Ultimate algorithm for quantum computers

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We first propose herein a novel parallel computation, even though today's algorithm methodology for quantum computing, for all of the combinations of values in variables of a logical function. Our concern so far has been to obtain an attribute of some function. In fact such a task is only for one task problem solving. However, we could treat positively the plural evaluations of some logic function in parallel instead of testing the function for finding out its attribute. In fact, these evaluations of the function are naturally included and evaluated, in parallel, in normal quantum computing discussing a function in a Boolean algebra stemmed from atoms in it. As is naturally understandable with mathematics, quantum computing with qubit systems naturally is included and exemplified by a Boolean algebra, which treats only both 0 and 1. Namely, the theory in this paper is quite natural in logical sense even though physics domain. Therefore, quantum computing has an ability to solve some mathematical problems described in a Boolean algebra. The reason why we positively introduce a Boolean algebra here is because we have multiple evaluations of a function in quantum computing general.

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I. INTRODUCTION

Articles on the history of research into quantum computing [1] are mentioned as follows: An implementation of a quantum algorithm to solve Deutsch's problem [2–4] on a nuclear magnetic resonance quantum computer is reported [5]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is reported [6]. Oliveira *et al.* implements Deutsch's algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [7]. Single-photon Bell states are prepared and measured [8]. The decoherence-free implementation of Deutsch's algorithm is introduced by using such a single-photon and by using two logical qubits [9]. A one-way based experimental implementation of Deutsch's algorithm is reported [10].

In 1993, the Bernstein-Vazirani algorithm was published [11, 12]. In 1994, Simon's algorithm [13] and Shor's algorithm [14] were discussed. In 1996, Grover [15] provided the motivation for exploring the computational possibilities offered by quantum mechanics. An implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement in an ensemble quantum computer is mentioned [16]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [17]. The question whether or not quantum learning is robust against noise is a subject of a study [18].

A quantum algorithm for approximating the influences of Boolean functions and its applications are studied [19]. Quantum computation with coherent spin states and the close Hadamard problem are reported [20]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is studied [21]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [22]. The dynamical analysis of Grover's search algorithm in arbitrarily high-dimensional search spaces is studied [23]. The relation between quantum computer and secret sharing with the use of quantum principles is discussed [24]. An application of quantum Gauss-Jordan elimination code to quantum secret sharing code is studied [25]. Quantum circuit by one step method and similarity with neural network are discussed [26].

There are many researches concerning quantum computing, quantum algorithm, and their experiments. However, a complete understanding of a fundamental structure of quantum computing is not given. The key of this paper is to develop the algorithms of quantum computers toward the ultimate parallel processing on them. The way to do is to find out the very true ultimate parallelism, thinking of the physical quantum phenomena. The algorithm developed here is toward the full uses of the features of quantum computers. The algorithm implies the ability of such computation based upon the concept of a Boolean algebra. Finally we have the ultimate computation for today's quantum computers.

In this contribution, we first propose herein a novel parallel computation, even though today's algorithm methodology for quantum computing, for all of the combinations of values in variables of a logical function. Our concern so far has been to obtain an attribute of some function. In fact such a task is only for one task problem solving. However, we could treat positively the plural evaluations of some logic function in parallel instead of testing the function for finding out its attribute. In fact, these evaluations of the function are naturally included and evaluated, in parallel, in normal quantum computing discussing a function in a Boolean algebra stemmed from atoms in it. As is naturally understandable with mathematics, quantum computing naturally meets the category of a Boolean algebra. The reason why we positively introduce a Boolean algebra here is because we have multiple evaluations of a function in quantum computing general.

II. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE 2^1 MAPPINGS OF A FUNCTION

Our discussion is based on Nielsen and Chuang [27]. Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the mappings of a function $f(x)$ for many different x simultaneously. Suppose

$$f : \{0,1\} \rightarrow \{0,1\} \tag{1}$$

is a function with a one-bit domain and range. A convenient way of computing the function on a quantum computer is to consider a two-qubit quantum computer that starts with the state $|x, y\rangle$, where x and y are variables used in mapping f. The abbreviation $|x, y\rangle$ stands for $|x\rangle \otimes |y\rangle$.

Like the Deutsch-Jozsa problem, we are given a black box quantum computer known as an oracle that implements some function $f : \{0,1\}^2 \rightarrow \{0,1\}$. For the quantum algorithms to work, the oracle computing $f(x)$ from x has to be a quantum oracle which doesn't decohere x. It also mustn't leave any copy of x lying around at the end of the oracle call. We have the function f implemented as a quantum oracle. The oracle maps the state $|x\rangle \otimes |y\rangle$ to $|x\rangle \otimes |y \oplus f(x)\rangle$, where $\oplus$ is addition modulo 2.

It is possible to transform the state $|x, y\rangle$ into

$$|x, y \oplus f(x)\rangle, \tag{2}$$

by applying the quantum oracle, where $\oplus$ indicates addition modulo 2. We denote the transformation U_f defined by the map

$$U_f : |x, y\rangle = |x, y \oplus f(x)\rangle. \tag{3}$$

Here (2) and (3) meet the category of a Boolean algebra. Later we see the fact by having the result of Section III using (2) and (3). Namely, the result meets the category of a Boolean algebra. Quantum computing meets the category of a Boolean algebra.

From the usual phase kick-back formation and the map U_f, we insert an imaginary number i and we can define the following formulas:

$$\begin{aligned} &U_f|0\rangle(|0\rangle - i|1\rangle)/\sqrt{2} = +|0\rangle(|f(0)\rangle - i|\overline{f(0)}\rangle)/\sqrt{2} \\ &= \begin{cases} (-i)^{f(0)}|0\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if} \quad f(0) = 0, \\ (-i)^{f(0)}|0\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if} \quad f(0) = 1. \end{cases} \end{aligned} \tag{4}$$

$$\begin{aligned} &U_f|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} = +|1\rangle(|f(1)\rangle - |\overline{f(1)}\rangle)/\sqrt{2} \\ &= \begin{cases} (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if} \quad f(1) = 0, \\ (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if} \quad f(1) = 1, \end{cases} \end{aligned} \tag{5}$$

where $|\overline{1}\rangle = |0\rangle$ and $|\overline{0}\rangle = |1\rangle$. We use a combination between a unitary transformation theory and logic theory. In other words, we use a combination between a Pauli operator σ_x and NOT operation.

The phase of the result of (4) is different from the phase of the result of (5). Let us take a summation, that is, (4) pluses (5). Then we have (6). Therefore, we can solve the problem if we define the input state as (6) because we define the map U_f. Here we use a phase effect, which is a quantum phenomenon. We define the input state as follows, using an imaginary number i:

$$\begin{aligned} &|\psi_0\rangle = \alpha|0\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], \\ &\langle\psi_0|\psi_0\rangle = 1 \Leftarrow |\alpha|^2 + |\beta|^2 = 1, \alpha \neq 0, \beta \neq 0. \end{aligned} \tag{6}$$

Applying $U_{f_i}, (i = 0, 1, 2, 3)$ to $|\psi_0\rangle$, $U_{f_i}|\psi_0\rangle = |\psi_1\rangle_i$, therefore leaves us with one of 2^{2^1} cases, where the power 1 of 2^{2^1} means the case of one qubit:

$$\begin{aligned} &|\psi_1\rangle_0 = \alpha|0\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ &\text{then} \quad f_0(0) = 0, f_0(1) = 0, \\ &|\psi_1\rangle_1 = -i\alpha|0\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ &\text{then} \quad f_1(0) = 1, f_1(1) = 1, \\ &|\psi_1\rangle_2 = \alpha|0\rangle \left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ &\text{then} \quad f_2(0) = 0, f_2(1) = 1, \\ &|\psi_1\rangle_3 = -i\alpha|0\rangle \left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + \beta|1\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \\ &\text{then} \quad f_3(0) = 1, f_3(1) = 0, \end{aligned} \tag{7}$$

where these equations have a property that the relation between each equation and the condition after "then" is regarded as a "if and only if" condition since we herein process all of the operations only under a unitary transformation. So, the conditions after "then" are regarded as the results.

We want to develop the algorithms of quantum computers toward the ultimate parallel processing on them. The way to do is to find out the very true ultimate parallelism, thinking of the physical quantum phenomena. If we have (7), we know both $f(0)$ and $f(1)$ by measuring the single output state, simultaneously. How do we have (7)? Note that we cannot solve it by using only the usual phase kick-back formation. It changes only the global phase and we cannot distinguish between them. We want to avoid this situation.

So, by measuring $|\psi_1\rangle_i$, we may determine all the 2^1 mappings of $f_i(x)$ for all x simultaneously. This is very interesting indeed: the quantum algorithm gives us the ability to determine a perfect property of $f_i(x)$, namely, $f_i(x)$ itself. This is faster than a classical apparatus, which would require at least 2^1 evaluations.

Our algorithm is as follows:

- Select a function f_i.
- Operate U_{f_i} to $|\psi_0\rangle$ in giving $|\psi_1\rangle_i$.
- From $|\psi_1\rangle_i$, obtain all the mappings concerning the function f_i.

III. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE 2^2 MAPPINGS OF A FUNCTION

We propose a quantum algorithm for determining the 2^2 mappings of a function.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the mappings of a function $f(x)$ for many different x simultaneously. Suppose newly

$$f : \{0,1\}^2 \to \{0,1\} \tag{8}$$

is a function. We want to know the 2^2 mappings $f(0,0), f(0,1), f(1,0)$, and $f(1,1)$, simultaneously. Later we see a complete matching between our result and Table 1 (a Boolean algebra F_2). In the Boolean algebra F_2, the function is a two-valuable function. For example, $f(x,y)$ is the function where x and y are variables used in mapping f. In what follows, the abbreviation $f(xy)$ stands for $f(x,y)$. We see a combination between a unitary transformation theory and logic theory.

We define the input state as follows, using an application of (6):

$$\begin{aligned}
&|\psi_0\rangle = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right],\\
&\langle\psi_0|\psi_0\rangle = 1 \Leftarrow |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1, a_1 \neq 0, a_2 \neq 0, a_3 \neq 0, a_4 \neq 0.
\end{aligned} \tag{9}$$

From the map U_f, we can define the following formulas:

$$\begin{aligned}
&U_f|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} = +|00\rangle(|f(00)\rangle - i|\overline{f(00)}\rangle)/\sqrt{2}\\
&= \begin{cases} (-i)^{f(00)}|00\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(00) = 0,\\ (-i)^{f(00)}|00\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(00) = 1. \end{cases}
\end{aligned} \tag{10}$$

$$\begin{aligned}
&U_f|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} = +|01\rangle(|f(01)\rangle - i|\overline{f(01)}\rangle)/\sqrt{2}\\
&= \begin{cases} (-i)^{f(01)}|01\rangle(|0\rangle - i|1\rangle)/\sqrt{2} & \text{if } f(01) = 0,\\ (-i)^{f(01)}|01\rangle(|0\rangle + i|1\rangle)/\sqrt{2} & \text{if } f(01) = 1. \end{cases}
\end{aligned} \tag{11}$$

$$\begin{aligned}
&U_f|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} = +|10\rangle(|f(10)\rangle - |\overline{f(10)}\rangle)/\sqrt{2}\\
&= \begin{cases} (-1)^{f(10)}|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(10) = 0,\\ (-1)^{f(10)}|10\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(10) = 1. \end{cases}
\end{aligned} \tag{12}$$

$$\begin{aligned}
&U_f|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} = +|11\rangle(|f(11)\rangle - |\overline{f(11)}\rangle)/\sqrt{2}\\
&= \begin{cases} (-1)^{f(11)}|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(11) = 0,\\ (-1)^{f(11)}|11\rangle(|0\rangle - |1\rangle)/\sqrt{2} & \text{if } f(11) = 1, \end{cases}
\end{aligned} \tag{13}$$

where $|\overline{1}\rangle = |0\rangle$ and $|\overline{0}\rangle = |1\rangle$. We use a combination between a unitary transformation theory and logic theory.

Applying $U_{f_i}, (i = 0, 1, 2, ..., 15)$ to $|\psi_0\rangle$, $U_{f_i}|\psi_0\rangle = |\psi_1\rangle_i$, therefore leaves us with one of 2^{2^2} cases:

$$|\psi_1\rangle_0 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_0(00) = 0, f_0(01) = 0, f_0(10) = 0, f_0(11) = 0, \tag{14}$$

$$|\psi_1\rangle_1 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_1(00) = 0, f_1(01) = 0, f_1(10) = 0, f_1(11) = 1, \tag{15}$$

$$|\psi_1\rangle_2 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_2(00) = 0, f_2(01) = 0, f_2(10) = 1, f_2(11) = 0, \tag{16}$$

$$|\psi_1\rangle_3 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_3(00) = 0, f_3(01) = 0, f_3(10) = 1, f_3(11) = 1, \tag{17}$$

$$|\psi_1\rangle_4 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_4(00) = 0, f_4(01) = 1, f_4(10) = 0, f_4(11) = 0, \tag{18}$$

$$|\psi_1\rangle_5 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_5(00) = 0, f_5(01) = 1, f_5(10) = 0, f_5(11) = 1, \tag{19}$$

$$|\psi_1\rangle_6 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_6(00) = 0, f_6(01) = 1, f_6(10) = 1, f_6(11) = 0, \tag{20}$$

$$|\psi_1\rangle_7 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$
$$\text{then } \; f_7(00) = 0, f_7(01) = 1, f_7(10) = 1, f_7(11) = 1, \tag{21}$$

$$|\psi_1\rangle_8 = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_8(00)=1, f_8(01)=0, f_8(10)=0, f_8(11)=0, \tag{22}$$

$$|\psi_1\rangle_9 = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_9(00)=1, f_9(01)=0, f_9(10)=0, f_9(11)=1, \tag{23}$$

$$|\psi_1\rangle_{10} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{10}(00)=1, f_{10}(01)=0, f_{10}(10)=1, f_{10}(11)=0, \tag{24}$$

$$|\psi_1\rangle_{11} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle-i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{11}(00)=1, f_{11}(01)=0, f_{11}(10)=1, f_{11}(11)=1, \tag{25}$$

$$|\psi_1\rangle_{12} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{12}(00)=1, f_{12}(01)=1, f_{12}(10)=0, f_{12}(11)=0, \tag{26}$$

$$|\psi_1\rangle_{13} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{13}(00)=1, f_{13}(01)=1, f_{13}(10)=0, f_{13}(11)=1, \tag{27}$$

$$|\psi_1\rangle_{14} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{14}(00)=1, f_{14}(01)=1, f_{14}(10)=1, f_{14}(11)=0, \tag{28}$$

$$|\psi_1\rangle_{15} = -ia_1|00\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - ia_2|01\rangle\left[\frac{|0\rangle+i|1\rangle}{\sqrt{2}}\right] - a_3|10\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$
$$\text{then}\quad f_{15}(00)=1, f_{15}(01)=1, f_{15}(10)=1, f_{15}(11)=1. \tag{29}$$

So, by measuring $|\psi_1\rangle_i$, we may determine all the 2^2 mappings of $f_i(x,y)$ for all x and y simultaneously. This is very interesting indeed: the quantum algorithm gives us the ability to determine a perfect property of $f_i(x,y)$, namely, $f_i(x,y)$ itself. This is faster than a classical apparatus, which would require at least 2^2 evaluations.

Later we discuss the relation between Set theory based upon atoms and our result in terms of a Boolean algebra. Especially the result reveals a complete matching between quantum computing and a Boolean algebra (See Table 1). As is naturally understandable with mathematics, quantum computing belongs to the category of a Boolean algebra. We positively mention that the fundamental structures of quantum computing and Von Neumann architecture are the same in terms of the category of a Boolean algebra. However, the main different is based on parallelism for determining all the mappings used especially in quantum computing.

A. Example

Two level systems are included and exemplified by a Boolean algebra, which treats only both 0 and 1. Namely, the theory in this paper is quite natural in logical sense even though physics domain. Therefore, quantum computing has an ability to solve some mathematical problems described in a Boolean algebra.

Let us consider the case where $i=1$. The logical function is as follows:

$$f_1(x,y) = A \wedge B. \tag{30}$$

where x and y are variables used in mapping f. $x(=0,1)$ is variable for A. $y(=0,1)$ is variable for B. We want to know all the mappings

$$f_1(0,0), f_1(0,1), f_1(1,0), f_1(1,1). \tag{31}$$

In the classical case we require 2^2 evaluations. In the quantum case we require just one query.

The input state is as folows:

$$|\psi_0\rangle = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] + a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{32}$$

Applying U_{f_1} to $|\psi_0\rangle$, $U_{f_1}|\psi_0\rangle = |\psi_1\rangle_1$, we have the following output state:

$$|\psi_1\rangle_1 = a_1|00\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_2|01\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + a_3|10\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] - a_4|11\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{33}$$

Therefore we obtain all the mappings of $f_1(x, y)$ simultaneously:

$$f_1(0,0) = 0, f_1(0,1) = 0, f_1(1,0) = 0, f_1(1,1) = 1. \tag{34}$$

This is faster than a classical apparatus, which would require at least 2^2 evaluations. Likewise, we can evaluate 16 functions in a Boolean algebra F_2.

IV. A NEW TYPE OF QUANTUM ALGORITHM FOR DETERMINING THE 2^N MAPPINGS OF A FUNCTION

We propose a quantum algorithm for determining the 2^N mappings of a function. N means the number of qubits for our algorithm.

Quantum superposition is a fundamental feature of many quantum algorithms. It allows quantum computers to evaluate the mappings of a function $f(x_1, x_2, ..., x_N)$ for many different $x_1, x_2, ..., x_N$ simultaneously. Suppose newly

$$f : \{0,1\}^N \to \{0,1\} \tag{35}$$

is a function.

We define the input state as follows, using an application of (6):

$$\begin{aligned} |\psi_0\rangle &= \sum_{j=0}^{2^{(N-1)}-1} a_j|j\rangle\left[\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right] + \sum_{k=2^{(N-1)}}^{2^N-1} a_k|k\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right], \\ \langle\psi_0|\psi_0\rangle &= 1 \Leftarrow |a_0|^2 + |a_1|^2 + ... + |a_{2^N-1}|^2 = 1, a_0 \neq 0, a_1 \neq 0, ..., a_{2^N-1} \neq 0. \end{aligned} \tag{36}$$

Applying $U_{f_i}, (i = 0, 1, 2, ..., 2^{2^N} - 1)$ to $|\psi_0\rangle$, $U_{f_i}|\psi_0\rangle = |\psi_1\rangle_i$, therefore leaves us with one of 2^{2^N} cases:

$$|\psi_1\rangle_i = \sum_{j=0}^{2^{(N-1)}-1} (-i)^{f_i(j)} a_j|j\rangle\left[\frac{|0\rangle - (-1)^{f_i(j)} i|1\rangle}{\sqrt{2}}\right] + \sum_{k=2^{(N-1)}}^{2^N-1} (-1)^{f_i(k)} a_k|k\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{37}$$

So, by measuring $|\psi_1\rangle_i$, we may determine all the 2^N mappings of $f_i(x_1, x_2, ..., x_N)$ for all $x_1, x_2, ..., x_N$ simultaneously. This is very interesting indeed: the quantum algorithm gives us the ability to determine a perfect property of $f_i(x_1, x_2, ..., x_N)$, namely, $f_i(x_1, x_2, ..., x_N)$ itself. This is faster than a classical apparatus, which would require at least 2^N evaluations.

V. THE RELATION BETWEEN THE ATOMS (SET THEORY) AND THE RESULT IN SECTION III

Let us discuss the relation between the atoms [28] (Set theory) and the result in Section III. These A and B are subsets which are constructed using the atoms $f1$ through $f4$ that are disjoint one another. For example, newly using fi as an element of a Boolean algebra F_2,

$$\begin{aligned} A &= f1 \vee f3 = \{f1, f3\}, \\ B &= f1 \vee f2 = \{f1, f2\}, \end{aligned} \tag{38}$$

where,

$$\begin{aligned} f1 &= A \wedge B, \\ f2 &= A' \wedge B, \\ f3 &= A \wedge B', \\ f4 &= A' \wedge B'. \end{aligned} \tag{39}$$

See FIG. 1 (Venn diagram for F_2).

We can introduce a Boolean algebra F_2 as a Power set of the atoms. See FIG. 2.

F_2 is based on the value "1" of the two-variable switching functions. An atom is a function including only one "1" as its mapped value, in the four combinations of the values of A and B for the two-variable function. See Table 1.

Clearly we notice a complete matching between Table 1 (the Boolean algebra F_2) and our result in Section III. In fact we can see that Eqs. (15), (16), (18), and (22) are regarded as the four atoms of the Boolean algebra F_2. For example, we notice (15) OR operation with (16) is equal to (17) and all elements are derived from the four atoms. (See FIG. 2).

We see that the relation between Set theory based upon atoms and our result in terms of a Boolean algebra.

The important point is that we obtain all the elements of F_2 by means of a Power set of atoms when we get the four atoms. (See FIG. 2).

So we can say that next our aim is of getting (15), (16), (18), and (22) simultaneously. That means we get (14)-(29) simultaneously (all 16 patterns!)

We positively stress that the fundamental structures of quantum computing and Von Neumann architecture are the same in terms of the category of a Boolean algebra. However, the main different is based on parallelism for determining all the mappings used especially in quantum computing. We hope our discussions conclude the very true ultimate importance of the quantum parallelism to construct quantum computers beyond Von Neumann architecture.

VI. CONCLUSIONS

In conclusion, we have first proposed herein a novel parallel computation, even though today's algorithm methodology for quantum computing, for all of the combinations of values in variables of a logical function. Our concern so far has been to obtain an attribute of some function. In fact such a task has been only for one task problem solving. However, we could have treated positively the plural evaluations of some logic function in parallel instead of testing the function for finding out its attribute. In fact, these evaluations of the function have been naturally included and evaluated, in parallel, in normal quantum computing discussing a function in a Boolean algebra stemmed from atoms in it. As is naturally understandable with mathematics, quantum computing naturally has met the category of a Boolean algebra. The reason why we positively introduce a Boolean algebra here has been because we have multiple evaluations of a function in quantum computing general.

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NOTE

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Table 1:　　　A Boolean algebra F_2

A B	0 0 1 1 0 1 0 1	Expressions A and B Representing the Function
f0	0 0 0 0	0
f1	0 0 0 1	$A \wedge B$
f2	0 0 1 0	$A \wedge B'$ or $A \not\Rightarrow B$
f3	0 0 1 1	A
f4	0 1 0 0	$A' \wedge B$ or $A \not\Leftarrow B$
f5	0 1 0 1	B
f6	0 1 1 0	$A \triangle B$ or Exclusive OR (A,B) $= (A \wedge B') \vee (A' \wedge B)$
f7	0 1 1 1	$A \vee B$
f8	1 0 0 0	$A' \wedge B'$ or NOR(A,B)
f9	1 0 0 1	$A' \triangle B'$ or $A \Leftrightarrow B$
f10	1 0 1 0	B’
f11	1 0 1 1	$A \vee B'$ or $A \Leftarrow B$
f12	1 1 0 0	A’
f13	1 1 0 1	$A' \vee B$ or $A \Rightarrow B$
f14	1 1 1 0	$A' \vee B'$ or NAND(A,B)
f15	1 1 1 1	1

Venn diagram for *F2*

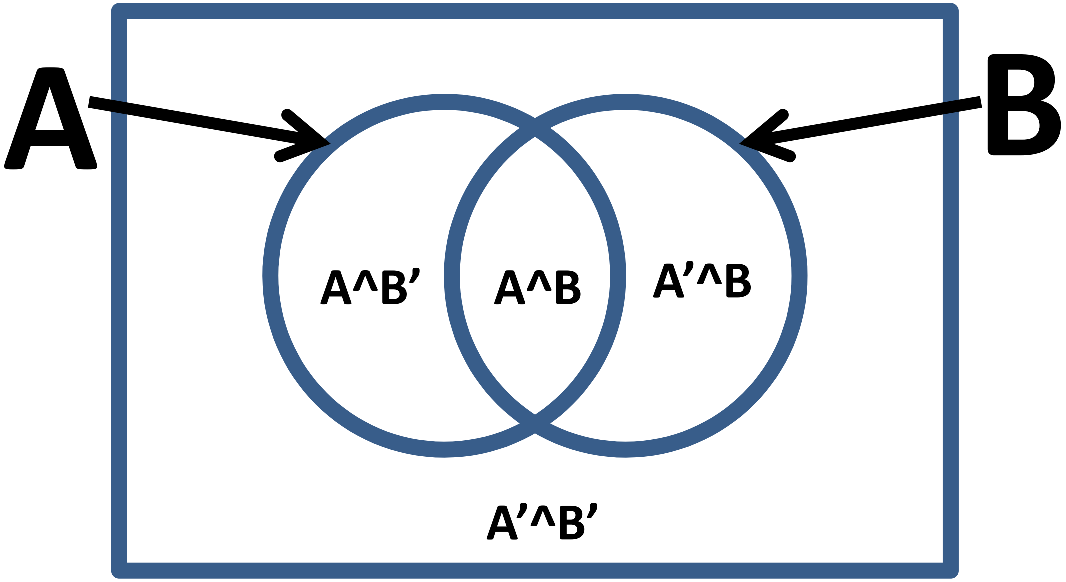

FIG. 1 NOTICE: *F2* is formed based on the value "1" in the domain and codomain of all the 16 two-variable A, B switching functions, where all atoms divide perfectly and independently the Venn diagram without overlapping.

Two-variable switching functions

Set *F*2 of two-variable switching functions fi(A, B)

*F*2={f0, f1, f2, . . . , f15,}

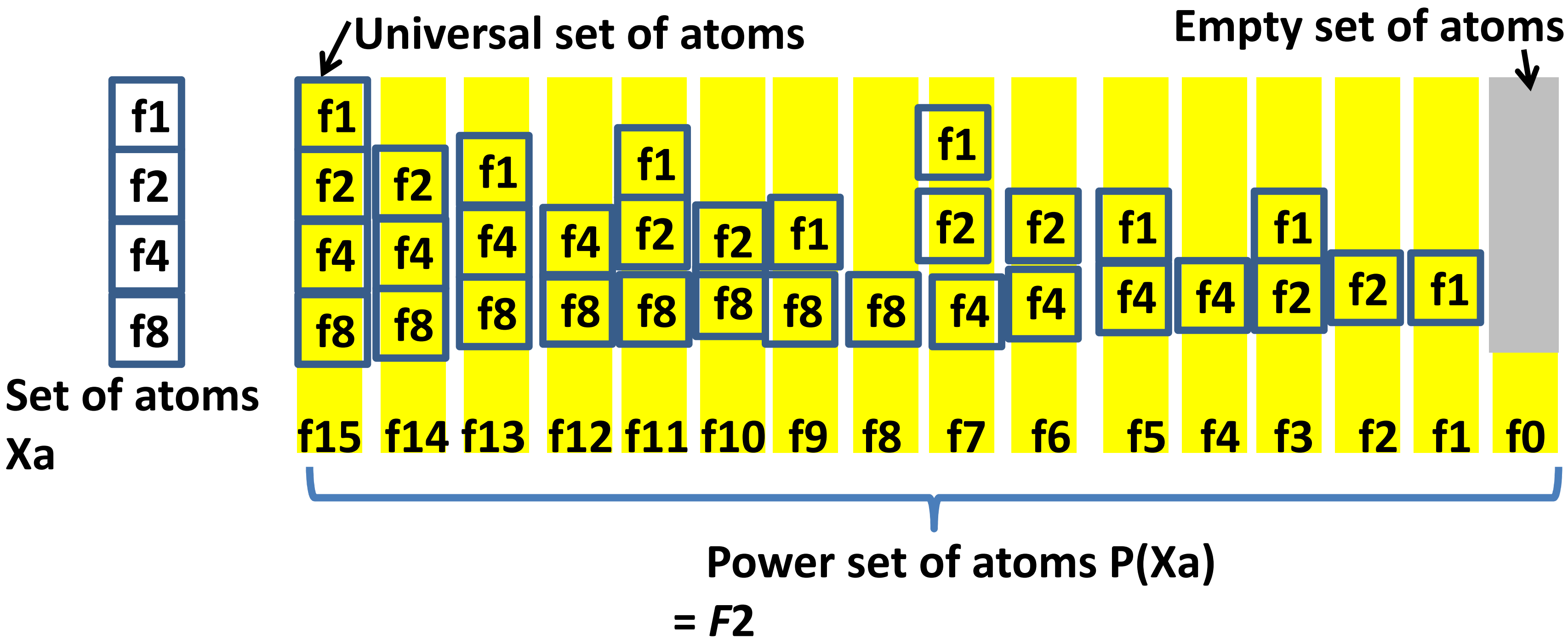

FIG. 2 NOTICE: *F*2 is based on the value "1" of the switching functions. An atom is a function including only one "1" as its mapped value, in the four combinations of the values of A and B for the two-variable function.