The Gravitational Quantum Limit of the Inverse Square Law

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Abstract

This paper attempts to give a theoretical foundation for the Modified Newtonian Dynamics equations developed by M. Milgrom Ref.1. It will show that there is a cross sectional limit, below which $1/r^2$ asymptotically changes to 1/r. When the cross section of a distant star falls below the minimum cross section of the geodesics of the paths of gravitons, then the inverse square law is no longer valid. Gravitational attraction then depends on the distance travelled by the gravitons and will result in the force of gravity varying as 1/r. Also, the mass, GM, used in Newton's Law of Universal Gravitation will change to $\sqrt{Ga_0M_a}$. This is due to multiple paths of gravitons merging into one path. Geodesics have very small cross sections which resonate and continuously adjust to changes in space-time.

Keywords: MOND, inverse square law, limit of inverse square law, Milgrom, graviton, a_0, 1/r, geodesic, quantum limit, gravity, Modified Newtonian Dynamics.

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Introduction

MOND explains the rotation curves of galaxies, at various luminosities, very well. "Milgrom's law fully specifies the rotation curve of a galaxy given only the distribution of its baryonic mass. In particular, MOND predicts a far stronger correlation between features in the baryonic mass distribution and features in the rotation curve than does the dark matter hypothesis (since dark matter dominates the galaxy's mass budget and is conventionally assumed not to closely track the distribution of baryons)." Quoted from Wikipedia Ref 1. MOND is a simpler solution to explain the rotation curves of galaxies.

However, most cosmologists support the Standard Cosmological Model, which includes cold dark matter Ref. 2. "Since the 1980s, it has been widely accepted that the baryonic matter in the Universe would not itself provide enough gravitational attraction to form the observed structures by the present age of the Universe. This problem can be circumvented by the introduction of non-baryonic dark matter, which provides the extra gravitational force to allow structures to form more quickly and is not inhibited by pressure effects. This view has been vindicated by modern precision measurements of the cosmic microwave background." Quoted from Andrew Liddle's Modern Cosmology Ref. 3.

Applicable Laws of Physics and Cosmological Observations

Kepler's Laws of Planetary Motion Ref. 10

Newton's law of Universal Gravitation Ref. 13

Newton's Second Law of Motion Ref. 13

M. Milgrom's Modified Newtonian Dynamics Ref. 1

V. Rubin's mapped rotation velocities of stars Ref. 11

R. Scarpa's Review of Modified Newtonian Dynamics Ref. 15

Gravity is not limited in distance Ref. 14

The graviton travels at the speed of light. Ref. 12

Gravity only pulls, unlike electro-magnetism Ref. 9

The diameter of the graviton is 8×10^{-65} cm^2 Ref. 16

Assumptions for this paper

The gravitational interactions occur on a particle level. Equations are integrals summing all the particle level interactions of suns, planets, moons, gases and black holes.

A probable gravitational interaction: a graviton is emitted by a proton, on sun A, and is absorbed by a proton on sun B. The graviton is re-emitted and pulls this proton a little closer towards sun A. The graviton travels at the speed of light to sun A and is absorbed by another proton. When it is re-emitted, it pulls this proton a little closer to sun B. Energy is conserved by adjustments in potential and kinetic energies of the protons.

The geodesics along which gravitons travel have very small cross sections. They constantly adjust their curvatures and diameters. There is a minimum cross section of geodesic tubes. Particles, such as protons, neutrons, photons and gravitons, are travelling along these tubes. These particles may make the geodesic tubes resonate, which implies that geodesic tubes carry energy.

As understood in this paper, mathematical points have no dimensions and mathematical lines have one dimension. Physical points have an extremely small volume which approaching a mathematical point. Physical lines, such as geodesics, have extremely small diameters, and follow the mathematical lines of geodesics. The above properties are due to quantum physics.

Geometric Calculations and a Hypothesis of Geodesic Streams

At $a_0 \approx 1.2 \pm 0.2 \times 10^{-8} cm\, s^{-2}$ (Ref. 4) is an inflection point, where $1/r^2$ changes to 1/r, according to Milgrom. From galactic observations of rotational velocities of stars, we know that there is a limit to the inverse square law. What could cause this? Quantum mechanics must be taking over, but there is no broadly accepted law of quantum gravity. Physicists suspect that a particle is involved in transmitting the force of gravity, the hypothesized graviton. Physicists calculated the cross section for absorption of a graviton to be 8 x 10^{-65} m^2 (Ref. 16), which is extremely small. The cross section of the stream of gravitons was calculated below as 1.07 x 10^{-42} m^2. This could be interpreted that there is a minimum cross-sectional limit of the stream of bosons from a distant star. If the image of this distant star falls below this limit, the cone of gravitons becomes a line. The frequency of interactions depend then on the distance particles need to travel between the stars. If a very distant star is even further away, the graviton needs to travel further resulting in a lower frequency of interactions. Of course, the particle mode of interaction is also true for a closer star, but then the cone becomes wider and a huge number of streams of gravitons flow between these two stars, which results again in $1/r^2$.

$$\left(\frac{\mathbf{1}}{\boldsymbol{r}}\right)^{\mathbf{2}} \rightarrow \boldsymbol{a_0} \rightarrow \left(\frac{\mathbf{1}}{\boldsymbol{r}}\right)^{\mathbf{1}} \qquad 1.0$$

On the left is the Newtonian law dependence on the inverse of distance squared and on the right is MOND regime dependence on the inverse of distance. Here the frequency of interactions is inversely proportional to the time that it takes gravitons to travel between the stars. The further stars are apart the lower the frequency. Exponent 2 indicates the area of the cone with many streams of particles at slightly different angles and exponent 1 indicates a line, that is a single stream of particles.

There is another effect: If the image of a star on a proton is smaller than the hypothesized cross-section of the geodesic, then the image of the star changes from an area to a point. This will result in

$$GM_a \rightarrow a_0 \rightarrow \sqrt{Ga_0M_a}$$

M_a is the mass generating the field. (The whole equation will be derived in section MOND Basics Derived.) The square root is the result of changing from an area of the base of the cone to a physical point.

In Fig. 1, stars A and B are stars with masses like our Sun. For simple calculations, there are no other stars or gas clouds near stars A and B. Using Newton's law of universal gravitation to find the distance between them when $a_0 = 1.2 \pm 0.2 \times 10^{-10} ms^{-2}\ Ref. 1$.

$$F = \frac{Gm_am_b}{r^2} \qquad \text{Newton's law of universal gravitation} \qquad 2.1$$

G = 6.674 x 10^{-11} N m^2 kg^{-2} Ref.5 Gravitational constant

$m_a = m_b = 1.989 \times 10^{30}$ kg Ref. 6 Mass of our Sun

r = distance between stars A and B

$$F = ma = m\frac{dv}{dt} \quad \text{Newton's Second law of motion} \qquad 2.2$$

Using equations 2.1 and 2.2 $\quad ma = \frac{Gm_a m_b}{r^2} \quad a = \frac{Gm_a}{r^2}$

Solving for r $\quad r = \sqrt{\frac{Gm}{a}}$ The distance between star A and B $\qquad 2.3$

$$r = \sqrt{\frac{6.67\times10^{-11}\times1.99\times10^{30}}{1.2\times10^{-10}}}$$

$$r = \sqrt{1.106 \times 10^{30}} = 1.05 \times 10^{15} \text{ m} = 0.111 \text{ ly}$$

Light year = 9.461×10^{15} m Ref. 7. This is a surprisingly short distance.

In Fig. 2 the tangent of the cone is shown (not to scale). Θ = the angle of the cone. The radius of the Sun is 6.957×10^8 m Ref. 6.

$$Tan\left(\frac{1}{2}\theta\right) = \frac{r_{sun}}{s_{ab}} = \frac{6.96\times10^{8}}{1.05\times10^{15}} = 6.63 \times 10^{-7} \qquad 2.4$$

In Fig. 3 the cross section σ of the streams gravitons on a proton is pictured (not to scale).

Radius of proton $r_p = 0.88 \times 10^{-15}$m Ref. 8; r_σ = radius of cross section of streams of gravitons.

$$Tan\left(\frac{1}{2}\theta\right) = \frac{r_\sigma}{r_p} \qquad 3.1$$

$$Tan\left(\frac{1}{2}\theta\right) = \frac{\sqrt{\frac{\sigma}{\pi}}}{r_P} \qquad 3.2$$

$$\sigma = \pi\left(tan\left(\frac{1}{2}\theta\right)r_p\right)^2 = \pi(6.63 \times 10^{-7} \times 0.88 \times 10^{-15})^2 = 1.07 \times 10^{-42} \text{ m}^2. \qquad 3.3$$

Acceleration at any point in space depends on the sum of the integrals from all directions.

$$\vec{a} = \int \frac{GM_a}{\vec{r}^2} + \int \frac{\sqrt{GM_a a_0}}{\vec{r}} \qquad 1.1$$

M_a is the mass generating the field. (Inclusion of a_0 will be derived further on.)

$\int \frac{GM_a}{\vec{r}^2}$ will approach zero as a star moves far away from any star or planet, at least further than 0.111 ly.

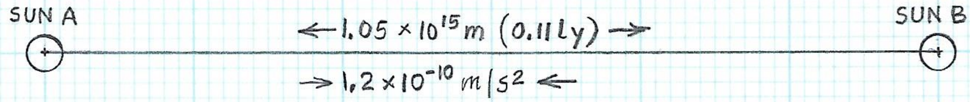
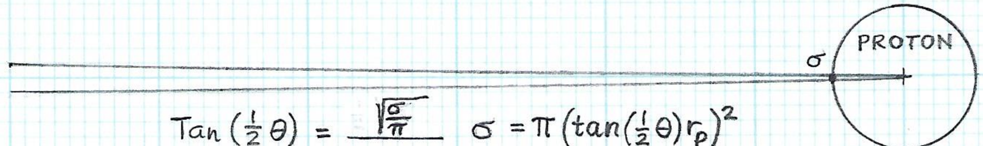


Fig. 1

Inflection point of Inverse Square law

$$\operatorname{Tan}\left(\tfrac{1}{2}\theta\right) = \frac{r_{\text{SUN A}}}{\text{Distance A} \rightarrow \text{B}}$$

Fig. 2



$$\operatorname{Tan}\left(\tfrac{1}{2}\theta\right) = \frac{\sqrt{\frac{\sigma}{\pi}}}{r_{\text{PROTON}}} \qquad \sigma = \pi\left(\tan\left(\tfrac{1}{2}\theta\right) r_p\right)^2$$

Fig.3

Cross section of a stream of gravitons on a proton

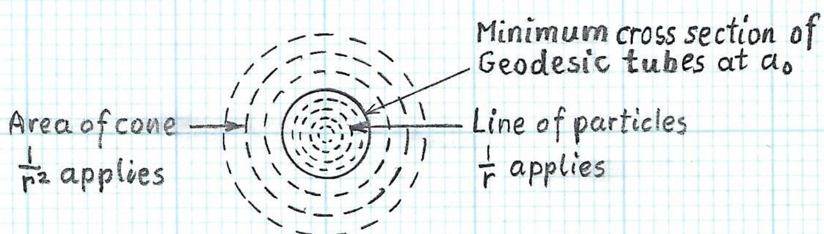


Fig. 4

Size of image of star on proton

MOND Basics Derived

Milgrom's law shows how the Newtonian acceleration a_N needs to be modified in very low acceleration space Ref. 1.

$$a_N = a\mu\left(\frac{a}{a_0}\right) \qquad 1.2$$

$a_0 = 1.2 \times 10^{-8}$ cm/s^2, which is a new constant empirically determined by Milgrom.

The interpolation function $\mu(a/a_0)$ results in the asymptotic behavior $\mu=1$ for $a>>a_0$, to retrieve the Newtonian expression in the strong field regime, and $\mu = a/a_0$ for $a<<a_0$ in the weak field regime Ref.15

The weak acceleration limit of gravity is

Substituting $\frac{GM_a}{r^2}$ for a_N and taking the square root

$$\mathrm{a} = \sqrt{a_N a_0} = \frac{\sqrt{GMa_0}}{r} \qquad 1.3$$

In a weak field regime, acceleration changes 1/r with distance r from mass M_a generating field.

Conclusions and Further Research

The main assertion of this paper is that the inverse square law changes from analog to digital at the inflection point a_0, where the acceleration due to gravity is equal to or less than 1.2 x 10^{-10} m/s^2. This is due to the geodesic tubes having minimum cross-sections. Particles, such as protons, neutrons, photons and gravitons, are travelling along these tubes. The baryons and bosons make the geodesic tubes resonate, which implies that geodesic tubes carry energy.

Th energy in geodesic tubes may be the mysterious dark energy of space-time.

The following three paragraphs are my opinion: Dark matter was needed to provide gravitational wells, 380,000 years after the big bang, to initiate and accelerate the formation of baryonic structures in our universe, including our Milky Way galaxy. Once stars had formed, the gravity of stars and black holes mostly took over.

Even with dark matter, there are vast regions in our universe where gravity is below a_0. In these regions MOND equations will apply. The distribution of dark matter is constrained very weakly, which makes it easy to make calculations come out to proof one's hypotheses, whereas a_0 is a constant.

Both ΛCDM and MOND are needed to explain astronomical observations in our universe. Gravitational equations will need to be modified in the weak field regimes.

If there is a limit to the inverse law, as claimed above, the following will need to be investigated further.

Gravity will decrease only by 1/r in the vast spaces of our Universe. This will make gravity much more powerful over the whole universe. The ratio of baryonic matter to dark matter needs to be recalculated.

References

1 https://en.wikipedia.org/wiki/Modified_Newtonian_dynamics

2 *An Introduction to Modern Cosmology* by A. Liddle, Chapter 15 Overview: The Standard Cosmological Model

3 *An Introduction to Modern Cosmology* by A. Liddle, Chapter 9, section 1.5 The formation of structure, page 71

4 www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics Where you will find a collection of detailed scholarly articles on MOND.

5 Wikipedia/wiki/Gravitational_constant

6 Wikipedia/wiki/Sun

7 Wikipedia/wiki/Light-year

8 Phys.org/new/2013-11-proton-radius-puzzle-quantum-gravity

9 https://www.newscientist.com/article/mg20227122-800-gravity-mysteries-why-does-gravity-only-pull/

10 https://en.wikipedia.org/wiki/Kepler%27s_laws_of_planetary_motion

11 https://en.wikipedia.org/wiki/Vera_Rubin

12 https://en.wikipedia.org/wiki/Speed_of_gravity

13 Physics text books by H. Ohanian, Rensselaer Polytechnic Institute

14 https://en.wikipedia.org/wiki/Gravity

15 Scarpa R., title of paper “Modified Newtonian Dynamics, an Introductory Review” European Southern Observatory, Chile;

16 Dyson F., title of paper “Is a Graviton Detectable?”