0/0=Nullity=refuted!

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Abstract

Objectives:

The scientific knowledge appears to grow by time. However, every scientific progress involves different kind of mistakes, which may survive for a long time. Nevertheless, the abandonment of partially true or falsified theorems, theories et cetera, for positions which approach more closely to the truth, is necessary. In a critical sense, a reduction of the myth in science demands the non-ending detection of contradictions in science and the elimination the same too.

Methods:

Nullity as one aspect of the trans-real arithmetic and equally as one of today's approaches to the solution of the problem of the division of zero by zero is re-analyzed. A systematic mathematical proof is provided to prove the logical consistency of Nullity.

Results:

There is convincing evidence that Nullity is logically inconsistent. Furthermore, the about 2000 year old rule of the addition of zero's (0+0+…+0 = 0) is proved as logically inconsistent and refuted.

Conclusion: Nullity is self-contradictory and refuted.

Keywords: Indeterminate forms, Classical logic, Zero divided by zero

1. Introduction

Dividing by zero on a computer causes several problems. No wonder that computer hardware manufacturers invented the concept of "*not a number*" to define circumstances that a meaningful result or number can't be returned. Anderson et al. (Anderson et al., 2007) are defined a new arithmetic which is claimed to have no arithmetical exceptions. Following Anderson et al. (Anderson et al., 2007) the transreal numbers include all of the real numbers, plus three other mathematical constructs: infinity (∞), negative infinity (-∞) and "nullity" (∅). In point to fact, Anderson et al. (Anderson et al., 2007) are treating infinity not as something real but as something trans (non-) real. What is "nullity"? Nullity as such as a number defined by the transreal axioms is not equal to any real or infinite number while it is equally the ratio of two numerical zeros (real numbers). At the end, Anderson et al. (Anderson et al., 2007) concept of "nullity" is not completely identical with the concept of "not a number" as already known in floating point arithmetic's on computers but effectively, it is. "Nullity" is just a reformulation of the basic mathematical concept of "undefined" and equally nothing else but "*not a number*". Still, the question is, is Nullity logically and mathematically consistent?

2. Material and Methods

2.1. Definitions

Definition 1. (Number +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579). The number +0 is defined as the expression

$$\begin{aligned} +0 &\equiv (c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \\ &\equiv +1 - 1 \\ &\equiv +i^2 - i^2 \end{aligned} \tag{1}$$

while "=" denotes the equals sign or equality sign (Robert Recorde, 1557) (Rolle, 1690) used to indicate equality and "-" (Widmann, 1489) (Pacioli, 1494) (Robert Recorde, 1557) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" (Widmann, 1489; Pacioli, 1494; Recorde, 1557) denotes the plus signs used to represent the operations of addition and the notions of positive as well.

Definition 2. (Number +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579). The number +0 is defined as the expression

$$+1 \equiv (c^2 \times \varepsilon_0 \times \mu_0) \equiv -i^2 \tag{2}$$

Definition 3. (Exponent Rules)

The base *b* raised to the power of *n* is equal to the multiplication of *b*, *n* times or

$$b^n \equiv \underbrace{(b \times b \times \dots b)}_{n\ times} \tag{3}$$

Definition 4. (Power Rule (Powers to Powers))

To raise a power to a power it is necessary to multiply the exponents. We obtain

$$(b^n)^m \equiv (b^{n \times m}) \tag{4}$$

Definition 5. (Anderson Et Al. Definition Of Nullity)

Anderson et al. (Anderson et al., 2007) defines Nullity as

$$\frac{0}{0} \equiv Nullity \tag{5}$$

DEFINITION 6. (ANDERSON ET AL. AXIOM'S WITH RESPECT TO NULLITY)

Some of Anderson et al. (Anderson et al., 2007) axioms especially with respect to Nullity are as follows.

$$Anderson\ et\ al.\ Axiom \quad [A4]: \quad Nullity + a \quad = \quad Nullity \tag{6}$$

$$Anderson\ et\ al.\ Axiom \quad [A7]: \quad -(-a) \quad = \quad a \tag{7}$$

$$Anderson\ et\ al.\ Axiom \quad [A9]: \quad -Nullity \quad = \quad Nullity \tag{8}$$

$$Anderson\ et\ al.\ Axiom \quad [A11]: \quad +\infty - \infty \quad = \quad Nullity \tag{9}$$

$$Anderson\ et\ al.\ Axiom \quad [A15]: \quad Nullity \times a \quad = \quad Nullity \tag{10}$$

$$Anderson\ et\ al.\ Axiom \quad [A16]: \quad \infty \times 0 \quad = \quad Nullity \tag{11}$$

$$Anderson\ et\ al.\ Axiom \quad [A20]: \quad \frac{+1}{+0} \quad = \quad +\infty \tag{12}$$

$$Anderson\ et\ al.\ Axiom \quad [A22]: \quad Nullity^{-1} \quad = \quad Nullity \tag{13}$$

DEFINITION 7. (NEGATION)

The validity of negation as a general notion cannot be restricted only to classical logic. Still, in classical logic, negation is an operation which takes truth to falsity and vice versa and is written many times as

$$Negation\ of\ 1: \quad \neg \quad \times 1 \quad = \quad 0 \tag{14}$$

$$Negation\ of\ 0: \quad \neg \quad \times 0 \quad = \quad 1 \tag{15}$$

DEFINITION 8. (NON-INFINITE AND INFINITE)

Let x_t denote something (a number, a mathematical object et cetera), let $\underline{\infty}_t$ denote the non-infinite complementary of x, let ∞_t denote the infinite. In general, it is

$$x_t \quad + \quad \underline{\infty}_t \quad = \quad \infty_t \tag{16}$$

In the case $x_t = 1$, it is $\underline{\infty}_t = \infty_t - 1$ and

$$1 \quad + \quad \infty_t - 1 \quad = \quad \infty_t \tag{17}$$

or

$$\frac{1}{\infty_t} \quad + \quad \frac{\infty_t - 1}{\infty_t} \quad = \quad \frac{\infty_t}{\infty_t} = +1 \tag{18}$$

2.2.1. Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

In general, it is

$$+1 \quad \equiv \quad +1 \tag{19}$$

or the superposition of +0 and +1 as one of **the foundations of quantum computing**

$$+1 \quad \equiv \quad (1+0) \times (1+0) \times (1+0) \times (\ldots) \times (1+0) \tag{20}$$

2.2.2. Axiom II (Lex contradictionis. Principium contradictionis. Contradiction Law)

Contradictions are an objective and important feature (Barukčić, 2019c) of objective reality. Still, contradictions in theorems, arguments and theories would allow us to conclude everything desired. In contrast to religion and other domains of human culture, one very important and at the end to some extent normative criteria to achieve some advances and progress in science is depended on detecting contradictions in science and eliminating the same too. The most important point is that even if we are surrounded by contradictions a co-moving observer (Barukčić, 2019c) will always find that something is *either* +1=+1 *or* +0=+0 but not both, i. e. it is not +1 = +0. The simplest form of Aristotle's law of contradiction (Barukčić, 2019a; Barukčić, 2019b; Barukčić, 2019c;) is defined as

$$+1 \quad \equiv \quad +0 \tag{21}$$

According to Popper, a philosopher of science of the 20th century, contradiction is the demarcation line between science and 'non-science'. "We see from this that if a theory contains a contradiction, then it entails everything, and therefore, indeed, nothing[...]. A theory which involves a contradiction is therefore entirely useless as a theory". (Popper, 2002, p. 429).

2.2. Methods

Unfortunately, not all published or proposed theorems or statement in science and mathematics are actually correct, and the wish to prove these theorems while using rigorous proof methods of science and mathematics which are acceptable beyond any shadow of doubt is more than justified.

2.2.1. Direct Proof

A direct (mathematical) proof is able to demonstrate the truth or falsehood of a given equation, statement by a straightforward combination of established facts.

2.2.2. Proof by contrapositive

Explicitly, the contrapositive of the statement P → Q ("**If *P is true*, then *Q is true***") is the known to be the statement or equation ¬Q → ¬P or in spoken language: "**If *Q is false*, then *P is false***". It is a matter of personal taste whether the a glass is half-full or half-empty. A statement and its contrapositive are logically equivalent or it is (P → Q) = (¬Q → ¬P).

2.2.3. Proof by inversion

Inversion is a valid rule of inference or a proof method "by which from a given proposition another is derived having for its *subject* the contradictory of the original subject and for its *predicate* the contradictory of the original predicate." (Toohey 1948, p. 51). In general, the inverse of the statement P → Q ("**If *P is true*, then *Q is true***") is known to be the statement or the equation ¬P → ¬Q (Table 1) or in spoken language: "**If *P is false*, then *Q is***

false" while the logical equivalent is viewed by table 2. In this context, it is worth to point out, that the basic relationship $(P \rightarrow Q) = (\neg\neg P \rightarrow \neg\neg Q)$ is valid. In other words, a direct proof provided without any technical errors which is grounded on something false must end up in something false.

Table 1. The inverse of the statement If P is true then Q is true.

If P is false **then** Q is false.		Q		
		IS FALSE	IS TRUE	
P	IS FALSE	1	0	
	IS TRUE	1	1	
				1

Table 2. The logical equivalent of the inverse statement **If** P is false **then** Q is false.

Without P is true **no** Q is true.		Q		
		IS TRUE	FALSE	
P	IS TRUE	1	1	
	IS FALSE	**0**	1	
				1

2.2.4. Proof by contradiction (Reductio ad absurdum)

In point of fact, it is difficult for scientists prove a theorem, a theory et cetera to be true for ever. Regardless of how many positive examples appear to support a theorem or a theory, one single counter-example or one single contradictory instance to a theory is sufficient enough to falsify the general validity of a theorem or of a theory et cetera. A proof by contradiction is such a scientific proof method which is able to proof the general the falsity or the truth of a statement, an equality, a principle (P) et cetera. "The proof ... *reductio ad absurdum*, which Euclid loved so much, is one of the mathematician's finest weapons" (Hardy 1992, p. 94). If the *goal* of a proof by contradiction is to prove that *P is not true*, then *assume* first that *P is true*. In the following, based on the assumption that P is true, it is necessary to be able to conclude or to derive something which is impossible or which is a contradiction. Under the conditions, that the logic of the proof by contradiction is sound (i.e. no technical errors et cetera), the only option is that the assumption that P is true is incorrect. Therefore, we must conclude that P is not true, which completes the proof. Something impossible or incorrect cannot be derived from something correct as long as there are nor technical or other errors inside a proof.

3. Results

THEOREM 3.1. (NULLITY (ANDERSON ET AL., 2007) AND PRINCIPIUM IDENTITATIS)

According to their axiom 7, Anderson et al. (Anderson et al., 2007) are respecting axiom 1 (*principium identitatis)* in the form –(-a) = a or a=a or +1=+1.

CLAIM.

The concept of Nullity is does not contradict the principium identitatis.

DIRECT PROOF.

In general, taking axiom 1 to be true, it is

$$+1 = +1 \tag{22}$$

Multiplying this starting point of this proof by Anderson et al. (Anderson et al., 2007) Nullity we obtain

$$+1 \times (Nullity) = +1 \times (Nullity) \tag{23}$$

According to Anderson et al. (Anderson et al., 2007) axiom 15 (Eq. 10) we obtain without any technical errors the result that

$$\text{Nullity} = \text{Nullity} \tag{24}$$

QUOD ERAT DEMONSTRANDUM.

Remark 1.

The concept of Nullity is claiming to be compatible with the principium identitatis. Thus far, it should not be possible to derived the same mathematical construct from something impossible. In other words, it is not possible to respect the principium identitatis and in the same case not to respect the principium identitatis. This is a contradiction.

THEOREM 3.2. (NULLITY (ANDERSON ET AL., 2007) IS SELF-CONTRADICTORY)

According to their axiom 7, Anderson et al. (Anderson et al., 2007) are respecting axiom 1 (*principium identitatis)* in the form –(-a) = a or a=a or +1=+1.

CLAIM.

The concept of Nullity is does not contradict the principium identitatis.

DIRECT PROOF.

In general, taking axiom 1 to be true, it is

$$+1 = +1 \tag{25}$$

The starting point is correct. In general, due to manipulations a proof may suffer some (technical) errors. A logically sound mathematical construct or logically sound mathematical rules should be able to “recognize” this. In this context, we add +2 on the left side of the equation and +3 on the right side of the equation

$$+1 + 2 = +1 + 3 \tag{26}$$

which is an **error** and do obtain at the end

$$+3 = +4 \tag{27}$$

Multiplying this contradictory and erroneous equation by Anderson et al. (Anderson et al., 2007) Nullity we obtain

$$+3 \times (Nullity) = +4 \times (Nullity) \tag{28}$$

According to Anderson et al. (Anderson et al., 2007) axiom 15 (Eq. 10) we obtain without any technical errors the result that

$$\text{Nullity} = \text{Nullity} \tag{29}$$

QUOD ERAT DEMONSTRANDUM.

Remark 2.

This proof is simple and very transparent thus that the error inside the proof can be found quickly which is not the case in general. Nullity itself has not been able to recognize that there is an error inside the proof performed, we obtained Nullity = Nullity. In other words, Nullity has the property to turn everything false into true. This is a contradiction and without any sense. It is not possible to rely on the behavior of Nullity.

THEOREM 3.3. (REFUTATION OF NULLITY (ANDERSON ET AL., 2007))

CLAIM.

Anderson et al. (Anderson et al., 2007) approach to the division by zero is based on the logical contradiction

$$+1 = +0 \tag{30}$$

DIRECT PROOF.

In general, taking axiom 1 to be true, it is

$$\mathbf{+1 = +1} \tag{31}$$

Any proof can be associated with an error which cannot be ignored by a logically sound mathematical object investigated. A logically sound mathematical object should be able to “recognize” such errors. In this context, we reduce erroneously the right side of the equation before by -1 to test how does Nullity behave under these circumstances and do obtain

$$+1 = +1 - 1 \tag{32}$$

which is an **error** or at the end that

$$+1 = +0 \tag{33}$$

Multiplying by (1/0) it is

$$+1 \times \left(\frac{+1}{+0}\right) = +0 \times \left(\frac{+1}{+0}\right) \tag{34}$$

or

$$+1 \times \left(\frac{+1}{+0}\right) = +1 \times \left(\frac{+0}{+0}\right) \tag{35}$$

or Saitoh’s self-contradictory equation (Barukčić, I. 2019b) as

$$\left(\frac{+1}{+0}\right) = \left(\frac{+0}{+0}\right) \tag{36}$$

Nullity is defined as (0/0) = Nullity. The equation can be rearranged as

$$\left(\frac{+1}{+0}\right) = \text{Nullity} \tag{37}$$

Multiplying by zero, it is

$$\left(\frac{+1}{+0}\right) \times (+0) = (+0) \times \text{Nullity} \tag{38}$$

or

$$\left(\frac{+0}{+0}\right) \times (+1) = \text{Nullity} \tag{39}$$

or

$$\text{Nullity} = \text{Nullity} \tag{40}$$

QUOD ERAT DEMONSTRANDUM.

Remark 3.

Nullity transfers something obviously false to something true. The concept of Nullity is logically inconsistent, self-contradictory and completely worthless.

THEOREM 3.4. (REFUTATION OF NULLITY (ANDERSON ET AL., 2007))

CLAIM.

Anderson et al. (Anderson et al., 2007) approach to the division by zero is based on the logical contradiction

$$+1 = +0 \tag{41}$$

DIRECT PROOF.

In general, taking axiom 2 to be true, it is

$$\mathbf{+1 = +0} \tag{42}$$

which as such is absolutely and obviously erroneous. This is equally the simplest mathematical form of Aristotle's law of contradiction (Barukčić, 2019). Anderson et al. (Anderson et al., 2007) are claiming that the transreal arithmetic as total arithmetic contains the real arithmetic without any arithmetical exceptions. In particular, Anderson et al. (Anderson et al., 2007) are pointing out by axiom 7 (Eq. 7) that **principium identitatis** is respected. In other words, Nullity as a mathematical construct of transreal arithmetic is not grounded on a logical contradiction. Thus far, Nullity taken as logical consistent cannot allow us nor is it possible to deduce something correct from such an incorrect and fallacious starting point. Multiplying the starting point of this proof by Anderson et al. (Anderson et al., 2007) Nullity we obtain

$$+1 \times (Nullity) = +0 \times (Nullity) \tag{43}$$

According to Anderson et al. (Anderson et al., 2007) axiom 15 (Eq. 10) we obtain without any technical errors the result that

$$\boldsymbol{Nullity = Nullity} \tag{44}$$

QUOD ERAT DEMONSTRANDUM.

Remark 4.

Define (P as +1=+1) and (Q as Nullity = Nullity). As demonstrated by theorem 3.1. Nullity is respecting the principium identitatis or in other words, if (P as +1=+1) is true then (Q as Nullity = Nullity) is true. In contrast to theorem 3.1., the theorem 3.2. is grounded on inversion too and we expect that if (P as +1=+0) is false then (Q as Nullity <> Nullity) is false. In contrast to expectation, we obtained a contradiction. We have been able to derive Nullity = Nullity without any mathematical contradiction from axiom 2, which is not allowed since it contradicts axiom 1. The concept of Nullity is self-contradictory and refuted.

THEOREM 3.5. (ANDERSON ET AL. AXIOM'S ARE SELF-CONTRADICTORY (ANDERSON ET AL., 2007))

CLAIM.

Anderson et al. axiom's are self-contradictory and based on a logical contradiction.

DIRECT PROOF.

In general, taking axiom 2 to be true, it is

$$(+\mathbf{1}) \quad = \quad (+\mathbf{0}) \tag{45}$$

In other words, we are starting with a contradiction. Whether a theorem or a theory grounded on such an axiom makes any sense, may be another question. Still, in the following of this proof no technical errors are allowed. Thus far, it is not allowed to derive Anderson et al. axiom or axiom's from such a disastrous starting point. In this case, such a ***direct proof*** provides evidence, that the same are grounded on a logical contradiction. Multiplying this equation by (+1/+0), we obtain according to our today's rules of mathematics that

$$\begin{aligned} (+1) \times \frac{(+1)}{(+0)} \quad &= \quad (+0) \times \frac{(+1)}{(+0)} \\ &or \\ (+1) \times \infty \quad &= \quad (+1) \times \frac{(+0)}{(+0)} \end{aligned} \tag{46}$$

according to Anderson's axiom 20 (Eq. 12). Nullity is defined as (+0/+0) = Nullity. The equation before can be rearranged and changes to

$$(+\infty) \quad = \quad (+Nullity) \tag{47}$$

Multiplying by (+0) we obtain

$$(+\infty) \times (+0) \quad = \quad (+Nullity) \times (+0) \tag{48}$$

According to Anderson et al axiom 15, this is equation is equivalent with

$$(+\infty) \times (+0) \quad = \quad (+Nullity) \tag{49}$$

or Anderson et al. axiom 16.

QUOD ERAT DEMONSTRANDUM.

Remark 5.

Anderson et al. axioms are logically inconsistent and self-contradictory. In a straightforward manner, it is possible to derive Anderson et al. axiom 16 without any technical errors while respecting Anderson et al. rules of trans-real arithmetic's from a logical contradiction. Anderson et al. axioms are grounded on a contradiction and worthless.

THEOREM 3.6. (REFUTATION OF TODAY'S RULE OF THE ADDITION OF ZERO'S)

Nicomachus of Gerasa (ca. 60 – ca. 120 AD), was born in Gerasa, a former Roman province of Syria, and is best known for his book *Introduction to Arithmetic*. Nicomachus of Gerasa (Nicomachus, pp. 48, 120, 237-238) claimed that *the sum of nothing added to nothing was nothing* or in other words it is 0+0+0 + …+ 0 = 0.

CLAIM.

Today's rule of the addition of zero's (+0 + 0 + … +0 = +0) is self-contradictory and based on a logical contradiction.

PROOF.

In general, taking axiom 2 to be true, it is

$$(\mathbf{1}) + (\mathbf{1}) + \cdots + (\mathbf{1}) \quad = \quad +(\mathbf{1}) \tag{50}$$

which is a non-acceptable contradiction. Multiplying this equation by 0, we obtain according to our today's rules of mathematics that

$$\begin{array}{ccc} (1 + 1 + \cdots + 1) \times 0 & = & +(1 \times 0) \\ & or & \\ (n\ times) \times 0 & = & +(1 \times 0) \end{array} \tag{51}$$

or that

$$(1 \times 0) + (1 \times 0) + \cdots + (1 \times 0) \quad = \quad +(0) \tag{52}$$

or today's rule of the addition of zero's as

$$(\mathbf{0}) + (\mathbf{0}) + \cdots + (\mathbf{0}) \quad = \quad +(\mathbf{0}) \tag{53}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.7. (THE CORRECT RULE OF THE ADDITION OF ZERO'S)

CLAIM.

The logically sound rule of the addition of zero's is n×0 = (+0 + 0 + … +0).

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) \quad = \quad +(\mathbf{1}) \tag{54}$$

Adding some terms we obtain

$$(1) + (1) + \cdots + (1) \quad = \quad (1) + (1) + \cdots + (1) \tag{55}$$

Multiplying this equation by 0, we obtain that

$$\begin{array}{ccc} (1 + 1 + \cdots + 1) \times 0 & = & (1 + 1 + \cdots + 1) \times 0 \\ & or & \\ (1 + 1 + \cdots + 1) \times 0 & = & (1 \times 0) + (1 \times 0) + \cdots + (1 \times 0) \end{array} \tag{56}$$

Let n = (1+1+…+1) and define n×0 = n_0 (Barukčić, 2018), we obtain

$$n \times 0 \quad = \quad \underbrace{+(0) + (0) + \cdots + (0)}_{n - times} = n\_0 \tag{57}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.8. (THE ADDITION OF INFINITE NUMBER OF ZERO'S)

CLAIM.

The addition of an infinite numbers of zero's is equivalent with $+\infty\times 0 = (+0 + 0 + 0 + \ldots)$.

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{58}$$

Define infinity as $+\infty = +1+1+1+\ldots$, we obtain

$$(1) + (1) + (1) + \cdots = (1) + (1) + (1) + \cdots \tag{59}$$

Multiplying this equation by 0, we obtain

$$\begin{aligned} (\infty) \times 0 &= (1 + 1 + 1 + \cdots) \times 0 \\ &\mathit{or} \\ \infty \times 0 &= (1 \times 0) + (1 \times 0) + (1 \times 0) + \cdots \end{aligned} \tag{60}$$

or

$$\infty \times 0 = \underbrace{+(0) + (0) + (0) + \cdots}_{\mathit{infinite\ number\ of\ zero's}} = \infty\_0 \tag{61}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.9. (THE INCORRECT ADDITION OF INFINITY)

CLAIM.

The addition of infinity as $(+\infty + \infty + \ldots +\infty = +\infty)$ is self-contradictory and based on a logical contradiction.

PROOF.

In general, taking axiom 2 to be true, it is

$$(\mathbf{1}) + (\mathbf{1}) + \cdots + (\mathbf{1}) = +(\mathbf{1}) \tag{62}$$

which is a non-acceptable contradiction. Multiplying this equation by $+\infty$, we obtain according to our today's rules of mathematics that

$$\begin{aligned} \underbrace{(1 + 1 + \cdots + 1)}_{n\ times} \times \infty &= +(1 \times \infty) \\ &\mathit{or} \\ (\quad n\ times \quad) \times \infty &= +\infty \end{aligned} \tag{63}$$

or that

$$(1 \times \infty) + (1 \times \infty) + \cdots + (1 \times \infty) = +(\infty) \tag{64}$$

At the end, *the incorrect rule of the addition of infinity* follows as

$$(\infty) + (\infty) + \cdots + (\infty) = +(\infty) \tag{65}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.10. (THE CORRECT RULE OF THE ADDITION OF INFINITY)

CLAIM.

The logically sound rule of the addition of infinity (∞) is $n \times \infty = (+\infty + \infty + \ldots +\infty)$.

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) \quad = \quad +(\mathbf{1}) \tag{66}$$

Adding some term, it follows that

$$(1) + (1) + \cdots + (1) \quad = \quad (1) + (1) + \cdots + (1) \tag{67}$$

Multiplying this equation by ∞, we obtain that

$$\begin{aligned} (1+1+\cdots+1) \times \infty \quad &= \quad (1+1+\cdots+1) \times \infty \\ &or \\ (1+1+\cdots+1) \times \infty \quad &= \quad (1 \times \infty) + (1 \times \infty) + \cdots + (1 \times \infty) \end{aligned} \tag{68}$$

Let $n = (1+1+\ldots+1)$ and define $n \times \infty = n\_\infty$ (Barukčić, 2018), we obtain

$$n \times \infty \quad = \quad \underbrace{+(\infty) + (\infty) + \cdots + (\infty)}_{n - times} \equiv n\_\infty \tag{69}$$

QUOD ERAT DEMONSTRANDUM.

Remark 6.

The theorem 3.7. is the first prove known which provides strict evidence that Euler's original position is correct: "Dieser Begriff von dem Unendlichen ist desto sorgfältiger zu bemerken, weil derselbe aus den ersten Gründen unserer Erkenntniß ist hergeleitet worden, und in dem folgenden von der größten Wichtigkeit seyn wird. Es lassen sich schon hier daraus schöne Folgen ziehen, welche unsere Aufmerksamkeit verdienen, da dieser Bruch **1/∞** den Quotus anzeigt, wann man das Dividend 1 durch den Divisor ∞ dividiret. Nun wissen wir schon, daß, wann man das Dividend 1 durch den Quotus, welcher ist **1/∞, oder 0** wie wir gesehen haben, dividiret, alsdann der Divisor nämlich ∞ heraus komme; daher erhalten wir einen neuen Begriff von dem Unendlichen, nämlich daß dasselbe herauskomme wann man 1 durch 0 dividiret; folglich kann man mit Grund sagen, daß **1 durch 0 dividiret eine unendlich große Zahl oder ∞ anzeige**. … Hier ist nöthig noch einen ziemlich gemeinen Irrthum aus dem Wege zu räumen, indem viele behaupten, ein unendlich großes könne weiter nicht vermehret werden. Dieses aber kann mit obigen richtigen Gründen nicht bestehen. Dann da 1/0 eine unendlich große Zahl andeutet, und 2/0 ohnstreitig zweymal so groß ist; so ist klar, daß **auch so gar eine unendlich große Zahl noch 2 mal größer werden könne**."

(Euler, 1771, p. 34). Euler's position stated in German can be translated (Barukčić, 2018) into English as follows:

"It is the more necessary to pay attention to this understanding of infinity, as it is derived from the first elements of our knowledge, and as it will be of the greatest importance in the following part of this treatise. We may here deduce from it a few consequences that are extremely nice and worthy of attention. The fraction 1/∞ represents the quotient resulting from the division of the dividend 1 by the divisor ∞. Now, we know, that if we divide the dividend l by **the quotient 1/∞, which is equal to 0** [*i.e. zero, author*], we obtain again the divisor ∞: hence we acquire a new understanding of infinity; and learn that it arises from the division of 1 by 0; so that we are thence authorized in saying, that **1 divided by 0 expresses a number infinitely great**, or ∞. ... It may be necessary also, in this place, to correct the mistake of those who assert, that a number infinitely great is not susceptible of increase. This position is inconsistent with the principles which we just have laid down; for 1/0 signifying a number infinitely great, and 2/0 being incontestably thee double of 1/0, **it is evident that a number, though infinitely great, may still become twice, thrice, or any number of times greater**."

THEOREM 3.11. (THE NORMALIZATION OF A FINITE AND INFINITY)

CLAIM.

Let x_t denote something (a number, a mathematical object et cetera), let $\underline{\infty}_t$ denote the non-infinite complementary of x, let ∞_t denote the infinite. In general, the normalization of the relationship between the non-infinite and the infinite follows as

$$\frac{x_t}{\infty_t} + \frac{\underline{\infty}_t}{\infty_t} = +1 \tag{70}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{71}$$

Multiplying by infinity ∞_t, it is

$$+1 \times +\infty_t = +1 \times +\infty_t \tag{72}$$

or

$$+\infty_t = +\infty_t \tag{73}$$

Adding x_t, it is

$$+\mathrm{x}_t + \infty_t = +\infty_t + \mathrm{x}_t \tag{74}$$

Rearranging equation, it is

$$+\mathrm{x}_t + \infty_t - \mathrm{x}_t = +\infty_t \tag{75}$$

and according to our definition

$$+x_t + \underline{\infty}_t = +\infty_t \tag{76}$$

Normalizing the relationship between the non-infinite $\underline{\infty}_t$ and the finite ∞_t, we obtain

$$\frac{+x_t}{+\infty_t} + \frac{+\underline{\infty}_t}{+\infty_t} = \frac{+\infty_t}{+\infty_t} = +1 \tag{77}$$

QUOD ERAT DEMONSTRANDUM.

Remark 7.

Under conditions where $+x_t = 0$, we obtain $+\underline{\infty}_t = +\infty_t - 0$ and or $+\underline{\infty}_t = +\infty_t$ and thus far

$$\frac{+0}{+\infty_t} + \frac{+\infty_t - 0}{+\infty_t} = \frac{+\infty_t}{+\infty_t} = +1 \tag{78}$$

or

$$\frac{+0}{+\infty_t} \quad = \quad +0 \tag{79}$$

Under conditions where $x_t = 1$, we obtain $\underline{\infty}_t = \infty_t - 1$ and thus far

$$\frac{1}{\infty_t} + \frac{\infty_t - 1}{\infty_t} = \frac{\infty_t}{\infty_t} = +1 \tag{80}$$

or

$$\frac{+1}{+\infty_t} = 1 - \left(\frac{+\infty_t - 1}{+\infty_t}\right) \approx +0 \tag{81}$$

THEOREM 3.12. (NEGATION AND INFINITY)

CLAIM.

From the standpoint of classical logic, negation is identical with 1/0.

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{82}$$

According to Barukčić (Barukčić and Barukčić, 2016; Barukčić, 2018), it is

$$\frac{+0}{+0} = +1 \tag{83}$$

Rearranging equation, we obtain

$$\frac{+1}{+1} \times \frac{+0}{+0} = +1 \tag{84}$$

or

$$\frac{+1}{+0} \times \frac{+0}{+1} = +1 \tag{85}$$

Simplifying, it is

$$\frac{+1}{+0} \times 0 = +1 \tag{86}$$

According to classical logic, ¬0=1. We obtain

$$\frac{+1}{+0} \times 0 = \neg \times 0 \tag{87}$$

The expressions on the left side of the equal sign denotes the same entity as the expression on the right side of the equal sign. Modify both sides of the equation before, at least according to optical criteria, it is

$$\frac{+1}{+0} = \neg \tag{88}$$

QUOD ERAT DEMONSTRANDUM.

Remark 8.

The theorem proves the logical meaning of 1/0. Assumed that classical logic is generally valid, such an approach to the problem of 1/0 could determine the foundation of a new understanding of indeterminate forms. Still, doubts about this relationship are allowed. It does not appear to be completely without any sense to assume that 1/0=∞. In this case, negation and infinity would be the same, which is difficult to accept. Furthermore, the theoretical question is not answered, is it possible at all, to reach infinity? In this case, it is possible that the infinite would be a finite, which is not.

THEOREM 3.13. (THE ORDER OF (MATHEMATICAL) OPERATIONS (RULES OF PRECEDENCE))

The order of operations used throughout science and technology and many computer programming languages is a way of reducing contradictions and the number of necessary parentheses. It is a matter of conventions about which procedures have to be performed first in order to evaluate a given mathematical equation. Symbols of grouping are used and must be used sometimes to override the usual order of operations. Today's order of precedence is something like the following.

1. Parentheses ( ) (sometimes replaced by brackets [ ] or braces { } for readability)
2. Negation
3. Exponents and roots
4. Multiplication and division
5. Addition and subtraction
6. …

CLAIM.

Until all problems related with indeterminate forms are solved, it is appropriate to use parentheses before dividing by zero

$$+1 = \frac{(+1-1)}{+0} \tag{89}$$

PROOF.

In general, taking axiom 1 <u>to be true</u>, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{90}$$

Subtracting -1, we obtain

$$+1-1 = +1-1 \tag{91}$$

or

$$+0 = +1-1 \tag{92}$$

Before dividing by zero, it is necessary to use the parentheses like

$$\frac{+0}{+0} = \frac{(+1-1)}{+0} \tag{93}$$

or

$$+1 = \frac{(+0)}{+0} \tag{94}$$

QUOD ERAT DEMONSTRANDUM.

Remark 9.

Ignoring the need to uses parenthesis could lead to unnecessary contradictions. Without the parenthesis, it is

$$\frac{+0}{+0} = \frac{+1}{+0} - \frac{+1}{+0} \tag{95}$$

or

$$+1 + \frac{+1}{+0} = \frac{+1}{+0} \tag{96}$$

or

$$+1 = +0 \tag{97}$$

which is a non-acceptable contradiction.

THEOREM 3.14. (0!=1 IS SELF-CONTRADICTORY)

CLAIM.

The definition of 0!=1 is self-contradictory.

DIRECT PROOF.

In general, taking axiom 2 to be true, it is

$$(+\mathbf{1}) = (+\mathbf{0}) \tag{98}$$

In other words, we are starting with a contradiction. The factorial of a positive integer n is denoted by n!. Taking the factorial on both sides, we obtain

$$(+\mathbf{1})! = (+\mathbf{0})! \tag{99}$$

The value of 1! =1 while the value of 0! is defined as 1. The equation before can be rearranged as

$$\mathbf{1} = \mathbf{1} \tag{100}$$

QUOD ERAT DEMONSTRANDUM.

Remark 10.

The proof is starting with a contradiction. In the following of the proof there are no technical errors. At the end, the proof provides a correct solution +1=+1. This is a contradiction, since from something incorrect follows something correct. The definition of 0!=1 is logically not consistent with the consequence that the use of factorials to solve the problem of indeterminate forms may not lead to correct solutions.

4. Discussion

Is it allowed to use the equation +1= +0 as a starting point of a theorem. Of course it is as long as it is assured the no further technical errors are made. In this case, we expect, even after several manipulations, that the result is again a contradiction. It is a problem, if a proof of a theorem starts with a contradiction and is ending with a logically sound result. In this case there must be a kind of an error located somewhere after the starting point (+1=+0) of the theorem proofed. Anderson's et al. (Anderson et al., 2007) Nullity is self-contradictory and refuted (theorem 3.2.). The proof is starting with a contradiction (+1=+0 or *P is false*) or something incorrect. In the following and according to the rules of trans-real arithmetic's, we obtain that Nullity = Nullity, which is correct. Still, if *P is false*, then it must be that *Q is false*, but it is not, Nullity = Nullity is correct. In other words, Nullity assures and demands that from something incorrect (+1=+0) follows something which is correct (Nullity = Nullity), which is a contradiction. Anderson et al. (Anderson et al., 2007) concept of Nullity conveys somehow the impression once the contact with Nullity is reached, there is no way back, everything which gets into contract with Nullity seems to be lost forever. Furthermore, Anderson et al. (Anderson et al., 2007) have not provided a method to convert the trans-real numbers into real numbers. The problem of the division of zero by zero is not solved by Anderson's et al. (Anderson et al., 2007) at all, but just redefined and transferred into a nebulous and purely subjective world full of contradictions and myths, the cold, dark and unknown world of trans-real arithmetic's. In other words, what is Nullity? However, the beauty of the division of zero by zero as it is, is independent of any trans-real arithmetic's or conscious awareness and cannot be treated as entirely subjective.

According to theorem 3.4., today's rule of the addition of zero's is logically inconsistent and refuted. We started with something which is not true or (+1 +1 + … + 1) = +1, something which is an apparent contradiction. Technical errors within the proof cannot be identified. Thus far, **if** (+1 +1 + … + 1) = +1 is false, **then** (+0+0+…+0) = +0 is false too, which completes our proof.

Today's understanding of the addition of infinity is inconsistent. According to theorem 3.6., if (+1+1+...+1) = +1 is not true then as (+∞ + ∞ + … +∞ = +∞) is not true. In contrast to this incorrect rule of the addition of infinity, Euler's position as proved by theorem 3.7. is right. If (+1=+1) is true then n×∞ = (+∞ + ∞ + … +∞) is true. This contradicts especially Anderson et al. (Anderson et al., 2007) axiom 5. In particular, the axioms of trans-real arithmetic's (i. e. axiom 16) are self-contradictory and worthless.

5. Conclusion

Anderson's et al. (Anderson et al., 2007) Nullity is logically inconsistent and refuted.

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