector becomes a real number.

We observe that there is a strong link between the above approach and Minkowski's approach to space-time; *cf.* space-time distance given as $\sqrt{c^2t^2 - x^2 - y^2 - z^2}$ in his four-dimensional space, (Minkowski, 1909). As given in Petkov (2012), Minkowski refers to our BC reading, *eq.* (1), as 'proper time, and our t as 'coordinate time'.

Further, we can express this time vector in terms of the event, (t, x), giving

$$\vec{t} = \sqrt{t^2 - (x/c)^2} + i \cdot (x/c)$$

Finally, in terms of (t^{BC}, w); or equivalently, by (t^{BC}, φ), we have

$$\vec{t} = (1 + i \cdot \tan\varphi)t^{BC}$$

4.2 Including another RF of velocity, *v*

Now consider the case that we also have an RF, K_v, moving relative to *K* at speed, v; *i.e.* the situation treated by the LT. We consider a specific event, (t, x) on *K*, corresponding to ('equivalent to') the event (t_v, x_v) on K_v, and still have K_w, carrying the BC of the event, see Fig. 5.

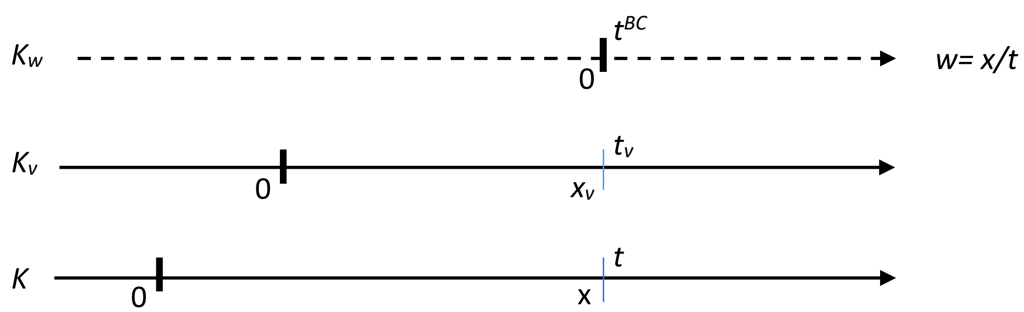

Figure 5 Two 'equivalent events' specified on *K* and K_v, respectively The RF, K_w carrying the BC of the event has velocity $w=x/t$ with respect to *K,* and velocity $u = x_v/t_v$ with respect to K_v.

Now there are three velocities involved

v = the velocity of K_v relative to the specific RF, K.

w = ***x/t*** = the velocity of RF, K_w, relative to the specific RF, K.

$u = x_v/t_v$ = the velocity of the RF, K_w, relative to K_v.

from $u = x_v/t_v$ it follows, using the LT, (2), (3) that

$$u = \frac{w-v}{1-\frac{w}{c}\frac{v}{c}} \tag{10}$$

which is the standard result for ‘adding’ velocities, (w and $-v$) in TSR. Now - in analogy with the results (6), (9) - we will give an expression for the time vector for the event (t_v, x_v) on K_v, *i.e.*

$$\overrightarrow{t_v} = t^{BC} + i(x_v/c)$$

In Appendix A we show that this time vector equals

$$\overrightarrow{t_v} = \left(\frac{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}{1-(w/c)\cdot(v/c)} + i\,\frac{w/c-v/c}{1-(w/c)\cdot(v/c)}\right) t_v \tag{11}$$

Now we further introduce ψ_u by

$$\sin\psi_u = u/c \tag{12}$$

and can write (10) as, (*cf.* (4), (8))

$$\sin\psi_u = \frac{\sin\varphi - \sin\theta_v}{1-\sin\varphi\sin\theta_v} \tag{13}$$

Then we also get

$$\cos\psi_u = \frac{\cos\varphi\cdot\cos\theta_v}{1-\sin\varphi\cdot\sin\theta_v} \tag{14}$$

Thus, we can also formulate (11) as

$$\overrightarrow{t_v} = t_v e^{i\psi_u} = t_v(\cos\psi_u + i\sin\psi_u) \tag{15}$$

We consider some special cases. First, by choosing $v = 0$ in (11), we are back to the result in Section 4.1 for the time vector on K, (see (9)), that is

$$\overrightarrow{t_0} = \vec{t} = (\sqrt{1-(w/c)^2} + i\cdot(w/c)t)$$

Next, choosing $w = 0$ in (11), meaning that the BC of the event is located at the origin of K, (so $t = t^{BC}$), gives

$$\overrightarrow{t_v} = (\sqrt{1-(v/c)^2} - i\cdot(v/c))t_v\,, \quad \text{(for } w = 0\text{)}$$

Finally, v=w, meaning that the BC of the event is located at the origin of K_v, (so $t_v = t^{BC}$) gives

$$\overrightarrow{t_v} = t_v\,, \quad \text{(for } v = w\text{)}$$

The expression (11) (or equivalently (15)) captures the essential features of the events (t_v, x_v); including the LT, see Appendix A.2 for further details.

5 Simultaneity

We now consider simultaneity of events and how this relates to the time vector, (6).

5.1 The simultaneity concept

For events having the same clock reading, t on this RF we talk about simultaneity ‘*in the perspective of K’*. This represents simultaneity in a rather narrow sense; as clocks on different RFs will not agree on the ‘time’ of ‘equivalent events’.

However, we can use the above concept of Basic Clocks (BCs) to define simultaneity in a stronger sense; (we refer to this as Simultaneity *Type II*). For an event (t_v, x_v) it is present a BC with the clock reading, t^{BC} (provided $t > |x|/c$). As all the BCs move relative to each other at constant speed, there is no way to claim that one goes faster than another does. We note that this BC reading is a feature of the event; being independent of the chosen RF.

Thus, our criterion for events being simultaneous (*Type II*), is simply that $Re\,\overrightarrow{t_v} = t^{BC}$ of the events are identical. In Section 4.2 (and Appendix A) we derived a general expression for $\overrightarrow{t_v}$; thus, the measure for simultaneity, *i.e.* t^{BC}= Const, can be written; see (11):

$$t^{BC} = \sqrt{t_v^2 - (x_v^2/c)^2} = \frac{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}{1-(w/c)\cdot(v/c)} t_v = \text{Const.} \tag{16}$$

All events (t_v, x_v) on any RF, K_v , satisfying (16) are simultaneous (*Type II*). This gives a consistent definition, which applies 'at a distance' and across RFs.

We exemplify this in Sections 5.2, 5.3 below by considering two special cases, illustrated in Figs. 5, 6.

5.2 Simultaneity on a single RF

As in Ch. 2 and Section 4.1 we consider a single RF, K, having a series of auxiliary RFs, K_w, ($-c<w<c$). Fig. 6 illustrates the situation that the BCs being present at the various locations (x) of K all show the same value, t^{BC}; see stippled blue vertical line; that is, the events are simultaneous *Type II*. We consider one specific of these events, (t, x), and as above, let $w = x/t$. Then φ is given by (8). So, the angle φ specifies the relative velocity of the RF, K_w carrying the BC of this event. In particular, for $\varphi = 0$, (x = 0), the relevant BC is that at the origin of K.

A time vector will have the angle φ with the (horizontal) t^{BC}–axis; see Fig. 6. Since the corresponding BC is located on K_w, we write t_w^{BC} for the BC reading of this time vector, and thus

$$t_w^{BC} = t\sqrt{1-(w/c)^2} = \sqrt{t^2 - (x/c)^2}$$

Now, events, (t, x) on K with BC reading, t_w^{BC}, which equals a specified value, t^{BC} are simultaneous (*Type II*). In the general criterion (16) this corresponds to letting v = 0, (*i.e.* the situation of Section 4.1).

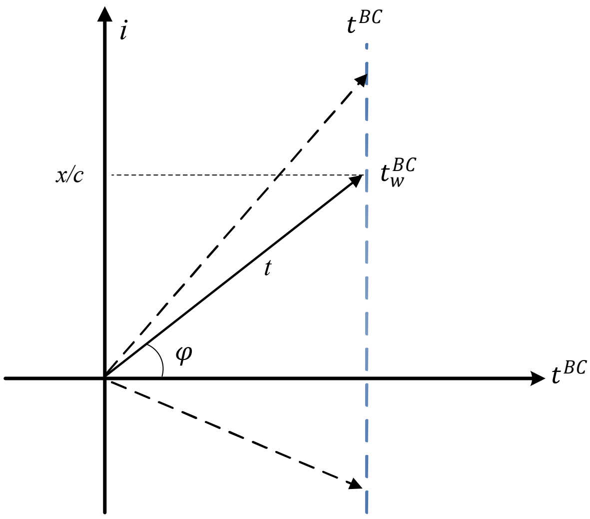

Figure 6 Time vectors of a single RF, *K*, with different BCs at various locations, but with the same BC reading, t^{BC}; thus, being simultaneous *Type II.* One of the time vector is fully drawn, two others are stippled.

5.3 A single BC and simultaneity

Next, in Fig. 7 we illustrate one single event, viewed in the perspective of various RFs; *i.e.* the situation described by the LT; *cf.* Ch. 3. So now there is just a single BC involved, and for simplicity, we assume that it is located on K. The angle θ_v defined in (4), corresponds to the event (t_v, x_v) on K_v. We can say that Fig. 6 illustrates the LT between K and K_v for all v. By choosing the velocity, v, of the second RF we also choose a θ_v, and then we directly find the time vector for the event, as specified on K_v.

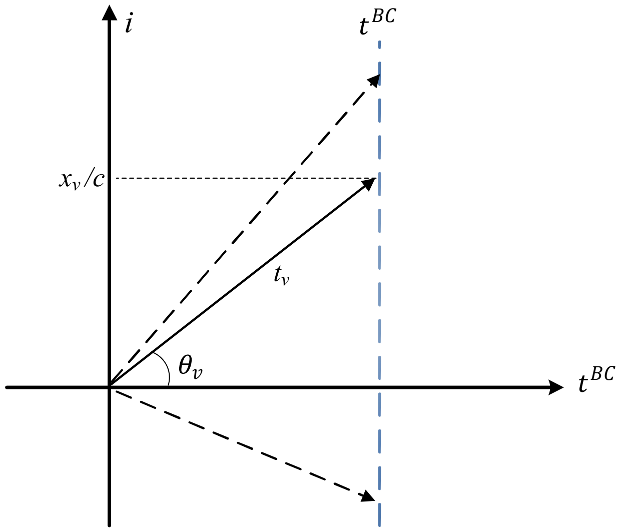

Figure 7 Time vectors of various RFs, related to a single BC reading; (*cf.* the LT). For convenience, we assume that this single BC is located on *K*.

On any RF, K_v the event (t_v, x_v) with the chosen BC reading, t^{BC} satisfies

$$t_v\sqrt{1-(v/c)^2} = \sqrt{t_v^2 - (x_v^2/c)^2} = t^{BC}$$

thus, having the same value. Here we have 'equivalent events', just being described by various RFs; therefore, we have strong reasons to claim simultaneity. In the general criterion (16) this corresponds to letting w=0, (*i.e.* the BC is located on K), but allowing any v value.

5.4 Restrictions regarding the use of our measure for simultaneity

Of course, defining simultaneity by applying t^{BC}, only, does not give the full picture. For instance, the first BC arriving (at speed, c) to a location, x, will read $t^{BC} = 0$; while the local clock at this position will read $t=x/c$. However, while t provides a good measure of simultaneity in the perspective of (restricting to) a given RF, we argue that the t^{BC} for $t > |x|/c$ will provide a good symmetric measure relative to ('in the perspective of') the 'point of initiation'.

So, when we define this simultaneity relative to a specific 'point of initiation' ('p.o.i.'); it does not represent an 'absolute simultaneity'. Now it would be of interest to relax on this assumption, for instance by allowing discontinuities in the chain of events, by specifying several 'p.o.i.'. We then go outside the framework of the TSR, but we can speculate on such an extension of the approach.

We give the following example; starting out with an event (t', x') on K with a BC reading, t^{BC}. The BC is located at the origin of a K_w, where $w = x'/t'$. However, at this event, conditions are changing: The velocity of K_w changes from w to w^*; or in other words, we 'replace' the RF, K_w with a new RF K_{w^*}. In doing so, we also perform a new calibration; *i.e.* specify a new 'p.o.i.'. Then we should go through the following steps:

1. First, we make a record of the current clock readings of the RFs at the given position. For K_w this equals the BC reading, $t_w = t^{BC}$, and for K it equals $t' = \sqrt{(t^{BC})^2 + (x'/c)^2}$. These values will be added to future clock readings, which we obtain after the new 'p.o.i.'. In particular, the BC reading, $t_w = t^{BC}$ on K_w will be added to the future clock reading on K_{w^*}.
2. We introduce new basic clocks, denoted BC*, at this new 'p.o.i.', and assign the values $t^{BC^*} = 0$.

3. All clocks on K and K_{w^*}, respectively are calibrated in the traditional way; based on the new t^{BC^*}.
4. We recalibrate the x-values on K, as the new origin is located at x'. For instance, the 'old' origin, ($x = 0$), on K is now assigned the value $-x'$, with the time vector, $\vec{t} = t^{BC*} - i \cdot x'/c$.

We note that such an updating/recalibration is required, *e.g.* for a proper handling of the travelling twin example; see next chapter.

6 Discussion. Examples

To illustrate our approach to time and simultaneity in STR we look at the two standard examples, the *travelling twin* and the *μ-mesons*.

6.1 The travelling twin

The so-called travelling twin paradox is frequently discussed, *e.g.* see Schuler and Robert (2014), and we also discussed it in Hokstad (2018a, b).

The earthbound twin has a RF, K with the origin on the earth, and the travelling twin has a RF, K_v with the origin at his rocket. The start of the travelling twin's journey is our 'point of initiation', and so both clocks read 0, and both twins are located at the origin of their RFs; thus, both clocks are BCs.

Now we use the numerical example of Mermin (2005). The distance from the earth to the star equals $x = 3$ light years, *i.e.* $x/c = 3$ years. Further, the velocity of the rocket is $v = 0.6c$, giving $\sqrt{1-(v/c)^2} = 0.8$. It follows that by the arrival to the star the clock at the star, which belong to the earthbound twin, will read $t = x/v = 3/0.6 = 5$ years, (assuming that the he has a synchronized clock, located on the star). At the same instant the clock of the travelling twin reads $t_v = t^{BC} = \sqrt{t^2-(x/c)^2} = t\sqrt{1-(v/c)^2} = 5 \cdot 0.8 = 4$ years. (We note that in the present case we have $w = v$. *i.e.* the travelling twin carries the BC for the events of main interest.)

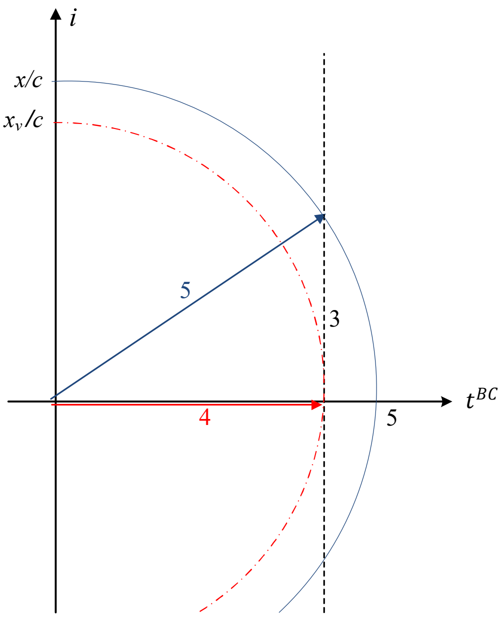

Figure 8. Time vectors by the arrival at the star: Blue for the earthbound twin. Red for the travelling twin. The relevant vectors are $\vec{t} = 4 + 3i$ (with absolute value 5), and $\vec{t} = 4$, respectively.

Fig. 8 illustrates the relevant time vectors by the arrival; the blue vector for the earthbound twin, and the red for the travelling twin, and. The semicircles indicate events (time vectors) that are simultaneous in *the perspective of* K and K_v, respectively

Further, when the travelling twin's clock shows 4 years by his arrival, this is simultaneous (*Type II*) with the event that the clock on the earth also shows 4 years. In the literature one often restricts to point out the simultaneity 'in the perspective of' K and K_v, respectively. However, considering the symmetry of the situation, 4 years is the only feasible result regarding this simultaneity at a distance, (Hokstad, 2018a, b).

The main problem of the paradox is rather how to handle the abrupt discontinuity in the chain of events caused by the turning of the travelling twin. This event contradicts the assumptions of TSR, and requires the introduction of a third RF, with a third BC, which is then brought back to the earth. So if we shall treat this within the framework of the TSR, it is not the twin himself that comes back to the earth, but rather a clock that had the identical reading to his own by the turning at the star. We gave an approach in Appendix A.3 of Hokstad (2018a), based on symmetry, *cf.* Fig. 3. However, we now apply a more direct, less detailed argument, following the discussion of Section 5.4.

So, at the instant when the travelling twin arrives to the star, we introduce a third RF, K_{-v}, which moves at a velocity $-v$ relative to K. We first make a record of the clock readings of the event; that is, $t' = |\vec{t}| = 5$ years on K, and $t^{BC} = 4$ years on K_v. The value 5 is 'kept for the record' for K, while the value, 4 on K_v is 'transferred' to K_{-v}.

Further, K and K_{-v} are assigned new BCs, located on the star, (at the new origins). These are denoted BC*, and are allocated the value $t^{BC^*} = 0$. Further, all clocks on the two RFs, K and K_{-v}, respectively, are calibrated according to these new BC*. Finally, positions, x on K are replaced by x-3, to account for the new origin on K. Then we are back to exactly the same situation as when the travel from the earth started.

We illustrate this in Fig. 9, (see Fig. 4 regarding the interpretation). After the departure, (a), the K_v moves to the right and K to the left, (or rather they move apart), ending up in position, (b);. After the recalibration, (c), K_{-v} moves to the left and K to the right, ending up with the final position (d); illustrating the return of the travelling twin.

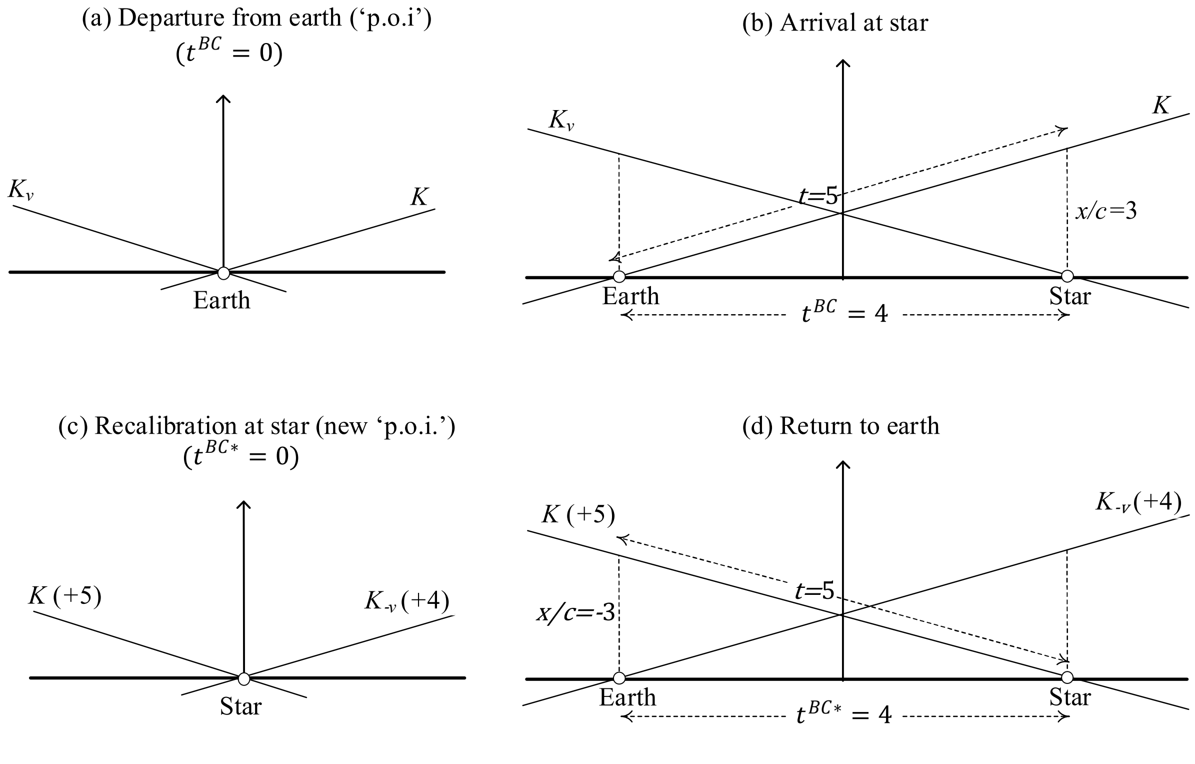

Figure 9. Illustrating the time vectors of the two twins in four steps. In (c) and (d), we also indicate the recorded clock readings at the arrival to the star in parentheses (5 and 4 years, respectively); to be added to clock readings of the recalibrated clocks, *BC.**

In (c) and (d) of Fig. 9 we add (+5) and (+4) to K and K_{-v}, respectively, referring to the clock readings, at the arrival, (b); so, these values are added to the clock readings on K and K_{-v} by the return, (d).

We note that the time vector at the earth after the change of p.o.i. (c) equals $\vec{t} = t^{BC*} - i \cdot 3$, as this position now has the distance x = 3 light years from the new origin. Thus, by the arrival of the BC* on K_{-v} to the earth, this time vector equals $\vec{t} = 4 - 3i$, with absolute value t=5. By adding the clock readings, recorded by the arrival to the star, we arrive at the result 5+5 = 10 years. Further, the clock on K_{-v} by the return reads t^{BC*} = 4 years, giving 4+4 = 8 years for the travel both ways. So, our approach provides the standard answer on the paradox.

6.2 The μ-mesons

The example of the μ-mesons is frequently referred, *e.g.* see Mermin (2005), where we have extracted the following text. The μ-mesons are produced by cosmic rays in the upper atmosphere. When 'at rest' they have a lifetime of about 2 microseconds, so if their internal clocks ran at a rate independent of their speed, even if they travelled at the speed of light about half of them would be gone after they had travelled 2.000 feet. Yet about half of the μ-mesons produced in the upper atmosphere (about 100.000 feet up) manage to make it all the way down to the ground. This is because they travel at speed so close to the speed of light that the slowing down factor equals 1/50, and they can survive for 50 times as long as they can when being stationary.

The phenomenon is easily described by the LT. We have the RF of the earth, K, and that of the μ-mesons, denoted K_v. The creation of the μ-mesons in the upper atmosphere, is denoted event A; then $x = x_v = 0$ and $t = t_v = 0$. So, this is the 'point of initiation', specifying two BCs; one on K, remaining in the upper atmosphere, one on K_v, going to the ground with the μ-mesons.

We refer to the arrival at the ground as Event B; then $x_v = 0$ and $t_v = 2\cdot10^{-6}$ sec. So this t_v equals the BC reading of event B, and $t^{BC} = t_v = 2\cdot10^{-6}$ sec.

Further, the RF of the μ-meson is moving relative to earth at a speed, $v \approx c$, with $\sqrt{1-(v/c)^2} \approx 0.02$. Thus, in event B, we have $t \approx t_v / 0.02 \approx 10^{-4}$ sec. The μ-mesons have then gone the distance $x = 10^5$ feet, (measured on K), giving $x/c \approx 10^{-4}$ sec. Thus, the time vectors of event B for the two RFs, respectively, equal

$$\vec{t} = t^{BC} + i \cdot (x/c) = (2 \cdot 10^{-6} + i\, 10^{-4}) sec; \text{ (thus, } |\vec{t}| = t \approx 10^{-4} \text{ sec)}$$

$$\vec{t_v} = t^{BC} = 2 \cdot 10^{-6}\, sec; \text{ (thus, } |\vec{t_v}| = t^{BC} = 2\; 10^{-6} \text{ sec)}$$

In summary, the theory (the LT) is in full agreement with the observations. Now, according to approach of the present work the BC reading of event B, ($2\; 10^{-6}$ sec) is simultaneous with the event that the BC in the upper atmosphere shows the same value. However, my main concern is how we should talk about/interpret the above result.

It is a fact that the clock following the μ-mesons goes slower by a factor 50, *when it is compared with two clocks,* which are *stationary relative to the earth*. However, I find it strange that Mermin (2005) formulates this as: "The atomic particles can go much further because their internal clocks that govern when they decay are running much more slowly in the frame in which they rush along at speed close to c. This is a real effect, and it plays a crucial role in the operation of such particles accelerators." We also note the above formulation: "…if their internal clocks ran at a rate independent of their speed …".

There are two statements here, that I should like to comment on. First, what does it actually mean that this is a *real effect*. Secondly, how should we interpret the statement that the experiment confirms that the *internal clocks* of the atomic particles are running more slowly, (that is, depends on their speed)? These are questions, perhaps of philosophical, rather than physical character, but are nevertheless of considerable interest.

First, regarding this being a *real effect*. It is often said that 'moving clock goes slower'. This occurs when a single clock is moving between two clocks which are at a fixed distance from each other. However, we should stress (*cf.* discussion in Hokstad 2018a), that it is the *single* clock that always goes slower, when compared to *two* clocks on another RF. This is the case, also if we see the single clock to be 'at rest', and the two other clocks as moving past. Thus, rather than being a real effect, we could say that it is a consequence of the chosen observational principle; *i.e.* the (relative) velocity of the RF which we choose as the basis for the observations (clock readings).

Further, what should be the physical meaning of the *internal clocks* of the atomic particles? Actually, if the observer is at rest (moves together) with respect to the phenomenon, then the average 'lifetime' of μ-mesons equals 2 microseconds. So, this is rather the duration which we should consider to express the 'inner clock' of the μ-mesons. Thus, provided the conditions of the TSR are valid, the μ-mesons should 'see' themselves as being at rest. Further, this 'inner clock' should hardly be affected by passing observers making observations. And the passing observers (at various speeds, *v*) will make 'all kind' of observations. Obviously, my view is that we should rather see $t^{BC} = 2 \cdot 10^{-6}\, sec$ to represents the 'true' lifetime also in the present experiment.

So, a main question is to what extent time dilation should be considered a 'true' physical phenomenon or an observational phenomenon. It is a fact that different observers obtain different results regarding the time duration for the same phenomenon. However, it seems unreasonable to claim that an observer will 'truly' affect the experiment / phenomenon as such.

7 Summary and Conclusions

In our approach we introduce an infinite set of reference frames (RFs), at any velocity. All of the (possibly imagined) RFs have a synchronized set clocks at virtually any position. Further, we synchronize the clocks at the origins of the RFs at a common instant and location, and we refer to this event as the 'point of initiation'. From symmetry, the clocks at the origins of the various RFs now remain synchronized, and we denote them as basic clocks (BCs).

It follows that one of these BCs will be present at any later event (t, x) on any RF, (provided $t>|x|/c$). Next we define time as a complex variable, where

1. The real part equals the clock reading (t^{BC}) of the BC currently at this position.
2. The imaginary part equals the time required for a light flash to go from the origin of the RF, (where another BC is located), to the current position,(x/c).
3. The absolute value of this time variable equals the clock reading, t, of the event, (when $t>|x|/c$).

It is well known that when events on the same RF have the same clock reading, t, then the events are simultaneous 'in the perspective' of this RF. From a holistic point of view, however, time variables with the same BC reading (t^{BC}) imply simultaneity in a stronger sense. They provide a form of symmetry relative to the given point of initiation, and this simultaneity is not affected by the RF(s) on which we specify the events. All events with the same t^{BC} will exhibit this form of simultaneity, also for events 'at a distance', possibly defined on different RFs.

The present work motivates further and elaborates on previous results. It provides a generic time vector and simultaneity criterion, valid for a RF of arbitrary speed. Alternative formulations of the Lorentz Transformation (LT) are provides as part of the argumentation; *cf.* Appendix A.

We discuss an extension of the approach, by allowing a sequence of 'points of initiation' *in series.* An example is provided by the travelling twin paradox; where the turning of the travelling twin obviously requires the introduction of such a second 'point of initiation'.

The approach discussed here may not provide essential new information, but should in my opinion affect the way we talk about/interpret time and simultaneity within the framework of STR. As an example, we also discuss the case of the μ-mesons.

References

Debs, Talal A. and Redhead, Michael L.G., The twin "paradox" and the conventionality of simultaneity. Am. J. Phys. **64** (4), April 1996, 384-392.

Giulini, Domenico, Special Relativity, A First Encounter, Oxford University Press, 2005.

Mermin. N. David, It's About Time. Understanding Einstein's Relativity. Princeton Univ. Press. 2005.

Minkowski, H., Raum und Zeit. Physikalische Zeitschrift 10, 75-88, 1909. English Translations in Wikisource: Space and Time

Hokstad, Per, An Approach for analysing Time Dilation in the TSR, **viXra:1706.0374**. Category Relativity and Cosmology, June 2018a.

Hokstad, Per, A time vector and simultaneity in TSR, **viXra:1711.0451.** Category Relativity and Cosmology, December 2018b.

Petkov, V., Introduction to Space and Time: Minkowski's papers on relativity; translated by Fritz Lewertoff and Vesselin Petkov. Minkowski Inst. Press, Montreal 2012, pp. 39-55, Free version online.

Shuler Jr., Robert L., The Twins Clock Paradox History and Perspectives. Journal of modern Physics, 2014, 5, 1062-1078. https://file.scirp.org/Html/3-7501845_47747.htm.

Appendix A Formulations of the Lorentz Transformation (LT) and the time vector

We look at alternative formulations of the LT, and relate this to the time vector defined above.

A.1 The LT as an orthogonal transformation

Again we start out with two RFs, *K,* and K_v moving at velocity *v* relative to *K.* As we consider one space coordinate, only, the LT, (2), (3) involves just four state variables: t, x, t_v and x_v. If we specify any two of these, the other two will be given by the LT.

The above standard version of the LT, (2), (3) gives (t_v, x_v) expressed by (t, x), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t, t_v) and (x, x_v). And – as a third possibility – we can formulate the LT as a relation between (t, x_v) and (t_v, x). Now we follow up on this third possibility. First, by combining (2) and (3), we also have

$$t_v = t \cdot \sqrt{1-(v/c)^2} - (x_v/c) \cdot (v/c) \tag{A1}$$

Now (3) and (A1) give a new version of the LT, which in matrix form becomes

$$\begin{pmatrix} t_v \\ x/c \end{pmatrix} = A_v \begin{pmatrix} t \\ x_v/c \end{pmatrix} \tag{A2}$$

The transformation matrix equals

$$A_v = \begin{bmatrix} \sqrt{1-(v/c)^2} & -v/c \\ v/c & \sqrt{1-(v/c)^2} \end{bmatrix} \tag{A3}$$

It is an orthogonal matrix as

$$A_v{}^{-1} = A_v{}^T = A_{-v}$$

Next, we introduce the time vectors

$$\overrightarrow{t_1(v)} = \begin{pmatrix} t \\ x_v/c \end{pmatrix} \tag{A4}$$

$$\overrightarrow{t_2(v)} = \begin{pmatrix} t_v \\ x/c \end{pmatrix} \tag{A5}$$

and then write the relation (A2) as

$$\overrightarrow{t_2(v)} = A_v \, \overrightarrow{t_1(v)} \tag{A6}$$

which provides an orthogonal version of the LT. We can further replace v by the angle, θ_v, which we - as before - define by, (see *eq.* (4)) $\sin\theta_v = v/c$.This implies that $\cos\theta_v = \sqrt{1-(v/c)^2}$, and so

$$A_v \;= \begin{bmatrix} \cos\theta_v & -\sin\theta_v \\ \sin\theta_v & \cos\theta_v \end{bmatrix}$$

represents a rotation, θ_v, of the (t, x_v/c) plane into the (t_v, x/c) plane.

A.2 Application of the orthogonal transformation.

A.2.1 Single RF, *K*, and a BC. (time vector)

In the present work, (chapters 2-4) it is observed that when we describe an event, it can be helpful to utilize the clock reading of the corresponding BC, which is located on the RF K_w (with $w=x/t$). Now (A6), represents a generalization of this result. Actually, by choosing $v = w = x/t$ in Appendix A.1, the RF, K_v is the RF of the BC for the event (t, x) on K. So by choosing $v=w$, we get $x_v = 0$ and $t_v = t^{BC}$, (*cf.* Fig. 1). That is

$$\overrightarrow{t_1(w)} = \binom{t}{0}.$$

Further, we can now can replace θ_v by φ. where our definition of φ (*eq.* (8)) equals

$$\sin\varphi = w/c$$

The relation (A6) now becomes

$$\overrightarrow{t_2(w)} = \binom{t^{BC}}{x/c} = A_w\binom{t}{0} = \binom{\cos\varphi}{\sin}t = \binom{\sqrt{1-(w/c)^2}}{w/c}t = \binom{\sqrt{t^2-(x/c)^2}}{x/c} \tag{A7a}$$

So, for this special case, $v = w = x/t$, (and thus, $\theta_v = \varphi$), actually $\overrightarrow{t_2(x/t)}$ specifies our time vector $\vec{t}$ defined on the RF, K. In complex form, the vector can be written for instance as

$$\vec{t} = (\sqrt{1-(w/c)^2} + i\cdot(w/c))t \tag{A7b}$$

e.g. see (9) in Ch. 4.

A.2.2 Two RFs, *K* and K_v. Time vector for K_v.

We can now carry out the same exercise for the time vector, $\overrightarrow{t_v} = \binom{t^{BC}}{x_v/c}$ defined on K_v, but still include K in our discussions; see Fig 5. Now we start by relating the two RFs K_w and K_v. First, we note that

$u = x_v/t_v$ = the velocity of K_w relative to K_v

Now using $u = x_v/t_v$, the LT (2), (3) immediately gives

$$u = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{A8}$$

which is also the standard results for 'adding' velocities in the TSR. Now we also introduce ψ_u by

$$\sin\psi_u = u/c \tag{A9}$$

Thus, we can write (A8) as

$$\sin\psi_u = \frac{\sin\varphi - \sin\theta_v}{1-\sin\varphi\,\sin\theta_v} \tag{A10}$$

also giving

$$\cos\psi_u = \frac{\text{co}\quad\cdot\cos\theta_v}{1-\sin\quad\cdot\sin\theta_v} = \frac{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}{1-(w/c)\cdot(v/c)} \tag{A11}$$

Now considering an event relative to the two RFs K_v and K_w, (rather than to K_v and K, as we did in Section A.2.1), the equations (A4)-(A6) give

$$\overrightarrow{t_v} = \begin{pmatrix} t^{BC} \\ x_v/c \end{pmatrix} = A_u \begin{pmatrix} t_v \\ 0 \end{pmatrix}$$

Using (A9) in the matrix A_u it follows that

$$\overrightarrow{t_v} = \begin{pmatrix} t^{BC} \\ x_v/c \end{pmatrix} = \begin{pmatrix} \cos \quad u \\ \sin \psi_u \end{pmatrix} t_v \tag{A12}$$

Thus, (A12), by also applying (A10) and (A11), specifies any time vector, $\overrightarrow{t_v}$ on K_v in terms of v and w. (*i.e.*, θ_v and φ). Further writing this time vector as a complex variable, we get

$$\overrightarrow{t_v} = t_v e^{i\psi_u} = t_v\ (\cos \psi_u + i \sin \psi_u) \tag{A13}$$

That is

$$\overrightarrow{t_v} = \left(\frac{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}{1-(w/c)\cdot(v/c)} + i\ \frac{w/c - v/c}{1-(w/c)\cdot(v/c)}\right) t_v \tag{A14}$$

So this gives the general expression of the time vector for event (t_v, x_v),.

A.2.3 Relating the time vectors of *K* and *K*$_v$. The LT.

The expression (A14) seems to capture all the essential features of the event. For instance, when an event on K_v is 'equivalent to' an event on *K,* (as described by the LT), then they have the same t^{BC}, and the real parts of (A7b) and (A14) are identical. This implies

$$\sqrt{1-(w/c)^2}\,t = \frac{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}{1-(w/c)\cdot(v/c)} t_v$$

That is, $\cos \varphi \cdot t = \cos \psi_u \cdot t_v$. Thus,

$$t_v = \frac{1-(w/c)\cdot(v/c)}{\sqrt{1-(v/c)^2}} t \tag{A15}$$

which is simply a version of the first equation of the LT. This allows us to replace t_v by t in (A14), giving

$$\overrightarrow{t_v} = \left(\sqrt{1-(w/c\,)^2} + i\ \frac{w/c - v/c}{\sqrt{1-(v/c)^2}}\right) t \tag{A16}$$

thus, expressing the time vector on K_v by the clock reading of the 'corresponding' clock reading on *K*. Now by equating the imaginary parts of (A14) and (A16), we also directly get the second equation of the LT, *i.e.* (3).

Further, another variant of the time vector, $\overrightarrow{t_v}$, is

$$\overrightarrow{t_v} = \left(1 + i \frac{w/c - v/c}{\sqrt{1-(w/c\,)^2}\sqrt{1-(v/c)^2}}\right) t^{BC} \tag{A17}$$

That is

$$\overrightarrow{t_v} = (1 + i\ \tan \psi_u)\ t^{BC}$$

Finally, we can now provide a formulation of the LT, expressed by the time vectors $\overrightarrow{t_v}$ and $\vec{t}$. We get

$$\overrightarrow{t_v} = \begin{pmatrix} 1 & 0 \\ a & b \end{pmatrix} \vec{t}$$

where

$$a = \frac{1}{\sqrt{1-(v/c)^2}}$$

$$b = \frac{-v/c}{\sqrt{1-(w/c)^2}\sqrt{1-(v/c)^2}}$$

The first line of the triangular matrix just states that the first component of both vectors are identical, (equal to t^{BC}). The second line (with a and b), provides an expression of x_v, given by t^{BC} and x. Also this relation we could write out in terms of complex variables. (and θ_v and φ).