Positive Energy of Gravitational Field

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Abstract

The issues, such as the negative energy density of the gravito-static field and the lack of an energy-momentum tensor in General Relativity (GR), have a long history. To provide possible resolutions for these issues, based on the U (1) gauge theory of gravity, the gravito-static and gravito-magnetic field lines are introduced for the positive/negative gravitational charge and current, respectively. It is shown that the direction of gravitational force acting on a negative gravitational charge is opposite to that of the gravitational field. Then we show that positive energy is stored in the gravitational field and that the exchanged energy between the gravitational field and gravitational charges/currents is always positive. The energy-momentum tensor，$T_g^{\mu\nu}$，of the vector gravitational field is introduced and has the following physical properties: (1) symmetric, $T_g^{\mu\nu} = T_g^{\nu\mu}$; (2) positive energy density, $T_g^{00} > 0$; (3) zero trace, $T_{g\mu}^{\mu} = 0$; (4) conserved, $\frac{\partial T_{g\nu}^{\mu}}{\partial x_{\mu}} = 0$; (5) localizable.

1. Introduction

The energy problems of gravity have been long-standing challenges for the vector field theories of gravity, in which the energy density of the gravito-static field is negative [1], and for General Relativity (GR), in which the energy-momentum of gravity is not a tensor, not symmetric and not conserved [2 and references wherein]. Moreover, the energy-momentum of gravity in GR is not localizable, which causes more conceptual difficulties in quantization and radiation. To resolve the energy issue in GR, the energy-momentum pseudo-tensor $t^{\mu\nu}$ was introduced. However, this $t^{\mu\nu}$ faces other difficulties [3].

Note the energy problems of gravity were discussed only for the situation where there is one kind of gravitational charges. Moreover, in discussion of the negative energy of the gravito-static field, one only considered the work done by the gravito-static field to assemble a collection of point masses, hereafter call this procedure "the assemble procedure". Any vector field theory of gravity has to address this negative energy issue.

Recently, the U (1) gauge theory of gravity has been established [4]. The benefits of this gauge theory are the following: it has been quantized, renormalized, unified with Electrodynamics in the framework of Abelian gauge theory and predicts the existences of negative gravitational charges, gravito-magnetic field and gravitational waves. It has been shown that the negative gravitational charges can explain naturally the accelerated expansion of the universe [5]. In the other word, the accelerated expansion of the universe might be the observational evidence of the existence of the negative gravitational charges.

This U (1) gauge theory of gravity is a vector field theory. We need to address the negative energy and energy-momentum tensor issues in the framework of this gauge theory of gravity. In order to discuss thoroughly, we take into account not only the positive and negative gravitational charges but also the assemble procedure and the dismantle procedure that is to dismantle the collection of gravitational charges. In this article we show that the energy the gravitational field gains or loses is positive. Then the energy-momentum tensor of the gravitational field is proposed and has required physical properties.

2. Gauge Theory of Gravity

For discussing the energy of the gravitational field, let's briefly review the U (1) gauge theory of gravity [4]. The gauge theory is based on the Precise Equivalence Principle (PEP), which states, "*The gravitational mass, m_g, of a body is equal to its*

inertial rest mass, m". The PEP is supported by all of torsion-balance-type and free-fall-type experiments [6]. The PEP is also valid for known astronomical matters. The PEP leads to the internal symmetry U (1) of gravity and allow us to study the nature of gravity from a perspective of physical fields that a physical theory for gravity would be an Abelian gauge theory, denoted as Gravitodynamics.

2.1. Gravitodynamics of Positive Gravitational charges

Based on the PEP, the Gravitodynamics was established by following the procedure used for constructing an Abelian gauge theory. The gravitational field equations were deduced,

$$\frac{\partial F_g^{\mu\nu}}{\partial x^\nu} = -4\pi J_g^\mu, \tag{1}$$

$$\frac{\partial F_g^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_g^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_g^{\beta\mu}}{\partial x^\alpha} = 0. \tag{2}$$

Where gravitational field strength $F_g^{\mu\nu} = \partial^\mu A_g^\nu - \partial^\nu A_g^\mu$, A_g^μ $(\phi_g, \mathbf{A}_g)$ is the gravitational 4-potential, J_g^μ $(\rho_g, \mathbf{J}_g)$ is the gravitational 4-current. Letters with subscript "g" denote the variables related with gravity.

2.2. Negative Gravitational Charge

For exploring the complete picture of gravity, we ask: is there a negative gravitational charge? The quantization of the Gravitodynamics predicts the existence of the negative gravitational charges as shown below.

To couple Gravitodynamics to fermions, let's consider Lagrangian,

$$\mathcal{L}_{QGD} = \frac{1}{4} F_g^{\mu\nu} F_{g\mu\nu} + \overline{\psi}(i\gamma^\mu \partial_\mu - m) \psi - Q_g \overline{\psi}\gamma^\mu A_{g\mu}\psi. \tag{3}$$

Then we have the gravitational charge Q_{gN} given by

$$Q_{gN} = Q_g \int d^3 x \, \overline{\psi}\gamma^0\psi.$$

Treating this as a quantum equation, we then have

$$Q_{gN} = Q_g \int d^3 p \sum_{s=1,2} (b_p^{s\,\dagger} b_p^s - c_p^{s\,\dagger} c_p^s). \tag{4}$$

Letters with subscripts "*N*" denote variables related to the net gravitational charges, $(Q_{g+} + Q_{g-})$. Eq. (4) indicates that the net gravitational charge is equal to the number of "gravitational particles", $b_p^{s\,\dagger} b_p^s$, minus the number of "gravitational antiparticles", $c_p^{s\,\dagger} c_p^s$. The "gravitational particles" carry "positive gravitational charge" and the "gravitational antiparticles" carry "negative gravitational charges". Letters with subscript

"+" or "−"denote the variables related with positive or negative gravitational charge, respectively. Once we define one gravitational charge as positive, then the gravitational charge that repels it would be negative.

The positive and negative gravitational charges have been defined as, respectively,

$$Q_{g+} = \sqrt{G}\, m,$$
$$Q_{g-} = -\sqrt{G}\, m. \tag{5}$$

Where G is the Newtonian gravitational constant. Eq. (5) implies that both the positive and negative gravitational charges have the positive rest mass m. We define the gravitational charges carried by the ordinary matter as positive.

2.3. Complete Gravitodynamics of Positive and Negative Gravitational Charges

In order to accommodate the possible existence of negative gravitational charges, we need the equations to describe various aspects of the gravitational fields of positive and negative gravitational charges. For this aim, we assume that the negative gravitational charges generate the gravitational fields in the same way as that of the positive gravitational charges does. Then we can write down the complete Gravitodynamics as,

$$\frac{\partial F_{gN}^{\mu\nu}}{\partial x^\nu} = -4\pi J_{gN}^\mu, \tag{6}$$

$$\frac{\partial F_{gN}^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_{gN}^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_{gN}^{\beta\mu}}{\partial x^\alpha} = 0. \tag{7},$$

Where $J_{gN}^\mu = \left(\rho_{gN}, \mathbf{J}_{gN}\right)$, $J_{gN}^\mu = J_{g+}^\mu + J_{g-}^\mu$, $\rho_{gN} = \rho_{g+} + \rho_{g-}$, $\mathbf{J}_{gN} = \mathbf{J}_{g+} + \mathbf{J}_{g-}$. The J_{gN}^μ, ρ_{gN} and $\mathbf{J}_{gN}$ are the net gravitational 4-current, the net density of the gravitational charges and the net gravitational current, correspondingly. The $F_{gN}^{\mu\nu}$ is the gravitational field strength generated by the net gravitational 4-current J_{gN}^μ. Letters with subscript "N" denote the variables related with net gravitational charge.

The Lagrangian for the equation of motion of a test body is

$$\mathcal{L}_{force} = -m_0 C^2 \sqrt{1 - \frac{v^2}{C^2}} + Q_{gt}\mathbf{A}_{gN} \cdot \mathbf{V} - Q_{gt}\phi_{gN},$$

which gives

$$\frac{d\mathbf{P}}{dt} = Q_{gt}\mathbf{g}_N + Q_{gt}\mathbf{V} \times \mathbf{B}_{gN}. \tag{8}$$

The Q_{gt} is either the positive or negative gravitational charge carried by the test body.

The set of equations (6, 7 and 8) forms the complete Gravitodynamics. It shows that the gravitational force between bodies carrying gravitational charges with the same

polarity is attractive and predicts that the gravitational force between bodies carrying gravitational charges with the opposite polarity is repulsive.

2.4. Observational Evidence of Negative Gravitational Charges

Are there the negative gravitational charges in nature? Where are the negative gravitational charges? Is there an evidence of the existence of the negative gravitational charges? To answer these questions, we argue that the accelerated expansion of the universe can be interpreted as the observational evidence of the existence of the negative gravitational charges. There is the profound discovery from the remarkable observations that the universe is in the accelerated expansion [7]. This phenomenon indicates that there is a mechanism that repels the ordinary mattes. To explain this fact, the dark energy models ([8] and reference wherein) have been proposed. However, the fundamental physical mechanism of the accelerated expansion of the universe remains unclear. Moreover, the origin of the cosmological constant suffers the fine-tuning problem.

Recently, a dynamic model is proposed [5] to not only explain the accelerated expansion of the positive universe but also resolve those problems encountered by the dark energy models. The dynamic model suggests that the positive and negative gravitational charges constitute the universe.

Let's consider the gravitational field of a spherical distribution of the positive and negative gravitational charges. Eqs. (6 and 5) give,

$$\nabla \cdot \mathbf{g}_N = -4\pi\sqrt{G}\rho_{m+} + 4\pi\sqrt{G}\,\rho_{m-}. \tag{9}$$

Where $\rho_{m+}\left(= \frac{\rho_{g+}}{\sqrt{G}} > 0\right)$ and $\rho_{m-}\left(= -\frac{\rho_{g-}}{\sqrt{G}} > 0\right)$ are the mass density of the

positive and negative gravitational charges, respectively. The second term on the right hand side of Eq. (9) represents the repulsive gravitational force, generated by the mass density ρ_{m-} of the negative gravitational charges.

For uniformly distributed positive and negative gravitational charges, Eqs. (8 and 9) give

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}\rho_{m+}(t) + \frac{4\pi G}{3}\rho_{m-}(t). \tag{10}$$

The dark energy models are established to explain the accelerated expansion of the universe [8]. Employing the FRW metric and taking K = 0, Einstein equation gives,

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\rho_{m+} + \frac{4\pi G}{3}(-3p). \tag{11}$$

The modified Einstein equation with the cosmological constant Λ, gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{m+} + \frac{4\pi G}{3}\left(\frac{\Lambda}{4\pi G}\right), \tag{12}$$

Comparing Eq. (10) with Eq. (11) and Eq. (12), respectively, we argue that the mass density ρ_{m-} of the negative gravitational charges is equivalent to either "the negative pressure p" or "Einstein's cosmological constant Λ". This physical interpretation of the cosmological constant Λ can resolve its fine-tuning problem encountered in explaining the accelerated expansion of the universe.

Therefore we argue that the accelerated expansion of the universe is an observational evidence of the existence of negative gravitational charges.

3. Energy of Gravitational Field

The existence of negative gravitational charges and the complete Gravitodynamics suggest us to restudy the energy issues of gravity. For this aim, we begin with the gravitational field line and the gravitational force between positive and negative gravitational charges. The gravitational field contains the gravito-electric and gravito-magnetic fields. For a source at rest, its gravito-static field is Newtonian gravitational field.

3.1. Gravito-static Field Line

The gravito-static field line of a gravitational charge at rest is determined by Eq. (9), which can be written as the following,

$$\mathbf{g}_N = \mathbf{g}_+ + \mathbf{g}_-,$$
$$\nabla \cdot \mathbf{g}_+ = -4\pi\sqrt{G}\rho_{m+}, \tag{13}$$
$$\nabla \cdot \mathbf{g}_- = +4\pi\sqrt{G}\,\rho_{m-}. \tag{14}$$

The gravito-static fields, $\mathbf{g}_+$ and $\mathbf{g}_-$, are generated by positive and negative gravitational charge, respectively. Eq. (13) is equivalent to both Newtonian equation of gravity and the weak field approximation of General Relativity.

Eq. (13) implies that the gravito-static field line of a positive gravitational charge points to the charge as illustrated in Fig. 1.
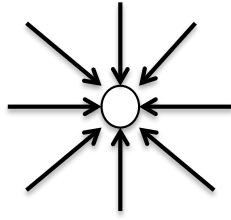
Fig. 1: The gravito-static field line of a positive gravitational charge

On the contrary, Eq. (14) implies that the gravito-static field lines of a negative gravitational charge point outward from the charge as illustrated in Fig. 2.
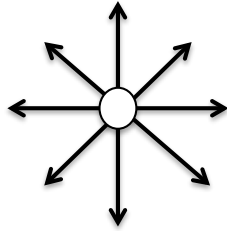

Fig. 2: The gravito-static field line of a negative gravitational charge

3.2. Gravito-magnetic Field Line

The gravito-magnetic field lines of a gravitational steady current are determined by Eq. (6), which can be written as the following,

$$\mathbf{B}_{gN} = \mathbf{B}_{g+} + \mathbf{B}_{g-},$$

$$\nabla \times \mathbf{B}_{g+} = -4\pi\rho_{m+}\mathbf{V}, \tag{15}$$

$$\nabla \times \mathbf{B}_{g-} = +4\pi\rho_{m-}\mathbf{V}. \tag{16}$$

To determine the directions of the positive gravitational current and gravito-magnetic field, Eq. (15) indicates us to use "the left-hand rule". The "left-hand rule" states: if one's thumb of left hand points to the direction of the positive gravitational current the fingers point to the direction of the gravito-magnetic field, $\mathbf{B}_{g+}$ (Fig. 3).
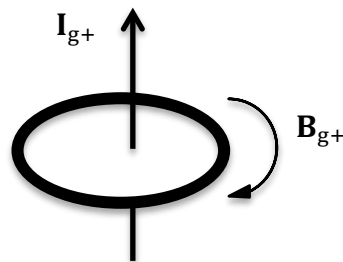

Fig.3: "The left-hand rule" for gravito-magnetic field $\mathbf{B}_{g+}$

On the contrary, Eq. (16) requires to use "the right-hand rule" is for the negative

gravitational current (Fig. 4). The "right-hand rule" is the same as that in the electromagnetism.
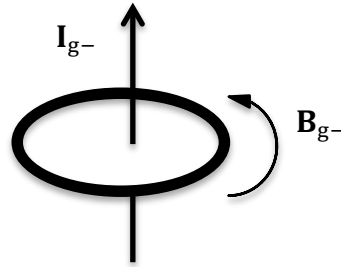

Fig. 4: "The right-hand rule" for gravito-magnetic field $\mathbf{B_{g-}}$

3.3. Attractive and Repulsive Gravito-static Forces

Now let's consider the gravitational force acting on a test body carrying either a positive gravitational charge Q_{t+} or a negative gravitational charge Q_{t-}. The gravito-static forces acting on the test body are determined by the equation of motion, Eq. (8) which gives

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = \begin{cases} Q_{t+}\, \mathbf{g_+} + \, Q_{t+}\mathbf{V} \times \mathbf{B_{g+}} \\ Q_{t+}\, \mathbf{g_-} + \, Q_{t+}\mathbf{V} \times \mathbf{B_{g-}} \end{cases} = \begin{cases} \sqrt{G}\, m\mathbf{g_+} + \sqrt{G}\, m\mathbf{V} \times \mathbf{B_{g+}} \\ \sqrt{G}\, m\mathbf{g_-} + \sqrt{G}\, m\mathbf{V} \times \mathbf{B_{g-}} \end{cases} \qquad (17)$$

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = \begin{cases} Q_{t-}\, \mathbf{g_+} + \, Q_{t-}\mathbf{V} \times \mathbf{B_{g+}} \\ Q_{t-}\, \mathbf{g_-} + \, Q_{t-}\mathbf{V} \times \mathbf{B_{g-}} \end{cases} = \begin{cases} -\sqrt{G}\, m\mathbf{g_+} - \sqrt{G}\, m\mathbf{V} \times \mathbf{B_{g+}} \\ -\sqrt{G}\, m\mathbf{g_-} - \sqrt{G}\, m\mathbf{V} \times \mathbf{B_{g-}} \end{cases} \qquad (18)$$

Eq. (17) shows that the gravito-static force acting on a positive gravitational charge has the same direction as that of the gravito-static field lines. Eq. (18) shows that the gravito-static force acting on a negative gravitational charge has the direction opposite to that of the gravito-static field lines.

Therefore, two positive gravitational charges are attractive to each other as shown in Fig. 5: $\mathbf{F} = Q_{t+}\, \mathbf{g_+} = \sqrt{G}\, m\mathbf{g_+}$.

Fig. 5: two positive gravitational charges are attractive to each other

Two negative gravitational charges are attractive to each other as shown in Fig. 6:

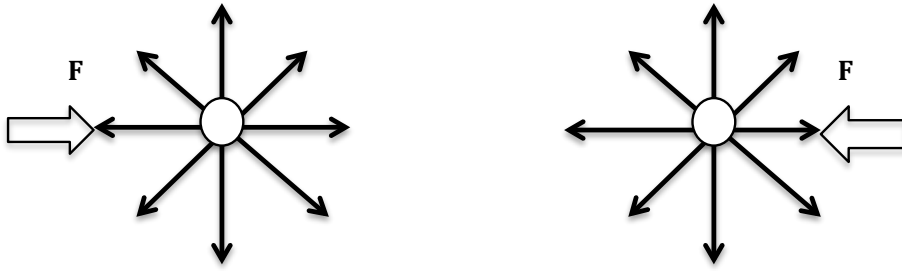$$\mathbf{F} = Q_{t-}\,\mathbf{g}_- = -\sqrt{G}\,m\mathbf{g}_-.$$

Fig. 6: two negative gravitational charges are attractive to each other

A positive and a negative gravitational charge repel each other as shown in Fig. 7: either $\mathbf{F} = Q_{t+}\,\mathbf{g}_- = \sqrt{G}\,m\mathbf{g}_-$ or $\mathbf{F} = Q_{t-}\,\mathbf{g}_+ = -\sqrt{G}\,m\mathbf{g}_+$.
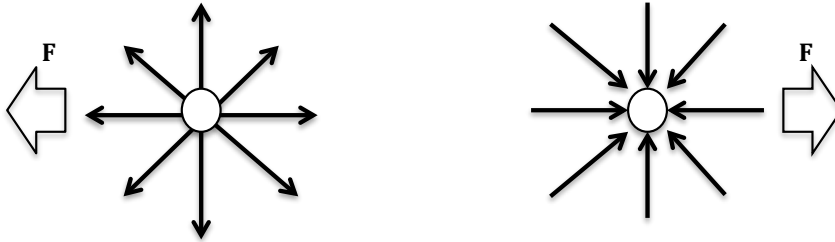

Fig. 7: a positive and a negative gravitational charge repel each other

The positive gravitational currents in same directions repel to each other (Fig. 8). The positive gravitational currents in opposite directions attract to each other (Fig. 9).
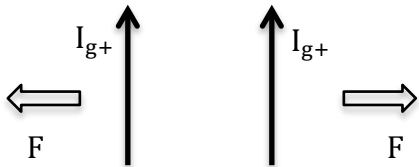

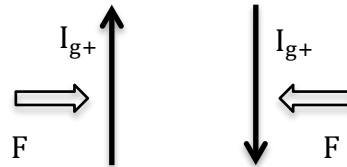Fig. 8   Fig. 9

The negative gravitational currents in same directions repel to each other (Fig. 10). The negative gravitational currents in opposite directions attract to each other (Fig. 11).
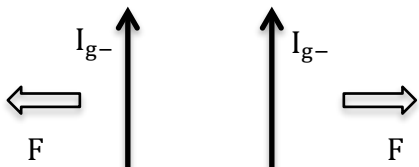

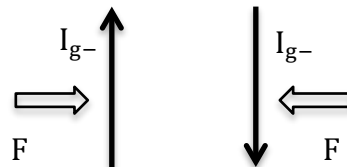Fig. 10   Fig. 11

A negative and a positive gravitational current in same directions attract to each other (Fig. 12). A negative and a positive gravitational current in opposite directions repel to each other (Fig. 13).
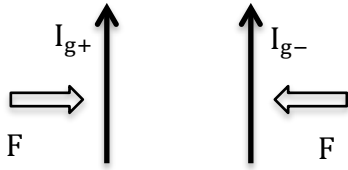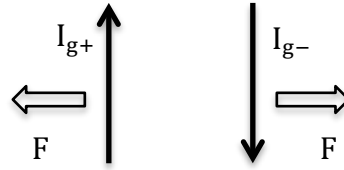
Fig. 12   Fig. 13

3.4. Energy of Gravito-static Field

The negative energy of the gravitational field is a serious issue [1]. After determining the gravitational field lines and the directions of the gravitational forces, we are able to discuss the negative energy issue from a different perspective. Traditionally, the potential energy of the field is related to the work done to *assemble a collection* of charges with same polarity by bringing them together. We call this "assemble procedure".

Let's consider the electrostatic potential energy. For assembling a collection of like electric changes, an external force is required and does the positive work that, then, is stored in their electrostatic fields. Note when the external force is removed and there is no mechanism to hold the electric charges together, the collection of electric charges will be dismantled by the repulsive force between the like electric charges, we call this "dismantle procedure". During the dismantle procedure, the electrostatic field does the work and, therefore loses positive energy. The collection of like electric charges alone is not stable.

Now let's consider the gravito-static energy. For assembling a collection of gravitational charges, there is no external force required. The gravitational field does the work required to assemble the collection of charges. On the other hand, an external force is needed to dismantle the collection of gravitational charges.

The comparisons of assembling and dismantling collections of gravitational and electric charges are shown in Table 1.

Table 1

	Collection of gravitational charges	Collection of Electric charges
Assemble procedure	by gravito-static field	by external force
Dismantle procedure	by external force	by electrostatic field

The Table 1 suggests us to discuss a complete procedure containing both the assemble

procedure and the dismantle procedure for studying the negative energy issue thoroughly.

Therefore we need an alternative approach: first, we consider the procedures in which the fields do the work of moving charges, i.e., the fields lose the positive energy. Second, we consider the procedures in which an external force is required to do the work of moving charges. Then we can conclude that the work done by the external force is store in the field, i.e., a positive energy is stored in the field.

Taking into account the negative gravitational charges, we will consider two situations: First situation is a collection of positive gravitational charges; Second situation is a collection of negative gravitational charges.

3.4.1. Work Done to Assemble Collection of Positive Gravitational Charges

Let's consider the assemble procedure of the first situation. The positive gravitational charges are attracted to each other. There is no need to apply an external force. Instead, the gravitational field $\mathbf{g}_+$ will do the work, W_{field}, to bring in positive gravitational charges Q_+ one by one. Imaging there is a positive gravitational charge, $Q_{source+}$, as the source of the original gravitational field. Now bring in a test charge, Q_{t+} from the point "a" to the point "b". In this case, the gravitational force of the $Q_{source+}$ and field line are in the same direction and are opposite to d$\mathbf{l}$, $\mathbf{F}_{field} // \mathbf{g}_+ // (-\mathrm{d}\mathbf{l})$. The d$\mathbf{l}$ is the displacement vector. Eqs. (13 and 17) gives,

$$W_{field} = \int_a^b \mathbf{F}_{field} \cdot \mathrm{d}\mathbf{l} = Q_{t+} \int_a^b \mathbf{g}_+ \cdot \mathrm{d}\mathbf{l} = -Q_{t+}\left[V_{g+}(b) - V_{g+}(a)\right].$$

For b $=$ r and $V_{g+}(a = \infty) = 0$, we have

$$W_{field} = -Q_{t+}V_{g+}(r). \tag{19}$$

Where $W_{field} = -Q_{t+}\left(-\frac{Q_{source+}}{r}\right) > 0$, i.e., the gravitational field does the work and loses positive energy. The gravitational potential energy is defined as the negative of the work done on a positive gravitational charge, $V_{g+}(r) \equiv -\frac{W_{field}}{Q_+}$. The potential energy,

$V_{g+}(r)$, is negative,

$$V_{g+}(r) = -\frac{Q_{source+}}{r} = -\frac{\sqrt{G}m_{source}}{r} < 0. \tag{20}$$

Traditionally one stops at this step and concludes that the energy of the gravito-static field is negative.

Next let's generalize this assemble procedure to the situation of building a continuous positive gravitational charge distribution. For a volume gravitational charge density ρ_{g+}, Eq. (19) gives

$W_{field} = -\frac{1}{2} \int \rho_{g+} V_{g+}(r) d\tau.$

Substituting $\rho_{g+} = -\frac{1}{4\pi} \nabla \cdot \mathbf{g}_+$, we obtain

$$W_{field} = \frac{1}{8\pi} \int g_+^2 d\tau. \tag{21}$$

This is how much work done by the gravitational field to assemble the continuous gravitational charge distribution.

Eqs. (19 and 21) imply that during the assembly procedure of the first situation, the work done by the gravito-static field to move the gravitational charge Q_{t+} is positive and is transformed to the kinetic energy of the test gravitational charge Q_{t+}. In this assemble procedure, the gravitational field loses positive energy. The total energy of the gravitational field decreases, i.e., the change of the energy of the gravitational fields is negative. Therefore, we argue that, precisely speaking, the assemble procedure implies only that the gravitational fields *lose positive energy*.

3.4.2. Work Done to Dismantle Collection of Positive Gravitational Charges

Let's assume that there is a collection of positive gravitational charges. We consider how to dismantle it. The dismantle procedure of the situation is the reverse procedure of the assemble procedure. To move a gravitational charge Q_{t+} from the point "b" to a far away point "a" in the field $\mathbf{g}_+$, an external force is required against the gravito-static field $\mathbf{g}_+$. The external force needs to be opposite to $\mathbf{g}_+$, $\mathbf{F}_{external} // d\mathbf{l} // (-\mathbf{g}_+).$

The work done by the external force $\mathbf{F}_{external}$ is

$W_{external-force} = \int_b^a \mathbf{F}_{external} \cdot d\mathbf{l} = Q_{t+} \int_b^a g_+ dl = Q_{t+} \left[V_{g+}(a) - V_{g+}(b) \right],$

For b = r and $V_{g+}(a = \infty) = 0$, we have

$$W_{external-force} = -Q_{t+} \left[V_{g+}(r) \right]. \tag{22}$$

Where $W_{external-force} = -Q_{t+}(-\frac{Q_{source+}}{r}) > 0$. That is how much work it required to separate the positive gravitational charge Q_{t+} from $Q_{source+}$.

To dismantle the continuous positive gravitational charge distribution, an external force against the gravitational field $\mathbf{g}_+$ of the distribution is required to move gravitational charges away. The work done by the external force, based on Eq. (22), is

$$W_{external-force} = -\frac{1}{2}\int \rho_{g+} \, V_{g+}(r)d\tau = \frac{1}{8\pi}\int g_+^2 d\tau. \tag{23}$$

Eq. (22) and Eq. (23) show the following: (1) the work done by the external force transforms to the gravitational field, i.e., the work is stored in the form of the energy in the gravitational field; (2) during the dismantle procedure, the gravitational field *gains positive energy.*

3.4.3. Work Done to Assemble Collection of Negative Gravitational Charges

Let's consider the assemble procedure of the second situation. The negative gravitational charges are attracted to each other. To move a negative gravitational charge, Q_{t-}, from the point "a" to point "b" in the gravitational field $\mathbf{g}_-$ of a negative gravitational source $Q_{source-}$, the gravitational force does the work W_{field}. For this situation, the gravitational field line and $d\mathbf{l}$ are in the same direction and are opposite to that of the gravitational force, $d\mathbf{l} \mathbin{/\mkern-5mu/} \mathbf{g}_- \mathbin{/\mkern-5mu/} (-\mathbf{F}_{field})$. Eqs. (14 and 18) gives,

$$W_{field} = \int_a^b \mathbf{F}_{field} \cdot d\mathbf{l} = -Q_{t-}\int_a^b \mathbf{g}_- \cdot d\mathbf{l} = -Q_{t-}\left[V_{g-}(b) - V_{g-}(a)\right].$$

For b = r and $V_{g-}(a = \infty) = 0$, we have

$$W_{field} = -Q_{t-}\left[V_{g-}(r)\right]. \tag{24}$$

Where $W_{field} = -Q_{t-}(-\frac{Q_{source-}}{r}) = \frac{(Q_{t-})(Q_{source-})}{r} > 0$. The work done by the gravitational field is positive and transforms to the kinetic energy of the test negative gravitational charge.

Following the definition of $V_{g+}(r)$, let's define the gravitational potential energy of a negative gravitational source as the negative of the work done on a negative gravitational charge,

$$V_{g-}(r) \equiv -\frac{W_{field}}{Q_-}.$$

Note the potential energy, $V_{g-}(r)$, is positive,

$$V_{g-}(r) = -\frac{Q_{source-}}{r} = \frac{\sqrt{G}m}{r} > 0. \tag{25}$$

The difference between $V_{g+}(r)$ and $V_{g-}(r)$ is caused by the negative charge.

Next let's consider the energy of a continuous negative gravitational charge distribution. For a volume negative gravitational charge density ρ_{g-}, Eq. (24) becomes

$$W_{field} = -\frac{1}{2}\int \rho_{g-}\, V_{g-}\,(r)d\tau.$$

Substituting $\rho_{g-} = -\frac{1}{4\pi}\nabla \cdot \mathbf{g}_{-}$, we obtain

$$W_{field} = \frac{1}{8\pi}\int g_{-}^2\, d\tau. \tag{26}$$

This is how much work done by the gravitational field to assemble a continuous negative gravitational charge distribution.

Eqs. (24 and 26) imply that during the assembly procedure of the second situation, the gravitational field *loses positive energy*.

3.4.4. Work Done to Dismantle Collection of Negative Gravitational Charges

Let's consider the dismantle procedure of the second situation. To separate two negative gravitational charges, an external force against the gravitational field $\mathbf{g}_{-}$ of the gravitational charge $Q_{source-}$ is required. For this situation, the external force, $d\mathbf{l}$ and $\mathbf{g}_{-}$ are in the same direction, $\mathbf{F}_{external}\ //\ d\mathbf{l}\ //\ \mathbf{g}_{-}.$ We have

$$W_{external-force}\ = \int_b^a \mathbf{F}_{external}\cdot d\mathbf{l} = Q_{t-}\int_a^b g_{-}dl = Q_{t-}\Big[V_{g-}\,(a)\ -\ V_{g-}(b)\Big].$$

For b = r and $V_{g-}\,(a = \infty) = 0$, we have

$$W_{external-force} = -\ Q_{t-}\Big[V_{g-}\,(b)\Big]. \tag{27}$$

Where, $W_{external-force} = -\ Q_{t-}\Big[V_{g-}\,(b)\Big] = \frac{G(m_t)(m_{source})}{r} > 0.$ The external force does positive work to move negative gravitational charges away.

To dismantle a continuous negative gravitational charge distribution, an external force against the gravitational field $\mathbf{g}_{-}$ of the distribution is required to move a gravitational charge Q_{t-} away. The work done by the external force, based on Eq. (27), is

$$W_{external-force}\ = -\ \frac{1}{2}\int \rho_{g-}\, V_{g-}\,(r)d\tau = \frac{1}{8\pi}\int g_{-}^2\, d\tau. \tag{28}$$

This is how much work done by the external force to dismantle a continuous negative gravitational charge distribution. Eqs. (27 and 28) imply that during the dismantle procedure of the second situation, the gravito-static field *gains positive energy*.

We conclude that the energies the gravito-static field gains in the dismantle procedure and loses in the assemble procedure are positive, i.e., the exchanged energy between the gravito-static field and gravitational charges is positive. One can still define the gravito-static potential energy for both positive and negative charges as the negative of

the work done by gravito-static field.

3.5. Energy of Gravito-magnetic Field

The gravito-magnetic field has been postulated since year 1893 [9]. Many effects of it have been predicted and experiments have been designed to detect it [10]. Eqs. (6 and 7) describe the gravito-magnetic field generated by positive gravitational current,

$$\nabla \times \mathbf{B}_{g+} = -4\pi \mathbf{J}_{g+} + \frac{\partial g_{+}}{\partial t}, \tag{29}$$

$$\nabla \times \mathbf{g}_{+} = -\frac{\partial \mathbf{B}_{g+}}{\partial t}. \tag{30}$$

These two equations have the form similar to that of Maxwell equations, thus we follow the same procedure of deriving the energy of the magneto-static field to study the energy of the gravito-magneto-static field.

Let's consider a rotating mass ring carrying the positive gravitational charges, which is the counterpart of an electric current loop. We introduce the gravito-magnetic flux,

$$\phi_{g+} = \int \mathbf{B}_{g+} \cdot d\mathbf{a} = \oint \mathbf{A}_{g+} \cdot d\mathbf{l}. \tag{31}$$

Based on the difference between Eq. (29) and the magnetic field equation

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t},$$

we assume that the gravito-magnetic field and, thus the flux, is proportional to the negative of the gravitational current, I_{g+},

$$\phi_{g+} = -L_g I_{g+}. \tag{32}$$

Where the L_g is the gravitational inductance of the ring. Note the "−" in Eq. (32) comes from the "−" in Eq. (29), i.e. the negative of the gravitational current, $-4\pi \mathbf{J}_{g+}$, generates the gravito-magnetic field.

According to Eqs. (30 and 32), let's define the gravitomotive force, gmf,

$$\varepsilon_{+} = -\frac{d\phi_{g+}}{dt} = L_g \frac{dI_{g+}}{dt}. \tag{33}$$

The work done by external force acting on a unit gravitational charge in the ring, against the back gravitomotive force, is $-\varepsilon_{+}$. If we start with a resting ring and rotates it up to a final value, I_{g+}, the work done by the external force is

$$\frac{dW}{dt} = -\varepsilon_{+} I_{g+}. \tag{34}$$

Applying Eqs. (31, 32 and 33), Eq. (34) gives

$$W = \frac{1}{2} I_{g+} \oint \mathbf{A}_{g+} \cdot d\mathbf{l}. \tag{35}$$

Note Eq. (29) requires applying the "left-hand rule" to determine the relation between

the directions of the line integral and the surface "$d\mathbf{a}$", instead of "right-hand rule in Electromagnetism. The "left-hand rule" states: if one's fingers of left hand point to the direction of the line integral, then the thumb fixed the direction of the surface $d\mathbf{a}$.
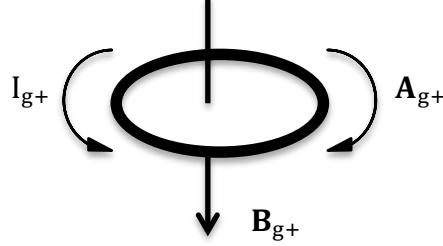

Fig. 14: Directions of I_{g+}, $\mathbf{A}_{g+}$ and $\mathbf{B}_{g+}$

On the other hand, since $\mathbf{B}_{g+} = \nabla \times \mathbf{A}_{g+}$, the relation between the directions of the $\mathbf{B}_{g+}$ and $\mathbf{A}_{g+}$ is determined by the "right-hand rule". Therefore $\mathbf{A}_{g+}$ and $\mathbf{J}_{g+}$ are in opposite directions along $d\mathbf{l}$ of the ring (Fig. 14). Eq. (35) becomes,

$$W = -\frac{1}{2}\oint(\mathbf{A}_{g+} \cdot \mathbf{I}_{g+})dl. \tag{36}$$

To generalize to the volume gravitational current, Eq. (36) becomes

$$W = -\frac{1}{2}\oint(\mathbf{A}_{g+} \cdot \mathbf{J}_{g+})d\tau. \tag{37}$$

Substituting Eq. (29) into Eq. (37), we have

$$W = \frac{1}{8\pi}\int B_{g+}^2\, d\tau. \tag{38}$$

Where we have let the surface integral over all of space to be zero. Eq. (38) implies that the work done by the external force is positive, transformed to energy of the gravito-magnetic field and stored there.

The above procedure can be applied to negative gravitational current to show that $W = \frac{1}{8\pi}\int B_{g-}^2\, d\tau$. There is an alternative approach to show this conclusion. The equations of the gravito-magnetic field generated by the negative gravitational current are

$$\nabla \times \mathbf{B}_{g-} = -4\pi\mathbf{J}_{g-} + \frac{\partial g_-}{\partial t}, \tag{39}$$

$$\nabla \times \mathbf{g}_- = -\frac{\partial \mathbf{B}_{g-}}{\partial t}. \tag{40}$$

From the definitions of the positive and negative gravitational charges, Eq. (5), we have

$$\mathbf{J}_{g-} = -\mathbf{J}_{g+}.$$

Therefore Eq. (39) becomes

$$\nabla \times \mathbf{B}_{g-} = 4\pi\mathbf{J}_{g+} + \frac{\partial g_-}{\partial t}. \tag{41}$$

Eq. (41) and Eq. (40) are in the form exactly same as that of Maxwell equations. Therefore, by following the same procure in the electromagnetism, we can readily show

that the energy of the gravito-magnetic field generated by the negative gravitational current is positive.

$$W = \frac{1}{8\pi} \int B_{g-}^2 \, d\tau. \tag{42}$$

As a summary, the energy stored in the gravitational field is positive,

$$W = \frac{1}{8\pi} \int (g^2 + B_g^2) d\tau. \tag{43}$$

The negative energy issue of the vector gravitational field is resolved.

3.6. Energy Density and Energy flux of Gravitational Field

Let's consider gravitational charges interacting with a gravitational field and moving with velocity $\mathbf{V_i}$. The change of the kinetic energy of the gravitational charges is

$$\frac{dK_g}{dt} = \sum_i Q_{gi} \mathbf{V_i} \cdot \mathbf{g}_N. \tag{44}$$

Eq. (44) can be expressed as an integral,

$$\frac{dK_g}{dt} = \int \mathbf{J_g} \cdot \mathbf{g_N} d\tau. \tag{45}$$

Substituting Eq. (6) into Eq. (45), we have

$$\frac{d}{dt} \int \frac{B_{gN}^2 + g_N{}^2}{8\pi} d\tau = \frac{dK_g}{dt} - \oint \mathbf{S} \cdot d\mathbf{a}. \tag{46}$$

Where

$$\mathbf{S} \equiv \frac{1}{4\pi} \mathbf{g_N} \times \mathbf{B_{gN}}. \tag{47}$$

K_g is the kinetic energy of all of the gravitational charges in the volume V. The $\mathbf{S}$ is the gravitational counterpart of the Poynting vector in Electrodynamics. Eq. (46) is consistent with Eq. (43). Based on the interpretations of Eq. (43), we interpret Eq. (46) as,

The positive energy that the gravitational field gained is equal to the work done on the gravitational charges/currents by external forces, less the energy that flowed out through the surface. The positive energy that the gravitational field lost is equal to the work done on the gravitational charges by the gravitational field, less the energy that flowed out through the surface.

When the kinetic energy of the gravitational charges is negligible, Eq. (46) indicates that the change of energy of the gravitational field is equivalent to the energy flowed out through the surface,

$$\frac{d}{dt} \int \frac{B_{gN}^2 + g_N{}^2}{8\pi} d\tau = - \oint \mathbf{S} \cdot d\mathbf{a}. \tag{48}$$

The density of momentum of the gravitational field store in the field itself is,

$$\mathbf{p}_g = \mathbf{S} = \frac{1}{4\pi}\, \mathbf{g}_N \times \mathbf{B}_{gN}. \tag{49}$$

The gravitational field also carries angular momentum,

$$\boldsymbol{l}_g = \mathbf{r} \times \mathbf{p}_g. \tag{50}$$

The application of Eq. (47) on the gravitational radiation is out of scope of this article.

3.7. Energy-Momentum Tensor of Gravitational Field

The energy-momentum $t^{\mu\nu}$ of the gravitational field in General Relativity is not a tensor and expressed as the energy-momentum pseudo-tensor, which is neither symmetric nor localizable [2]. After showing that the energy of the vector gravitational field is positive, let's consider the energy-momentum.

The U (1) gauge theory of gravity shows that the gravitational field is similar to the Electromagnetic field [4]. This similarity allows us to define the energy-momentum tensor $T_{g\mu}^{\nu}$ for the gravitational field as,

$$T_{g\mu}^{\nu} = \frac{\partial A_{g\alpha}}{\partial x^{\mu}} \frac{\partial \mathcal{L}_g}{\partial \left(\frac{\partial A_{g\alpha}}{\partial x^{\nu}} \right)} - \delta_{\mu}^{\nu} \mathcal{L}_g. \tag{51}$$

Substituting $\mathcal{L}_{gN} = -\frac{1}{4} F_{gN}^{\mu\nu} F_{gN\mu\nu}$ into Eq. (51), the energy-momentum tensor is,

$$T_g^{\mu\nu} = -F_g^{\mu\alpha} F_{g\alpha}^{\nu} + \frac{1}{4} \eta^{\mu\nu} F_{g\alpha\beta} F_g^{\alpha\beta}. \tag{52}$$

By analogous to Electrodynamics, this energy-momentum tensor obeys the following conditions required by a standard field theory:

(1) It is symmetric,

$$T_g^{\mu\nu} = T_g^{\nu\mu}. \tag{53}$$

(2) It has zero trace,

$$T_{g\mu}^{\mu} = 0. \tag{54}$$

(3) The energy-momentum of the gravitational field is conserved,

$$T_{g\nu,\mu}^{\mu} = 0. \tag{55}$$

(4) The component T_g^{00} gives the positive energy density of the gravitational field.

(5) The components T_g^{i0} is the momentum density of the gravitational field.

(6) This tensor is localizable, i.e., we can detect its effects on a gravitational charge and current. The localizability of the $T_g^{\mu\nu}$ is consistent with that in the U (1) gauge theory of gravity, the gravitational field is quantized.

4. Conclusions and Discussion

In this article, we propose the complete procedure containing the assembly procedure and the dismantle procedure. We show that the gravito-static field either transforms its positive energy to gravitational charges or gains the positive energy from the external force acting of the gravitational charge. In both the assemble procedure and the dismantle procedure, the exchanged energy of the gravito-static field is always positive (Table 2).

Table 2

	Work done by gravito-static field	Work done by external force
Assemble procedure	Gravito-static field *loses* *positive* energy	
Dismantle procedure		Gravito-static field *gains* *positive* energy:

Also we show that the energy store in the gravito-magnetic field is positive.

Let's discuss the origin of the negative potential energy of the gravitational field (Table 3).

Table 3

	Positive Gravitational charge	Negative Gravitational charge
Definition	$Q_{g+} \equiv \sqrt{G}\, m$	$Q_{g-} \equiv -\sqrt{G}\, m$
Potential Energy	$V_{g+}(r) \equiv -\dfrac{W_{field}}{Q_{g+}}$ $V_{g+}(r) = -\dfrac{Q_{g/source+}}{r}$ $= -\dfrac{\sqrt{G}m}{r} < 0$	$V_{g-}(r) \equiv -\dfrac{W_{field}}{Q_{g-}}$ $V_{g-}(r) = -\dfrac{Q_{g/source-}}{r}$ $= \dfrac{\sqrt{G}m}{r} > 0$
Field strength	$\mathbf{g}_+ = -\dfrac{Q_{g+}}{r^2}\,\hat{\mathbf{r}}$ $= -\dfrac{\sqrt{G}m}{r^2}\,\hat{\mathbf{r}}$	$\mathbf{g}_- = -\dfrac{Q_{g-}}{r^2}\,\hat{\mathbf{r}}$ $= \dfrac{\sqrt{G}m}{r^2}\,\hat{\mathbf{r}}$
Field Equation	$\nabla \cdot \mathbf{g}_+ = -4\pi Q_{g+}$ $= -4\pi\sqrt{G}\rho_{m+},$	$\nabla \cdot \mathbf{g}_- = -4\pi Q_{g-}$ $= +4\pi\sqrt{G}\,\rho_{m-}$
Energy Exchange	$W_{field-exchanged} = \dfrac{1}{8\pi}\int g_+^2\, d\tau$	$W_{field-exchanged} = \dfrac{1}{8\pi}\int g_-^2\, d\tau$

Table 3 shows that although the definitions of the gravitational potential energies, $V_{g+}(r)$ and $V_{g-}(r)$, are the same, we have $V_{g+}(r) < 0$ and $V_{g-}(r) > 0$. This difference between the $V_{g+}(r)$ and $V_{g-}(r)$ originates from definition of the Q_{g+} and Q_{g-}. Imaging that we defined the gravitational charge of the ordinary matter is negative, and then we would have the positive gravitational potential energy for our ordinary world. Moreover, as shown in this article, the exchanged energy of the gravitational field with gravitational charge/current is always positive. Therefore either the negative or positive gravitational potential energy is just a matter of notation of bookkeeping.

We conclude that the energy stored in the vector U (1) gauge field of gravity is positive, therefore, resolves the negative energy issue. Moreover, this is consistent with the fact that the gravitational waves carry positive energy.

The energy-momentum tensor deduced from the U (1) gauge theory of gravity satisfies the requirements for a physical field theory and, therefore, resolves the energy-momentum issue encountered in GR.

We suggest a possible phenomenon that when a positive and a negative gravitational charge collide and annihilate, their gravitational charges vanish, but the total masses of two gravitational charges converted to radiation energy, which might be observable. The detail discussion is out of the scope of this article.

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