A Mechanism of Accelerating and Jerking Expansion of Universe

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Abstract

In 2016, scientists report that the acceleration of the expansion of the universe could be accelerating, denote as jerk universe. We notice that the acceleration of the expanding universe is attributed to linear-effect of gravity, and believe that the jerk is linear-effect as well. We face two tasks: (1) A single dynamic mechanism is required to explain both acceleration and jerk; (2) higher order derivative theories of gravity is needed to govern jerk. We propose a theory-independent dynamic mechanism and extend Newton's and linearized Einstein's theory to contain high order differential evolution equations for time-varying source, gravitational field, and movement. We show that the mechanism, (1) predicts repulsive gravitational force, explain the accelerating expansion without fine-tuning problem, and provide an explanation of the jerk; (2) provides a candidate for driving inflation; (3) suggests a new understanding of the nature of dark energy, and postulates gravitational-charge/dark energy duality. Generalized linearized Einstein's theory predicts that the accelerated receding objects in the universe emit gravitational wave.

Key words: accelerating universe, accelerated acceleration of universe, jerking universe, dynamic mechanism, repulsive gravity, dark energy, inflation, gravitational wave

Contents

1. Expansion of Universe

In the 1929, Hubble discovered the expansion of the universe. The receding velocity is assumed to be uniform, which is reflected in the "distance-velocity relation" of Hubble law. In the 1998, Scientists reported the profound observation that the expansion of the universe is accelerating [1], $\ddot{a} > 0$, which is explained by dark energy model, and the acceleration is assumed to be unchanged, $\dddot{a} = 0$. In the 2016, Scientists report that even that acceleration is faster than expected. If the 2016 observation is confirmed, there is a discrepancy in Hubble constant obtained from the 2016 observation and from the 2013 data of European Planck mission, which could mean that the acceleration is accelerating, $\dddot{a} > 0$, denote as jerk, i.e., that dark energy has grown in strength over time. "In the case of dark energy, there is no simple explanation at present, leaving direct measurements as the only guide among numerous complex or highly tuned explanations" [2]. Moreover, the nature of dark energy remains ambiguous. "Breakthroughs typically arise from a change of perspective, and the ability to discover simplicity in the hitherto superficially complicated problem" [3].

Above observations indicate: (1) Hubble law needs to be generalized to reflect the effects of acceleration and jerk; (2) to explain the 2016 observation we need both a mechanism and higher order derivative theory of gravity. The existing theories of gravity, e.g., Newton and Einstein theories, are second order derivative.

In this article we postulate a mechanism and generalize the existing theories of gravity to contain the mechanism and higher order differential equations to explain the 1998/2016 observations and the discrepancy, and to disclose a possible nature of dark energy. The generalized linearized Einstein theory predicts that the accelerating recessional objects in the universe emit gravitational wave (GW), which exerts a damping force.

2. Kinematics: Extended Hubble Law

Before studying dynamical theory of the accelerating and jerking universe, let's study kinematical description of them. For this aim, we revisit Hubble law, the distance-velocity relation, $r = \dot{r}\left(\frac{1}{H}\right)$, which plays the fundamental role in cosmology. Recently, Hubble law has been extended mathematically to relate distance to not only velocity but also acceleration and jerk [4]. Let's briefly review it by starting with Taylor series of radial distance,

$$r(t_1) = r(t) + \dot{r}(t)(t_1 - t) + \frac{1}{2!}\ddot{r}(t)(t_1 - t)^2 + \frac{1}{3!}\dddot{r}(t)(t_1 - t)^3 + \cdots. \qquad (1)$$

The history of discovering the expansion of universe matches Eq. (1).

Static ---1929	Uniformly expansion: 1929	Accelerating expansion: 1998	Jerking expansion: 2016	Jouncing Expansion: ?
$r(t)$	$\dot{r}(t)$	$\ddot{r}(t)$	$\dddot{r}$	$r^{(4)}$

Before Hubble, it was believed that the universe is static, $r(t_1) = r(t)$, which corresponds to the first term of right hand side (RHS) of Eq. (1). Hubble law corresponds to the second term and assumed velocity to be constant, $\dot{r}(t_1) = \dot{r}(t)$.

In the 1998, scientists reported that the expansion of the universe is accelerating, which corresponds to the third term of RHS. It was assumed that the acceleration is uniform, $\ddot{r}(t_1) = \ddot{r}(t)$. Kinematics gives,

$$r(t_1) = \dot{r}(t)(t_1 - t) + \frac{1}{2!}\ddot{r}(t)(t_1 - t)^2.$$

The faster an object is originally receding and uniformly accelerating, the farther it is away. It implies *the universality of free rising of accelerating objects* in a repulsive gravitational field.

In the 2016, Riess reports that the acceleration of the expansion is accelerating, which corresponds to the fourth term of RHS. Kinematics gives,

$$r(t_1) = \dot{r}(t)(t_1 - t) + \frac{1}{2!}\ddot{r}(t)(t_1 - t)^2 + \frac{1}{3!}\dddot{r}(t)(t_1 - t)^3.$$

Assume jerk is constant, $\dddot{r}(t_1) = \dddot{r}(t)$. This equation implies *the universality of the free rising of jerking objects* in a repulsive time-varying gravitational field.

The history of discovering the expansion of the universe proves, one term at time, the capability of Eq. (1) in describing the expansion of the universe. So the extended Hubble law has been proposed as the "*distance-velocity-acceleration-jerk*" relation, denote as the "*distance-motion*" relation for short, in the following equivalent forms,

$$\left.\begin{aligned} 1 &= -q_1 - \frac{1}{2}q_2 - \frac{1}{3!}q_3 \\ \frac{\dot{r}(t)}{r(t_1)} &= H\left\{1 + \frac{1}{2}q_2 + \frac{1}{3!}q_3\right\} \\ \frac{\ddot{r}(t)}{r(t_1)} &= 2H^2\left\{1 + q_1 + \frac{1}{3!}q_3\right\} \\ \frac{\dddot{r}(t)}{r(t_1)} &= 3!\,H^3\left(1 + q_1 + \frac{1}{2}q_2\right) \end{aligned}\right\}, \qquad (2)$$

where q_n is the generalized "decelerate parameter", denoted as the "*motion parameter*", that judges the motion status at time t, and is defined as,

$$q_n \equiv -\frac{r^{(n)}(t)}{r(t_1)H^n}. \tag{3}$$

Converting parameters at time t to that at the same time t_1, the distance-motion relation is

$$\left.\begin{aligned} 1 &= -q_{s1} + \tfrac{1}{2}q_{s2} + \tfrac{5}{6}q_{s3} \\ \frac{\dot{r}(t_1)}{r(t_1)} &= H\left\{1 - \tfrac{1}{2}q_{s2} - \tfrac{5}{6}q_{s3}\right\} \equiv \overline{H} \\ \frac{\ddot{r}(t_1)}{r(t_1)} &= -2H^2\left\{1 + q_{s1} - \tfrac{5}{6}q_{s3}\right\} \\ \frac{\dddot{r}(t_1)}{r(t_1)} &= -\tfrac{6}{5}H^3\left\{1 + q_{s1} - \tfrac{1}{2}q_{s2}\right\} \end{aligned}\right\}, \tag{4}$$

where $\overline{H}$ is the extended Hubble parameter and would replace the regular Hubble parameter in the accelerating/jerking universe. $1/\overline{H}$ is an effective time that takes into account the effects of not only velocity but also acceleration and jerk. The "motion parameter" q_{sn}, which judging the motion status at the same time t_1, is

$$q_{sn} \equiv -\frac{r^{(n)}(t_1)}{r(t_1)H^n}. \tag{5}$$

Note the distance-motion relation only provides a kinematical description of accelerating and jerking universe. Next we will establish dynamical theories for them.

3. Nature of Expansion

Before searching for a mechanism, let's study the nature of the expansion of the universe. We notice that the accelerating expansion of the universe is result of linear effects of gravity. To show it, let's start with curvatures in FLRW metric,

$$R_0^0 = \frac{3\ddot{a}}{a}, \qquad R_j^i = \left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right)\delta_j^i, \qquad R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right). \tag{6}$$

Eq. (6) shows that there are non-linear structures of spacetime.

However, when substituting Eq. (6) into modified Einstein's equation, one obtains the linear evolution equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{7}$$

During derivation of Eq. (7), all non-linear terms cancel each other exactly. Eq. (7) indicates a hitherto un-emphasized fact that non-linear effect of Einstein's theory doesn't account for the acceleration. We believe this fact is not a coincidence. Thus linear theories of gravity should explain the 1998 observations as well. We suggest substituting a postulated mechanism into linear theories to test it first, which should be simpler to calculate and to compare with both observations and predictions of other models, be clearer in physical concept that may help to understand the nature of dark energy, and be heuristic for further study. Moreover the concepts and results of linear theories can be introduced and/or converted to cosmological study.

4. Conditions of Dynamic Mechanism

We believe that the 1998/2016 observations and other acceleration related phenomena should be explained by the same mechanism. Since $p(< 0)$ and Λ of dark energy cannot explain the 2016 observation, following observational guidance, we replace them by a new mechanism X(t) that satisfies the following requirement:

(1) X(t) represents repulsive gravitational force without fine-tune problem, i.e., has opposite sign to that of ρ, but doesn't relate to negative mass [5] which leads to the Runaway motion [6] and violate the Energy Condition.

(2) X(t) is time-dependent, leads to higher order derivative theories of gravity, explains the 1998/2016 observations, and provides a dynamic interpretation for the extended Hubble law.

(3) X(t) should be theory-independent, i.e., it should be able to be added to different linear gravitational theories, e.g., Newton's theory, naturally and gives:

Newton's theory	Newton's Theory with X(t)	Linearized Einstein's Theory with X(t)
$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}\rho$	$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}[\rho + X(t)]$ $\frac{\dddot{r}}{r} \sim f(X(t))$	$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho + X(t)]$ $\frac{\dddot{a}}{a} \sim f(X(t))$

5. Dynamic Mechanism: Negative Gravitational Charge (g-Charge)

The positive g-charge, $Q_g \equiv +\sqrt{G}m$, was proposed [7]. Along this line, we propose a mechanism: the negative g-charge Q_{g-}. The argument is the following. The concepts of symmetry and duality have played important roles in the development of physical theories. The underlying idea of duality is that the apparent analogy between physical laws is not a mere coincidence in Nature; those laws may have same symmetry. The Coulomb/Newton correspondence suggests that, analogy to electric charge conjugation symmetry, there may be g-charge conjugation symmetry.

We define negative g-charges as $Q_{g-} \equiv -\sqrt{G}m_{-}$ [8]. The negative g-charges emit no light and distribute homogeneously and isotropically in the universe. With its physical nature, Q_{g-} avoids the fine-tuning problem. The subscripts "g" and "−" denote the variables related with g-charge and negative g-charge, respectively. $m_{-}(> 0)$ is mass of an object carrying negative g-charge. The g-charge and gravitational mass are conceptually different quantities. A positive mass object can carry either positive or negative g-charge, thus Energy Condition holds. Ordinary objects carry positive g-charge. The 1998/2016 observations indicates that the negative g-charge is dominant, i.e., $\rho_g < |\rho_{g-}|$ or $\rho_m < \rho_{m-}$, thus we adopt it throughout the article. In local region of the universe positive g-charges are dominant.

6. Dynamics: the g-charge Newton Theory

Let's first introduce the mechanism into Newton theory, denote as the g-charge Newton's Theory. Assume that the universe is uniformly filled with positive and negative g-charges, denote as Positive and Negative g-charge Universe, respectively. The positive g-charge universe corresponds to the conventional expanding universe. The g-charge Newton's theory is governed by following Rules.

6.1.g-Charge Conjugation (gC) and gC-PT Symmetry

We generalize Newton theory to have the same form and contain negative g-charge/density, Q_{g-}/ρ_{g-}, as,

$$\left.\begin{aligned} &\nabla \cdot \mathbf{g}_{Net} = -4\pi\sqrt{G}(\rho_g + \rho_{g-}) > 0 \\ &m\ddot{\mathbf{r}} = Q_g \mathbf{g}_{Net} \qquad m_{-}\ddot{\mathbf{r}}_{-} = Q_{g-}\mathbf{g}_{Net} \end{aligned}\right\}. \tag{8}$$

Where $\rho_g \equiv \sqrt{G}\rho_m$, $\rho_{g-} \equiv -\sqrt{G}\rho_{m-}$. The ρ_m and $\rho_{m-}(> 0)$ are mass densities of objects carrying positive and negative g-charges, respectively. Subscript "Net" represents variables related with net g-charges.

Eq. (8) is invariant under simultaneous transformations of g-charge conjugation, Parity, and Time reversal. Eq. (8) indicates the following,

(A) Gravitational field lines of positive (Fig. 1) and negative g-charges (Fig. 2) are,

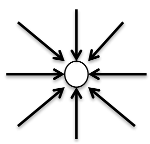

Fig. 1

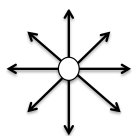

Fig. 2

(B) Gravitational potentials of positive and negative g-charges are, respectively,

$$V_g = \frac{\left(-\frac{4\pi G r^3}{3}\rho_g\right)}{r} < 0 \qquad \text{and} \qquad V_{g-} = \frac{\left(-\frac{4\pi G r^3}{3}\rho_{g-}\right)}{r} > 0.$$

(C) Interaction: like g-charges attract each other, Fig. 3 and Fig.4, which causes the *universality of free fall* in attractive gravitational fields and

Fig. 3

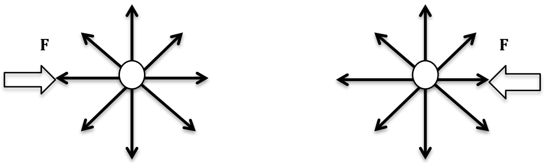

Fig. 4

unlike g-charges repel each other, Fig. 5, which causes the *universality of free rise* in repulsive gravitational fields.

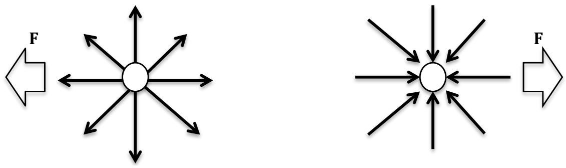

Fig. 5

6.2.Energy Conservation and Work

In a time-varying net $\mathbf{g}_{\mathrm{Net}}$ field, an object gains kinematic energy from the work done by net $\mathbf{g}_{\mathrm{Net}}$ field,

$$\left.\begin{aligned} \frac{1}{2} m\dot{r}^2 &= Q_g \int \mathbf{g}_{\mathrm{Net}} \cdot d\mathbf{r} \\ \frac{1}{2} m_-\dot{r}_-^2 &= Q_{g-} \int \mathbf{g}_{\mathrm{Net}} \cdot d\mathbf{r} \end{aligned}\right\}. \tag{9}$$

For a conservative $\mathbf{g}_{\mathrm{Net}}$ field, the kinematic energy is equal to net potential. Positive potential of negative g-charges contributes to and negative potential of positive g-charges consumes objects' kinematic energy, respectively. Eq. (9) becomes Friedmann-type equation,

$$\left.\begin{aligned} \frac{1}{2}\dot{r}^2 &= \frac{\left(\frac{4\pi G r^3}{3}\rho_{m-}\right)}{r} + \frac{\left(-\frac{4\pi G r^3}{3}\rho_m\right)}{r} \\ \frac{\dot{r}^2}{r^2} &= \frac{8\pi G}{3}\rho_{m-} - \frac{8\pi G}{3}\rho_m \end{aligned}\right\}. \tag{10}$$

6.3. g-Charge Conservation

Now considering two comoving spheres, one is attached to receding positive g-charges, denote as positive sphere, other is attached to falling negative g-charges, denote as negative sphere. At an arbitrary time t_{arb}, the positive sphere passes through the imaginary sphere to continuously expand, while the negative sphere passes through the imaginary sphere to continuously shrink.

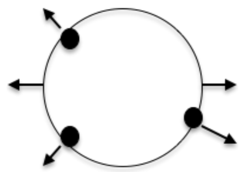

Fig. 6

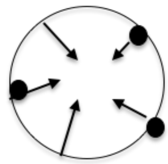

Fig. 7

Fig. 6 shows the positive sphere. Inside the positive sphere the total number of positive g-charges doesn't change; but the density ρ_g decreases. The positive g-charge conservation law, $\dot{\rho}_g = -\nabla \cdot (\rho_g \mathbf{V})$, gives,

$$\left.\begin{array}{ll} \dot{\rho}_g = -3\overline{H}\rho_g & \rho_g = \rho_{g0}e^{-3(\overline{H})t} \\ \overline{H} \equiv \dot{r}/r & r = r_0 e^{\overline{H}t} \end{array}\right\}. \qquad (11)$$

Fig. 7 shows the negative sphere crunching with collapsing negative g-charged objects. According to the conservation law, $\dot{\rho}_{g-} = -\nabla \cdot \left(\rho_{g-}\mathbf{V}_-\right)$, the negative g-charge density ρ_{g-} increases as,

$$\left.\begin{array}{ll} \dot{\rho}_{g-} = 3\overline{H}_-\rho_g - & \rho_{g-} = \rho_{g0-}e^{3(\overline{H}_-)t} \\ \overline{H}_- \equiv \dot{r}_-/r_- & r_- = r_{-0}e^{-(\overline{H}_-)t} \end{array}\right\}. \qquad (12)$$

$\overline{H}_-$ is generalized Hubble parameter for negative g-charge.

The radial distances of positive and negative g-charges are exponentially increasing and decreasing, respectively. So the g-charge conservation leads to de Sitter Universe. The densities of positive and negative g-charges are exponentially decreasing and increasing, respectively. Thus repulsive gravitational force grows in strength over the eons.

To understand it conceptually, the following approach is equivalent to the comoving approach above and provides a comprehensive picture. Imaging a sphere with an arbitrary observer as origin and "fixed" radius r_0 (Fig. 8), denote as the imaginary sphere. Two events simultaneously take place: interior negative g-charges continuously repel interior positive g-charges out and attract exterior negative g-charges into the imaginary sphere.

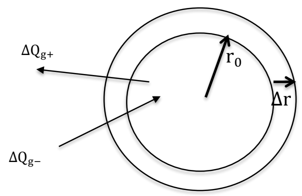

Fig. 8

The total numbers of positive and negative g-charges are conserved, respectively. In flat space, this implies that the positive and negative g-charge densities satisfy respectively the continuity equation, we obtain Eq. (11 and 12) again.

6.4. Higher Order Differential Evolution Equation

In Newton's classical mechanics of rigid body, there are third $r^{(3)}$ and fourth $r^{(4)}$ order derivatives of position, denote as jerk and jounce, respectively. However, theoretically, existing theories of gravity don't have a mechanism to drive jerk and higher order derivative, i.e., there are no forces associated with the higher order derivatives of the position. In Newton's theory of gravity, there are only second order derivative of position and corresponding gravitational force. We need a high order derivative theory of gravity.

Fortunately, Exponential behavior of "r", "r_-", "ρ_g", and "ρ_{g-}" leads and allows us to establish higher order differential equations to describe evolutions of those parameters. The evolution of g-charge density governs that of $\mathbf{g}_{\mathrm{Net}}$ field that, in turn, governs that of movement. High order differential equations of gravity are,

$$\left.\begin{array}{ll} \rho_g^{(n+1)} = \rho_{g0}\frac{d^{n+1}}{dt^{n+1}}e^{-3\bar{H}t} & \rho_{g-}^{(n+1)} = \rho_{g0-}\frac{d^{n+1}}{dt^{n+1}}e^{3(\bar{H}_-)t} \\ \nabla\cdot\mathbf{g}_{\mathrm{Net}}^{(n)} = -4\pi\sqrt{G}\left(\rho_g^{(n)}+\rho_{g-}^{(n)}\right) & n = 1, 2, 3, 4 \ldots \\ mr^{(n+2)} = Q_g\mathbf{g}_{\mathrm{Net}}^{(n)} & m_-r_-^{(n+2)} = Q_{g-}\mathbf{g}_{\mathrm{Net}}^{(n)} \end{array}\right\}. \quad (13)$$

Superscript "(n)" denotes n^{th} order derivative with respect to time.

In addition, we introduce high order derivative formula for angular momentum,

$$\mathbf{L}^{(n)} = (\mathbf{r}\times\mathbf{p})^{(n)} \qquad \mathbf{L}_-^{(n)} = (\mathbf{r}_-\times\mathbf{p}_-)^{(n)}, \quad (14)$$

where $\mathbf{L}$ and $\mathbf{L}_-$ are angular momentum of objects carrying positive and negative g-charge, respectively. Eq. (14) may be applied to describe behaviors of spiral galaxies, where positive g-charges are dominant.

Eq. (8-14) form g-charge Newton Theory.

6.5. Test of Mechanism and Theory

We suggest that the following explanations/predictions verify the mechanism and g-charge theory.

6.5.1. The 1998/2016 Observations

Positive g-charges experience net repulsive gravitational force. Eq. (8) produces Friedmann-type equation,

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}(\rho_m - \rho_{m-}) > 0, \tag{15}$$

which explains the 1998 observation [9]. For time-varying net g-charge density, Eq. (13) implies that the acceleration is inevitably accelerating,

$$\frac{\dddot{r}}{r} = -\frac{4\pi G}{3}\{(\rho_m - \rho_{m-})\overline{H} + (\dot{\rho}_m - \dot{\rho}_{m-})\}. \tag{16}$$

Eq. (16) provides a possible explanation for the 2016 observation and for the discrepancy between the 2013 and the 2016 observations.

6.5.2. Motion Parameter

For judging a status of motion, we define a "*motion-parameter*" as,

$$q_0^n \equiv -\frac{r^{(n)}(t_0)r^{n-1}(t_0)}{\dot{r}^n(t_0)}. \tag{17}$$

The q_0^2 is the regular deceleration parameter.

Substituting Eq. (10 and 15) into the motion parameter with $n = 2$, q_0^2 is the regular deceleration parameter, we obtain,

$$q_0^2 = -\frac{1}{2} < 0, \tag{18}$$

which implies that the expansion of the universe is accelerating.

For judging the status of the acceleration of the universe, substituting Eq. (10 and 16) into Eq. (17) with $n = 3$, we obtain,

$$q_0^3 \equiv -\frac{\dddot{r}(t_0)r^2(t_0)}{\dot{r}^3(t_0)} < 0, \tag{19}$$

which implies that the acceleration is accelerating, i.e., it is in a jerking movement.

6.5.3. Redshift

Based on uniformly expanding universe, Hubble presented the linear distance-redshift relation [10], $Z_H = H_0 r/c$. For accelerating/jerking universe, this relation is extended to including the effects of acceleration and jerk [4], which has the same form as that of the cosmological redshift,

$$1 + Z = \frac{r(t_0)}{r(t)}, \tag{20}$$

with the following equivalent expressions of Z,

$$Z = Z_{H0} + \frac{1}{2}q_0^2 Z_{H0}^2 - \frac{1}{3!}q_0^3 Z_{H0}^3 + \frac{1}{4!}q_0^4 Z_{H0}^4 - \cdots, \tag{21}$$

$$Z = Z_{H0} - \frac{\dot{Z}_{H0}}{2H_0} + \frac{\ddot{Z}_{H0}}{3!H_0^2} - \frac{\dddot{Z}_{H0}}{4!H_0^3} + \cdots, \tag{22}$$

where $Z_{H0} = \frac{H_0 r(t_0)}{c}$. The redshift of an object drifts due to its acceleration. The accelerating acceleration causes accelerating drift.

6.5.4. Inflation

The g-charge Newton's theory predicts repulsive gravity. Thus negative g-charge is an alternative mechanism for driving inflation. Slow-roll parameters are corresponding to different order accelerations as,

$$\left.\begin{array}{lcl} \varepsilon \equiv -\frac{\dot{\bar{H}}}{\bar{H}^2} & \leftrightarrow & \frac{\ddot{r}}{r} = (1-\varepsilon)\bar{H}^2 \\ \eta \equiv -\frac{\ddot{\bar{H}}}{2\bar{H}\dot{\bar{H}}} & \leftrightarrow & \frac{\dddot{r}}{r} = \left(-\eta + \frac{3}{2} - \frac{1}{2\varepsilon}\right) 2\bar{H}\dot{\bar{H}} \\ \xi \equiv \frac{\dddot{\bar{H}}}{2\bar{H}^2\dot{\bar{H}}} - 2\eta^2 & \leftrightarrow & \frac{r^{(4)}}{r} = \left(\xi - 4\eta - \frac{3}{2}\varepsilon + 3 + \frac{1}{2\varepsilon} + 2\eta^2\right) 2\bar{H}^2\dot{\bar{H}} \end{array}\right\}. \tag{23}$$

Different order accelerations may start and end at different slow-roll conditions.

6.5.5. Conservation of Angular Quantities

We take higher order derivatives of the angular momentum of positive g-charge, Eq. (14) gives,

$$\left.\begin{array}{l} \frac{d\mathbf{L}}{dt} \equiv L^{(1)} = \dot{\mathbf{r}}\times m\boldsymbol{v} + \mathbf{r}\times m\dot{\boldsymbol{v}} = \mathbf{r}\times m\dot{\boldsymbol{v}} = \mathbf{r}\times\mathbf{F} \equiv \boldsymbol{\tau}_1 \\ \frac{d^2 L}{dt^2} \equiv L^{(2)} = \dot{r}\times m\dot{v} + r\times m\ddot{v} = \boldsymbol{v}\times\mathbf{F} + \mathbf{r}\times\dot{\mathbf{F}} \equiv \boldsymbol{\tau}_2 \\ \frac{d^3\mathbf{L}}{dt^3} \equiv L^{(3)} = \dot{\boldsymbol{v}}\times\mathbf{F} + \boldsymbol{v}\times\dot{\mathbf{F}} + \dot{\mathbf{r}}\times\dot{\mathbf{F}} + \mathbf{r}\times\ddot{\mathbf{F}} = 2\boldsymbol{v}\times\dot{\mathbf{F}} + \mathbf{r}\times\ddot{\mathbf{F}} \equiv \boldsymbol{\tau}_3 \end{array}\right\}, \tag{24}$$

Following the rule of naming the angular momentum, let's name those quantities: $L^{(1)}$ as angular force; $L^{(2)}$ as angular jerk-force; $L^{(3)}$ as angular jounce-force. When $\boldsymbol{\tau}_3 = 0$, we obtain a *new conservative quantity*, the angular jerk-force $L^{(2)}$. When $\boldsymbol{\tau}_2 = 0$, the angular force, $\mathbf{L}_1$, is conserved. For Newton gravity, $\mathbf{L}_1 = \mathbf{r}\times\mathbf{F}$ is always zero, thus we have the regular conservative angular momentum **L**.

6.5.6. Universality of Free rising and falling

Eq. (8) predicts *universality of free rising* and *universality of free falling* with constant acceleration in a constant $\mathbf{g}$ or $\mathbf{g}_-$ field:

	Gravitational field of Positive g-charge: $\mathbf{g}$	Gravitational field of Negative g-charge: $\mathbf{g}_-$
Positive testing g-charge Q_g	Free falling	Free rising
Negative testing g-charge Q_{g-}	Free rising	Free falling

The *Einstein's elevator and observer in it will free rise in the gravitational field of a negative g-charge*, while an elevator made of negative g-charge will free fall with the same magnitude. Eq. (13) predicts the same with time-varying accelerations in a time-varying $\mathbf{g}$ field.

7. Dynamics: the g-Charge Linearized Einstein theory

7.1. g-Charge Linearized Einstein Equation

For describing the jerk universe, third order derivative theory is necessary. Einstein's geometric theory of gravity is a second order derivative theory and can't be generalized to higher order derivative. However, the universe is flat, thus as shown above, a linear theory is sufficient to describe the movement of objects in the universe.

In the linearized Einstein theory, gravity has been expressed as physical gravito-electric and gravito-magnetic field [11], $G^{\mu\nu\alpha}$, and satisfies,

$$\left.\begin{array}{l} \frac{\partial G^{\mu\nu\alpha}}{\partial x^\alpha} = -4\pi\, G T^{\mu\nu} \\ G^{\alpha\mu\nu,\lambda} + G^{\alpha\nu\lambda,\mu} + G^{\alpha\lambda\mu,\nu} = 0 \end{array}\right\}. \tag{25}$$

Where $G^{\mu\nu\alpha} \equiv \frac{1}{4}(\bar{h}^{\mu\nu,\lambda} - \bar{h}^{\mu\lambda,\nu})$, $g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}$, and $\bar{h}^{\alpha\beta}{}_{,\beta} = 0$.

Let's generalize the linearized Einstein equation to contain negative g-charge,

$$\left.\begin{array}{l} \frac{\partial G^{\mu\nu\alpha}}{\partial x^\alpha} = -4\pi\sqrt{G}\, T^{\mu\nu}_{gNet} = -4\pi\sqrt{G}\,(T^{\mu\nu}_{g} + T^{\mu\nu}_{g-}) \\ G^{\alpha\mu\nu,\lambda} + G^{\alpha\nu\lambda,\mu} + G^{\alpha\lambda\mu,\nu} = 0 \end{array}\right\}. \tag{26}$$

Where $T^{\mu\nu}_{g} = \sqrt{G}\, T^{\mu\nu}$, $T^{\mu\nu}_{g-} = -\sqrt{G}\, T^{\mu\nu}_{-}$.

For situations of ignoring T^{ij}_{g} and T^{ij}_{g-}, Eq. (26) becomes,

$$\left.\begin{array}{ll} \nabla\cdot\mathbf{g}_{Net} = -4\pi\sqrt{G}(\rho_g + \rho_{g-}) & \nabla\times\mathbf{g}_{Net} = -\frac{\partial \mathbf{B}_{gNet}}{\partial t} \\ \nabla\times\mathbf{B}_{gNet} = -4\pi\sqrt{G}(\mathbf{J}_g + \mathbf{J}_{g-}) + \frac{\partial \mathbf{g}_{Net}}{\partial t} & \nabla\cdot\mathbf{B}_{gNet} = \mathbf{0} \end{array}\right\}. \tag{27}$$

We are not in a comoving frame, so $J^i_g = T^{0i}_g = \rho_g V^i$ and $J^i_{g-} = T^{0i}_{g-} = \rho_{g-} V^i_-$.

Eq. (26 and 27) is invariant under simultaneous transformations of g-charge conjugation, Parity, and Time reversal.

7.2. g-charge linearized Einstein/Maxwell Duality

Eq. (27) shows that there is *g-charge linearized Einstein/Maxwell duality*, which is the generalization of Newton/Coulomb duality:

Maxwell theory	Mapping	g-charge linearized Einstein theory
$\nabla\cdot\mathbf{E}=4\pi(\rho_e+\rho_{e-})$		$\nabla\cdot\mathbf{g}=-4\pi\sqrt{G}(\rho_g+\rho_{g-})$
$\nabla\times\mathbf{B}=4\pi(\mathbf{J}_e+\mathbf{J}_{e-})+\frac{\partial\mathbf{E}}{\partial t}$	$Q_e\leftrightarrow Q_g,\ Q_{e-}\leftrightarrow Q_{g-}$ $\mathbf{E}\leftrightarrow\mathbf{g}/\sqrt{G},\ \mathbf{B}\leftrightarrow\mathbf{B}_g/\sqrt{G}$	$\nabla\times\mathbf{B}_g=-4\pi\sqrt{G}(\mathbf{J}_g+\mathbf{J}_{g-})+\frac{\partial\mathbf{g}}{\partial t}$
$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$	$x^\alpha\leftrightarrow -x^\alpha$	$\nabla\times\mathbf{g}=-\frac{\partial\mathbf{B}_g}{\partial t}$
$\nabla\cdot\mathbf{B}=0$		$\nabla\cdot\mathbf{B}_g=0$

Where $\rho_e(\mathbf{J}_e)$ and $\rho_{e-}(\mathbf{J}_{e-})$ are the densities (currents) of positive and negative electric charges (e-charge), respectively. In tensor form, the duality is as

Maxwell theory	Mapping	g-charge linearized Einstein theory
$\frac{\partial F^{\mu\alpha}}{\partial x^\alpha}=4\pi J^\mu$	$Q_e\leftrightarrow Q_g,\ Q_{e-}\leftrightarrow Q_{g-}$ $F^{\mu\nu}\leftrightarrow G^{\alpha\mu\nu}$	$\frac{\partial G^{\mu\nu\alpha}}{\partial x^\alpha}=-4\pi T_g^{\mu\nu}$
$F^{\mu\nu,\lambda}+F^{\nu\lambda,\mu}+F^{\lambda\mu,\nu}=0$	$x^\alpha\leftrightarrow -x^\alpha$	$G^{\alpha\mu\nu,\lambda}+G^{\alpha\nu\lambda,\mu}+G^{\alpha\lambda\mu,\nu}=0$

7.3. Ultra symmetry

Theoretically, the g-charge linearized Einstein/Maxwell Duality leads to *the Ultra Symmetry* [12]: under the simultaneous transformations of g-charge/e-charge conjugation, Parity, and Time reversal, g-charge linearized Einstein theory converts to Maxwell theory, and vice versa.

Practically, the analysis in electrodynamics can be transferred to gravity, which makes the concepts clear and calculations easy, e.g., radiations of moving charges.

7.4. g-Charge Geodesic Equation

Let's generalize the geodesic equation, $\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau}=0$, to describe the movements of positive and negative testing g-charge, respectively,

$$\left.\begin{aligned}\frac{d^2x^\mu}{d\tau^2}+\frac{Q_g}{\sqrt{G}m}\Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau}=0\\ \frac{d^2x^\mu}{d\tau^2}+\frac{Q_{g-}}{\sqrt{G}m_-}\Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau}=0\end{aligned}\right\}. \tag{28}$$

For weak gravitational field, Eq. (28) gives,

$$\left.\begin{aligned} m\ddot{\mathbf{r}} &= Q_g\mathbf{g}_{Net} + 4Q_g V\times B_{gNet} \\ m_-\ddot{\mathbf{r}}_- &= Q_{g-}\mathbf{g}_{Net} + 4Q_{g-}V_-\times B_{gNet} \end{aligned}\right\}. \tag{29}$$

7.5. High Order Derivative Equation

If we adopt the physical interpretation, the g-charge linearized Einstein theory may be extended to third order derivative theory. The exponential behaves of the g-charge density lead to the high order derivative of gravitational fields and, thus the high order acceleration, which are,

$$\left.\begin{aligned} &\rho_g^{(n+1)} = \rho_{g0}\frac{d^{n+1}}{dt^{n+1}}e^{-3Ht} && \rho_{g-}^{(n+1)} = \rho_{g0-}\frac{d^{n+1}}{dt^{n+1}}e^{3(H_-)t} \\ &\nabla\cdot\mathbf{g}_{\mathbf{Net}}^{(n)} = -4\pi\sqrt{G}(\rho_g^{(n)} + \rho_{g-}^{(n)}) && \nabla\times\mathbf{g}_{\mathbf{Net}}^{(n)} = -\frac{\partial\mathbf{B}_{\mathbf{gNet}}^{(n)}}{\partial\mathbf{t}} \\ &\nabla\times\mathbf{B}_{\mathbf{gNet}}^{(n)} = -4\pi\sqrt{G}(\mathbf{J}_{\mathbf{g}}^{(n)} + \mathbf{J}_{\mathbf{g-}}^{(n)}) + \frac{\partial\mathbf{g}_{\mathbf{Net}}^{(n)}}{\partial t} && \nabla\cdot\mathbf{B}_{\mathbf{gNet}}^{(n)} = 0 \end{aligned}\right\}, \tag{30}$$

$$\left.\begin{aligned} &m\mathbf{r}^{(n+2)} = Q_g\mathbf{g}_{\mathbf{Net}}^{(n)} + 4Q_g\left(V\times B_{gNet}\right)^{(n)} && n = 1, 2, 3, \dots \\ &m_-r_-^{(n+2)} = Q_{g-}\mathbf{g}_{\mathbf{Net}}^{(n)} + 4Q_{g-}\left(V_-\times B_{gNet}\right)^{(n)} && \end{aligned}\right\}. \tag{31}$$

Eq. (26-31) has gC-PT symmetry and form g-charge linearized Einstein theory. Eq. (30 and 31) is the generalization of Eq. (13). The scenarios requiring Eq. (13, 30, 31) exist only in cosmic phenomena.

If ignoring effects of moving source, g-charge linearized Einstein theory produces the same results as that of g-charge Newton theory. Indeed the g-charge mechanism is theory-independent.

Since Eq. (26-31) also describe the effects of moving sources, g-charge linearized Einstein theory has higher predictive power than either g-charge Newton theory or linearized Einstein theory. We now turn our attention to gravitational wave (GW).

7.6. GW Emitted by Accelerating Recessional Objects

According to linearized Einstein theory, GW's sources need to have a time-variable quadrupole (or higher-order) mass moment. A point positive or negative g-charged object moving with acceleration along radial direction has zero quadrupole momentum. So objects in the accelerating universe will not emit quadrupole GW.

On the contrary, however, the g-charge linearized Einstein/Maxwell Duality predicts that an accelerating positive or negative g-charge radiates GW [13] with the radiation power,

$$\left.\begin{aligned} I &= \frac{2Q_g^2}{3C^3}\ddot{\mathbf{r}}^2 \\ I_- &= \frac{2Q_{g-}^2}{3C^3}\ddot{\mathbf{r}}_-^2 \end{aligned}\right\}. \tag{32}$$

Substituting Eq. (15 and 20), Eq. (32) becomes,

$$\left.\begin{aligned} I &= \frac{Q_g^2}{6C^3}\overline{H}^4\frac{r_0^2}{(1+Z)^2} \\ I_- &= \frac{Q_{g-}^2}{6C^3}\overline{H}_-^4\frac{r_{-0}^2}{(1+Z_-)^2} \end{aligned}\right\}. \tag{33}$$

Where the non-radiation term, $\sim 1/r^2$ and $\sim 1/r_-^2$, have been ignored. Radiation powers, I and I_-, are positive, therefore not distinguishable. Denote Eq. (32) as the gravitational Larmor formula. We would expect Eq. (32), since the radiation field is dependent upon acceleration, either circular acceleration, e.g., binary black holes, or linear acceleration, e.g., receding objects in the accelerating universe. GWs are strongest in the directions perpendicular to the particle's linear acceleration.

The relativistic generalization of the Larmor formula, denote as the gravitational Lienard-type formula, is

$$I = \frac{2Q_g^2}{3C^3}\frac{1}{(1-\beta^2)^3}[\ddot{r}^2 - (\boldsymbol{\beta}\times\ddot{\mathbf{r}})^2]. \tag{34}$$

Where $\boldsymbol{\beta} = \frac{\overline{H}}{c}\frac{r_0}{1+Z}$. For receding objects, $\dot{\mathbf{r}} \,//\, \ddot{\mathbf{r}}$, we have

$$I = \frac{2Q_g^2}{3C^3}\frac{\ddot{r}^2}{(1-\beta^2)^3}. \tag{35}$$

Or in terms of Hubble parameter and Redshift,

$$I = \frac{Q_g^2}{6C^3}\overline{H}^4 r_0^2\frac{(1+Z)^4}{\left((1+Z)^2 - \frac{\overline{H}^2 r_0^2}{c^2}\right)^3}. \tag{36}$$

7.7. The 1998/2016 observations with the GW Damping

GW radiation from a g-charge carries energy and momentum. In order to satisfy energy and momentum conservation, the g-charge must experience recoil at the time of emission. The radiation must exert an additional force on the g-charge. Namely, GWs generate damping. For a non-relativistic g-charge, the reaction force of GW radiation acting on the g-charge is,

$$\mathbf{F} = -\frac{2Gm^2}{3C^3}\dddot{\mathbf{r}}. \tag{37}$$

Averaging over time, the work done by this force is,

$$\overline{\mathbf{F}\cdot\mathbf{V}} = \frac{2Gm^2}{3C^3}\overline{\ddot{\mathbf{r}}^2}. \tag{38}$$

This work is equal to the intensity of radiation of the g-charge; call it *GW damping*.

Taking into account the GW damping, the acceleration and jerk of the expansion of the universe are reduced, respectively,

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}(\rho_m - \rho_{m-}) - \frac{2Gm}{3C^3}\frac{\dddot{\mathbf{r}}}{\mathbf{r}}. \tag{39}$$

$$\frac{\dddot{r}}{r} = -\frac{4\pi G}{3}\{(\rho_m - \rho_{m-})\overline{H} + (\dot{\rho}_m - \dot{\rho}_{m-})\} - \frac{2Gm}{3C^3}\frac{\mathbf{r}^{(4)}}{\mathbf{r}}. \tag{40}$$

where $r^{(4)}$ is the 4th order derivative of r. ρ_m and GW damping play the same role in reducing the acceleration of the expansion.

Substituting Eq. (40) into Eq. (39) and ignoring $r^{(4)}$, we obtain the acceleration,

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}(\rho_m - \rho_{m-}) + \frac{8\pi G^2 m}{9C^3}\{(\rho_m - \rho_{m-})H + (\dot{\rho}_m - \dot{\rho}_{m-})\}. \tag{41}$$

The g-charge linearized Einstein theory predicts a significant phenomenon that the *accelerating objects of the universe emits GW*s, and that the GW damping reduces the acceleration and the jerk.

7.8. GW in the Expanding Universe

The total power of GW radiated by all the non-relativistic accelerating objects of the expanding universe is, in terms of acceleration,

$$I = \sum_i \frac{2Q_{gi}^2}{3C^3}\ddot{\mathbf{r}}_i^2 \qquad \text{and} \qquad I_- = \sum_i \frac{2Q_{gi-}^2}{3C^3}\ddot{\mathbf{r}}_{i-}^2, \tag{42}$$

or in terms of the extended Hubble parameter and redshift,

$$I = \sum_i \frac{Q_{gi}^2}{6C^3}\overline{H}_i^4 \frac{r_{i0}^2}{(1+z_i)^2} \qquad \text{and} \qquad I_- = \sum_i \frac{Q_{gi-}^2}{6C^3}\overline{H}_{i-}^4 \frac{r_{i0-}^2}{(1+z_{i-})^2}. \tag{43}$$

Where the summations are over all of positive and negative g-charges in the positive and negative g-charge universes, respectively. To conclude, the gravitational potential energy is transported to the kinematic energy of g-charges, and part of which is transported to GW energy.

The total power of GW radiated by all the relativistic accelerating objects, positive g-charges, of the expanding universe is

$$I = \frac{\bar{H}^4}{6C^3}\sum_i \frac{Q_{gi}^2 r_{i0}^2 (1+z_i)^4}{\left((1+z_i)^2 - \frac{\bar{H}^2 r_{i0}^2}{c^2}\right)^3}. \tag{44}$$

8. Nature of Dark Energy and Duality

In g-charge Newton and linearized Einstein's theories, negative g-charge generates repulsive gravity that causes the accelerating movement of ordinary objects,

Negative g-charge → *repulsive gravity* → receding movement.

In dark energy model, either p (< 0), Λ, or scalar field ϕ, generate repulsive gravity,

Dark energy → *repulsive gravity* → receding movement

Although hypothesized origins of repulsive gravity in different models are conceptually different, they explain the 1998 observation equally well. Thus we suggest that dark energy and negative g-charge are equivalent,

negative g-charge = dark energy

With this nature, dark energy varies with time and explains the 2016 observation with no fine turn problem. Denote above correspondences as g-charge/dark-energy duality:

	g-charge linear theories	Mapping	Dark energy
Observations	$\ddot{r} > 0,\ \dddot{r} > 0$	$r \leftrightarrow a$	$\ddot{a} > 0,\ \dddot{a} > 0$
Mechanism	ρ_g, ρ_{g-}	$\rho_g, \rho_{g-} \leftrightarrow \rho_m, \Lambda, p, \phi$	ρ_m, Λ, p, ϕ
Measurement	$r(t_0)/r(t) = 1 + z$	$r \leftrightarrow a$	$a(t_0)/a(t) = 1 + z$

9. Summary

The negative g-charge mechanism combining with linear theories of gravity captures several essential aspects of cosmic dynamics: Friedmann equations, expansion of universe, inflation, Redshift, dark energy, and de Sitter universe. The g-charge linear theories of gravity have gC-PT symmetry and predicts higher order differential evolution equations, motion-parameter, relation between slow-roll parameters and different order accelerations, g-Charge/Dark Energy Duality. Moreover, the g-charge linearized Einstein's theory predicts that the accelerating universe radiates GW which it expands, and the GW damping.

Let's summarize the observations/explanations of the expansion of the universe.

	Hubble (1929)	Perlmutter, Schmidt, Riess (1998)	Riess, et al (2016)
Law	Hubble law	Dark Energy Cosmology	g-charge linear Theories
$\dddot{a}/a$			$\frac{\dddot{r}}{r} = -\frac{4\pi G}{3}\{(\rho_m - \rho_{m-})\bar{H} + (\dot{\rho}_m - \dot{\rho}_{m-})\}$
$\ddot{a}/a$		$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$	$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}(\rho_m - \rho_{m-})$
$\dot{a}/a$	$\dot{a}/a = H$	$\dot{a}/a = H$	$\dot{r}/r = H$

We argue that the negative g-charge mechanism and higher order derivative theories of gravity are worth to pursue and that the expansion of the universe may be evidence of negative g-charge and of higher order derivative theories of gravity.

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