ON THE WAVE MOTION AND INERTIAL MASS OF QUANTUM PARTICLES

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work we will discuss the possibility that if a quantum particle possesses a fluid state, as shown in our work on the fluid state of Dirac quantum particles, then the motion of the physical particle in an ambient space may be directly related to its fluid state and therefore it can be explained in terms of the wave motion of physical objects that move in a substrate space with the mechanism similar to that of peristaltic locomotion. In particular, we will also discuss the possibility that the so-called inertial mass of a particle in classical physics may be identified as a form of gripping connection between the particle and the substrate, which is the ambient space itself, that is needed for a particle to move in space. This is similar to a tight grip that is needed for a physical object to move on a surface.

In physics, normally, an analytical dynamics of a physical particle is formulated relying on the perception and examination of the motion of the particle in space under the influence of physical fields. And it has been shown that the dynamics is determined not only by the external physical fields but also by the physical properties associated with the particle itself, such as an inertial mass m and a charge q. An inertial mass m given to a physical object is considered as a physical quantity that manifests the inertial resistance to the acceleration $\mathbf{a}$ of the object under the net action of applied forces $\mathbf{F}$ acting on it. This is represented mathematically by Newton's second law of motion as $m\mathbf{a} = \mathbf{F}$. Consider the case of an elementary particle of an inertial mass m. If the elementary particle is regarded as a mass-point with no substructures then the inertial mass m is no more than a proportional constant that connects a physical entity to a mathematical object. However, if the elementary particle in fact possesses substructures, such as it is constructed by actual mass points that join together to form a physical object that has the mathematical structure of a differentiable manifold, then it is possible to speculate that the inertial mass of the elementary particle is the result that arises from a mechanism that determines the way how a physical object moves in space [1-5]. Since different physical fields have different influences on the motion of a physical object but they all assume the same inertial mass for the object when it moves in space, this leads to the reasonable conclusion that if there is a mechanism that can give an explanation to the inertial mass of an object then such mechanism must be related directly to some form of interaction between the object and the ambient space itself. For example, if a particle has an inertial mass m and a charge q is placed in a region of space with an electric field $\mathbf{E}$ then it has been established from experimental observations that the relationship between the quantities m, q and $\mathbf{E}$ is given by the equation $m\mathbf{a} = q\mathbf{E}$. We may suggest that the quantity $q\mathbf{E}$ provides a mechanism for the charged particle to move in space. As shown in our previous work on the fluid state of the electromagnetic field that the electric field **E** can be identified with the velocity of a fluid flow and as a consequence the electrical interaction can be explained mechanically using the Stokes' law in fluid dynamics. Furthermore, from a dimensional analysis, the new dimension of the charge q is $[q] = M/T$. Therefore the charge q is a change of mass with respect to time. If the charge is constant then the particle which possesses the charge changes an equal amount of mass per unit time [6]. In this case the motion of the particle may be directly related to its fluid state and therefore it can be explained in terms of the wave motion of physical objects that move in space with the mechanism in the form of peristaltic locomotion. In this work we will discuss how the peristaltic locomotion can be applied to Dirac quantum particles. We first give a brief account on how the motion of a physical object can be mathematically formulated in physics.

In classical physics, the action of a physical field on the motion of a physical object in space is represented by a mathematical expression that is used to determine the acceleration and velocity so that the path of the particle can be established. Furthermore, except for physical constants that are associated with a particular elementary particle and whose values are determined by experiments, the mathematical analysis of the motion of a particle normally assumes that the particle is a mass-point which has no substructures that may have direct effect on its motion in space. It should be emphasised here that mass-point is a relative concept that depends on the scale of the physical system under consideration. For example, the Earth can be considered as a mass-point when its motion is investigated in the solar system and an electron as a mass-point in the Bohr model of the hydrogen atom. However, even though the physical structure of the Earth must be taken into account when we investigate the motion of physical objects within its scale, at the present state of physical development the investigation of physical behaviour on the scale of an electron is a question that needs to be investigated. On the other hand, since experiments have shown that a quantum particle such as an electron can behave like a wave, which, according to classical physics, is the motion of a medium, therefore it is reasonable to suggest that an electron is not a mass-point but rather a physical compound which is formed by mass points, and, as shown in our work on the fluid state of Dirac quantum particles, it may be in a fluid state [7]. Mathematically, the motion of physical objects in an ambient space can be described by geometric transformations under which the properties of the configuration of the objects remain unchanged, such as isometric transformations that preserve the distance from a configuration space onto itself. This purely mathematical description can be applied into classical dynamics in which the motion of solid objects can be described by the Poincaré group. On the other hand, in our previous works on the quantum structures of elementary particles we suggested that instead of viewing elementary particles as mass-points we may consider elementary particles as three-dimensional differentiable manifolds, therefore we would need to extend the description of the dynamics of elementary particles in classical physics as mass-points to the dynamics of elementary particles as three-dimensional differentiable manifolds in an ambient space. In particular, we may describe the evolution of elementary particles as a change of their geometric structures through evolution processes, such as the Ricci flow, rather than their motion in an ambient space, and in this case we may assume that the motion of elementary particles is an isometric transformation, which is a continuous isometric embedding into spacetime. The continuous isometric embedding of three-dimensional Riemannian manifolds can also be viewed as geometric solitons which are formed by a continuous process of materialising spacetime structures rather than the motion of a solid physical object through space with respect to time as described in classical physics. The dynamics of an elementary particle described as a soliton can be formulated by a covariant Ricci flow using the Lie differentiation given by the equation $L_X g_{\alpha\beta} = \kappa R_{\alpha\beta}$, where κ is a dimensional constant, $g_{\alpha\beta}$ is a covariant metric tensor and $R_{\alpha\beta}$ is the Ricci curvature tensor. The fundamental problem that emerges from the consideration on how a three-dimensional Riemannian manifold can be isometrically embedded in an ambient Euclidean space can be outlined as follows [8]. Let (M^n, g) and (N^m, h) be two Riemannian manifolds. It is shown in differential geometry that the manifolds M^n and N^m are isometric if there exists a diffeomorphism f which preserves the distances. Let $x^\alpha,\ \alpha = 1,2,\ldots,n$ be the local coordinates on the manifold M^n with the Riemannian metric $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ and $y^\beta = f(x^\alpha),\ \beta = 1,2,\ldots,m$ be the local coordinates in the manifold N^m with the Riemannian metric $ds^2 = h_{\alpha\beta}dy^\alpha dy^\beta$, then the two manifolds M^n and N^m are said to be locally isometric if the following condition holds $g_{\alpha\beta} = (\partial y^\mu/\partial x^\alpha)(\partial y^\nu/\partial x^\beta)h_{\mu\nu}$. In topology, a topological embedding between topological spaces is a homeomorphism, which is an injective continuous transformation. If the topological spaces are smooth manifolds then the topological embedding is a diffeomorphism and the image is a submanifold of the codomain manifold. According to the Whitney embedding theorem, a manifold of dimension n can be smoothly embedded in the Euclidean space of dimension $2n$. Consider a smooth embedding f of a Riemannian manifold (M^n, g) in the Euclidean space R^m whose Euclidean metric is given by $ds^2 = \sum_{\beta=1}^{m}\left(dy^\beta\right)^2$. Then we obtain $g_{\alpha\beta} = \sum_{\mu=1}^{n}(\partial y^\mu/\partial x^\alpha)(\partial y^\mu/\partial x^\beta)$. This is a system of $n(n+1)/2$ non-linear partial differential equations in m unknown functions. It is also conjectured in differential geometry that any Riemannian manifold of dimension n can be isometrically embedded in a Euclidean space of dimension $n(n+1)/2$. If we consider elementary particles as Riemannian manifolds of dimension three then the required ambient Euclidean space must have a dimension six. As discussed in our previous works, a six-dimensional Euclidean space can be formulated as the union of a three-dimensional spatial manifold and a three-dimensional temporal manifold. However, there are two possibilities that can be considered when we attempt to formulate such a unified spacetime, because a spacetime may be endowed either with a Euclidean metric or a pseudo-Euclidean metric. If the ambient Euclidean space is a pseudo-Euclidean spacetime of signature (3,3) then there remains the question of whether it is possible to embed three-dimensional quantum particles with positive definite metric in such ambient space. It is interesting to note that if a three-dimensional Riemannian spatial manifold can be embedded in a six-dimensional pseudo-Euclidean spacetime of signature (3,3) then there would also exist three-dimensional temporal manifolds that could also be embedded and these temporal manifolds could also exist as physical objects [9-10]. On the other hand, if the ambient Euclidean space is a Euclidean spacetime then we also showed that it is possible to formulate a Euclidean relativity so that spacetime has a positive definite Euclidean metric instead of a pseudo-Euclidean metric so that the Whitney embedding theorem can be applied [11].

Even though the above purely mathematical description of the motion of quantum particles is satisfactory with regard to the metric of the mathematical structure of a quantum particle, it does not specify a physical mechanism for the motion of a quantum particle in space. We may ask the question of how a quantum particle is displaced from one position to the next in space. If we assume quantum particles are formed from mass points by contact forces then these quantum particles can be connected to space by some form of physical grips also by the contact forces. From this physical mechanism then we may infer that the physical grip of a quantum particle into the ambient space determines its inertial mass. Furthermore, we may also conclude that all physical objects that can move in space must have a composite structure. In the following we will discuss a possible mechanism for the motion of a quantum particle that also manifests the feature of the wave-particle duality based on our previous suggestion that Dirac quantum particles possess a fluid state. It is well-known that physical objects that have a hydrostatic structure can move in space with the mechanism in the form of peristaltic locomotion. In order to perform a peristaltic locomotion a quantum particle that possesses a fluid state must grip the substrate, which is the ambient space, to increase sliding resistance on the contact area. This could be the source of the so-called inertial mass defined in classical physics. In one form of peristaltic locomotion, the object moves forwards when the peristaltic wave also travels forwards, and in another different form of peristaltic locomotion, the object moves forwards when the peristaltic wave travels backwards. This could be related to the charge of an elementary particle. We would like to mention here one particular work that provides a simple mechanical model for peristaltic locomotion [12]. The authors of the paper discuss the simple but instructive problem of peristalsis-like locomotion driven by elongation-contraction waves that propagate along the body axis of the physical object under investigation. They showed that the basic equation that describes the peristaltic locomotion is a one-dimensional linear diffusion equation whose coefficient and source term express the biological action that drives the motion. Furthermore, they also showed that a perturbation analysis of a more general equation reveals that adequate control of friction with the substrate on which locomotion occurs is required for a translation of the internal motion into directional migration of a body. They arrived at a differential equation of the form $\left(1+\varepsilon p(s,t)\right)u_t = D_0\left(u_{ss}+ke_0 cos(\omega t-ks)\right)$, which gives rise to a series of diffusion equations of the form $f_t = D_0 f_{ss} + g(s,t)$, whose formal solution can be expressed in an integral form as $f(s,t) = f(s,0) + \int_0^t dt' \int_{-\infty}^{\infty} ds' G(s-s', t-t') g(s',t')$, where $G(s,t) = \left(1/\sqrt{4\pi D_0 t}\right) exp(-s^2/4D_0 t)$ is the Gaussian Green's function. It is interesting to note that using a diffusion equation to describe the dynamics of peristaltic locomotion is similar our formulation of quantum physics in terms of differential geometry and topology. For example, we showed that the Ricci scalar curvature that describes the geometrical structure of a quantum particle satisfies a three-dimensional diffusion equation $\partial_t R = k\nabla^2 R$, whose solutions can be found as $R(x,y,z,t) = \left(M/\left(\sqrt{4\pi kt}\right)^3\right) e^{-(x^2+y^2+z^2)/4kt}$, which determines the probabilistic distribution of an amount of geometrical substance M which manifests as observable matter. We would like to mention that the geometric and topological structures of quantum particles can also be expressed in terms of wavefunctions for the case of one and two dimensions. We showed that in one dimension, the geometric structure of a 1D differentiable manifold that is represented by the curvature κ can be expressed in terms of a wavefunction ψ as $\kappa = (1 + (d\psi/dx)^2)^{-3/2}(d^2\psi/dx^2)$, and in two dimensions the Ricci scalar curvature R of a 2D differentiable manifold can be expressed in terms of a wavefunction ψ as $R = 2(\psi_{11}\psi_{22} - \psi_{12}^2)^2/(1 + \psi_1^2 + \psi_2^2)^2$, where $\psi_\mu = \partial\psi/\partial x^\mu$ and $\psi_{\mu\nu} = \partial^2\psi/\partial x^\mu \partial x^\nu$. The question now is how to express the motion of quantum particles as peristaltic locomotion which is a continuous process of the internal propagation of the elongation-contraction wave along the direction of migration.

We have shown in our work on the fluid state of Dirac quantum particles that Dirac equation can be used to describe the state of fluid flow formulated in the theory of classical fluids. For free particles, Dirac equation is given as

$$\left(i\gamma^\mu \partial_\mu - m\right)\psi = 0 \tag{1}$$

with the matrices γ_i are given as

$$\gamma_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma_3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{2}$$

By expansion, we obtain

$$-\frac{\partial\psi_1}{\partial t} - im\psi_1 = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_4 + \frac{\partial\psi_3}{\partial z} \tag{3}$$

$$-\frac{\partial\psi_2}{\partial t} - im\psi_2 = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 - \frac{\partial\psi_4}{\partial z} \tag{4}$$

$$\frac{\partial\psi_3}{\partial t} - im\psi_3 = \left(-\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_2 - \frac{\partial\psi_1}{\partial z} \tag{5}$$

$$\frac{\partial\psi_4}{\partial t} - im\psi_4 = \left(-\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_1 + \frac{\partial\psi_2}{\partial z} \tag{6}$$

It is observed from the field equations given in Equations (3-6) that the change of the field (ψ_1, ψ_2) with respect to time generates the field (ψ_3, ψ_4), and the change of the field (ψ_3, ψ_4) with respect to time generates the field (ψ_1, ψ_2). This is similar to the case of Maxwell field equations of the electromagnetic field in which the change of the electric field generates the magnetic field, and vice versa. Then it may be suggested that Dirac field of qunatum particles may also be viewed as being composed of two different physical fields, namely the field (ψ_1, ψ_2) and the field (ψ_1, ψ_2). Furthermore, Dirac field equations given in Equations (3-6) can be rewritten as a system of real equations as follows

$$-\frac{\partial\psi_1}{\partial t} = \frac{\partial\psi_4}{\partial x} + \frac{\partial\psi_3}{\partial z} \tag{7}$$

$$-\frac{\partial\psi_2}{\partial t} = \frac{\partial\psi_3}{\partial x} - \frac{\partial\psi_4}{\partial z} \tag{8}$$

$$-\frac{\partial\psi_3}{\partial t} = \frac{\partial\psi_2}{\partial x} + \frac{\partial\psi_1}{\partial z} \tag{9}$$

$$-\frac{\partial\psi_4}{\partial t} = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z} \tag{10}$$

$$\frac{\partial\psi_4}{\partial y} = m\psi_1 \tag{11}$$

$$\frac{\partial\psi_3}{\partial y} = -m\psi_2 \tag{12}$$

$$\frac{\partial\psi_2}{\partial y} = -m\psi_3 \tag{13}$$

$$\frac{\partial\psi_1}{\partial y} = m\psi_4 \tag{14}$$

From Equations (7-14), we derive the following system of equations for all components of the Dirac field

$$\frac{\partial^2\psi_i}{\partial^2 y} - m^2\psi_i = 0 \tag{15}$$

$$\frac{\partial^2\psi_i}{\partial t^2} - \frac{\partial^2\psi_i}{\partial x^2} - \frac{\partial^2\psi_i}{\partial z^2} = 0 \tag{16}$$

Solutions to Equation (15) can be written as

$$\psi_i = c_{1i}(x,z,t)e^{my} + c_{2i}(x,z,t)e^{-my} \tag{17}$$

where c_{1i} and c_{2i} are undetermined functions of (x,z,t). The solutions given in Equation (17) give a distribution of a physical quantity along the *y*-axis. On the other hand, Equation (16) can be used to describe the dynamics, for example, of a vibrating membrane in the (x,z)-plane. If the membrane is a circular membrane of radius a then the domain D is given as $D = \{x^2 + z^2 < a^2\}$. In the polar coordinates given in terms of the Cartesian coordinates (x,z) as $x = r\cos\theta$, $z = r\sin\theta$, the two-dimensional wave equation given in Equation (16) becomes

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = 0 \tag{18}$$

The general solution to Equation (18) for the vibrating circular membrane with the condition $\psi = 0$ on the boundary of D can be found as

$$\psi(r,\theta,t) = \sum_{m=1}^{\infty} J_0\left(\sqrt{\lambda_{0m}}r\right)\left(C_{0m}\cos\sqrt{\lambda_{0m}}ct + D_{0m}\sin\sqrt{\lambda_{0m}}ct\right) + \sum_{m,n=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right)(A_{nm}\cos n\theta + B_{nm}\sin n\theta)\left(\left(C_{nm}\cos\sqrt{\lambda_{nm}}ct + D_{nm}\sin\sqrt{\lambda_{nm}}ct\right)\right) \quad (19)$$

where $J_n\left(\sqrt{\lambda_{nm}}r\right)$ is the Bessel function of order n and the quantities A_{nm}, B_{nm}, C_{nm} and D_{nm} can be specified by the initial and boundary conditions. As has been mentioned above, at each moment of time the vibrating membrane appears as a 2D differentiable manifold which is a geometric object whose geometric structure can be constructed using the wavefunction given in Equation (19). Even though elementary particles may have the geometric and topological structures of a 3D differentiable manifold, it is seen from the above descriptions via Dirac equation that they also appear as 3D physical objects that embedded in three-dimensional Euclidean space. For steady states, the field (ψ_1, ψ_2) and the field (ψ_3, ψ_4) satisfy the Cauchy-Riemann equations in the (x, z)-plane and therefore it is possible to consider Dirac quantum particles as physical systems which exist in a fluid state as defined in the classical fluid dynamics as substances that retain a definite volume, have the ability to flow and deform continually, hence they can exhibit a wave motion in which the field (ψ_1, ψ_2) may also be identified as the stream function and the velocity potential of one fluid flow and the field (ψ_3, ψ_4) with another fluid flow. We now show that the two fields (ψ_1, ψ_2) and (ψ_3, ψ_4) are connected and, most importantly, how such connection would lead to the prospect of using them to describe the motion of a Dirac quantum particle in an ambient space as a peristaltic locomotion. If the physical quantity m, which is identified with the inertial mass of a quantum particle, is assumed to be positive, $m > 0$, then it is observed that it is possible to describe the physical structure of a Dirac quantum particle as a spinning top if we consider solutions to Equation (15) as hybrid functions of the form

$$\psi_i = \begin{cases} c_{1i}(x,z,t)e^{my} & for \quad y < 0 \\ c_{2i}(x,z,t)e^{-my} & for \quad y \geq 0 \end{cases} \quad (20)$$

Solutions given in Equation (20) can be rewritten in the following forms

$$\psi_1 = c_{21}(x,z,t)e^{-my}, \qquad \psi_2 = c_{22}(x,z,t)e^{-my} \quad (21)$$

$$\psi_3 = c_{23}(x,z,t)e^{-my}, \qquad \psi_4 = c_{24}(x,z,t)e^{-my} \quad (22)$$

Using the equations given in Equations (11-14), we further obtain the conditions $c_{24} = -c_{21}$ and $c_{23} = c_{22}$. If we write $c_{21} = f(x,z,t)$ and $c_{22} = g(x,z,t)$ then we have

$$\psi_1 = f(x,z,t)e^{-my}, \qquad \psi_2 = g(x,z,t)e^{-my} \quad (23)$$

$$\psi_3 = g(x,z,t)e^{-my}, \qquad \psi_4 = -f(x,z,t)e^{-my} \quad (24)$$

From the above forms of solutions given to the components ψ_i of the wavefunction ψ we can show how a standing wave can be established from the superposition of a wave associated with the field (ψ_1, ψ_2) and a wave associated with the field (ψ_3, ψ_4). Let $\psi_1 = f(x,z,t)e^{-my}$ be identified with the velocity potential and $\psi_2 = g(x,z,y)e^{-my}$ with the stream function of one fluid flow. Now we have two different descriptions that can be given to the field (ψ_3, ψ_4). If we identify the component $\psi_3 = g(x,z,t)e^{-my}$ with the velocity potential and $\psi_4 = -f(x,z,t)e^{-my}$ with the stream function of another fluid flow then we have the stream function of the first flow equals the velocity potential of the second flow, and the stream function of the second flow is a reflection of the velocity of the first flow. Even though this kind of identification may be used to describe a particular type of fluid flow of Dirac quantum particles, it does not give rise to the physical structure that we are looking for, that is a standing wave. However, if we now identify the component $\psi_3 = g(x,z,t)e^{-my}$ with the stream function and $\psi_4 = -f(x,z,t)e^{-my}$ with the velocity potential of the second flow then the two flows that are identical except for their flow directions, which are opposite to each other, and in this case they can form a required standing wave. If a Dirac quantum particle is formed from mass points by contact forces then the particle can also be connected to an ambient space and performs a peristaltic locomotion, which then manifests the feature of the wave-particle duality.

References

[1] Vu B Ho, *Spacetime Structures of Quantum Particles*, ResearchGate (2017), viXra 1708.0192v2, Int. J. Phys. vol 6, no 4 (2018): 105-115.

[2] Vu B Ho, *A Classification of Quantum Particles*, ResearchGate (2018), viXra 1809.0249v2, GJSFR-A. vol 18, no 9 (2018): 37-58.

[3] Vu B Ho, *Quantum Particles as 3D Differentiable Manifolds,* ResearchGate (2018), viXra 1808.0586v2, Int. J. Phys. vol 6, no 5 (2018): 139-146.

[4] Vu B Ho, *On the Principle of Least Action*, ResearchGate (2016), viXra 1708.0226v2, Int. J. Phys, vol. 6, no. 2 (2018): 47-52.

[5] Vu B Ho, *Topological Transformation of Quantum Dynamics*. Preprint, ResearchGate, 2018, viXra 1810.0300v1, *Journal of Modern Physics* (2019), **10**, 102-127.

[6] Vu B Ho, *Fluid State of the Electromagnetic Field*, ResearchGate (2018), viXra 1812.0298v1.

[7] Vu B Ho, *Fluid State of Dirac Quantum Particles*, ResearchGate (2018), viXra 1811.0217v2, *Journal of Modern Physics* (2018), **9**, 2402-2419.

[8] Vu B Ho, *On the motion of quantum particles and Euclidean relativity*. Preprint, ResearchGate, 2017, viXra 1710.0253v1.

[9] Vu B Ho, *A Temporal Dynamics: A Generalised Newtonian and Wave Mechanics*. Preprint, ResearchGate, 2016, viXra 1708.0198v1.

[10] Vu B Ho, *A Theory of Temporal Relativity*. Preprint, ResearchGate, 2017, viXra 1708.0196v1.

[11] Vu B Ho, *Euclidean Relativity* (Preprint, ResearchGate, 2017), viXra 1710.0302v1.

[12] Yoshima Tanaka, Kentaro Ito, Toshiyuki Nakagaki and Ryo Kobayashi, *Mechanics of peristaltic locomotion and role of anchoring*. *J. R. Soc. Interface* **9**, 222-233 (2011). ResearchGate (2014).