Negative Gravitational Charge and Theories of Gravity

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Abstract

The purpose of this article is to identify laws that would describe gravitation if negative gravitational mass/gravitational charge (g-charge) exists. WEP is generalized to establish correlations between mass and positive/negative g-charge (Q_{g+}/Q_{g-}), although Q_{g-} on Earth would fall upward. Newton's theory and Einstein's theory are generalized to describe behaviors of objects carrying Q_{g-}. A significant conclusion is that, as long as Q_{g-} is introduced, all of Newton's theory, Gravitodynamics and Einstein's theory explain the accelerated expansion of Universe equally well without needs of "negative pressure" and "cosmological constant" that has fine-tuning issue. This implies that this mechanism of the cosmological phenomenon is universal and theory independent, which strongly supports the existence of negative g-charges. Moreover the acceleration is increasing with time. Gravitodynamics and generalized Einstein's theory predict: (1) gravitational wave (GW_-) emitted by Q_{g-}; (2) GW_- cannot be distinguished from GW_+ emitted by Q_{g+}.

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Key words: repulsive gravitation, accelerated expansion of Universe, Newton's theory of gravity, Gravitodynamics, Einstein's theory of gravity

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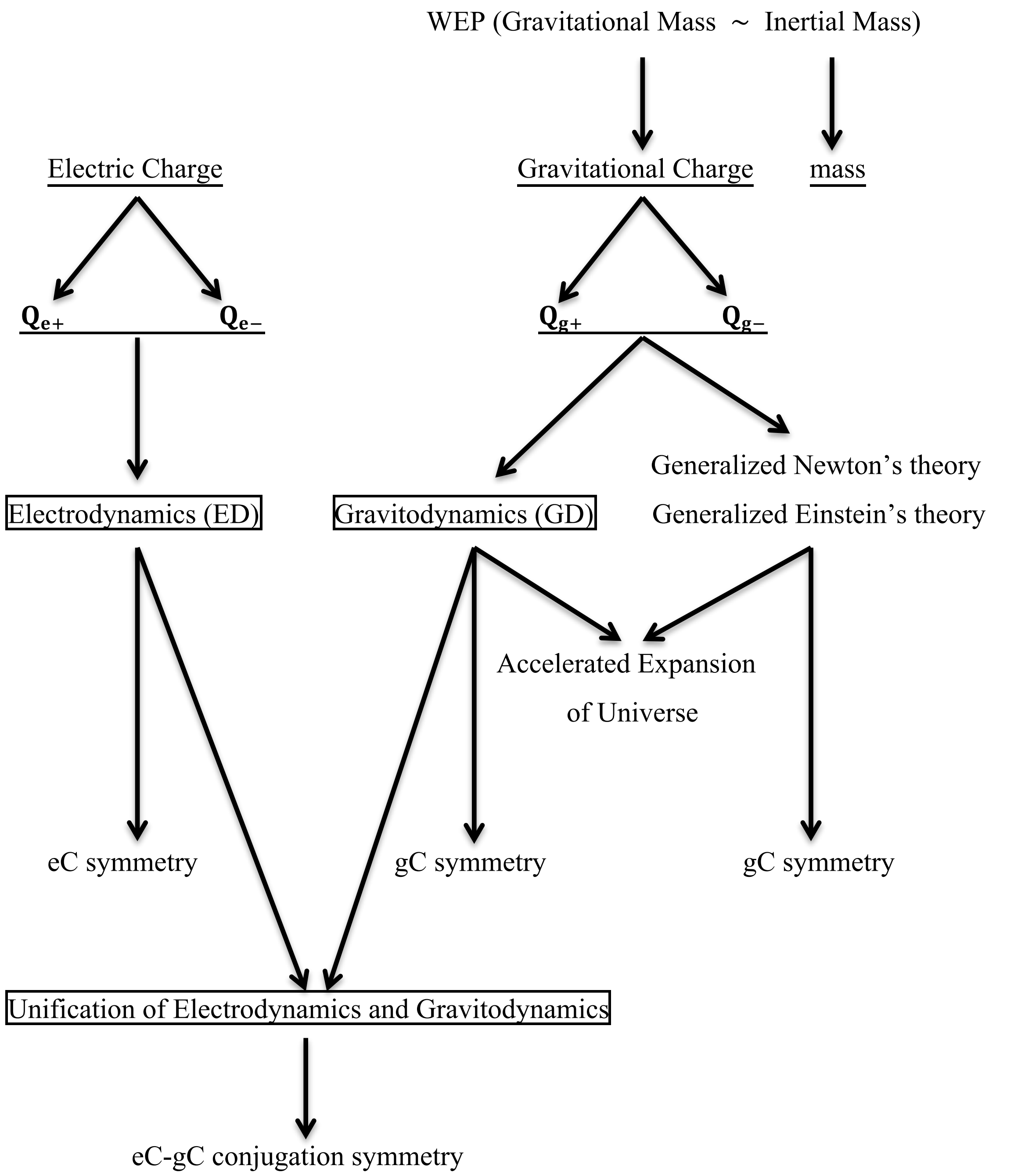
The Development of the Concepts of this Article



1. Introduction

In 1998, scientists have discovered that the Universe's expansion is accelerated [1]. To explain this mystery, the dark energy and cosmological constant models have been proposed. There is lack of a commonly accepted mechanism [2]. The repulsive gravitation as a physical mechanism has been proposed to explain this phenomenon. Among them there are Villata's model [3], Hajdukovic's model [4, 5], and Peng's model [6]. According to Villata's and Hajdukovic's models, there exist gravitation repulsion between matter and antimatter. Villata argues that the C conjugation of particle-antiparticle leads to that there is gravitation repulsion between matter and antimatter in Einstein's theory of gravity. On the contrary, Cabbolet argues that the CPT theorem excludes the repulsive gravitation [7]. Denoting this controversy as "CPT issue". Moreover Weak Equivalence Principle (WEP) states that positive gravitational mass (g-mass) is equivalent to positive inertial mass, thus excludes the repulsive gravitation between positive mass matter and positive mass antimatter. Denoting this as "WEP issue". The gravitational interactions between matter and antimatter have been studied, but not established experimentally yet [8].

The concept of negative gravitational mass (g-mass) has been proposed [9] and leads to "Runaway paradox" [10]. With negative g-mass, the bi-metric theory postulate that there are two parallel universes described respectively by two metrics, one generated by positive g-mass of matter and the other generated by negative g-mass of antimatter [11]. In the bi-metric theory, negative g-mass is equivalent to negative inertial mass.

Based on quantum Gravitodynamics [12], Peng's model postulates that negative gravitational charges (g-charge) carried by objects (not necessarily antimatter) repel positive g-charges. The most significant implication is that the g-charge and inertial mass *are conceptually separated*. Therefore the signs of g-charges may be either the same or opposite to that of mass. Let's compare those three models:

	Villata's Model	Hajdukovic model	Peng's model
Carrier (*Positive*)	Matter carries positive g-mass	Matter carries positive g-mass	Matter carries positive g-charge
Carrier (*Negative*)	Antimatter carries negative g-mass	Antimatter carries negative g-mass	Matter carries negative g-charge
Derivation of *Repulsive* Gravity	by CPT Symmetry	by Quantum Vacuum	by Quantum Gravitodynamics

Note there is no concept of g-charge, thus no correlation between mass and positive/negative g-charge in Newton and Einstein's theories. The questions are:

(1) Experimentally: how can one directly detect the negative g-charge, although it has been suggested that the accelerated expansion of the Universe may be an evidence of the existence of the negative g-charge [6].

(2) Theoretically: How to resolve the CPT issue, WEP issue and Runaway issue? How to generalize Newton and Einstein's theory of gravity to describe negative g-charge? Is there g-charge conjugation symmetry in theories of gravity? What are new predictions?

To answer the first question, the AEgIS experiment at CERN with higher precision is designed to reveal whether antiparticle/antimatter repelled by ordinary matter [8]. If the repulsive gravitation detected experimentally, then a new era in studying gravity begins.

This article tries to answer the second question by postulating the correlations between positive/negative g-charge and mass. Based on the concepts of g-charge, WEP and definition of antimatters are extended, which provide possible resolutions of the WEP issue and CPT issue respectively. Newton's theory and Einstein's theory are generalized respectively for describing negative g-charge. It is shown that Gravitodynamics and those two generalized theories have gC symmetry and predict: (1) there is repulsive gravitation; (2) there are sub-Multiverses including our accelerated Universe. The Gravitational wave emitted by negative g-charges is systematically investigated in the framework of both Gravitodynamics and generalized Einstein's theory. Finally, we show that the unified Gravitodynamics and Electrodynamics have gC-eC symmetry.

2. Positive/Negative Gravitational Charge and gC symmetry

2.1. Positive/Negative Gravitational Charge (g-Charge) vs. Mass

There are two distinct masses: inertial mass and gravitational mass (g-mass). The WEP postulates that the ratio of g-mass to inertial mass is constant. The mystery is why g-mass is equivalent to inertial mass or restated as: why one kind of positive mass is equivalent to another kind of positive mass. It is not conceptually allowed that one kind of negative mass is equivalent to another kind of positive mass. Motz [13] postulated positive g-charge. It has been shown that, in the natural units $\hbar = C = 1$, the g-charge is dimensionless and acts as a generator of the symmetry group of gravity [14]. Applying the concept of g-charge, let's redefine quantities: there is only one kind of mass, inertial mass; gravitational mass, g-mass, is gravitational charge, g-charge. We will use these definitions.

Although these concepts of mass and g-charges appear to be only a formality, it clearly distinguishes g-charge from mass and, thus has conceptual importance, e.g., it is conceptually allowed that positive mass objects may carry either positive or negative g-charge. Now the mystery becomes: why g-charge is proportional to mass. In other word, the mystery now is charge vs. mass.

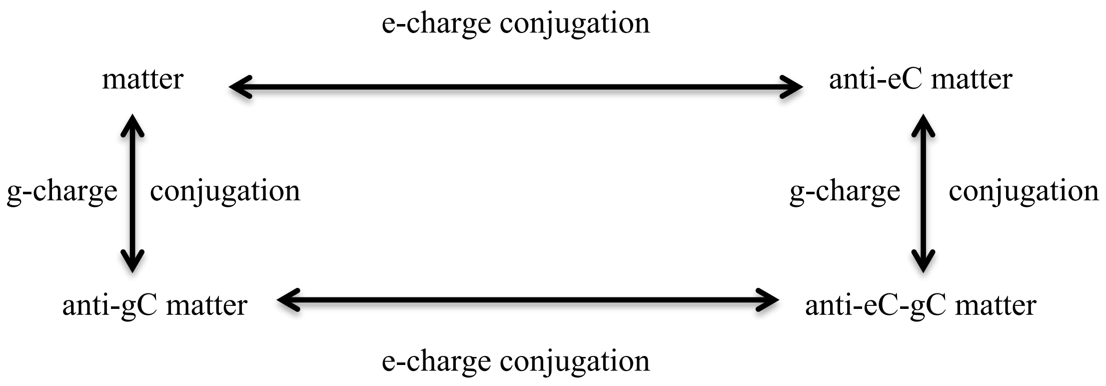
2.2. Re-defining Antimatter with g-Charge Conjugation (gC)

Let's introduce g-charge to redefine matter and antimatter (Table 1).

Table 1: Definitions of matter and its corresponding antimatter

	Sign of e-Charge	Sign of g-Charge
Matter	+ (−)	+ (−)
Anti-eC-matter	− (+)	+ (−)
Anti-gC-matter	+ (−)	− (+)
Anti-eC-gC-matter	− (+)	− (+)

Where, anti-eC- and anti-gC-matters represent antimatters obtained by single transformation of "electric charge (e-charge)" and "g-charge" conjugations respectively; anti-eC-gC-matter represents antimatters obtained by simultaneous transformations of "e-charge and g-charge" conjugations. Note both matter and antimatter may carry either positive or negative g-charge. The transformations of gC and eC conjugations are illustrated bellow:



Let's classify matters by characters of mass, e-charge, and g-charge. We have:

Matter 1 group: + mass, + e-charge, + g-charge; e.g., proton.

Matter 2 group: + mass, − e-charge, + g-charge; e.g., electron.

For brevity, particles without e-charge, e.g., neutron, are out of scope of this article.

According to this classification, the traditional antimatters belong to either anti-eC-matter 1 group, e.g. antiproton, or anti-eC-matter 2 group, e.g. positron. We suggest that objects of anti-gC-matter 1 and 2 groups, anti-eC-gC-matter 1 and 2 groups, carry negative g-charges.

2.3. Hypotheses, Generalized WEP, and Generalized CPT

Before introducing negative g-charge into the theories of gravity, let's postulate the following correlations/hypotheses:

Hypothesis 1: *An object having* $m > 0$ $(m = \int \rho_m d^3x)$ *may carry either positive or negative g-charge* defined as bellow,

$$Q_{g+} \equiv +\sqrt{G}m, \qquad \rho_{g+} = +\sqrt{G}\rho_m, \tag{1}$$

$$Q_{g-} \equiv -\sqrt{G}m, \qquad \rho_{g-} = -\sqrt{G}\rho_m. \tag{2}$$

Or $m \equiv +\frac{Q_{g+}}{\sqrt{G}}$, $m \equiv -\frac{Q_{g-}}{\sqrt{G}}$. Eq. (1) and Eq. (2) can be combined as,

$$Q_{g\pm} \equiv \pm\sqrt{G}m, \qquad \rho_{g\pm} = \pm\sqrt{G}\rho_m. \tag{3}$$

The quantities with subscript "$+$", "$-$", and "$\pm$" are related with positive, negative, and either positive or negative g-charge, respectively.

Hypothesis 2: *the laws governing gravitational interaction generated respectively by positive and negative g-charges have the same form.*

gC symmetry: by analogy to C symmetry, the gC symmetry is that under transformation of g-charge conjugation, gravitation laws are unchanged. Repulsive gravity then exists, since that would imply a difference in sign of the g-charges between matter and its corresponding Anti-gC-matter or Anti-eC-gC-matter.

Generalized WEP and Resolution of WEP issue: The normal WEP leads to the Universality of Free Fall (UFF). However WEP/UFF is valid only for positive g-charge and mass, which excludes repulsive gravitation. On the contrary, Eq. (3) leads to a generalized WEP to govern positive/negative g-charges and mass:

The absolute value of the ratio of g-charge to mass is constant.

Now either positive or negative g-charge, thus, either attractive or repulsive gravitations, satisfies the generalized WEP, which provides a resolution to the WEP issue.

Generalized CPT and a resolution of CPT issue: Taking into account negative g-charge, in addition to CPT symmetry, there are alternative symmetries: (1) gC-PT symmetry: g-charge replaces e-charge in CPT symmetry; (2) gC+CPT symmetry: gC symmetry and CPT symmetry independently hold; (3) gCCPT symmetry: simultaneous transformations of gC conjugation and CPT. Now CPT symmetry is extended to include g-charge, which clarifies the CPT arguments between Villata and Cabbolet.

Now let's reinterpret the physics significance of the AEgIS experiment: it will determine, when g-charge is involved, whether there is CPT symmetry (e.g., positron has positive g-charge), or gCCPT symmetry (e.g., positron has negative g-charge).

3. Generalized Newton's Theory of Negative g-Charge

3.1. Field Equation

Newton's theory of gravity has the field equation,

$$\nabla \cdot \mathbf{g}_{+} = -4\pi G\, \rho_{m+}. \tag{4}$$

where ρ_{m+} is the g-mass density; the subscript "+" reflects the fact that at Newton's time, the g-mass is positive. G is Newtonian gravitational constant. No negative g-mass (negative g-charge) exists in Newton's theory.

Based on the <u>*hypothesis 1 and 2*</u>, let's generalize Newton's theory of gravity to govern the gravitational fields generated by positive/negative g-charges, $Q_{g\pm}$, respectively:

$$\nabla \cdot \mathbf{g}_{+} = -4\pi\rho_{g+}, \tag{5}$$

$$\nabla \cdot \mathbf{g}_{-} = -4\pi\rho_{g-}. \tag{6}$$

The field lines of gravitational fields, $\mathbf{g}_{+}$ and $\mathbf{g}_{-}$, are shown in Fig. 1 and Fig.2, respectively [15].

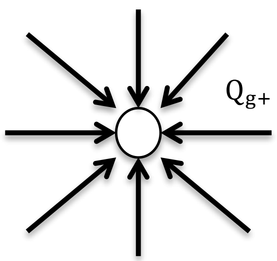


Fig. 1: field line of Q_{g+}

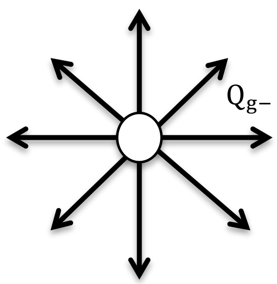


Fig. 2: field line of Q_{g-}

For a system of g-charges containing both positive and negative g-charges, we have

$$\nabla \cdot \mathbf{g}_{net} = -4\pi(\rho_{g+} + \rho_{g-}), \qquad \mathbf{g}_{net} \equiv \mathbf{g}_{+} + \mathbf{g}_{-}. \tag{7}$$

Eq. (5-7) form the complete set of field equations describing gravitational fields generated by $Q_{g\pm} = \int \rho_{g\pm} d^3 x$.

3.2. Equation of Motion

Newton's equation of motion of a non-relativistic test body having positive inertial rest mass, m_{i+}, and positive g-mass, m_{g+}, is

$$m_{i+}\frac{d\mathbf{V}}{dt} = m_{g+}\,\mathbf{g}_{+}. \tag{8}$$

The WEP states that inertial rest mass and g-mass are equivalent, $m_{i+} = m_{g+}$. Then Eq. (8) predicts the Universality of Free Fall (UFF), $\mathbf{a} = \mathbf{g}_{+}$.

However for a relativistic test body, the Newton's equation of motion is,

$$\frac{d\mathbf{P}}{dt} = \frac{d(\gamma m_{i+}\mathbf{V})}{dt} = m_{g+}\,\mathbf{g}_{+}, \tag{9}$$

where $\gamma = \left(\sqrt{1 - V^2/C^2}\right)^{-1}$. Therefore, WEP implies that a relativistic test body violates the UFF [16]. No negative g-charge exists in original Newton's equation of motion.

Based on the *hypothesis 2*, let's generalize Newton's equation of motion, Eq. (9), to,

$$\frac{d\mathbf{P}}{dt} = \frac{d(\gamma m\mathbf{V})}{dt} = Q_g\,\mathbf{g}, \tag{10}$$

where the g-charge Q_g may be either positive or negative, denoted as $Q_{g+} > 0$ and $Q_{g-} < 0$, respectively; the gravitational field $\mathbf{g}$ may be generated by either positive/ negative g-charges, or net g-charges of a system of positive/negative g-charges, denoted as $\mathbf{g}_{+}$, $\mathbf{g}_{-}$, or $\mathbf{g}_{net}$, correspondingly. Substituting those notations and Eq. (1, 2) into Eq. (10), correspondingly, we obtain explicit equations of motion as below:

1. $\frac{d(\gamma\mathbf{V})}{dt} = \frac{Q_{g+}}{m}\,\mathbf{g}_{+}$, or $\frac{d(\gamma\mathbf{V})}{dt} = \sqrt{G}\mathbf{g}_{+}$. (11)

This is relativistic Newton's equation of motion in terms of g-charge. For a non-relativistic test body, Eq. (11) reduces to Newton's equation, Eq. (8).

2. $\frac{d(\gamma\mathbf{V})}{dt} = \frac{Q_{g-}}{m}\,\mathbf{g}_{+}$, or $\frac{d(\gamma\mathbf{V})}{dt} = -\sqrt{G}\mathbf{g}_{+}$, (12)

i.e., the gravitational force acting on the test body is repulsive, which implies that an object having Q_{g-} on Earth falls upward and violates UFF, but obeys generalized WEP.

3. $\frac{d(\gamma\mathbf{V})}{dt} = \frac{Q_{g+}}{m}\,\mathbf{g}_{-}$, or $\frac{d(\gamma\mathbf{V})}{dt} = \sqrt{G}\mathbf{g}_{-}$, (13)

i.e., the gravitational force, $Q_{g+}\,\mathbf{g}_{-}$, acting on the test body is repulsive.

4. $\frac{d(\gamma\mathbf{V})}{dt} = \frac{Q_{g-}}{m}\,\mathbf{g}_{-}$, or $\frac{d(\gamma\mathbf{V})}{dt} = -\sqrt{G}\mathbf{g}_{-}$, (14)

i.e., the gravitational force, $Q_{g-}\,\mathbf{g}_{-}$, acting on the test body is attractive.

The equations of motion for a relativistic test body moving in a net gravitational field are:

$$m\frac{d(\gamma\mathbf{V})}{dt} = Q_{g\pm}\, \mathbf{g}_{net}, \qquad \text{or} \qquad \frac{d(\gamma\mathbf{V})}{dt} = \pm\sqrt{G}\, \mathbf{g}_{net}. \tag{15}$$

Eq. (10-15) form a complete set of generalized Newton's equations of motion governing motion of test bodies carrying either Q_{g+} or Q_{g-}, and shows that with either positive or negative g-charge, there is no "Runaway motion".

3.3. Generalized Newton's Theory has gC Symmetries

The introduction of positive/negative g-charge leads naturedly to a question: whether there is g-charge conjugation (gC) symmetry in theories of gravity? Eq. (5-7) and Eq. (10-15) show that under the transformations of g-charge conjugations, the generalized Newton's field equations and equations of motion have gC conjugation symmetry.

3.4. Accelerated Expansion of Universe

It is straightforward to show that the generalized Newton's theory can explain the accelerated expansion of the Universe. Let's assume that objects in our Universe: (1) have positive mass; (2) carry either positive or negative g-charges, and form positive or negative sub-Universe, respectively.

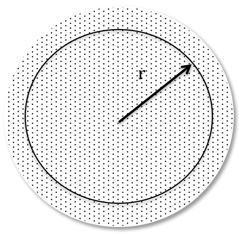


Fig. 3: Accelerated Expansion of Universe

Let's consider gravitational fields of a spherical distribution (Fig. 3) of mixed negative and positive g-charges. Eq. (7) gives

$$\nabla \cdot \mathbf{g}_{net} = -4\pi\sqrt{G}[\rho_{m+}(r,t) - \rho_{m-}(r,t)]. \tag{16}$$

Where $\rho_{g-} \equiv -\sqrt{G}\,\rho_{m-}$, $\rho_{g+} \equiv \sqrt{G}\,\rho_{m+}$. $\rho_{m-}\ (>0)$ and $\rho_{m+}\ (>0)$ are the mass densities of objects in negative and positive sub-Universes, respectively.

Eq. (16) gives the gravitational field at r,

$$\mathbf{g}_{net} = -\frac{\sqrt{G}\int[\rho_{m+}(r,t) - \rho_{m-}(r,t)]d^3x}{r^2}. \tag{17}$$

For uniform positive and negative sub-Universes, the gravitational field $\mathbf{g}_{net}$ at r is,

$$\mathbf{g}_{\text{net}} = -\frac{4\pi\sqrt{G}}{3}\, r\, [\rho_{m+}(t) - \rho_{m-}(t)]. \tag{18}$$

The motion of a non-relativistic object carrying positive g-charge at a given r is described by Eq. (15). Substituting Eq. (18) into Eq. (15), we obtain

$$\frac{1}{r}\frac{d^2r}{dt^2} = -\frac{4\pi G}{3}\rho_{m+}(t) + \frac{4\pi G}{3}\rho_{m-}(t). \tag{19}$$

To solve Eq. (19), we consider three different situations:

First situation: $\rho_{m+} > \rho_{m-}$, the positive (negative) sub-Universe is in accelerated collapse (expansion).

Second situation: $\rho_{m-} = \rho_{m+}$, $\mathbf{g}_{\text{net}} = 0$. This case corresponds to static positive and negative sub-Universes.

Third situation: $\rho_{m-} > \rho_{m+}$, we obtain

$$\mathbf{g}_{\text{net}} = \frac{4\pi G}{3}\, r\, [\rho_{m-}(t) - \rho_{m+}(t)]\,. \tag{20}$$

And Eq. (19) becomes,

$$\frac{\ddot{r}}{r} = \frac{4\pi G}{3}[\rho_{m-}(t) - \rho_{m+}(t)] > 0. \tag{21}$$

This result agrees with observation data, but no need of negative pressure and cosmological constant, which may distinguish negative g-charge model from dark energy /cosmological constant model.

Eq. (20, 21) implies that an object carrying positive g-charge is pushed away from the center of the distribution with acceleration, i.e., the negative g-charge provides physics mechanism for the expansion of positive sub-Universe.

The mass density, ρ_{m+}, of objects carrying positive g-charges in the ball of radius r decreases continuously, thus the gravitational field $\mathbf{g}_+$ decreases, and the acceleration is increasing with time.

The Hubble parameter H is time dependent as,

$$H^2 \equiv \left(\frac{\dot{r}}{r}\right)^2 = \frac{8\pi G}{3}[\rho_{m-}(t) - \rho_{m+}(t)], \tag{22}$$

The comparison between Eq. (22) and the current (t_0) observational data of the accelerated expansion and the flatness of the positive universe suggests that the current ratio of the negative to the positive g-charges ranges around 3, $\frac{\rho_{m-}(t_0)}{\rho_{m+}(t_0)} \approx 3$. Then Eq. (21) becomes

$$\frac{\ddot{r}}{r} = \frac{8\pi G}{3}\rho_{m+}(t_0), \tag{23}$$

$$\frac{\ddot{r}}{r} = \frac{8\pi G}{9}\rho_{m-}(t_0). \tag{24}$$

Then Eq. (22) gives the present Hubble constant,

$$H^2(t_0) = \frac{16\pi G}{3}\rho_{m+}(t_0). \tag{25}$$

Moreover, for the third situation, Eq. (16) for a non-relativistic object of negative g-charge gives,

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}\rho_{m-}(t) + \frac{4\pi G}{3}\rho_{m+}(t) < 0. \tag{26}$$

Eq. (26) implies that a body carrying negative g-charge is attracted toward to the center of the distribution, which leads to the accelerated collapse of the negative sub-Universe. Thus, the mass density, ρ_{m-}, of objects carrying negative g-charge in the ball of radius r continuously increases with time. Therefore the gravitational field $\mathbf{g}_-$ increases with time.

Eq. (16-26) form a dynamical model that explains the accelerated expansion of our observed Universe and predict that the acceleration is increase with time, that the negative sub-Universe is accelerate collapse, and that Hubble parameter is time dependent.

4. Gravitodynamics of Negative g-Charge

Gravitodynamics predicts negative g-charge [12,15]. In this section, we study the gC symmetry and gravitational wave generated by negative g-charge.

4.1. Field Equation

Based on the *hypothesis 1 and 2*, the field equations of Gravitodynamics are,

$$\frac{\partial F_{g\pm}^{\mu\nu}}{\partial x^\nu} = -\frac{4\pi}{c}J_{g\pm}^{\mu}, \quad \text{and} \quad \frac{\partial F_{g\pm}^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_{g\pm}^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_{g\pm}^{\beta\mu}}{\partial x^\alpha} = 0, \tag{27}$$

$$\frac{\partial F_{g,net}^{\mu\nu}}{\partial x^\nu} = -\frac{4\pi}{c}J_{g,net}^{\mu}, \quad \text{and} \quad \frac{\partial F_{g,net}^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_{g,net}^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_{g,net}^{\beta\mu}}{\partial x^\alpha} = 0. \tag{28}$$

Where $F_{g\pm}^{\mu\nu} \equiv \partial^\mu A_{g\pm}^\nu - \partial^\nu A_{g\pm}^\mu$ is the gravitational field tensor; $F_{g,net}^{\mu\nu} \equiv F_{g+}^{\mu\nu} + F_{g-}^{\mu\nu}$; $A_{g\pm}^\mu$ is four-potential generated by either positive or negative g-charge; $J_{g\pm}^\mu$ is either positive or negative four-current; $J_{g,net}^\mu = J_{g+}^\mu + J_{g-}^\mu$. Subscript "g" denote variables related to gravity.

Let's introduce definitions of vector gravitational field strength and potentials,

$$\mathbf{g}_\pm \equiv -\frac{1}{c}\frac{\partial \mathbf{A}_{g\pm}}{\partial t} - \nabla V_{g\pm}, \qquad \mathbf{B}_{g\pm} \equiv \nabla \times \mathbf{A}_{g\pm}, \tag{29}$$

$$\mathbf{g}_{net} \equiv -\frac{1}{c}\frac{\partial \mathbf{A}_{g,net}}{\partial t} - \nabla V_{g,net}, \qquad \mathbf{B}_{g,net} \equiv \nabla \times \mathbf{A}_{g,net}. \tag{30}$$

Then, Eq. (27, 28) can be written in the vector form as the following:

$$\left.\begin{array}{ll}\nabla\cdot\mathbf{g}_{\pm}=-4\pi\rho_{g\pm}=\mp4\pi\sqrt{G}\,\rho_m, & \nabla\times\mathbf{g}_{\pm}=-\frac{1}{c}\frac{\partial\mathbf{B}_{g\pm}}{\partial t}\\ \nabla\cdot\mathbf{B}_{g\pm}=0,\ \ \nabla\times\mathbf{B}_{g\pm}=-\frac{4\pi}{c}\mathbf{J}_{g\pm}+\frac{1}{c}\frac{\partial\mathbf{g}_{\pm}}{\partial t}=\mp\frac{4\pi}{c}\sqrt{G}\rho_m\mathbf{V}+\frac{1}{c}\frac{\partial\mathbf{g}_{\pm}}{\partial t} & \end{array}\right\}, \quad (31)$$

$$\left.\begin{array}{ll}\nabla\cdot\mathbf{g}_{net}=-4\pi\rho_{g,net}, & \nabla\times\mathbf{g}_{net}=-\frac{1}{c}\frac{\partial\mathbf{B}_{g,net}}{\partial t}\\ \nabla\cdot\mathbf{B}_{g,net}=0, & \nabla\times\mathbf{B}_{g,net}=-\frac{4\pi}{c}\mathbf{J}_{g,net}+\frac{1}{c}\frac{\partial\mathbf{g}_{net}}{\partial t}\end{array}\right\}. \quad (32)$$

Eq. (27-32) form a complete set of field equations of Gravitodynamics.

4.2. Equation of Motion

The equation of motion describing a test body is [15],

$$mc\frac{dV^{\mu}}{ds}=\frac{Q_g}{c}F_g^{\mu\nu}V_{\nu}. \quad (33)$$

Eq. (33) can be written explicitly as the following:

(1) For a test body carrying either Q_{g+} or Q_{g-} and moving in fields, $\mathbf{g}_+$ and $\mathbf{B}_{g+}$,

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_+ +\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g+}, \quad \text{or} \quad \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_+ +\mathbf{V}\times\mathbf{B}_{g+}). \quad (34)$$

(2) For a test body carrying either Q_{g+} or Q_{g-} and moving in fields, $\mathbf{g}_-$ and $\mathbf{B}_{g-}$,

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_- +\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g-}, \quad or \quad \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_- +\mathbf{V}\times\mathbf{B}_{g-}). \quad (35)$$

(3) For net gravitational fields, $\mathbf{g}_{net}$ and $\mathbf{B}_{g,net}$, we have,

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_{net}+\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g,net} \ \text{or}\ \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_{net}+\mathbf{V}\times\mathbf{B}_{g,net}). \quad (36)$$

Eq. (33-36) form a complete set of equations of motion. For a non-relativistic test body, $\gamma\approx1$, $\mathbf{V}\times\mathbf{B}_{g,net}\approx0$, Eq. (33-36) reduce correspondingly to generalized Newton's Eq. (10-15).

4.3. Gravitational Wave (GW_-) Radiated by Negative g-Charge

Einstein predicted gravitational wave (GW). All theories of gravity thereafter predict GW. Note GW in all of exist theories is radiated by **positive** g-charge, denote it as $\mathbf{GW}_+$. In the framework of Gravitodynamics, (1) the issue of positive/negative energy of vector theories of gravity has been resolved [15]; (2) the compatibility between $\mathbf{GW}_+$ and quantum mechanics, and wave-particle duality of $\mathbf{GW}_+$ have been studied [17]. In the framework of Gravitodynamics, let's study $\mathbf{GW}_-$ emitted by negative g-charges.

4.3.1. $\mathbf{GW_-}$

Based on Eq. (31), the results of $\mathbf{GW_-}$ can be directly written down as in Table 2:

Table 2: $\mathbf{GW_+}$ and $\mathbf{GW_-}$

	$\mathbf{GW_+}$	$\mathbf{GW_-}$
g-Field Strength	$F_{g+}^{\mu\nu} \equiv \partial^\mu A_{g+}^\nu - \partial^\nu A_{g+}^\mu$	$F_{g-}^{\mu\nu} \equiv \partial^\mu A_{g-}^\nu - \partial^\nu A_{g-}^\mu$
g-Field Equation	$\frac{\partial F_{g+}^{\mu\nu}}{\partial x^\nu} = -\frac{4\pi}{C} J_{g+}^\mu$	$\frac{\partial F_{g-}^{\mu\nu}}{\partial x^\nu} = -\frac{4\pi}{C} J_{g-}^\mu$
g-Wave Equation	$\frac{\partial^2 V_{g+}}{C^2\,\partial t^2} - \nabla^2 V_{g+} = -4\pi\rho_{g+}$ $\frac{\partial^2 \mathbf{A}_{g+}}{C^2\,\partial t^2} - \nabla^2 \mathbf{A}_{g+} = -4\pi\mathbf{J}_{g+}$	$\frac{\partial^2 V_{g-}}{C^2\,\partial t^2} - \nabla^2 V_{g-} = -4\pi\rho_{g-}$ $\frac{\partial^2 \mathbf{A}_{g-}}{C^2\,\partial t^2} - \nabla^2 \mathbf{A}_{g-} = -4\pi\mathbf{J}_{g-}$
g-Retarded Potential	$V_{g+} = -\sqrt{G}\int \frac{\rho_{m,t-\left(\frac{R}{C}\right)}}{R} d^3x$ $\mathbf{A}_{g+} = -\frac{\sqrt{G}}{C}\int \frac{J_{m,t-\left(\frac{R}{C}\right)}}{R} d^3x$	$V_{g-} = \sqrt{G}\int \frac{\rho_{m,t-\left(\frac{R}{C}\right)}}{R} d^3x$ $\mathbf{A}_{g-} = +\frac{\sqrt{G}}{C}\int \frac{J_{m,t-\left(\frac{R}{C}\right)}}{R} d^3x$
g-Lienard-Wiechert Potential	$V_{g+} = -\frac{\sqrt{G}m}{R(1-\boldsymbol{\beta}\cdot\mathbf{n})}$, $\mathbf{A}_{g+} = -\frac{\sqrt{G}m\boldsymbol{\beta}}{R(1-\boldsymbol{\beta}\cdot\mathbf{n})} = \boldsymbol{\beta}V_{g+}$	$V_{g-} = +\frac{\sqrt{G}m}{R(1-\boldsymbol{\beta}\cdot\mathbf{n})}$, $\mathbf{A}_{g-} = +\frac{\sqrt{G}m\boldsymbol{\beta}}{R(1-\boldsymbol{\beta}\cdot\mathbf{n})} = \boldsymbol{\beta}V_{g-}$
g-Field Strength (vector)	$\mathbf{g}_+ = \frac{1}{C}(\dot{\mathbf{A}}_{g+}\times\mathbf{n})\times\mathbf{n}$ $\mathbf{B}_{g+} = \frac{1}{C}(\dot{\mathbf{A}}_{g+}\times\mathbf{n})$	$\mathbf{g}_- = \frac{1}{C}(\dot{\mathbf{A}}_{g-}\times\mathbf{n})\times\mathbf{n}$ $\mathbf{B}_{g-} = \frac{1}{C}(\dot{\mathbf{A}}_{g-}\times\mathbf{n})$
g-Poynting flux	$\mathbf{S}_{g+} = \frac{C\mathbf{g}_+^2\mathbf{n}}{4\pi} = \frac{C\mathbf{B}_{g+}^2\mathbf{n}}{4\pi}$	$\mathbf{S}_{g-} = \frac{C\mathbf{g}_-^2\mathbf{n}}{4\pi} = \frac{C\mathbf{B}_{g-}^2\mathbf{n}}{4\pi}$
g-Field Energy	$W_{g+} = \frac{1}{8\pi}(\mathbf{g}_+^2 + \mathbf{B}_{g+}^2)$	$W_{g-} = \frac{1}{8\pi}(\mathbf{g}_-^2 + \mathbf{B}_{g-}^2)$

Where $J_{g\pm}^\mu = \pm\sqrt{G}J_m^\mu$; $J_m^\mu = (\rho_m, \mathbf{J}_m)$, $\mathbf{J}_m = \rho_m\mathbf{V}$. In tables 2-5, We use "g-…" to denote "gravitational…". The signs of both potentials and field strengths generated by negative g-charge/current are opposite of that generated by positive g-charge/current. However, the g-Poynting flux $\mathbf{S_g}$ and g-field Energy density W_g are equal to the square of the field strengths. Therefore, $\mathbf{S}_{g+} = \mathbf{S}_{g-}$, $\mathbf{W}_{g+} = \mathbf{W}_{g-}$. Since we don't know the detail of motion of a cosmological GW source, e.g., direction of motion; thus, we cannot distinguish $\mathbf{A}_{g+}$ from $\mathbf{A}_{g-}$. Table 2 shows that the laws of $\mathbf{GW_+}$ and $\mathbf{GW_-}$ have the same form.

4.3.2. Propagation of $\mathbf{GW_-}$

By analogy to $\mathbf{GW_+}$, it is obvious that $\mathbf{GW_-}$ is transverse wave, has the same polarity property and has the same propagation properties as that of $\mathbf{GW_+}$. We can directly write the properties of propagation of $\mathbf{GW_-}$ (Table 3).

Table 3: Properties of Propagation of $\mathbf{GW_+}$ and $\mathbf{GW_-}$

	$\mathbf{GW_+}$	$\mathbf{GW_-}$
g-Doppler effect	$\lambda_{go+} = (\lambda_{gs+})\frac{1-\beta\cos\alpha}{\sqrt{1-\beta^2}}$ $\lambda_{go+} = (\lambda_{gs+})(1+\mathbb{Z}_{GW+})$ $\lambda_{go+} = \frac{(\lambda_{gs+})(1-\frac{H_0R_0}{C}\cos\alpha)}{\sqrt{1-\left(\frac{H_0R_0}{C}\right)^2}}$	$\lambda_{go-} = (\lambda_{gs-})\frac{1-\beta\cos\alpha}{\sqrt{1-\beta^2}}$ $\lambda_{go-} = (\lambda_{gs-})(1+\mathbb{Z}_{GW-})$ $\lambda_{go-} = \frac{(\lambda_{gs-})(1-\frac{H_0R_0}{C}\cos\alpha)}{\sqrt{1-\left(\frac{H_0R_0}{C}\right)^2}}$
Absorption of GW	$\mathbf{B}_{g+} = \mathbf{B}_{g0+}e^{-(\kappa_+)z}e^{-i(\omega_+)t}$ $\mathbf{g}_+ = \mathbf{g}_{0+}e^{-(\kappa_+)z}e^{-i(\omega_+)t}$	$\mathbf{B}_{g-} = \mathbf{B}_{g0-}e^{-(\kappa_-)z}e^{-i(\omega_-)t}$ $\mathbf{g}_- = \mathbf{g}_{0-}e^{-(\kappa_-)z}e^{-i(\omega_-)t}$
Refraction of GW	$\frac{\sin\theta_T}{\sin\theta_i} = \frac{n_{g1}}{n_{g2}}$	$\frac{\sin\theta_T}{\sin\theta_i} = \frac{n_{g1}}{n_{g2}}$
g-Thomson cross section-1	$\sigma_{GW+} = \frac{8\pi}{3}\left(\frac{G}{C^2}\right)^2 m^2$	$\sigma_{GW-} = \frac{8\pi}{3}\left(\frac{G}{C^2}\right)^2 m^2$
g-Thomson cross section-2	$d\sigma_{GW+} = \frac{G^2m^2(\omega_+^4)}{C^4(\omega_0^2-\omega_+^2)^2}\sin^2\theta d\Omega$	$d\sigma_{GW-} = \frac{G^2m^2(\omega_-^4)}{C^4(\omega_0^2-\omega_-^2)^2}\sin^2\theta d\Omega$
g-Thomson cross section-3	$d\sigma_{GW+} = \left(\frac{G}{C^2}\right)^2 \frac{m^2(\omega_+^4)\sin^2\theta d\Omega}{(\omega_0^2-\omega_+^2)^2+(\omega_+^2)\gamma^2}$	$d\sigma_{GW-} = \left(\frac{G}{C^2}\right)^2 \frac{m^2(\omega_-^4)\sin^2\theta d\Omega}{(\omega_0^2-\omega_-^2)^2+(\omega_-^2)\gamma^2}$
Change of frequency	$\omega'_+ = (\omega_+)\frac{1-\beta\cos\theta}{1-\beta\cos\theta'}$	$\omega'_- = (\omega_-)\frac{1-\beta\cos\theta}{1-\beta\cos\theta'}$

In the g-Doppler effect of $\mathbf{GW_+}$ and $\mathbf{GW_-}$, H_0 is Hubble constant, $\mathbb{Z}_{GW\pm}$ is the redshift of $\mathbf{GW_+}$ and $\mathbf{GW_-}$ respectively.

Table 3 shows the propagation of $\mathbf{GW_+}$ and $\mathbf{GW_-}$ and implies that by investigating the propagation properties, one cannot distinguish between $\mathbf{GW_+}$ and $\mathbf{GW_-}$.

4.3.3. Radiation of $\mathbf{GW_-}$

Based on Table 2 and follow the calculation for $\mathbf{GW_+}$, the radiations of $\mathbf{GW_-}$ can be directly obtained as shown in Table 4.

Table 4: Radiations of $\mathbf{GW_+}$ and $\mathbf{GW_-}$

	Radiation of $\mathbf{GW_+}$	Radiation of $\mathbf{GW_-}$
Intensity	$dI_+ = \frac{C}{4\pi}(B_{g+}^2)R_0^2 d\Omega$	$dI_- = \frac{C}{4\pi}(B_{g-}^2)R_0^2 d\Omega$
g-Larmor Radiation	$I_+ = \frac{2Gm^2}{3C^3}\mathbf{a}^2, \quad \mathbf{a} = \dot{\mathbf{V}}$	$I_- = \frac{2Gm^2}{3C^3}\mathbf{a}^2, \quad \mathbf{a} = \dot{\mathbf{V}}$
g-Lienard Radiation	$I_+ = \frac{2Gm^2}{3C^3}\frac{[\dot{\mathbf{V}}^2 - (\boldsymbol{\beta}\times\dot{\mathbf{V}})^2]}{(1-\beta^2)^3}$	$I_- = \frac{2Gm^2}{3C^3}\frac{[\dot{\mathbf{V}}^2 - (\boldsymbol{\beta}\times\dot{\mathbf{V}})^2]}{(1-\beta^2)^3}$
Non-relativistic g-Dipole Radiation:	$I_{NR,+} = \sum_k \frac{2}{3C^3}(Q_{g+,k}\ddot{\mathbf{r}}_k)^2$	$I_{NR,-} = \sum_k \frac{2}{3C^3}(Q_{g-,k}\ddot{\mathbf{r}}_k)^2$
g-Dipole Radiation: a pair of positive and negative g-charges		$I_{NR,+/-} = \frac{2}{3C^3}\ddot{\mathbf{d}}_{+/-}^2$; $\mathbf{d}_{+/-} \equiv \sum_i Q_{g+,i}\mathbf{r}_{i+} + \sum_i Q_{g-,i}\mathbf{r}_{i-}$
Relativistic g-Dipole radiation	$I_{r+} = \frac{4Gm^2}{3C^3(1-\beta^2)^3}[\mathbf{a}^2 - \lvert\boldsymbol{\beta}\times\mathbf{a}\rvert^2]$	$I_{r-} = \frac{4Gm^2}{3C^3(1-\beta^2)^3}[\mathbf{a}^2 - \lvert\boldsymbol{\beta}\times\mathbf{a}\rvert^2]$
Non-relativistic Quadrupole Radiation (Binary)	$I_{NR,+} = \frac{2Gm^2}{5C^5}(\omega_+^6)R^4$	$I_{NR,-} = \frac{2Gm^2}{5C^5}(\omega_-^6)R^4$
Relativistic Quadrupole Radiation (Binary)	$I_{R,+} = \frac{2Gm^2}{5C^5\omega_+^2}\frac{[\mathbf{a}^2 - (\boldsymbol{\beta}\times\mathbf{a})^2]^2}{(1-\beta^2)^6}$	$I_{R,-} = \frac{2Gm^2}{5C^5\omega_-^2}\frac{[\mathbf{a}^2 - (\boldsymbol{\beta}\times\mathbf{a})^2]^2}{(1-\beta^2)^6}$
g-Bremsstrahlung	$I_+ = \frac{2Gm^2\mathbf{a}^2}{3C^3(1-\beta^2)^3}$	$I_- = \frac{2Gm^2\mathbf{a}^2}{3C^3(1-\beta^2)^3}$
g-synchrotron Radiation	$I_+ = \frac{2G^2B_{g+}^2\mathbf{P}^2}{3C^5}, \quad \mathbf{P} = m\gamma\mathbf{V}$	$I_- = \frac{2G^2B_{g-}^2\mathbf{P}^2}{3C^5}$
g-Tully-Fisher law	$I_+ = \frac{2Gm^2}{3C^3r^2}\mathbf{V}^4$	$I_- = \frac{2Gm^2}{3C^3r^2}\mathbf{V}^4$

Where $D_{\alpha\beta\pm} \equiv \pm\sqrt{G}D_{\alpha\beta,m} = \pm\sqrt{G}\int \rho_m\left(x_\alpha x_\beta - \frac{1}{3}\delta_{\alpha\beta}r^2\right)d^3x$ has been adopted. For point g-charge, we have $D_{\alpha\beta\pm} \equiv \pm\sqrt{G}D_{\alpha\beta,m} = \pm\sqrt{G}\sum m\left(x_\alpha x_\beta - \frac{1}{3}\delta_{\alpha\beta}r^2\right)$.

4.3.4. Particle Nature of $\mathbf{GW}_-$

By analogy to the derivation of wave-particle duality of $\mathbf{GW}_+$ [17], let's derive the wave-particle duality of $\mathbf{GW}_-$. Denoting quanta of $\mathbf{GW}_+$ and $\mathbf{GW}_-$ as $(\text{Gravito}-\text{photon})_+$ and $(\text{Gravito}-\text{photon})_-$, respectively. Table 5 compares particle natures.

Table 5: Comparison of particle natures of $\mathbf{GW}_+$ and $\mathbf{GW}_-$

	$\mathbf{GW}_+$	$\mathbf{GW}_-$
Energy of Gravito-photon	$E_+ = h_g\nu_{g+} > 0$ $H_{GW+} = \sum_{\mathbf{k_g}}\sum_{\sigma=1}^{2} \hbar\omega_{g\mathbf{k_g},+}\left[N_{\boldsymbol{\sigma},\mathbf{k_g},+} + \frac{1}{2}\right]$	$E_- = h_g\nu_{g-} > 0$ $H_{GW-} = \sum_{\mathbf{k_g}}\sum_{\sigma=1}^{2} \hbar\omega_{g\mathbf{k_g},-}\left[N_{\boldsymbol{\sigma},\mathbf{k_g},-} + \frac{1}{2}\right]$
Momentum of Gravito-photon	$\frac{h_g}{\lambda_{g+}} > 0$	$\frac{h_g}{\lambda_{g-}} > 0$
g-Beer-Lambert Law	$I_{rem,+}(z) = I_{inc,+}e^{-z/\ell}$	$I_{rem,-}(z) = I_{inc,-}e^{-z/\ell}$
Absorption of Gravito-photon	$K_{max} = \hbar_g(\omega_{g/ph+}) - \phi$	$K_{max} = \hbar_g(\omega_{g/ph-}) - \phi$
g-Compton scattering by particle	$E_{g+} = \frac{h_g C}{\lambda_{g+}}$, $\lambda_{g+} = \lambda_{g0+} + \lambda_{gC+}(1-\cos\theta)$, $\lambda_{gC+} \equiv \frac{h_g}{mC}$	$E_{g-} = \frac{h_g C}{\lambda_{g-}}$, $\lambda_{g-} = \lambda_{g0-} + \lambda_{gC-}(1-\cos\theta)$, $\lambda_{gC-} \equiv \frac{h_g}{mC}$

Where h_g is g-Planck constant; $\lambda_{g\pm}$ and $\nu_{g\pm}/\omega_{g\mathbf{k_g},\pm}/\omega_{g/ph\pm}$ are wavelength and frequency of $\mathbf{GW}_+$ and $\mathbf{GW}_-$, respectively. $\mathbf{GW}_+$ and $\mathbf{GW}_-$ are respectively radiated by Q_{g+} and Q_{g-} in the same way. We cannot distinguish $\mathbf{GW}_+$ and $\mathbf{GW}_-$ by their particle natures.

4.4. Gravitodynamics has gC Symmetrie

As shown in section 4.1 and 4.2, there is the gC symmetry for both field equations and equations of motion, i.e., Gravitodynamics, has the gC symmetry. Eq. (27-36) has the form exactly same as that of Electrodynamics, and imply that two long-range interactions in nature are more symmetry than one thought before.

4.5. Accelerated Expansion of Universe

For the non-relativistic case, $\gamma \approx 1$ and $\mathbf{V}\times\mathbf{B}_{g,net} \approx 0$, we have shown that Eq. (32 and 36) explain the accelerated expansion of the universe without fine-tune issue [6].

5. Generalized Einstein's Theory of Negative g-Charge

5.1. Generalized Einstein's Equation

Einstein's equation,

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{C^4} T^{\mu\nu}, \tag{37}$$

describes spacetime, $R^{\mu\nu}$, curved by sources, $T^{\mu\nu}$, of only positive g-mass.

Introducing the concept of g-charge, for a system containing positive g-charges only, Einstein's equation, Eq. (37) can be rewritten as,

$$R_{+}^{\mu\nu} - \frac{1}{2} g_{+}^{\mu\nu} R_{+} = \frac{8\pi\sqrt{G}}{C^4} T_{g+}^{\mu\nu}. \tag{38}$$

According to the traditional interpretation, Eq. (38) implies that spacetime $(+)$ is curved by, Q_{g+}, as shown in Fig. 4.

Fig. 4: Spacetime $(+)$ curved by positive g-charge Q_{g+}

Writing Einstein's equation in this form, Eq. (38), gives us an indication how to generalize Einstein's theory to describe spacetime curved by negative g-charge. For this aim, based on hypothesis 2, we generalize Einstein's theory to contain additional equations having the same form as that of Eq. (38) to describe spacetime curved either by negative g-charge alone or by a combination of positive and negative g-charges.

For source(s) carrying negative g-charge, $T_{g-}^{\mu\nu}$, we propose Einstein-like equation,

$$R_{-}^{\mu\nu} - \frac{1}{2} g_{-}^{\mu\nu} R_{-} = \frac{8\pi\sqrt{G}}{C^4} T_{g-}^{\mu\nu}. \tag{39}$$

Let's interpret geometrically Einstein-like equations. Following the interpretation of Eq. (38), Eq. (39) can be interpreted as that spacetime $(-)$ is curved by Q_{g-} (Fig. 5).

Fig. 5: Spacetime $(-)$ curved by negative g-charge Q_{g-}

From geometric point of view, spacetimes curved respectively by either Q_{g-} or Q_{g+} are the same. It is obvious that the Einstein's equations, Eq. (38) and Einstein-like equation, Eq. (39), have gC conjugation symmetry. Note objects having positive mass, $T_{m+}^{\mu\nu}$, carry either $T_{g+}^{\mu\nu}$ or $T_{g-}^{\mu\nu}$.

We postulate that for a two compound system as source, by analogy to electrodynamics, its gravitational field should be determined by net g-charges. In geometric term, even there are positive and negative g-charges, spacetime is still described by one metric. Next let's consider a two compound system containing first and second kinds of object(s). The former has positive energy-momentum $T_{m1}^{\mu\nu}$ and carrying positive g-charge $T_{g+}^{\mu\nu}$; the latter has positive energy-momentum $T_{m2}^{\mu\nu}$ and carrying negative g-charge $T_{g-}^{\mu\nu}$. To describe this two compound system, we propose Einstein-like equation,

$$R_{net}^{\mu\nu} - \frac{1}{2} g_{net}^{\mu\nu} R_{net} = \frac{8\pi\sqrt{G}}{C^4} T_{g,net}^{\mu\nu} = \frac{8\pi G}{C^4}(T_{m1}^{\mu\nu} - T_{m2}^{\mu\nu}). \quad (40)$$

Where $T_{g,net}^{\mu\nu} \equiv T_{g+}^{\mu\nu} + T_{g-}^{\mu\nu}$, $T_{g+}^{\mu\nu} \equiv +\sqrt{G} T_{m1}^{\mu\nu}$, $T_{g-}^{\mu\nu} \equiv -\sqrt{G} T_{m2}^{\mu\nu}$, $T_{m1}^{\mu\nu}$ and $T_{m2}^{\mu\nu}$ are related to masses m1 and m2, respectively.

There are three situations:

(1) $T_{g+}^{\mu\nu} > T_{g-}^{\mu\nu}$, spacetime $R_{net}^{\mu\nu}$ is the same as $R_{+}^{\mu\nu}$, thus Eq. (40) can be written as

$$R_{+}^{\mu\nu} - \frac{1}{2} g_{+}^{\mu\nu} R_{+} = \frac{8\pi G}{C^4}(T_{m1}^{\mu\nu} - T_{m2}^{\mu\nu}); \quad (41)$$

(2) $T_{g+}^{\mu\nu} < T_{g-}^{\mu\nu}$, spacetime $R_{net}^{\mu\nu}$ is the same as $R_{-}^{\mu\nu}$, thus Eq. (40) can be written as

$$R_{-}^{\mu\nu} - \frac{1}{2} g_{-}^{\mu\nu} R_{-} = -\frac{8\pi G}{C^4}(T_{m2}^{\mu\nu} - T_{m1}^{\mu\nu}). \quad (42)$$

(3) $T_{g+}^{\mu\nu} = T_{g-}^{\mu\nu}$, spacetime is flat Minkowski spacetime.

Eq. (41) and (42) can be combined as

$$R_{\pm}^{\mu\nu} - \frac{1}{2} g_{\pm}^{\mu\nu} R_{\pm} = \pm\frac{8\pi G}{C^4}\left|T_{m1}^{\mu\nu} - T_{m2}^{\mu\nu}\right|. \quad (43)$$

Spacetime curved by $T_{g,net}^{\mu\nu}$ is either same as spacetime (+) or same as spacetime (−). Eq. (40-43) has similar form as that of bi-metric theories [11] as shown in Table 6.

Table 6: Comparison between Generalized Einstein's and Bi-metric Theories

Source	Generalized Einstein's Theory	Bi-metric Theory
Compound 1: $m1 > 0$ Compound 2: $m2 < 0$		$R_{+}^{\mu\nu} - \frac{1}{2} g_{+}^{\mu\nu} R_{+} = \chi\,(T_{m1}^{\mu\nu} + T_{m2-}^{\mu\nu})$ $R_{-}^{\mu\nu} - \frac{1}{2} g_{-}^{\mu\nu} R_{-} = -\chi\,(T_{m1+}^{\mu\nu} + T_{m2-}^{\mu\nu})$
Compound 1: $T_{g1+}^{\mu\nu} > 0$ and $m1 > 0$ Compound 2: $T_{g2-}^{\mu\nu} < 0$ and $m2 > 0$	$R_{net}^{\mu\nu} - \frac{1}{2} g_{net}^{\mu\nu} R_{net}$ $= \pm\frac{8\pi G}{C^4}\left|T_{m1}^{\mu\nu} - T_{m2}^{\mu\nu}\right|$	

But the main conceptual difference is whether there is only one metric or bi-metric caused by a two compound system containing both positive and negative g-charges. Another difference is that, in bi-metric theory, negative gravitational mass is equivalent to negative inertial mass; thus, bi-metric theories have the WEP issue and the Runaway issue.

5.2. Generalized Linearized Einstein's Equation

Linearized Einstein's equation has been written in the form same as that of Maxwell's equation [18],

$$\frac{\partial F_{g+}^{\gamma\mu\nu}}{\partial x^{\nu}} = -\frac{4\pi G}{c} T_{m+}^{\gamma\mu}, \quad \text{and} \quad \frac{\partial F_{g+}^{\gamma\mu\alpha}}{\partial x^{\beta}} + \frac{\partial F_{g+}^{\gamma\alpha\beta}}{\partial x^{\mu}} + \frac{\partial F_{g+}^{\gamma\beta\mu}}{\partial x^{\alpha}} = 0, \tag{44}$$

where $F_{g+}^{\gamma\mu\nu} \equiv \partial^{\mu} A_{g+}^{\gamma\nu} - \partial^{\nu} A_{g+}^{\gamma\mu}$ is the tensor gravitational field; $A_{g\pm}^{\gamma\mu}$ is potential generated by positive g-charge; $T_{m+}^{\gamma\mu}$ is positive energy momentum of a source. Eq. (44) is only for positive $T_{m+}^{\gamma\mu}$.

Following the same procedure of last section, based on hypothesis 2, let's generalize it to describe negative g-charge, as the following,

$$\frac{\partial F_{g-}^{\gamma\mu\nu}}{\partial x^{\nu}} = -\frac{4\pi\sqrt{G}}{c} T_{g-}^{\gamma\mu}, \quad \text{and} \quad \frac{\partial F_{g-}^{\gamma\mu\alpha}}{\partial x^{\beta}} + \frac{\partial F_{g-}^{\gamma\alpha\beta}}{\partial x^{\mu}} + \frac{\partial F_{g-}^{\gamma\beta\mu}}{\partial x^{\alpha}} = 0, \tag{45}$$

$$\frac{\partial F_{g,net}^{\gamma\mu\nu}}{\partial x^{\nu}} = -\frac{4\pi\sqrt{G}}{c} T_{g,net}^{\mu}, \quad \text{and} \quad \frac{\partial F_{g,net}^{\gamma\mu\alpha}}{\partial x^{\beta}} + \frac{\partial F_{g,net}^{\gamma\alpha\beta}}{\partial x^{\mu}} + \frac{\partial F_{g,net}^{\gamma\beta\mu}}{\partial x^{\alpha}} = 0. \tag{46}$$

Where $F_{g-}^{\gamma\mu\nu} \equiv \partial^{\mu} A_{g-}^{\gamma\nu} - \partial^{\nu} A_{g-}^{\gamma\mu}$; $F_{g,net}^{\gamma\mu\nu} \equiv F_{g+}^{\gamma\mu\nu} + F_{g-}^{\gamma\mu\nu}$; $A_{g\pm}^{\gamma\mu}$ is potential generated by either positive or negative g-charge, $T_{g\pm}^{\gamma\mu} = \sqrt{G} T_{m\pm}^{\gamma\mu}$; $T_{g,net}^{\mu} = T_{g+}^{\gamma\mu} + T_{g-}^{\gamma\mu}$. For the component $T_{g\pm}^{0\mu} = \pm\sqrt{G} T_{m\pm}^{0\mu}$, Eq. (44-46) reduce to Eq. (31, 32) of Gravitodynamics.

5.3. Generalized Geodesic Equation

Now, let's study the motion of test bodies in both spacetime (+) of Fig. 4 and spacetime (−) of Fig. 5, respectively. Put test Body 1 (TB1) carrying Q_{g+} in spacetime (+). The gravitational force acting on it is attractive, and can be illustrated as Fig. 6.

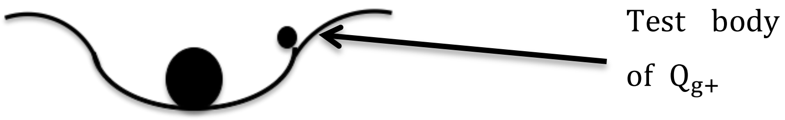


Fig. 6: TB1 of Q_{g+} moving in spacetime (+)

Put Test Body 2 (TB2) carrying Q_{g-} in spacetime (−). The gravitational force acting on TB2 is attractive; it can be illustrated as Fig. 7.

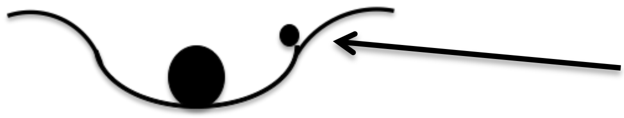


Fig. 7: TB2 of Q_{g-} moving in spacetime $(-)$

At first glance, so far, everything seems the same as original Einstein's theory. However as we have shown that g-charges having opposite polarities repel each other. Therefore negative g-charge violates the concept that metric is determined only by the source, i.e., negative g-charge leads to a "Metric Paradox". To resolve this paradox, we argue that motion of test body or, in geometric terms, metric should not be determined by source alone, but by combination of source and test body.

The original geodesic equation describing the motion of a test body is

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta+}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0. \quad (47)$$

Note Eq. (47) is a pure geometric formula, i.e., in a curved spacetime, the motions of objects, regardless of the nature of their g-charge and of whether they are non-relativistic or relativistic, are exactly the same. Geometrically, this holds only if UFF is valid. However, after introducing negative g-charge, we have shown that there is repulsive gravitation [15], thus negative g-charge violates UFF. Physically, Eq. (47) needs to be generalized to describe a relativistic test body carrying negative g-charge.

For this aim, the pure geometric equation needs to be generalized to include g-charge. Let's introduce the physical terms, $Q_{g\pm}$ and m_{ir}, into the geodesic equation, Eq. (47). We obtain the equation of motion,

$$\frac{d^2(m_{ir}x^\mu)}{ds^2} + \frac{Q_g}{\sqrt{G}}\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \quad (48)$$

where m_{ir} and Q_g are the relativistic mass and g-charge of the test body, respectively; Q_g may be either positive or negative. Note the relativistic mass, $m_{ir} = \gamma m$ and rest mass m is positive. $\Gamma^\mu_{\alpha\beta}$ may be generated by either positive g-charge, or negative g-charge, or net g-charges of a source, denoted as $\Gamma^\mu_{\alpha\beta+}$ or $\Gamma^\mu_{\alpha\beta-}$ or $\Gamma^\mu_{\alpha\beta,net}$, correspondingly. We call Eq. (48) the generalized equation of motion of generalized Einstein's theory. Now the motion of a test body depends not only on the curved spacetime, $\Gamma^\mu_{\alpha\beta}$, but also on the polarity of its g-charge.

Therefore, the generalized equations of motion, Eq. (48), can be rewritten explicitly as the following for different situations:

(1) A test body carrying Q_{g+} in field $\Gamma^{\mu}_{\alpha\beta+}$,

$$\left.\begin{aligned} m\frac{d^2(\gamma x^{\mu})}{ds^2} + (\frac{Q_{g+}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta+})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \\ \text{or} \quad \frac{d^2(\gamma x^{\mu})}{ds^2} + (\Gamma^{\mu}_{\alpha\beta+})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \end{aligned}\right\}, \tag{49}$$

i.e., the gravitational force, $(\frac{Q_{g+}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta+})$, is attractive. The motion of the test body can be interpreted as in Fig. 6. For a non-relativistic test body, Eq. (49) reduces to Eq. (47).

(2) A test body carrying Q_{g-} in field $\Gamma^{\mu}_{\alpha\beta+}$,

$$\left.\begin{aligned} m\frac{d^2(\gamma x^{\mu})}{ds^2} + (\frac{Q_{g-}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta+})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \\ \text{or} \quad \frac{d^2(\gamma x^{\mu})}{ds^2} - (\Gamma^{\mu}_{\alpha\beta+})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \end{aligned}\right\}, \tag{50}$$

thus, the gravitational force, $(\frac{Q_{g-}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta+})$, is repulsive.

(3) A test body carrying Q_{g+} in field $\Gamma^{\mu}_{\alpha\beta-}$,

$$\left.\begin{aligned} m\frac{d^2(\gamma x^{\mu})}{ds^2} + (\frac{Q_{g+}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta-})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \\ \text{or} \quad \frac{d^2(\gamma x^{\mu})}{ds^2} + (\Gamma^{\mu}_{\alpha\beta-})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \end{aligned}\right\}, \tag{51}$$

i.e., the gravitational force, $(\frac{Q_{g+}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta-})$, is repulsive.

(4) A test body carrying Q_{g-} in field $\Gamma^{\mu}_{\alpha\beta-}$,

$$\left.\begin{aligned} m\frac{d^2(\gamma x^{\mu})}{ds^2} + (\frac{Q_{g-}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta-})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \\ \text{or} \quad \frac{d^2(\gamma x^{\mu})}{ds^2} - (\Gamma^{\mu}_{\alpha\beta-})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \end{aligned}\right\}, \tag{52}$$

i.e., the gravitational force, $(\frac{Q_{g-}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta-})$, is attractive as illustrated in Fig. 7.

The equations of motion for a test body moving in a net gravitational field, $\mathbf{g}_{net}$, are:

(1) for a test body carrying Q_{g+},

$$\left.\begin{aligned} m\frac{d^2(\gamma x^{\mu})}{ds^2} + (\frac{Q_{g+}}{\sqrt{G}})(\Gamma^{\mu}_{\alpha\beta,net})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \\ \text{or} \quad \frac{d^2(\gamma x^{\mu})}{ds^2} + (\Gamma^{\mu}_{\alpha\beta,net})\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \end{aligned}\right\}. \tag{53}$$

(2) for a test body carrying Q_{g-},

$$\left.\begin{aligned} m\frac{d^2(\gamma x^\mu)}{ds^2}+\left(\frac{Q_{g-}}{\sqrt{G}}\right)(\Gamma^\mu_{\alpha\beta,net})\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=0 \\ \text{or}\quad \frac{d^2(\gamma x^\mu)}{ds^2}-(\Gamma^\mu_{\alpha\beta,net})\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=0 \end{aligned}\right\}. \qquad (54)$$

For a non-relativistic test body moving in a weak gravitational field, Eq. (49-54) reduces respectively to the generalized Newton's Eq. (10-15).

5.4. Generalized Linearized Geodesic Equation

For a weak gravitational field and only consider the component of $T_g^{0\mu}$, Eq. (49-54) reduce to,

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_{+}+4\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g+},\quad \text{or}\quad \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_{+}+4\mathbf{V}\times\mathbf{B}_{g+});\qquad (55)$$

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_{-}+4\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g-},\quad \text{or}\quad \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_{-}+4\mathbf{V}\times\mathbf{B}_{g-});\qquad (56)$$

$$\frac{d(\gamma\mathbf{V})}{dt}=\frac{Q_{g\pm}}{m}\mathbf{g}_{net}+4\frac{Q_{g\pm}}{m}\mathbf{V}\times\mathbf{B}_{g,net}\quad \text{or}\quad \frac{d(\gamma\mathbf{V})}{dt}=\pm\sqrt{G}(\mathbf{g}_{net}+4\mathbf{V}\times\mathbf{B}_{g,net}).\qquad (57)$$

Eq. (55-57) have the form same as that of Eq. (34-36) of Gravitodynamics, except a factor of 4, which can be used to distinguish Einstein's theory from Gravitodynamics by detecting gravitomagnetic force, $\frac{Q_g}{m}\mathbf{V}\times\mathbf{B}_g$.

5.5.Generalized Einstein's Theory has gC Symmetry

Eq. (38-40, 48-54) form a complete set of generalized Einstein's theory and show that the generalized Einstein-like equations and the generalized geodesic equations have the gC symmetry. Now let's apply the gC conjugation. There are different possibilities:

(1) if traditional antimatter is anti-eC-matter, then it carries positive g-charge.

(2) if traditional antimatter is anti-eC-gC-matter, then it carries negative e-charge and negative g-charge. This is the antimatter in Villata's model.

(3) if antimatter is anti-gC-matter, then it carries the same e-charge as its corresponding matter, but carries g-charge opposite to its corresponding matter.

The polarity of g-charge of antimatter is unknown and needs to be determined empirically. For directly detecting negative g-charge, the behaviors of antimatter in a gravitational field draw attentions [8].

5.6. Accelerating Expansion of Universe

Now let's apply the generalized Einstein equation to explain the accelerated expansion of Universe. Assume that there are negative g-charge and positive g-charges in Universe, the former forms negative sub-Universe, and the latter forms positive sub-Universe. Eq. (40) gives

$$R^{\mu\nu}_{net} - \frac{1}{2} g^{\mu\nu}_{net} R_{net} = 8\pi\sqrt{G}\,(T^{\mu\nu}_{g+} + T^{\mu\nu}_{g-}). \quad (58)$$

Applying the FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2\right], \quad (59)$$

where a(t) is the scale factor; K is a constant. Substituting Eq. (59) into Eq. (58) and (53), we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi\sqrt{G}}{3}\left[\left(\rho_{g+} + 3p_{g+}\right) - \left(\rho_{g-} + 3p_{g-}\right)\right], \quad (60)$$

and Hubble parameter,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\sqrt{G}}{3}\left[\rho_{g+} - \rho_{g-}\right] - \frac{K}{a^2}. \quad (61)$$

In the case of $3p_{g+} \ll \rho_{g+}$ and $3p_{g-} \ll \rho_{g-}$, then we obtain,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_{m+} - \rho_{m-}\right]. \quad (62)$$

Eq. (62) implies that for $\rho_{g-} > \rho_{g+}$, the positive g-charges carried by regular objects will be repelled away. With negative g-charge, the generalized Einstein's theory can explain the accelerated expansion of the universe without the need of both "negative pressure" and the cosmological constant that has issue of fine-turn.

5.7. Gravitational Wave (GW_-) Radiated by Negative g-charge

Einstein's equations, Eq. (37, 38), have predicted and described GW_+ emitted by positive g-charges. Einstein-like equation Eq. (39) has the same form as that of Eq. (38). The similar results about GW_- can be obtained straightforward (Table 7). Table 7 shows that it is impossible to distinguish GW_- from GW_+.

Table 7: $\mathbf{GW_+}$ and $\mathbf{GW_-}$

	$\mathbf{GW_+}$	$\mathbf{GW_-}$
Source	$T^{\mu\nu}_{g+} > 0$	$T^{\mu\nu}_{g-} < 0$
Field Equation	$R^{\mu\nu}_{+} - \frac{1}{2} g^{\mu\nu}_{+} R_{+} = 8\pi\sqrt{G}\, T^{\mu\nu}_{g+}$	$R^{\mu\nu}_{-} - \frac{1}{2} g^{\mu\nu}_{-} R_{-} = 8\pi\sqrt{G}\, T^{\mu\nu}_{g-}$
Wave Equation	$\nabla^2\bar{h}^{\mu\nu}_{+} - \frac{\partial^2\bar{h}^{\mu\nu}_{+}}{C^2\,\partial t^2} = -16\pi G T^{\mu\nu}_{m}$	$\nabla^2\bar{h}^{\mu\nu}_{-} - \frac{\partial^2\bar{h}^{\mu\nu}_{-}}{C^2\,\partial t^2} = +16\pi G T^{\mu\nu}_{m}$
Retarded Potential	$\bar{h}^{\mu\nu}_{+} = -4G\int \frac{T^{\mu\nu}_{m}(t-\frac{R}{C},\ x)}{R} d^3x$	$\bar{h}^{\mu\nu}_{-} = +4G\int \frac{T^{\mu\nu}_{m}(t-\frac{R}{C},\ x)}{R} d^3x$
g-Quadrupole moment	$Q^{\mu\nu}_{TT,m} = \rho_m \int \left(x^\mu x^\nu - \frac{\delta^{\mu\nu} r^2}{3}\right) d^3x$ $\bar{h}^{\mu\nu}_{+} = \frac{2G}{RC^4}\ddot{Q}^{\mu\nu}_{TT,m},\ \dot{\bar{h}}^{\mu\nu}_{+} = \frac{2G}{RC^4}\dddot{Q}^{\mu\nu}_{TT,m}$	$Q^{\mu\nu}_{TT,m} = \rho_m \int \left(x^\mu x^\nu - \frac{\delta^{\mu\nu} r^2}{3}\right) d^3x$ $\bar{h}^{\mu\nu}_{-} = -\frac{2G}{RC^4}\ddot{Q}^{\mu\nu}_{TT,m},\ \dot{\bar{h}}^{\mu\nu}_{-} = -\frac{2G}{RC^4}\dddot{Q}^{\mu\nu}_{TT,m}$
g-Quadrupole Radiation	$L_{GW+} = \frac{G}{5C^5}\langle \dddot{Q}^{\mu\nu}_{TT,m}\dddot{Q}^{TT}_{\mu\nu,m}\rangle$	$L_{GW-} = \frac{G}{5C^5}\langle \dddot{Q}^{\mu\nu}_{TT,m}\dddot{Q}^{TT}_{\mu\nu,m}\rangle$
g-Quadrupole Radiation (Binary System)	$L_{GW+} = \frac{32G}{5C^5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 r^4\omega^4$	$L_{GW-} = \frac{32G}{5C^5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 r^4\omega^4$

6. Multiverse

Taking into account positive/negative g-charges, we break Multiverse into sub-Multiverses. The generalized Newton's theory, Gravitodynamics, and generalized Einstein's theory predict the following single compound sub-Multiverses and two compound sub-Multiverses.

(A) ***The single compound sub-Multiverses:*** all objects have like g-charges. There are two sub-Multiverses as following:

sub-Multiverses A1: all objects have positive g-charges. This model can be described by Newton's theory, Einstein's theory, and Gravitodynamics.

sub-Multiverses A2: all objects have negative g-charges. This model is predicted by generalized Newton's theory, Gravitodynamics, and generalized Einstein's theory.

All objects in either sub-Multiverses A1 or sub-Multiverses A2 will attract each other and eventually collapse.

(B) ***Two Compound sub-Multiverses B***: this sub-Multiverse B contains coexisting *sub-Multiverses B1 and B2.* All objects in B1 carry positive g-charges; objects in B2 carry negative g-charges. The objects carrying same polarities g-charges attract and move toward to each other; the objects carrying opposite polarities g-charges repel and move away from each other.

One of *sub-Multiverses B1* and *B2* collapse; the other expands; which is determined by the ratio of densities of positive to negative g-charges.

When the density of negative g-charge of B2 is larger than that of positive g-charge density of B1, B1 is in accelerated expansion, and B2 is in accelerated collapse.

It is possible, although not demonstrated directly yet, that sub-Multiverses A1, A2, and B coexist somewhere in Multiverse.

7. Unified Electrodynamics and Gravitodynamics has gC-eC Symmetry

Electrodynamics has field equations,

$$\frac{\partial F_e^{\mu\nu}}{\partial x^\nu} = \frac{1}{c} J_e^\mu, \qquad \text{and} \qquad \frac{\partial F_e^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_e^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_e^{\beta\mu}}{\partial x^\alpha} = 0, \qquad (63)$$

and has e-charge conjugation (eC) symmetry. Since Maxwell field equations are independent of g-charge, thus eC symmetry can be extended to eC-gC symmetry.

Gravitodynamics has field equations,

$$\frac{\partial F_g^{\mu\nu}}{\partial x^\nu} = -\frac{4\pi}{C} J_g^\mu, \qquad \text{and} \qquad \frac{\partial F_g^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_g^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_g^{\beta\mu}}{\partial x^\alpha} = 0. \qquad (64)$$

which have gC symmetry and can be extended to eC-gC symmetry.

The field equations of Electrodynamics and Gravitodynamics can be expressed in one formula [12],

$$\frac{\partial U_a^{\mu\nu}}{\partial x^\nu} = \frac{1}{c} J_a^\mu, \qquad \text{and} \qquad \frac{\partial U_a^{\mu\alpha}}{\partial x^\beta} + \frac{\partial U_a^{\alpha\beta}}{\partial x^\mu} + \frac{\partial U_a^{\beta\mu}}{\partial x^\alpha} = 0, \qquad (65)$$

where, $U_1^{\mu\alpha} \equiv F_g^{\mu\alpha} = \partial^\mu A_g^\alpha - \partial^\alpha A_g^\mu$, $U_2^{\mu\alpha} \equiv F_e^{\mu\alpha} = \partial^\mu A_e^\alpha - \partial^\alpha A_e^\mu$, $J_1^\mu = -J_g^\mu$, and $J_2^\mu = J_e^\mu$. The Lagrangian density L_T of unified Electrodynamics and Gravitodynamics includes two parts, Gravitodynamics Lagrangian, L_g, and Electrodynamics Lagrangian, L_e,

$$L_T = L_g + L_e = -\frac{1}{4} U_a^{\mu\nu} U_{\mu\nu}^a + A_a^\mu J_\mu^a. \qquad (66)$$

The field equations of unified Electrodynamics and Gravitodynamics, Eq. (65, 66), have eC-gC symmetry.

The equation of motion for a test body can be generalized as

$$mc\frac{dV^{\mu}}{ds} = \frac{Q_a}{C}U_a^{\mu\nu}V_{\nu}. \quad (67)$$

The Lagrangian for the test body is

$$L = -mC^2\sqrt{1-\frac{\mathbf{v}^2}{C^2}} - \frac{Q^{a}}{\gamma C}V^{\mu}A_{\mu a}, \quad (68)$$

where, $Q^{a}\, V^{\mu}A_{\mu a} \equiv Q^1V^{\mu}A_{\mu 1} + Q^2V^{\mu}A_2$, $Q^1 \equiv Q_g$, $Q^2 \equiv Q_e$, $Q_e = (Q_{e+}, Q_{e-})$, $Q_g = (Q_{g+}, Q_{g-})$, $A_{\mu 1} \equiv A_{g\mu}$, $A_{\mu 2} \equiv A_{e\mu}$, $a = 1, 2$ is the internal index. The unified equation of motion describes a test body carrying positive/negative e-charge and positive /negative g-charge, and moving in external electromagnetic and gravitational fields. Eq. (67 and 68) have eC symmetry, gC symmetry and eC-gC symmetry, where eC-gC symmetry is under simultaneous transformations of eC conjugation and gC conjugation.

8. Conclusions and Discussion

The purpose of this article is to identify laws of gravity if negative g-charge exists. Based on Gravitodynamics, it has been pointed out that the accelerated expansion of universe may be an evidence of the existence of negative g-charge [6]. In this article a significant conclusion is reached that all of theories including scalar Newton's theory, vector Gravitodynamics and tensor Einstein's theory can explain the accelerated expansion of Universe equally well without needs of "negative pressure" and "cosmological constant", as long as negative g-charge is introduced. This implies that these explanations of this cosmological phenomenon are not theory dependent, which strongly supports the existence of negative g-charge.

We:

(1) distinguish conceptually gravitational mass from inertial mass, and define gravitational mass as g-charge, inertial mass as mass; these definitions allow the existence of negative g-charge carried by positive mass objects;

(2) postulate the correlations between mass and positive/negative g-charge, generalize WEP to represent these correlations, and propose hypothesis to guide the establishment of laws of gravitation of negative g-charge;

(3) redefine antimatter based not only on e-charge conjugation but also on g-charge conjugation, which is expected to provides a base for studying the gravitational interaction between matter and antimatter.

Based on above preparation works, then, we:

(1) resolve the WEP issue and CPT issue; show that there is no Runaway issue for negative g-charge;
(2) derive the generalized Newton's theory and generalized Einstein's theory for describing negative g-charge; both theories predict gravitation repulsion; show that the generalized Newton's theory, Gravitodynamics and generalized Einstein's theory have gC conjugation symmetry;
(3) predict GW_- generated by negative g-charges, and show that the detection GW on Earth cannot distinguish GW_- from GW_+;
(4) propose different sub-Multiverses based on positive/negative g-charge;
(5) show that the Unified Electrodynamics and Gravitodynamics has eC-gC symmetry.

The negative mass is out of scope of this article.

Appendix: Explanations of Accelerated Expansion of Universe

To explicitly show that the explanation of the accelerated expansion of Universe is theory-independent, let's compare the laws in different theories (Table 8).

Table 8:

	Equation of Motion	Hubble Parameter
Einstein's theory (Original) (Dark energy, negative pressure)	$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[(\rho_{m+} + 3p)]$	$H^2 = \frac{8\pi G}{3}\rho_{m+} - \frac{K}{a^2}$
Einstein's theory (Modified) (Cosmological constant)	$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[(\rho_{m+} + 3p)] + \frac{\Lambda}{3}$	$H^2 = \frac{8\pi G}{3}\rho_{m+} - \frac{K}{a^2} + \frac{\Lambda}{3}$
Einstein's theory (Generalized) (Negative g-Charge)	$\frac{\ddot{a}}{a} = -\frac{4\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}]$	$H^2 = \frac{8\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}] - \frac{K}{a^2}$
Newton' theory (Generalized) (Negative g-Charge)	$\frac{\ddot{a}}{a} = -\frac{4\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}]$	$H^2 = \frac{8\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}]$
Gravitodynamics (Negative g-Charge)	$\frac{\ddot{a}}{a} = -\frac{4\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}]$	$H^2 = \frac{8\pi\sqrt{G}}{3}[\rho_{g+} + \rho_{g-}]$

Where $\rho_{g\pm} = \pm\sqrt{G}\rho_{m\pm}$ and $\rho_{m\pm} > 0$. Table 8 shows that all of theories explain mathematically this cosmological phenomenon equally well, but with quit different physics interpretations. Negative g-charge is a universal mechanism in explaining this cosmological phenomenon. Once one identifies "negative pressure" and/or the "cosmological constant" with negative g-charge, $\rho_{g\pm}$, then all explanations are the same.

Appendix B: Geometric Illustration of Repulsive Gravitation

Let's put a test body of Q_{g+} in negative spacetime ($-$). The gravitational force acting on it is repulsive, which can be illustrated geometrically as in Fig. 8.

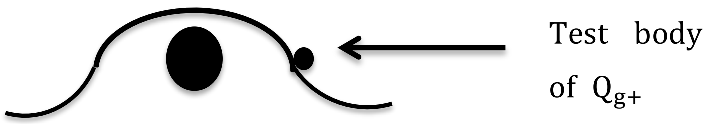


Fig. 8: TB1 of Q_{g+} moving in spacetime ($-$)

Similarly, Let's put a test body of Q_{g-} in positive spacetime ($+$); the gravitational force acting on it is repulsive, which can be illustrated geometrically as in Fig. 9.

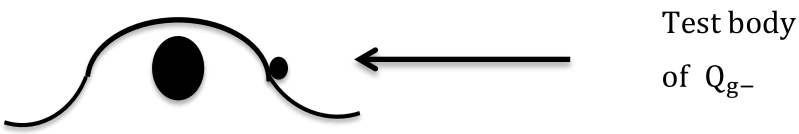


Fig. 9: TB2 of Q_{g-} moving in spacetime ($+$)

We can geometrically illustrate attractive force by Fig. 6/7 and repulsive force by Fig. 8/9.

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