Classical logic and the division by zero

Ilija Barukčić[#1]

[#]Internist

Horandstrasse, DE-26441 Jever, Germany

[1]Barukcic@t-online.de

Abstract — The division by zero turned out to be a long lasting and not ending puzzle in mathematics and physics. An end of this long discussion appears not to be in sight. In particular zero divided by zero is treated as indeterminate thus that a result cannot be found out. It is the purpose of this publication to solve the problem of the division of zero by zero while relying on the general validity of classical logic. A systematic re-analysis of classical logic and the division of zero by zero has been undertaken. The theorems of this publication are grounded on classical logic and Boolean algebra. There is some evidence that the problem of zero divided by zero can be solved by today's mathematical tools. According to classical logic, zero divided by zero is equal to one.

Keywords — *Indeterminate forms, Classical logic, Zero divided by zero, Infinity*

I. Introduction

Aristotle's unparalleled influence on the development of scientific knowledge in western world is documented especially by his contributions to classical logic too. Besides of some serious limitations of Aristotle's logic, Aristotle's logic became dominant and is still an adequate basis of our understanding of science to some extent, since centuries. In point of fact, some authors are of the opinion that Aristotle himself has discovered everything there was to know about classical logic. After all, classical logic as such is at least closely related to the study of objective reality and deals with absolutely certain inferences and truths. In general, classical logic describes the most general, the simplest, the most abstract laws of objective reality. Under conditions of Aristotle's classical logic, there is no uncertainty. In contrast to classical logic, probability theory deals with uncertainties. This raises questions concerning whether there is an overlap between classical logic and probability theory at all. Without attempting to be comprehensive, it may help to sketch view words on this matter in this publication. Classical logic is closely allied with probability theory and vice versa. As such, classical logic has no meaning apart from probability theory and vice versa. It should therefore come as no surprise that there are trials to combine logic and probability theory within one and the same mathematical framework, denoted as dialectical logic. However, as already published, there are natural ways in which probability theory is treated as an extension of classical logic to the values between +0 and +1 where probability of an event is treated as its truth value. In this context, Fuzzy logic is of no use and already refuted [1]. In particular, the relationship between classical logic and probability theory [2] is the same as between Newtonian mechanic's and Einstein's special theory of relativity. The one passes over into the other and vice versa without any contradictions.

II. Material and methods

A. Definitions

DEFINITION 1. (NUMBER +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let *i* denote an imaginary number [3]. The number +0 is defined as the expression

$$\begin{aligned} +0 &\equiv \left(c^2 \times \varepsilon_0 \times \mu_0\right) - \left(c^2 \times \varepsilon_0 \times \mu_0\right) \\ &\equiv +1-1 \\ &\equiv +i^2 - i^2 \end{aligned} \tag{1}$$

while "=" denotes the equals sign or equality sign [4, 5] used to indicate equality and "-" [4, 6, 7] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus [6] signs used to represent the operations of addition and the notions of positive as well.

DEFINITION 2. (NUMBER +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number [3]. The number +0 is defined as the expression

$$+1 \equiv \left(c^2 \times \varepsilon_0 \times \mu_0\right) \equiv -i^2 \tag{2}$$

Remark 1. Quantum computing

Quantum mechanical processes can enable some new types of computation [8]. Soon, Benjamin Schumacher replaced "the classical idea of a binary digit with a quantum two-state system, such as the spin of electron. These quantum bits, or 'qubits', are the fundamental units of quantum information." [9]. A qubit is one of the simplest quantum mechanical systems. Examples: the spin of the electron (spin up and spin down), the polarization of a single photon (vertical polarization and the horizontal polarization).

DEFINITION 3. (THE SAMPLE SPACE)

Let ${}_RC_t$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature, at a (random) Bernoulli trial t. Let ${}_0x_t$ denote an event, a subset of the sample space ${}_RC_t$. Let ${}_0x_t$ denote the negation of an event ${}_0\underline{x}_t$, another, complementary subset of the sample space ${}_RC_t$. In general, we define the sample space ${}_RC_t$ as

$${}_RC_t \equiv \left\{ {}_0x_t \quad , \quad {}_0\underline{x}_t \right\} \tag{3}$$

or equally as

$${}_RC_t \equiv {}_0x_t + {}_0\underline{x}_t \tag{4}$$

In other words, and according to quantum theory, the sample space ${}_RC_t$ at one certain Bernoulli trial t is in a state of superposition of ${}_0x_t$ and ${}_0x_t$. Under conditions of classical logic, it is $({}_0x_t + {}_0\underline{x}_t) = {}_RC_t = +1$.

DEFINITION 4. (THE COMPLEX CONJUGATE SAMPLE SPACE ${}_RC_T^*$)

Let ${}_RC_t^*$ denote the complex conjugate of the sample space ${}_RC_t$, the set of all the possible outcomes of a random experiment et cetera. In general, we define

$${}_RC_t \times {}_RC_t^* \equiv +1 \tag{5}$$

with the consequence that

$$ {}_{R}C_t^{*} \equiv \frac{+1}{{}_{R}C_t} \tag{6} $$

DEFINITION 5. (THE EIGEN-VALUES OF ${}_{0}X_{T}$)

Under conditions of classical logic, ${}_{0}x_t$ can take only one of the values

$$ {}_{0}x_t \equiv \{+0 \ , \ +1\} \tag{7} $$

DEFINITION 6. (THE EIGEN-VALUES OF ${}_{0}\underline{X}_{T}$)

Under conditions of classical logic, ${}_{0}\underline{x}_t$ can take only one of the values

$$ {}_{0}\underline{x}_t \equiv \{+0 \ , \ +1\} \tag{8} $$

DEFINITION 7. (THE SIMPLE FORM OF NEGATION)

Let ${}_{0}\underline{x}_t$ denote the negation of an event/outcome/eigenvalue ${}_{0}x_t$ (i. e. anti ${}_{0}x_t$). In general, we define the negation ${}_{0}\underline{x}_t$ of an event/outcome/eigenvalue ${}_{0}x_t$ as

$$ {}_{0}\underline{x}_t \equiv {}_{R}C_t - {}_{0}x_t \tag{9} $$

Under conditions of classical logic 'anti ${}_{0}x_t$' passes over to 'not ${}_{0}x_t$'. Negation is a very important concept in philosophy [10] and classical logic. In classical logic, negation converts only false to true and true to false. In other words, it is

$$ {}_{0}\underline{x}_t \equiv \neg \ {}_{0}x_t \tag{10} $$

or

$$ {}_{0}x_t \equiv \neg \ {}_{0}\underline{x}_t \tag{11} $$

where $\neg$ denotes the sign of negation of classical logic. So, if ${}_{0}x_t = +1$ (or true), then $\neg \ {}_{0}x_t = {}_{0}\underline{x}_t$ (pronounced 'not ${}_{0}x_t$' or equally 'anti ${}_{0}x_t$') would therefore be ${}_{0}\underline{x}_t = +0$ (false); and conversely, if ${}_{0}\underline{x}_t = +1$ (true) then $\neg \ {}_{0}\underline{x}_t = {}_{0}x_t = +0$ would be false. Determination and negation are related [11]. In particular, Benedict de Spinoza (1632 –1677) addressed these notions in his lost letter of June 2, 1674 to his friend *Jarig Jelles* [12] by the discovery that "*determinatio negatio est*" [13]. The German philosopher Hegel extended Spinoza's slogan to "*Omnis determinatio est negatio*" [14]. The relationship between ${}_{0}x_t$ and ${}_{0}\underline{x}_t$ is illustrated by the following table (**Table 1**).

Table 1. The relationship between ${}_{0}x_t$ and ${}_{0}\underline{x}_t$

Bernoulli trial t	${}_{0}x_t = \neg \ {}_{0}\underline{x}_t$	${}_{0}\underline{x}_t = \neg \ {}_{0}x_t$	${}_{0}\underline{x}_t + {}_{0}\underline{x}_t = {}_{R}C_t$	${}_{R}C_t$
1	+1	+0	+1 +0 = +1	+1
2	+1	+0	+1 +0 = +1	+1
3	+0	+1	+0 + 1 = +1	+1
4	+0	+1	+0 +1 = +1	+1
…	…	…	…	…

The first mathematically or algebraically formulation of *the notion negation* was provided to us by Georg Boole. In general, following Boole, negation in terms of algebra, can be expressed as ${}_{0}\underline{x}_t = 1 - {}_{0}x_t$. According to Boole, "*whatever … is represented by the symbol x, the contrary … will be expressed by 1 - x*" [15]. Under conditions of classical logic, it is ${}_{R}C_t = 1$, but not in general. In a slightly different way, we generalize Boole's negation to a simple and general form of Boole's negation as

$$ {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} - {}_{0}x_{t} \tag{12} $$

Equally, it is in the same respect that

$$ {}_{0}x_{t} \equiv {}_{R}C_{t} - {}_{0}\underline{x}_{t} \tag{13} $$

DEFINITION 8. (THE RIGHT-ANGLED TRIANGLE)

A right-angled triangle is a triangle in which one angle is 90-degree angle. Let ${}_{R}C_{t}$ denote the *hypotenuse*, the side opposite the right angle (side ${}_{R}C_{t}$ in the figure). The sides a_t and b_t are called legs. In a right-angled triangle ABC, the side AC, which is abbreviated as b_t, is the side which is *adjacent* to the angle α, while the side CB, denoted as a_t, is the side *opposite* to angle α. The following figure ([16], p. 117) may illustrate a right-angled triangle.

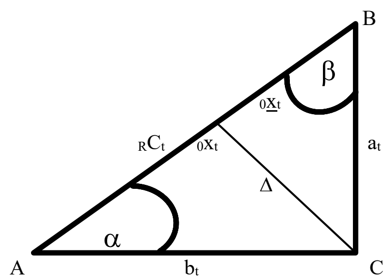

Figure 1. A right-angled triangle

DEFINITION 9. (THE RELATIONSHIP BETWEEN ${}_{0}X_{T}$ AND ANTI ${}_{0}\underline{X}_{T}$)

In general, we define

$$ \left({}_{0}x_{t} \right) + \left({}_{0}\underline{x}_{t} \right) \equiv {}_{R}C_{t} \tag{14} $$

and

$$ a_t^2 \equiv {}_{0}x_{t} \times {}_{R}C_{t} \tag{15} $$

and

$$ b_t^2 \equiv {}_{0}\underline{x}_{t} \times {}_{R}C_t^2 \tag{16} $$

Remark 2.

The equation ${}_{R}C_{t} = {}_{0}x_{t} + {}_{0}\underline{x}_{t}$ is valid even under conditions of classical (bivalent) logic. Under conditions of classical bivalent logic, it is ${}_{R}C_{t} = +1$ while ${}_{0}x_{t}$ takes the only values *either* +0 *or* +1. Since ${}_{0}\underline{x}_{t} = {}_{R}C_{t} - {}_{0}x_{t}$, ${}_{0}\underline{x}_{t}$ itself takes also only the values *either* +0 *or* +1. However, if ${}_{0}x_{t} = 0$ then ${}_{0}\underline{x}_{t} = 1$and vice versa. If ${}_{0}x_{t} = 1$ then ${}_{0}\underline{x}_{t} = 0$.

DEFINITION 10. (EUCLID'S THEOREM)

Euclid's (ca. 360 - 280 BC) derived the so called geometric mean theorem or *right triangle altitude theorem* or Euclid's theorem, published in his book Elements [17] in a corollary to proposition 8 in Book VI, used in proposition 14 of Book II to square a rectangle too, as

$$ \Delta_t^2 \equiv \frac{(a_t^2) \times (b_t^2)}{{}_{R}C_t^2} = {}_{R}C_t^2 \times \left(\sin^2(\alpha) \times \cos^2(\alpha) \right) = \left({}_{0}x_{t} \right) \times \left({}_{0}\underline{x}_{t} \right) \tag{17} $$

DEFINITION 11. (PYTHAGOREAN THEOREM)

The Pythagorean theorem is defined as

$$\left({}_{R}C_t^2 \times \left(\left({}_{0}x_t\right) + \left({}_{0}\underline{x}_t\right)\right)\right) = \left({}_{R}C_t^2 \times {}_{0}x_t\right) + \left({}_{R}C_t^2 \times {}_{0}\underline{x}_t\right) = \left(a_t^2 + b_t^2\right) \equiv {}_{R}C_t^2 \tag{18}$$

DEFINITION 12. (THE NORMALIZATION OF THE PYTHAGOREAN THEOREM)

The *normalization* [18, 19] of the Pythagorean theorem is defined as

$$\left(\frac{a_t^2}{{}_{R}C_t^2}\right) + \left(\frac{b_t^2}{{}_{R}C_t^2}\right) = \sin^2(\alpha) + cos^2(\alpha) \equiv +1 \tag{19}$$

DEFINITION 13. (THE VARIANCE OF A RIGHT-ANGLED TRIANGLE)

The variance σ^2 of a right-angled triangle [18, 19] is defined as

$$\sigma_t^2 \equiv \frac{\left(a_t^2\right) \times \left(b_t^2\right)}{\left({}_{R}C_t^2\right) \times \left({}_{R}C_t^2\right)} \equiv \sin^2(\alpha) \times \sin^2(\beta) = \sin^2(\alpha) \times \cos^2(\alpha) = \frac{\Delta_t^2}{\left({}_{R}C_t^2\right)} \tag{20}$$

DEFINITION 14. (SINE FUNCTION)

The sine function of an acute angle α denoted as $\sin(\alpha)$ is a trigonometric function which relates an angle of a right-angled triangle and the ratios of two side lengths. The sine function is defined ([16], p. 117) as

$$\sin(\alpha) = \frac{\left(a_t\right)}{\left({}_{R}C_t\right)} = \sqrt[2]{\frac{\left(a_t^2\right)}{\left({}_{R}C_t^2\right)}} = \sqrt[2]{\frac{\left({}_{R}C_t \times {}_{0}x_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)}} = \sqrt[2]{\frac{\left({}_{0}x_t\right)}{\left({}_{R}C_t\right)}} = \sqrt[2]{\frac{\left({}_{R}C_t - {}_{0}\underline{x}_t\right)}{\left({}_{R}C_t\right)}} = \sqrt[2]{1 - \frac{\left({}_{0}\underline{x}_t\right)}{\left({}_{R}C_t\right)}} \tag{21}$$

and as

$$\sin(\beta) = \frac{\left(b_t\right)}{\left({}_{R}C_t\right)} = \sqrt[2]{\frac{\left(b_t^2\right)}{\left({}_{R}C_t^2\right)}} = \sqrt[2]{\frac{\left({}_{R}C_t \times {}_{0}\underline{x}_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)}} = \sqrt[2]{\frac{\left({}_{0}\underline{x}_t\right)}{\left({}_{R}C_t\right)}} = \sqrt[2]{1 - \frac{\left({}_{0}x_t\right)}{\left({}_{R}C_t\right)}} \tag{22}$$

In a right-angled triangle it is $\alpha + \beta = 90°$. The *Lorentz factor* [20], defined as $(1-(v^2)/(c^2))^{1/2}$ appears to be only a special case of $\sin(\beta)$ in the form $(1-(v^2)/(c^2))^{1/2} = 1/\sin(\beta)$.

DEFINITION 15. (SINE-SQUARED FUNCTION)

The sine-squared function is defined as

$$\sin^2(\alpha) = \left(sin(\alpha) \times sin(\alpha)\right) = \frac{\left(a_t^2\right)}{\left({}_{R}C_t^2\right)} = \frac{\left({}_{R}C_t \times {}_{0}x_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)} = \frac{\left({}_{0}x_t\right)}{\left({}_{R}C_t\right)} \tag{23}$$

and as

$$\sin^2(\beta) = \left(sin(\beta)\right) \times \left(sin(\beta)\right) = \frac{\left(b_t^2\right)}{\left({}_{R}C_t^2\right)} = \frac{\left({}_{R}C_t \times {}_{0}\underline{x}_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)} = \frac{\left({}_{0}\underline{x}_t\right)}{\left({}_{R}C_t\right)} \tag{24}$$

The sine function for complex arguments z is defined as $\sin(z) = (\sinh(i \times z)/i)$ where $i^2 = -1$ and sinh is hyperbolic sine.

DEFINITION 16. (COSINE FUNCTION)

The sine, the cosine, and the tangent are the most familiar trigonometric functions. The cosine function abbreviated as *cos* is defined ([16], p. 117) as

$$\cos(\alpha) = \frac{(b_t)}{({}_RC_t)} = \sqrt[2]{\frac{(b_t^2)}{({}_RC_t^2)}} = \sqrt[2]{\frac{({}_RC_t \times {}_0\underline{x}_t)}{({}_RC_t \times {}_RC_t)}} = \sqrt[2]{\frac{({}_0\underline{x}_t)}{({}_RC_t)}} \tag{25}$$

and

$$\cos(\beta) = \frac{(a_t)}{({}_RC_t)} = \sqrt[2]{\frac{(a_t^2)}{({}_RC_t^2)}} = \sqrt[2]{\frac{({}_RC_t \times {}_0x_t)}{({}_RC_t \times {}_RC_t)}} = \sqrt[2]{\frac{({}_0x_t)}{({}_RC_t)}} \tag{26}$$

DEFINITION 17. (COSINE-SQUARED FUNCTION)

The cosine-squared function is defined as

$$\cos^2(\beta) = \left(\cos(\beta) \times \cos(\beta)\right) = \frac{(a_t^2)}{({}_RC_t^2)} = \frac{({}_RC_t \times {}_0x_t)}{({}_RC_t \times {}_RC_t)} = \frac{({}_0x_t)}{({}_RC_t)} \tag{27}$$

and as

$$cos^2(\alpha) = \left(\cos(\alpha)\right) \times \left(\cos(\alpha)\right) = \frac{(b_t^2)}{({}_RC_t^2)} = \frac{({}_RC_t \times {}_0\underline{x}_t)}{({}_RC_t \times {}_RC_t)} = \frac{({}_0\underline{x}_t)}{({}_RC_t)} \tag{28}$$

DEFINITION 18. (TANGENT FUNCTION)

The tangent function is defined ([16], p. 117) as

$$tan(\alpha) \equiv \frac{(a_t)}{(b_t)} \tag{29}$$

and as

$$\text{cotan}(\alpha) \equiv \frac{(b_t)}{(a_t)} = \frac{1}{tan(\alpha)} \tag{30}$$

DEFINITION 19. (SIMPLE ALGEBRAIC VALUES)

The following table provides an overview [21] about some simplest algebraic values of trigonometric functions.

Table 1. Simple algebraic values

Degree Function:	0°	90°	180°	270°	360°
sin	0	1	0	-1	0
cos	1	0	-1	0	1
tan	0	∞	0	0	0

DEFINITION 20. (THE NORMALIZATION OF THE SAMPLE SPACE)

Let A_t denote a (random) variable or any mathematical object, let $\Psi(A_t)$ denote the wave function as associated with A_t. Let $\underline{A}_t$ denote the other, the complementary part of a (random) variable et cetera, let $\Psi(\underline{A}_t)$ denote the wave function as associated with $\underline{A}_t$. Let ${}_RC_t$ denote the sample space of A_t while $\Psi({}_RC_t)$ denotes the wave function as associated with ${}_RC_t$. Let ${}_RC_t^*$ denote the complex conjugate of the sample space ${}_RC_t$ while $\Psi({}_RC_t^*)$ denotes the wave function as associated with ${}_RC_t^*$. In general, it is $A_t + A_t = {}_RC_t$ or $\Psi(A_t) + \Psi(\underline{A}_t) = \Psi({}_RC_t)$. The normalization of the sample space is defined

$$\Psi\left({}_RC_t\right) \times \Psi\left({}_RC_t^*\right) \equiv +1 \tag{31}$$

or $\Psi({}_RC_t^*) = 1/(\Psi({}_RC_t))$. The probability $p(A_t)$ and $p(\underline{A}_t)$ is defined according to *Born's rule* [22] as

$$p\left({}_RA_t\right) \equiv \Psi\left({}_RA_t\right) \times \Psi\left({}_RC_t^*\right) \tag{32}$$

and as

$$p\left({}_R\underline{A}_t\right) \equiv \Psi\left({}_R\underline{A}_t\right) \times \Psi\left({}_RC_t^*\right) \tag{33}$$

In general, it is $p(A_t) + p(\underline{A}_t) = +1$. Thus far, two different random variables A_t and B_t can possess the same wave function and the same complex conjugate wave function of the sample space. However, a joint wave function of the two different random variables A_t and B_t in this context is possible but not necessary.

B. Axioms

There have been many attempts to define the foundations of logic in a generally accepted manner. However, besides of an extensive discussion in the literature it is far from clear whether the truth as such is a definable notion. As generally known, axioms and rules of a publication have to be chosen carefully especially in order to avoid paradoxes and inconsistency. Thus far, for the sake of definiteness and in order to avoid paradoxes the theorems of this publication are based on the following axiom.

1) Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

In general, it is

$$+1 \equiv +1 \tag{34}$$

or the superposition of +0 and +1 as one of the foundations of quantum computing

$$+1 \equiv (1+0) \times (1+0) \times (1+0) \times (\ldots) \times (1+0) \tag{35}$$

2) Axiom II (Lex contradictionis. Principium contradictionis. Contradiction Law)

The (logical) contradiction is expressed mathematically as

$$+1 \equiv +0 \tag{36}$$

3) Axiom III (Principium negationis)

In general, it is

$$\frac{+1}{+0} \approx +\infty \tag{37}$$

III. RESULTS

THEOREM 3.1 (THE DETERMINATION OF $_{R}C_{T}$ I)

CLAIM.

From the standpoint to $_{0}x_{t}$ and due to our definitions before, $_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{38}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{39}$$

Multiplying by $_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} \equiv {}_{R}C_{t} \tag{40}$$

Adding zero, the relationship does not change as such. It is

$${}_{R}C_{t} + 0 \equiv {}_{R}C_{t} \tag{41}$$

According to mathematical requirements, it is $+{}_{0}\underline{x}_{t} - {}_{0}\underline{x}_{t} = +0$ is. Rearranging equation we obtain

$${}_{R}C_{t} - {}_{0}\underline{x}_{t} + {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} \tag{42}$$

In particular, due to our definition ${}_{0}x_{t} = {}_{R}C_{t} - {}_{0}\underline{x}_{t}$, the equation changes to

$$+{}_{0}x_{t} + {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} \tag{43}$$

Normalizing the relationship, it is

$$+\frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} \equiv \frac{{}_{R}C_{t}}{{}_{R}C_{t}} = +1 \tag{44}$$

or

$$+\frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} \equiv \frac{{}_{R}C_{t}}{{}_{R}C_{t}} = +1 \tag{45}$$

or

$$+\frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \equiv \left(1-\left(\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)\right) \tag{46}$$

or

$${}_{0}x_{t} = {}_{R}C_{t} \times \left(1-\left(\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)\right) \tag{47}$$

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{48}$$

QUOD ERAT DEMONSTRANDUM.

Remark 3. (Example).

Under condition of classical logic, it is

$${}_{0}x_{t} + {}_{0}\underline{x}_{t} = {}_{R}C_{t} = +1 \tag{49}$$

Under circumstances where ${}_{0}x_{t} = +1$ it is equally $+{}_{0}\underline{x}_{t} = 0$ or

$$+1 + 0 = {}_{R}C_{t} = +1 \tag{50}$$

and we obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{1}{\left(1-\left(\frac{0}{1}\right)\right)} = \frac{1}{(1)} = {}_{R}C_{t} = 1 \tag{51}$$

a correct result.

THEOREM 3.2 (THE DETERMINATION OF ${}_{R}C_{T}$ II)

CLAIM.

From the standpoint to ${}_{0}\underline{x}_{t}$ and due to our definitions before, ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{52}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{53}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} \equiv {}_{R}C_{t} \tag{54}$$

Adding zero, the relationship does not change as such. It is

$${}_{R}C_{t} + 0 \equiv {}_{R}C_{t} \tag{55}$$

According to mathematical requirements, it is $+{}_{0}\underline{x}_{t} - {}_{0}\underline{x}_{t} = +0$ is. Rearranging equation we obtain

$${}_{R}C_{t} - {}_{0}\underline{x}_{t} + {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} \tag{56}$$

In particular, due to our definition ${}_{0}x_{t} = {}_{R}C_{t} - {}_{0}\underline{x}_{t}$, the equation changes to

$$+ {}_0x_t \quad + {}_0\underline{x}_t \quad \equiv \quad {}_RC_t \tag{57}$$

Normalizing the relationship, it is

$$+\frac{{}_0x_t}{{}_RC_t} \quad +\frac{{}_0\underline{x}_t}{{}_RC_t} \quad \equiv \quad \frac{{}_RC_t}{{}_RC_t} = +1 \tag{58}$$

or

$$+\frac{{}_0x_t}{{}_RC_t} \quad +\frac{{}_0\underline{x}_t}{{}_RC_t} \quad \equiv \quad \frac{{}_RC_t}{{}_RC_t} = +1 \tag{59}$$

or

$$+\frac{{}_0\underline{x}_t}{{}_RC_t} \quad + \quad \equiv \quad \left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right) \tag{60}$$

or

$${}_0\underline{x}_t \quad = \quad {}_RC_t \times \left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right) \tag{61}$$

or

$$\frac{{}_0\underline{x}_t}{\left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} \quad = \quad {}_RC_t \tag{62}$$

QUOD ERAT DEMONSTRANDUM.

Remark 4. (Example).

Under condition of classical logic, it is

$${}_0x_t \quad + \quad {}_0\underline{x}_t \quad = \quad {}_RC_t \quad = +1 \tag{63}$$

Under circumstances where ${}_0x_t = +0$ it is equally $+{}_0\underline{x}_t = +1$ or

$$+0 \quad + \quad 1 \quad = \quad {}_RC_t \quad = +1 \tag{64}$$

and we obtain

$$\frac{{}_0\underline{x}_t}{\left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} = \frac{1}{\left(1-\left(\frac{0}{1}\right)\right)} = \frac{1}{(1)} \quad = \quad {}_RC_t \quad = 1 \tag{65}$$

a correct result.

THEOREM 3.3 (CLASSICAL LOGIC AND THE DIVISION ZERO BY ZERO I)

CLAIM.

In general, ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{66}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{67}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} = {}_{R}C_{t} \tag{68}$$

According to theorem 3.1., ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{69}$$

According to theorem 3.2. the same ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{70}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.4 (CLASSICAL LOGIC AND THE DIVISION ZERO BY ZERO II)

CLAIM.

According to classical logic, it is

$$\frac{+0}{+0} = +1 \tag{71}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{72}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} = {}_{R}C_{t} \tag{73}$$

or according to theorem 3.3.

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{74}$$

Under conditions of classical logic, it is ${}_{R}C_{t} = +1$. We obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = +1 \tag{75}$$

Under conditions of classical logic were ${}_{0}x_{t} = +1$ it is equally ${}_{0}\underline{x}_{t} = +0$. We obtain

$$\frac{+1}{\left(1-\left(\frac{+0}{+1}\right)\right)} = \frac{+0}{\left(1-\left(\frac{+1}{+1}\right)\right)} = +1 \tag{76}$$

or

$$\frac{+1}{(1-0)} = \frac{+0}{(1-1)} = +1 \tag{77}$$

or

$$\frac{+1}{+1} = \frac{+0}{+0} = +1 \tag{78}$$

or

$$\frac{+0}{+0} = +1 \tag{79}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.5 (CLASSICAL LOGIC AND THE DIVISION OF ZERO BY ZERO III)

CLAIM.

According to classical logic, it is

$$\frac{+0}{+0} = +1 \tag{80}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{81}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} = {}_{R}C_{t} \tag{82}$$

or according to theorem 3.3.

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{83}$$

Under conditions of classical logic, it is ${}_{R}C_{t} = +1$. We obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = +1 \tag{84}$$

Under conditions of classical logic were ${}_0x_t = +0$ it is equally ${}_0\underline{x}_t = +1$, we obtain

$$\frac{+0}{\left(1-\left(\frac{+1}{+1}\right)\right)} = \frac{+1}{\left(1-\left(\frac{+0}{+1}\right)\right)} = +1 \tag{85}$$

or

$$\frac{+0}{(1-1)} = \frac{+1}{(1-0)} = +1 \tag{86}$$

or

$$\frac{+0}{+0} = \frac{+1}{+1} = +1 \tag{87}$$

or

$$\frac{+0}{+0} = +1 \tag{88}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.6 (THE DIVISION OF ZERO BY ZERO AND AXIOM 1)

Let +X denote any (mathematical) object thus that +X-X = + 0. Let +Y denote any (mathematical) object thus that +Y-Y = + 0. In point of fact, even +X=+Y is possible.

CLAIM.

As long as (+0 /+0) = + 1, it is equally true that

$$+1 \equiv +1 \tag{89}$$

PROOF.

In general, if it is true that

$$\frac{+0}{+0} = +1 \tag{90}$$

we have to face some consequences? Especially, it should not be possible to derive any logical contradiction out of this relationship. As defined above, it is +X-X=+ 0 and +Y-Y=+0. Substituting into equation, we obtain

$$\frac{+X-X}{+Y-Y} = +1 \tag{91}$$

or

$$+X-X = +Y-Y \tag{92}$$

or

$$+X+Y = +Y+X \tag{93}$$

or

$$+X = +X \tag{94}$$

Thus far, especially if +X=+1, it follows that

$$+1 \equiv +1 \tag{95}$$

QUOD ERAT DEMONSTRANDUM.

Remark 5.

This theorem has not been able to derive a logical contradiction from the equation (+0/+0) = +1.

THEOREM 3.7 (EINSTEIN'S NORMALIZED MASS ENERGY EQUIVALENCE RELATIONSHIP)

Let m_0 denote the "rest-mass" as measured by the *co-moving observer* at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the *stationary observer* at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Einstein's mass-energy equivalence relationship can be normalized as

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{96}$$

PROOF.

According to Einstein's special theory of relativity, it is

$$m_0 = \sqrt[2]{1 - \frac{v^2}{c^2}} \times m_R \tag{97}$$

Rearranging equation, it is

$$(m_0)^2 = \left(1 - \frac{v^2}{c^2}\right) \times (m_R)^2 \tag{98}$$

or

$$\frac{(m_0)^2}{(m_R)^2} = \left(1 - \frac{v^2}{c^2}\right) \tag{99}$$

and at the end

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{100}$$

QUOD ERAT DEMONSTRANDUM.

Remark 6.

The equation before can be extended in a more general way [18, 19] to a relativistic energy-momentum relation as

$$\frac{(m_0)^2 \times (c)^2 \times (c)^2}{(m_R)^2 \times (c)^2 \times (c)^2} + \frac{(v)^2 \times (m_R)^2 \times (c)^2}{(c)^2 \times (m_R)^2 \times (c)^2} = +1 \tag{101}$$

or to formulate "*the particle wave duality*" [19] as

$$\frac{(E_0)^2}{(E_R)^2} + \frac{(p_R)^2 \times (c)^2}{(E_R)^2} = \frac{(E_0)^2}{(E_R)^2} + \frac{(E_{Electro-mag.\ wave})^2}{(E_R)^2} = +1 \tag{102}$$

THEOREM 3.8 (THE DEFINITION OF THE NUMBER +1 BY EINSTEIN'S SPECIAL THEORY OF RELATIVITY I)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum. Einstein's special relativity theory has its own understanding of the number +1. Independent of the circumstances, the number +1 is that what it is.

CLAIM.

Einstein's special relativity theory defines and determines the number +1 as

$$\frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{103}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{104}$$

According to one of our theorems before [19], it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{105}$$

or equally

$$(v)^2 = (c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right) \tag{106}$$

Under *conditions* were a division is possible and allowed, we obtain

$$\frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{107}$$

QUOD ERAT DEMONSTRANDUM.

Remark 7.

Einstein's mass–energy equivalence arose originally from a paradox described by Henri Poincaré [23]. In his publication on 21 November 1905 entitled as "Does the inertia of a body depend upon its energy-content?" Einstein proposed to consider the following: "Gibt ein Körper die Energie L in From von Strahlung ab, so verkleinert sich seine Masse um L/V^2" [24] as the first reference to Einstein's famous $E = mc^2$.

THEOREM 3.9 (THE DEFINITION OF THE NUMBER +1 BY EINSTEIN'S SPECIAL THEORY OF RELATIVITY II)

Let m_0 denote the "rest-mass" as measured by the *co-moving observer* at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the *stationary observer* at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Einstein's special relativity theory defines and determines the number +1 in another way too as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{108}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{109}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{110}$$

or

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{111}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} = 1 - \frac{(v)^2}{(c)^2} \tag{112}$$

Under conditions were a division is possible and allowed, we obtain

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{113}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.10 (EINSTEIN'S SPECIAL THEORY OF RELATIVITY UNDER CONDITIONS WHERE V=0)

Let m_0 denote the "rest-mass" as measured by the *co-moving observer* at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the *stationary observer* at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = 0, Einstein's special relativity theory does not collapse. Under these circumstances it is

$$\frac{(m_0)^2}{(m_R)^2} = +1 \tag{114}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{115}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{116}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = 0, it is

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(0)^2}{(c)^2} = +1 \tag{117}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} = +1 \tag{118}$$

QUOD ERAT DEMONSTRANDUM.

Remark 8.

The rule of Guillaume François Antoine, Marquis de L'Hospital (1661–1704) published in his 1696 book "*Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes*" [25] covers many but not all possible cases. In point of fact, even indeterminate forms such as 0/0, 1/0, 1^∞, 0^0, ∞^0, $0\times\infty$, ∞/∞ and $+\infty-\infty$ are sometimes evaluated using L'Hôpital's rule. However, applying L'Hôpital's rule with respect to indeterminate forms can lead to inconsistencies. In contrast to classical logic, L'Hôpital's rule [26] applies only to view certain situations [27, 28] but not [29] to every situation possible. Classical logic as generally valid is of more use to solve the problem of indeterminate forms then l'Hôpital's rule.

THEOREM 3.11 (EINSTEIN'S SPECIAL THEORY OF RELATIVITY UNDER CONDITIONS WERE V=C)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = c, Einstein's special relativity theory does not collapse. Under these circumstances it is

$$\frac{(m_0)^2}{(m_R)^2} = +0 \tag{119}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{120}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{121}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = c, it is

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(c)^2}{(c)^2} = +1 \tag{122}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} + 1 = +1 \tag{123}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = c, it is equally valid that

$$\frac{(m_0)^2}{(m_R)^2} = +0 \tag{124}$$

QUOD ERAT DEMONSTRANDUM.

Remark 9.

In other words, due to Einstein's relativistic energy-momentum relation and the particle wave duality [18, 19]

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{125}$$

Something can reach the speed of the light only if the rest mass $m_0 = 0$, otherwise not.

$$\frac{0^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = \frac{0^2}{(E_R)^2} + \frac{(E_{Electro-mag.\ wave})^2}{(E_R)^2} = +1 \tag{126}$$

In this case, there is no rest-mass or rest-energy and the whole energy of the same physical entity is contained within an electromagnetic wave.

THEOREM 3.12 (EINSTEIN'S SPECIAL THEORY OF RELATIVITY AND THE DIVISION OF ZERO BY ZERO I)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions of Einstein's special relativity were v = 0, it is

$$\frac{+0}{+0} \equiv +1 \tag{127}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{128}$$

Einstein's special relativity defines the number +1 on the one hand as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{129}$$

However, in the same respect, Einstein's special relativity defines the number +1 in its own way as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{130}$$

Thus far, as proofed previously, under conditions where v = 0, it is $((m_0^2)/(m_R^2)) = 1$. The equation above can be rearranged as

$$\frac{1}{\left(1 - \frac{(0)^2}{(c)^2}\right)} = +1 = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{131}$$

or

$$\frac{1}{\left(1 - \frac{(0)^2}{(c)^2}\right)} = +1 = \frac{(0)^2}{(c)^2 \times (1-1)} = \frac{(0)^2}{(c)^2 \times (0)} = +1 \tag{132}$$

Finally, it is

$$\frac{+0}{+0} \equiv +1 \tag{133}$$

QUOD ERAT DEMONSTRANDUM.

Remark 10.

Einstein's special relativity and thus far the division zero by zero can be tested by **(physical) experiments**. Einstein's special relativity allows and demands the division zero by zero [30]. According to Einstein's special relativity it is ((+0)/ (+0)) = +1.

THEOREM 3.13 (EINSTEIN'S SPECIAL THEORY OF RELATIVITY AND THE DIVISION OF ZERO BY ZERO II)

Let m_0 denote the "rest-mass" as measured by the *co-moving observer* at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the *stationary observer* at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions of Einstein's special relativity were v = c, it is

$$\frac{+0}{+0} \equiv +1 \tag{134}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{135}$$

Einstein's special relativity defines the number +1 on the one hand as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{136}$$

and on the other hand, equally as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{137}$$

Under conditions, were v = c it is it is $((m_0^2)/(m_R^2)) = 0$ and the equation above can be rearranged as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 = \frac{(c)^2}{(c)^2 \times (1-0)} = +1 = +1 \tag{138}$$

Under conditions, were v = c, it is $((m_0^2)/(m_R^2)) = 0$ and it is equally

$$(+0)\frac{+1}{\left(1 - \frac{(c)^2}{(c)^2}\right)} = \frac{+0}{+0} = \frac{(c)^2}{(c)^2 \times (1-0)} = +1 = +1 \tag{139}$$

and at the end

$$\frac{+0}{+0} \equiv +1 \tag{140}$$

QUOD ERAT DEMONSTRANDUM.

Remark 11.

Einstein's special relativity demands the division of zero by zero even under conditions were v = c. Under these conditions, we must consider the pure electro-magnetic energy/wave. According to Einstein's special relativity it is again ((+0)/ (+0)) = +1.

THEOREM 3.14 (THE GENERALLY NORMALIZED FROM OF THE PYTHAGOREAN THEOREM)

The Pythagorean theorem, more or less attributed to Pythagoras (ca. 570 BC-ca. 495 BC), was already known by the Old Babylonians (ca. 1900-1600 B.C.E) more than a millennium before Pythagoras [31] who used this relation to solve some geometric problems. The Pythagorean theorem is still one of the fundamental relations in Euclidean geometry and equally one of the most famous statements in mathematics, and is defined itself as $(a_t)^2 + (b_t)^2 = ({}_RC_t)^2$, where ${}_RC_t$ represents the length of the hypotenuse of a right-angled triangle and a_t and b_t the lengths of the triangle's other two sides. According to *Euclid's Elements, Book I, Proposition 47* it is: "In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle." [32]

CLAIM.

The generally normalized form of the Pythagorean theorem [19] is given by

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{141}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{142}$$

Multiplying by ${}_RC_t^2$, the length of the hypotenuse of a right-angled triangle squared, we obtain $1\times {}_RC_t^2 = 1\times {}_RC_t^2$ or

$${}_RC_t^2 = {}_RC_t^2 \tag{143}$$

The Pythagorean theorem can be expressed as the Pythagorean equation as

$$a_t^2 + b_t^2 = {}_RC_t^2 \tag{144}$$

Finally, rearranging equation, it is

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = \frac{{}_RC_t^2}{{}_RC_t^2} = +1 \tag{145}$$

QUOD ERAT DEMONSTRANDUM.

Remark 12.

According to the generally normalized form of the Pythagorean theorem we must consider the following situations. Pythagorean theorem is defined as $\mathbf{a_t^2 + b_t^2 = {}_RC_t^2}$. If $a_t^2 = 0$ then it is $0 + b_t^2 = {}_RC_t^2$ or

$$\frac{0}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{146}$$

Pythagorean theorem is defined even if $b_t^2 = 0$, In this case it is $a_t^2 = {}_RC_t^2$ or

$$\frac{a_t^2}{{}_RC_t^2} + \frac{0}{{}_RC_t^2} = +1 \tag{147}$$

In other words, the Pythagorean theorem is defined even under these conditions and demands that $0/_0x_t = 0$. This equation is equivalent with $(({}_RC_t \times 0)/({}_RC_t \times_0 x_t)) = 0$. Multiplying equation $0/_0x_t = 0$ by $(0/0)$ it is $((0\times 0)/(0\times_0 x_t))=0^2$.

THEOREM 3.15 (THE DEFINITION OF THE NUMBER +1 BY THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the number +1 as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{148}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{149}$$

According to the theorem before, the Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{150}$$

Rearranging equation, we obtain

$$\frac{a_t^2}{{}_RC_t^2} = 1 - \frac{b_t^2}{{}_RC_t^2} \tag{151}$$

or

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{152}$$

QUOD ERAT DEMONSTRANDUM.

Remark 13.

In this context, applying a kind of l'Hôpital's rule [25] or something like

$$\lim_{b_t \to {}_RC_t} \left(\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} \right) \to +1 \tag{153}$$

would complicate the matter and is of no use at all. *Per definitionem*, l'Hôpital's rule demands that something can only approach unity while in reality it has to be equal to unity.

THEOREM 3.16 (THE DEFINITION OF THE NUMBER +1 BY THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the number +1 equally as

$$\frac{b_t^2}{\left({}_R C_t^2\right) \times \left(1 - \frac{a_t^2}{{}_R C_t^2}\right)} = +1 \tag{154}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{155}$$

According to the theorem before, the Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{{}_R C_t^2} + \frac{b_t^2}{{}_R C_t^2} = +1 \tag{156}$$

Rearranging equation, we obtain

$$\frac{b_t^2}{{}_R C_t^2} = 1 - \frac{a_t^2}{{}_R C_t^2} \tag{157}$$

or

$$\frac{b_t^2}{\left({}_R C_t^2\right) \times \left(1 - \frac{a_t^2}{{}_R C_t^2}\right)} = +1 \tag{158}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.17 (THE DIVISION OF ZERO BY ZERO ACCORDING TO THE PYTHAGOREAN THEOREM I)

CLAIM.

The Pythagorean theorem determines the division of zero by zero as

$$\frac{+0}{+0} = +1 \tag{159}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{160}$$

The Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{\left({}_R C_t^2\right) \times \left(1 - \frac{b_t^2}{{}_R C_t^2}\right)} = +1 \tag{161}$$

and equally as

$$\frac{a_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{b_t^2}{{}_RC_t^2}\right)} = \frac{b_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{a_t^2}{{}_RC_t^2}\right)} = +1 \tag{162}$$

Under conditions, where $a_t^2=0$, it is equally $b_t^2 = {}_RC_t^2$. The equation before changes to

$$\frac{a_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{b_t^2}{{}_RC_t^2}\right)} = \frac{{}_RC_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{0^2}{{}_RC_t^2}\right)} = \frac{+1}{+1} = +1 \tag{163}$$

or to

$$\frac{0^2}{\left({}_RC_t^2\right)\times\left(1-\frac{{}_RC_t^2}{{}_RC_t^2}\right)} = \frac{0}{\left({}_RC_t^2\right)\times(1-1)} = \frac{+0}{+0} = \frac{{}_RC_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{0^2}{{}_RC_t^2}\right)} = \frac{+1}{+1} = +1 \tag{164}$$

or to

$$\frac{+0}{+0} = +1 \tag{165}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.18 (THE DIVISION OF ZERO BY ZERO ACCORDING TO THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the division of zero by zero as

$$\frac{+0}{+0} = +1 \tag{166}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{167}$$

The Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{168}$$

and equally as

$$\frac{a_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{b_t^2}{{}_RC_t^2}\right)} = \frac{b_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{a_t^2}{{}_RC_t^2}\right)} = +1 \tag{169}$$

Under conditions, where $b_t^2=0$, it is equally $a_t^2 = {}_RC_t^2$. The equation before changes to

$$\frac{a_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{b_t^2}{{}_RC_t^2}\right)} = \frac{+0}{\left({}_RC_t^2\right)\times\left(1-\frac{{}_RC_t^2}{{}_RC_t^2}\right)} = \frac{+0}{+0} = +1 \tag{170}$$

or to

$$\frac{{}_RC_t^2}{\left({}_RC_t^2\right)\times\left(1-\frac{0^2}{{}_RC_t^2}\right)} = \frac{+1}{1-0} = \frac{+1}{+1} = \frac{+0}{\left({}_RC_t^2\right)\times\left(1-\frac{{}_RC_t^2}{{}_RC_t^2}\right)} = \frac{+0}{+0} = +1 \tag{171}$$

or to

$$\frac{+0}{+0} = +1 \tag{172}$$

QUOD ERAT DEMONSTRANDUM.

Remark 14.

According to the Pythagorean theorem it is (+0)/(+0) = +1.

THEOREM 3.19 (THE CONSEQUENCES OF COS(α=0) = 1)

The knowledge of the true origins of memorable discoveries [33] even if found by accident is of help to recognize the relativity even of mathematical knowledge. In this context, the early studies of triangles can be traced back to the 2nd millennium BC to Egyptian (*Rhind Mathematical Papyrus*) and Babylonian mathematics. However, a systematic study of trigonometric functions began with the Hellenistic mathematics during the second half of the 2nd century BC. The Hellenistic astronomer *Hipparchus of Nicaea* (ca. 180–ca. 125 BC) has been the first to compile a trigonometric table and is known as "*the father of trigonometry*" [34]. The Hellenistic mathematics reached India [34] where significant developments of trigonometry are ascribed especially *Aryabhata* (sixth century CE), who discovered *the sine function*. Finally, Aryabhata's table of sines reached China in 718 AD [35] during the Tang Dynasty. In the following, the studies of trigonometry continued in the Middle Ages by Islamic mathematicians [36] and led to the discovery of all six trigonometric functions. Latin translations of accumulated Arabic knowledge inspired trigonometry to be adopted in western Europe. In 1342, *Levi ben Gershon (1288-1344)* [37], known as Gersonides too, worked On Sines, Chords and Arcs [38]. Finally, the western *Age of Enlightenment* inspired and accelerated the development of modern trigonometry by Jost Bürgi (1552-1632) [39], Henry Briggs (1561-1630) [40], Isaac Newton (1643 - 1727) [41], Roger Cotes (1682–1716) [42], James Stirling (1692-1770) [43], Leonhard Euler (1707-1783) [44] and other too.

CLAIM.

Under conditions where cos (α = 0) = 1 it is

$$a_t = 0 \tag{173}$$

PROOF.

In general, according to the rules of trigonometry, the cosine function is defined as

$$\text{cosine}(\alpha) \equiv \frac{(b_t)}{\left({}_RC_t\right)} \tag{174}$$

However, it is accepted as correct that **cos (α = 0) = 1** [21]. In this case it is

$$\text{cosine}(\alpha=0) \equiv \frac{(b_t)}{\left({}_RC_t\right)} = +1 \tag{175}$$

In other words, it is

$$\frac{(b_t\ )}{({}_RC_t\ )} = +1 \tag{176}$$

or

$$(b_t\ ) = ({}_RC_t\ ) \tag{177}$$

or

$$(b_t^2) = ({}_RC_t^2) \tag{178}$$

Even under these circumstances, Pythagorean theorem as

$$a_t^2 + b_t^2 \equiv {}_RC_t^2 \tag{179}$$

is valid. Rearranging, we obtain

$$a_t^2 + {}_RC_t^2 \equiv {}_RC_t^2 \tag{180}$$

or

$$a_t^2 \equiv 0 \tag{181}$$

or

$$a_t \equiv 0 \tag{182}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.20 (THE DEFINITION COS(α=0) = 1 LEADS TO CONTRADICTIONS)

CLAIM.

The definition **cos (α = 0) = 1** is logically inconsistent because the same reduces the value of b_t only to

$$b_t = +1 \tag{183}$$

PROOF.

In general, according to the rules of trigonometry it is

$$\frac{cos(\alpha)}{sin(\alpha)} \equiv \frac{\frac{b_t}{{}_RC_t}}{\frac{a_t}{{}_RC_t}} = \frac{b_t}{a_t} \tag{184}$$

This relationship is claimed to be valid even if α = 0. In this case, it is

$$\frac{cos(0)}{sin(0)} = \frac{+1}{+0} = \frac{\frac{b_t}{{}_RC_t}}{\frac{a_t}{{}_RC_t}} = \frac{b_t}{a_t} \tag{185}$$

In other words, it is

$$\frac{+1}{+0} = \frac{b_t}{a_t} \tag{186}$$

However, if **cos (α = 0) = 1** then $a_t = 0$ as proofed by the theorem before and we obtain

$$\frac{+1}{+0} = \frac{b_t}{+0} \tag{187}$$

Under these circumstances, the division by zero doesn't matter at all. In particular, whatever the division by 0 may be, cos (α = 0) = 1 demands that

$$b_t = +1 \tag{188}$$

QUOD ERAT DEMONSTRANDUM.

Remark 15.

Trigonometric functions [45] are widely used in science and our trust into the same appears to be limitless. In one way or another trust is important in science but can be dangerous too. What we risk while trusting without a clear proof of the correctness of something is among other that contradictions may take root in science to such an extent that one definition after the other is necessary to rescue what can and must be rescued.

In this context, it is important to note, that we cannot rely on trigonometric functions any longer to the extent which is necessary. Especially under conditions, where **cos(α = 0) = 1,** there is a contradiction. The side b_t of a right-angled triangle can take values different from 1, especially if **α = 0**. However, *cos(α = 0) = 1* demands under these conditions that b_t must be equal to +1, which is a non-acceptable contradiction.

THEOREM 3.21 (THE COSINE FUNCTION IS INCONSISTENT)

CLAIM.

The trigonometric cosine function is logically inconsistent.

PROOF BY MODUS INVERSUS.

In general, it is false that

$$0 = 360 \tag{189}$$

Taking the cosine of both sides, we obtain

$$cos(0) = cos(360) \tag{190}$$

In general, according to the rules of trigonometry it is

$$+1 = +1 \tag{191}$$

QUOD ERAT DEMONSTRANDUM.

Remark 16.

According to modus inversus, if the premise is false then the conclusion is false. The proof by modus inversus was started with a false premise 0 = 360, In the following we obtained a true conclusion +1=+1 and not a false conclusion. This is a contradiction.

THEOREM 3.22 (TODAY'S MULTIPLICATION BY ZERO IS SELF-CONTRADICTORY)

Following the lead of the principles of classical logic, it is appropriate to focus on what it means that from something which is obviously wrong cannot follow something which is obviously true assumed that there are no technical errors. Particularly, technically correct and allowed logical or mathematical operations cannot result in true statements being deduced from false statements. In this sense, today's rules concerning the multiplication by zero are completely useless and must be abandoned.

CLAIM.

Todays' understanding of the multiplication by zero is logically and mathematically inconsistent because the same can change a statement which is obviously wrong (+1=+0) into a statement which is which is obviously true.

PROOF BY MODUS INVERSUS.

In general, our starting statement is

$$+\mathbf{1} \equiv +\mathbf{0} \tag{192}$$

and as such obviously not true. It should not be possible in the absence of any technical errors to deduce a true statement from such a false one. Adding +2 on both sides of the equation, it is

$$+1+2 \equiv +0+2 \tag{193}$$

or

$$+3 \equiv +2 \tag{194}$$

Multiplying equation by +0, we obtain

$$(+3) \times (+0) \equiv (+2) \times (+0) \tag{195}$$

According to today's rule of the multiplication by zero this is identical with

$$(+0) \equiv (+0) \tag{196}$$

or

$$(+1-1) \equiv (+1-1) \tag{197}$$

Today's rules of the multiplication by zero enables that a statement which is obviously wrong (+1=+0) can be changed without any technical errors into a statement which is obviously true or

$$+\mathbf{1} \equiv +\mathbf{1} \tag{198}$$

QUOD ERAT DEMONSTRANDUM.

Remark 17.

A consistent logical or mathematical operation is one that does not entail any contradiction. Consistently with the theorem above is that from contradictory premises or statements (+1=+0), anything follows (*ex contradictione sequitur quodlibet* (ECSQ)). In other words, whatever is claimed, its contradiction is also true. The more from a theorem or a theory containing a true contradiction, everything as true as well as everything as false can be deduced the more such theorems and theories must be identified and labeled with a contradiction. Historically, ex contradictione sequitur quodlibet (or the Principle of Explosion) is ascribed to William of Soissons, a 12th century French logician who lived in Paris. Karl Popper made similar claims in a different context: "*We see from this that if a theory contains a contradiction, then it entails everything, and therefore, indeed, nothing [...]. A theory which involves a contradiction is therefore entirely useless as a theory*" [46]. Today's rules concerning the multiplication by zero are logically inconsistent. New techniques which remove today's inconsistency as associated with the rules of the multiplication by zero are necessary.

THEOREM 3.23 (THE NEW RULE OF THE MULTIPLICATION BY ZERO)

CLAIM.

The multiplication by zero is logically and mathematically consistent if the same does not change a statement which is obviously wrong (+1=+0) into a statement which is which is obviously true.

PROOF.

In general, our starting statement is

$$+\mathbf{1} \equiv +\mathbf{0} \tag{199}$$

and as such obviously not true. It should not be possible in the absence of technical errors to deduce a true statement from such a false one. Adding +2 on both sides of the equation, it is

$$+1+2 \equiv +0+2 \tag{200}$$

or

$$+3 \equiv +2 \tag{201}$$

Multiplying equation by +0, we obtain

$$(+3)\times(+0) \equiv (+2)\times(+0) \tag{202}$$

The new rule of the multiplication by zero is that

$$(+3)\times(+0) \equiv (+2)\times(+0) \tag{203}$$

stays that what it is and does not collapse into +0 = +0. Dividing by zero, it is

$$(+3)\times\frac{(+0)}{(+0)} \equiv (+2)\times\frac{(+0)}{(+0)} \tag{204}$$

or

$$(+3)\times(+1) \equiv (+2)\times(+1) \tag{205}$$

We obtain

$$(+3) \equiv (+2) \tag{206}$$

or

$$+3-2 \equiv +2-2 \tag{207}$$

The new rule of the multiplication by zero assures *the preservation of contradictions*. A statement which is obviously wrong (+1=+0) stays in the absence of any technical errors that what it is, obviously wrong or

$$+\mathbf{1} \equiv +\mathbf{0} \tag{208}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.24 (REFUTATION OF TODAY'S RULE OF THE ADDITION OF ZERO'S)

Nicomachus of Gerasa (ca. 60 – ca. 120 AD), was born in Gerasa, a former Roman province of Syria, and is best known for his book *Introduction to Arithmetic*. Nicomachus of Gerasa [47] claimed that *the sum of nothing added to nothing was nothing* or in other words it is 0+0+0 + …+ 0 = 0.

CLAIM.

Today's rule of the addition of zero's (+0 + 0 + … +0 = +0) is self-contradictory and based on a logical contradiction.

PROOF BY MODUS INVERSUS.

In general, with a false premise like

$$(\mathbf{1})+(\mathbf{1})+\cdots+(\mathbf{1}) = +(\mathbf{1}) \tag{209}$$

which is a non-acceptable contradiction, we must reach at a false conclusion. Multiplying this equation by 0, we obtain according to our today's rules of mathematics that

$$\begin{aligned} (1+1+\cdots+1)\times 0 &= +(1\times 0) \\ &or \\ (n\ times)\times 0 &= +(1\times 0) \end{aligned} \tag{210}$$

or that

$$(1\times 0)+(1\times 0)+\cdots+(1\times 0) = +(0) \tag{211}$$

or today's rule of the addition of zero's as

$$(\mathbf{0})+(\mathbf{0})+\cdots+(\mathbf{0}) = +(\mathbf{0}) \tag{212}$$

According to our today's rule of addition of zero's, this equation is identical with

$$+(\mathbf{0}) = +(\mathbf{0}) \tag{213}$$

or with

$$+(\mathbf{1})-(\mathbf{1}) = +(\mathbf{1})-(\mathbf{1}) \tag{214}$$

and at the end with

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{215}$$

QUOD ERAT DEMONSTRANDUM.

Remark 18.

According to a proof by modus inversus, if a premise is false then a conclusion must be false too. The premise (1+1+…+1) =+1 is false while the conclusion +1=+1 is true. This is a contradiction. Today's rule of the addition of zero's does not preserve the contradiction and has the potential to lead to logical inconsistencies. The same is refuted.

THEOREM 3.25 (THE CORRECT RULE OF THE ADDITION OF ZERO'S)

CLAIM.

The logically sound rule of the addition of zero's is n×0 = (+0 + 0 + … +0).

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{216}$$

Adding some terms we obtain

$$(1)+(1)+\cdots+(1) = (1)+(1)+\cdots+(1) \tag{217}$$

Multiplying this equation by 0, we obtain that

$$\begin{aligned} (1+1+\cdots+1)\times 0 &= (1+1+\cdots+1)\times 0 \\ &or \\ (1+1+\cdots+1)\times 0 &= (1\times 0)+(1\times 0)+\cdots+(1\times 0) \end{aligned} \tag{218}$$

Let n = (1+1+…+1) and define n×0 = n_0 [48], we obtain

$$\underbrace{n \times 0 = +(0) + (0) + \cdots + (0)}_{n - times} = n\_0 \tag{219}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.26 (THE ADDITION OF INFINITE NUMBER OF ZERO'S)

CLAIM.

The addition of an infinite numbers of zero's is equivalent with $+\infty\times0 = (+0 + 0 +0 + \ldots)$.

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{220}$$

Define infinity as $+\infty= +1+1+ 1+\ldots$, we obtain

$$(1) + (1) + (1) + \cdots = (1) + (1) + (1) + \cdots \tag{221}$$

Multiplying this equation by 0, we obtain

$$\begin{aligned} (\infty) \times 0 &= (1 + 1 + 1 + \cdots) \times 0 \\ &or \\ \infty \times 0 &= (1 \times 0) + (1 \times 0) + (1 \times 0) + \cdots \end{aligned} \tag{222}$$

or

$$\infty \times 0 = \underbrace{+(0) + (0) + (0) + \cdots}_{infinite\ number\ of\ zero's} = \infty\_0 \tag{223}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.27 (THE INCORRECT ADDITION OF INFINITY)

CLAIM.

The addition of infinity as $(+\infty + \infty + \ldots +\infty = +\infty)$ is self-contradictory and based on a logical contradiction.

PROOF BY MODUS INVERSUS.

According to modus inversus, we start this proof with a false premise like

$$(\mathbf{1}) + (\mathbf{1}) + \cdots + (\mathbf{1}) = +(\mathbf{1}) \tag{224}$$

which is a non-acceptable contradiction. Multiplying this equation by $+\infty$, we obtain according to our today's rules of mathematics that

$$\begin{aligned} (1 + 1 + \cdots + 1) \times \infty &= +(1 \times \infty) \\ &or \\ (\underbrace{1 + 1 + \cdots + 1}_{n\ times}) \times \infty &= +\infty \end{aligned} \tag{225}$$

or that

$$(1 \times \infty) + (1 \times \infty) + \cdots + (1 \times \infty) = +(\infty) \tag{226}$$

According to the proof by modus inversus, *if* the premise is false, *then* the conclusion is false too. Our premise above is false, technical errors are not apparent. Thus far, at the end, *the incorrect rule of the addition of infinity* follows as

$$(\infty) + (\infty) + \cdots + (\infty) \quad = \quad +(\infty) \tag{227}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.28 (THE CORRECT RULE OF THE ADDITION OF INFINITY)

CLAIM.

The logically sound rule of the addition of infinity (∞) is n×∞ = (+∞ + ∞ + … +∞).

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) \quad = \quad +(\mathbf{1}) \tag{228}$$

Adding some term, it follows that

$$(1) + (1) + \cdots + (1) \quad = \quad (1) + (1) + \cdots + (1) \tag{229}$$

Multiplying this equation by ∞, we obtain that

$$\begin{array}{ccc} (1+1+\cdots+1)\times\infty & = & (1+1+\cdots+1)\times\infty \\ & or & \\ (1+1+\cdots+1)\times\infty & = & (1\times\infty)+(1\times\infty)+\cdots+(1\times\infty) \end{array} \tag{230}$$

Let n = (1+1+…+1) and define n×∞ = n_∞ [48], we obtain

$$n \times \infty \quad = \quad \underbrace{+(\infty) + (\infty) + \cdots + (\infty)}_{n-times} \equiv n\_\infty \tag{231}$$

QUOD ERAT DEMONSTRANDUM.

Remark 19.

The theorem 3.25. is the first prove known which provides strict evidence that Euler's original position is correct: "Dieser Begriff von dem Unendlichen ist desto sorgfältiger zu bemerken, weil derselbe aus den ersten Gründen unserer Erkenntniß ist hergeleitet worden, und in dem folgenden von der größten Wichtigkeit seyn wird. Es lassen sich schon hier daraus schöne Folgen ziehen, welche unsere Aufmerksamkeit verdienen, da dieser Bruch **1/**∞ den Quotus anzeigt, wann man das Dividend 1 durch den Divisor ∞ dividiret. Nun wissen wir schon, daß, wann man das Dividend 1 durch den Quotus, welcher ist **1/∞, oder 0** wie wir gesehen haben, dividiret, alsdann der Divisor nämlich ∞ heraus komme; daher erhalten wir einen neuen Begriff von dem Unendlichen, nämlich daß dasselbe herauskomme wann man 1 durch 0 dividiret; folglich kann man mit Grund sagen, daß **1 durch 0 dividiret eine unendlich große Zahl oder** ∞ **anzeige**. … Hier ist nöthig noch einen ziemlich gemeinen Irrthum aus dem Wege zu räumen, indem viele behaupten, ein unendlich großes könne weiter nicht vermehret werden. Dieses aber kann mit obigen richtigen Gründen nicht bestehen. Dann da 1/0 eine unendlich große Zahl andeutet, und 2/0 ohnstreitig zweymal so groß ist; so ist klar, daß **auch so gar eine unendlich große Zahl noch 2 mal größer werden könne**." ([49], p. 34).

Euler's position stated in German can be translated into English as follows:

"It is the more necessary to pay attention to this understanding of infinity, as it is derived from the first elements of our knowledge, and as it will be of the greatest importance in the following part of this treatise. We may here deduce from it a few consequences that are extremely nice and worthy of attention. The fraction 1/∞ represents the quotient resulting from the division of the dividend 1 by the divisor ∞. Now, we know, that if we divide the dividend l by **the quotient 1/∞, which is equal to 0** [*i.e. zero, author*], we obtain again the divisor ∞: hence we acquire a new understanding of infinity; and learn that it arises from the division of 1 by 0; so that we are thence authorized in saying, that **1 divided by 0 expresses a number infinitely great**, or ∞. ... It may be necessary also, in this place, to correct the mistake of those who assert, that a number infinitely great is not susceptible of increase. This position is inconsistent with the principles which we just have laid down; for 1/0 signifying a number infinitely great, and 2/0 being incontestably thee double of 1/0, **it is evident that a number, though infinitely great, may still become twice, thrice, or any number of times greater**."

THEOREM 3.29 (THE NORMALIZATION OF A FINITE AND AN INFINITE)

CLAIM.

Let x_t denote something (a number, a mathematical object et cetera), let $\underline{\infty}_t$ denote the non-infinite complementary of x, let ∞_t denote the infinite. In general, the normalization of the relationship between the non-infinite and the infinite assumed to be possible, it follows that

$$\frac{x_t}{\infty_t} + \frac{\underline{\infty}_t}{\infty_t} = +1 \tag{232}$$

PROOF.

In general, taking axiom 1 <u>to be true</u>, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{233}$$

Multiplying by infinity ∞_t, it is

$$+1 \times +\infty_t = +1 \times +\infty_t \tag{234}$$

or

$$+\infty_t = +\infty_t \tag{235}$$

Adding x_t, it is

$$+\mathrm{x}_t + \infty_t = +\infty_t + \mathrm{x}_t \tag{236}$$

This approach to infinite is based on Euler's ([49], p. 34) understanding of infinity. Rearranging equation, it is

$$+\mathrm{x}_t + \infty_t - \mathrm{x}_t = +\infty_t \tag{237}$$

and according to our definition

$$+x_t + \underline{\infty}_t = +\infty_t \tag{238}$$

Normalizing the relationship between the non-infinite $\underline{\infty}_t$ and the infinite ∞_t, we obtain

$$\frac{+x_t}{+\infty_t} + \frac{+\underline{\infty}_t}{+\infty_t} = \frac{+\infty_t}{+\infty_t} = +1 \tag{239}$$

QUOD ERAT DEMONSTRANDUM.

Remark 20.

Under conditions where $+x_t = 0$, we obtain $+\underline{\infty}_t = +\infty_t - 0$ and or $+\underline{\infty}_t = +\infty_t$ and thus far

$$\frac{+0}{+\infty_t} + \frac{+\infty_t - 0}{+\infty_t} = \frac{+\infty_t}{+\infty_t} = +1 \tag{240}$$

or

$$\frac{+0}{+\infty_t} = +0 \tag{241}$$

Under conditions where $x_t = 1$, we obtain $\underline{\infty}_t = \infty_t - 1$ and thus far

$$\frac{1}{\infty_t} + \frac{\infty_t - 1}{\infty_t} = \frac{\infty_t}{\infty_t} = +1 \tag{242}$$

In particular, following the thoughts of Wallis, Newton, Euler [49–51] and other authors to some extent, we will have to accept that

$$\frac{+1}{+\infty_t} = 1 - \left(\frac{+\infty_t - 1}{+\infty_t}\right) \approx +0 \tag{243}$$

The term

$$\left(\frac{+\infty_t - 1}{+\infty_t}\right) \approx +1 \tag{244}$$

in approximately equal to +1. However, the term before is not completely identical to +1 and exactly equal to +1, which is necessary to be considered very precisely in the further course of the arguments [18, 52] and proofs provided.

THEOREM 3.30 (DIVISION 1/0 AND NEGATION)

CLAIM.

From the standpoint of classical logic, negation is identical with 1/0.

PROOF.

In general, taking axiom 1 to be true, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{245}$$

According to Barukčić [30, 48] and the results of this publication, it is (+0/+0) = +1. Substituting this relationship into equation before, it is

$$\frac{+0}{+0} = +1 \tag{246}$$

Rearranging equation, we obtain

$$\frac{+1}{+1} \times \frac{+0}{+0} = +1 \tag{247}$$

or

$$\frac{+1}{+0} \times \frac{+0}{+1} = +1 \tag{248}$$

Simplifying, it is

$$\frac{+1}{+0} \times 0 = +1 \tag{249}$$

According to classical logic, it is $\neg \times 0 = 1$ while the sign $\neg$ denote negation. Substituting, we obtain

$$\frac{+1}{+0} \times \mathbf{0} \quad = \quad \neg \times \mathbf{0} \tag{250}$$

The expressions on the left side of the equal sign denotes the same entity as the expression on the right side of the equal sign. Modifying both sides of the equation before, it is

$$\frac{+1}{+0} \quad = \quad \neg \tag{251}$$

QUOD ERAT DEMONSTRANDUM.

Remark 21.

Classical logic demands too that ¬×1=0. The theorem proves the logical meaning of 1/0. Assumed that classical logic is generally valid, such an approach to the problem of 1/0 could determine the foundation of a new understanding of indeterminate forms. Still, doubts about this relationship are allowed. It does not appear to be completely without any sense to assume that 1/0 ≈ ∞. In this case, negation and infinity would be closely related. Furthermore, the theoretical question is not answered, is it possible at all, to reach infinity? In this case, it is possible that an infinite can change to a finite and vice versa, which is associated with some theoretical problems.

THEOREM 3.31 (ZERO POWER ZERO)

CLAIM.

In general, it is

$$0^0 \quad = \quad 1 \tag{252}$$

PROOF.

$$0^0 \quad = \quad 0^{+1-1} \quad = \quad \frac{0^{+1}}{0^{+1}} \quad = \quad \frac{0}{0} \quad = \quad 1 \tag{253}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.32 (ZERO POWER 2)

CLAIM.

In general, it is

$$0^0 \quad = \quad +0-0 \tag{254}$$

PROOF.

$$0^2 \quad = \quad 0 \times 0 \quad = \quad (+1-1) \times 0 \quad = \quad \big((+1) \times 0\big) - \big((+1) \times 0\big) \quad = \quad +0-0 \tag{255}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.33 (INFINITY MINUS INFINITY)

CLAIM.

In general, it is

$$+\infty - \infty \quad = \quad 0 \times \infty \tag{256}$$

PROOF.

$$+\infty - \infty = (+1 \times \infty) - (+1 \times \infty) = (+1 - 1) \times \infty = 0 \times \infty \tag{257}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.34 (INFINITY DIVIDED BY INFINITY)

CLAIM.

In general, it is

$$\frac{\infty}{\infty} = 1 \tag{258}$$

PROOF.

$$\frac{\infty}{\infty} = \infty \times \frac{1}{\infty} = \infty \times \left(1 - \frac{\infty}{\infty}\right) = \infty \times \left(\frac{\infty - (\infty - 1)}{\infty}\right) = \infty - (\infty - 1) = 1 \tag{259}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.35 (INFINITY POWER ZERO)

CLAIM.

In general, it is

$$\infty^{0} = 1 \tag{260}$$

PROOF.

$$\infty^{0} = \infty^{+1-1} = \frac{\infty^{+1}}{\infty^{+1}} = \frac{\infty}{\infty} = 1 \tag{261}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.36 (ONE POWER INFINITY)

CLAIM.

In general, it is

$$1^{\infty} = 1 \tag{262}$$

PROOF.

$$1^{\infty} = 1^{\infty-1} \times 1^{1} = 1^{\infty-1+1} = 1 \times 1 \times \ldots 1 = 1 \tag{263}$$

QUOD ERAT DEMONSTRANDUM.

Remark 22.

A possible consequence could be that

$$log_{1}(1^{\infty}) = \infty \tag{264}$$

where *log* denotes the logarithm [39, 53].

THEOREM 3.37 (THE DIVISION 1 BY 0)

CLAIM.

According to classical logic, it is

$$\frac{+1}{+0} = \neg \tag{265}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{266}$$

Dividing by zero, we obtain

$$\frac{+1}{+0} = \frac{+1}{+0} \tag{267}$$

Under conditions of classical logic, it is ${}_{R}C_{t} = {}_{0}x_{t} + {}_{0}\underline{x}_{t} = +1$. Thus far, it is is ${}_{0}\underline{x}_{t} = {}_{R}C_{t} - {}_{0}x_{t} = +1 - {}_{0}x_{t}$. In other words, if ${}_{0}x_{t} = 1$ then ${}_{0}\underline{x}_{t} = {}_{R}C_{t} - {}_{0}x_{t} = +1 - {}_{0}x_{t} = 1\text{-}1 = 0$. Under these assumptions, the equation before changes to

$$\frac{+1}{+0} = \frac{{}_{0}x_{t}}{{}_{0}\underline{x}_{t}} \tag{268}$$

As proofed previously, it is equally

$${}_{0}x_{t} = {}_{R}C_{t} \times \left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right) \tag{269}$$

or

$${}_{0}\underline{x}_{t} = {}_{R}C_{t} \times \left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right) \tag{270}$$

Substituting these relationships into the equation before, we obtain

$$\frac{+1}{+0} = \frac{{}_{0}x_{t}}{{}_{0}\underline{x}_{t}} = \frac{{}_{R}C_{t} \times \left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right)}{{}_{R}C_{t} \times \left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right)} \tag{271}$$

Equation simplifies as

$$\frac{+1}{+0} = \frac{{}_{0}x_{t}}{{}_{0}\underline{x}_{t}} = \frac{\left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right)}{\left(\left(1 - \left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)\right)} = \neg \tag{272}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.38 (MINUS TIMES MINUS IS MINUS)

Unfortunately, the question why a *minus times a minus is a plus* is associated with curiosities even if mathematicians from ancient times were aware of numbers with negative sign. The ancient Greeks did not really address the problem of negative numbers, because the mathematics of the ancient Greeks was founded on geometrical axioms. The ancient Greeks did not really work on the problem of negative numbers and the rules of the manipulation of the same. In point of fact, Diophantus [54] rejected any equation like x + 4 = 0. In about 200 BC the Chinese number rod system represented negative numbers in black and positive numbers in red. In India, negative numbers [55] appeared at least about 620 CE in the work of Brahmagupta (598 - 670). In Europe, Griolamo Cardano (1501-1576) provided on of the first treatment of negative numbers in his *Ars magna* [56] in 1545.

CLAIM.

According to classical logic, minus times minus is minus or

$$(-1) \times (-1) \quad = \quad (-1)^2 \tag{273}$$

PROOF BY MODUS INVERSUS.

In general, we are starting with an incorrect premise that

$$+0 \quad \equiv \quad +2 \tag{274}$$

with the consequence that an incorrect conclusion must be achieved. Subtraction of -1 leads to

$$+0 - 1 \quad = \quad +2 - 1 \tag{275}$$

or too

$$-1 \quad = \quad +1 \tag{276}$$

Squaring equation before according to today's rules of mathematics, it is

$$(-1)^2 \quad = \quad (+1)^2 \tag{277}$$

or, according to today's rules,

$$(+1)^2 \quad = \quad (+1)^2 \tag{278}$$

Subtracting, we obtain

$$(+1)^2 - (+1)^2 \quad = \quad (+1)^2 - (+1)^2 \tag{279}$$

or

$$+0 \quad \equiv \quad +0 \tag{280}$$

or

$$+1 - 1 \quad \equiv \quad +1 - 1 \tag{281}$$

and at the end

$$+1 \quad \equiv \quad +1 \tag{282}$$

QUOD ERAT DEMONSTRANDUM.

Remark 23.

The proof before is based on modus inversus [57], which has to be regarded as generally valid. Thus far and according to modus inversus, *if* a premise (i.e. P_t: +0 = +2) is false *then* the conclusion (C_t: +1 = +1) is false too. The premise P_t: +0 = +2 is false but the conclusion C_t: +1 = +1 is not false, which is a contradiction. Thus far, *either* we must abandon the general validity of modus inversus *or* today's rule that minus times a minus is plus. One of the consequences would be that we had to accept again the general validity of the *principle ex contradictione sequitur quodlibet* as given. However, the general validity of the principle ex contradictione sequitur quodlibet [57] has been refuted. The only option left appears to be that minus times a minus is minus. Are we forced to abandon the general validity of modus inversus [57]? *Example*. A *witness X* testifies before a court that he was in about 2 meters personally away from the criminal scene in New York (USA) where he observed how the *defendant Y* murdered the *victim Z*. However, the judge knows absolutely for sure that witness X was in Paris (France) at the time of the murder of victim Z. Now the judge applies modus inversus and asks himself the following question: *if* it is false that witness X was in New York (USA) at the time of the crime, *then* it is also false that witness X has observed directly the murder of the victim Z in New York (USA). The premise: the witness X has been in New York (USA) at the time of the crime is false for sure. The judge must conclude that the statement that the witness X has directly observed how the defended Y murdered the victim Z is false too. Finding the truth is not always an easy task, neither before a court nor in science as such. However, a general ban on using *modus inversus* [57] to establish the truth would open the door and gate before a court and in science as such for untruth and injustice with inhumane, bestial and completely unacceptable consequences.

THEOREM 3.39 (THE ORDER OF (MATHEMATICAL) OPERATIONS (RULES OF PRECEDENCE))

The order of operations used throughout science and technology and many computer programming languages is a way of reducing contradictions and the number of necessary parentheses. It is a matter of conventions about which procedures have to be performed first in order to evaluate a given mathematical equation. Symbols of grouping are used and must be used sometimes to override the usual order of operations. Today's order of precedence is something like the following.

1. Parentheses ( ) (sometimes replaced by brackets [ ] or braces { } for readability)
2. Negation
3. Exponents and roots
4. Multiplication and division
5. Addition and subtraction
6. …

CLAIM.

Until all problems related with indeterminate forms are solved, it is appropriate to use parentheses before dividing by zero

$$+1 = \frac{(+1-1)}{+0} \tag{283}$$

PROOF.

In general, taking axiom 1 <u>to be true</u>, it is

$$+(\mathbf{1}) = +(\mathbf{1}) \tag{284}$$

Subtracting -1, we obtain

$$+1-1 = +1-1 \tag{285}$$

or

$$+0 = +1 - 1 \tag{286}$$

Before dividing by zero, it is appropriate to use the parentheses like

$$\frac{+0}{+0} = \frac{(+1-1)}{+0} \tag{287}$$

or

$$+1 = \frac{(+0)}{+0} \tag{288}$$

QUOD ERAT DEMONSTRANDUM.

Remark 24.

Ignoring the need to uses parenthesis could lead to unnecessary contradictions. *Without* the parenthesis, it is

$$\frac{+0}{+0} = \frac{+1}{+0} - \frac{+1}{+0} \tag{289}$$

or

$$+1 + \frac{+1}{+0} = \frac{+1}{+0} \tag{290}$$

or

$$+1 = +0 \tag{291}$$

which is a non-acceptable contradiction. In general, it is appropriate to define something like

$$\_0\_x \equiv \frac{+0}{+x} \tag{292}$$

which is different from the definition n_0.

IV. DISCUSSION

The prohibited, division by zero is an unresolved issue in pure mathematics and has a very long history. The issue of division by zero is documented in literature at least since the times of Aristotle. The concept of zero and the symbol of zero appears to have travelled from the Mesopotamians via the Greeks to India. Historically, especially the Hindu mathematicians of ancient India like Aryabhatta (476 - 550 AD), Brahmagupta (598-665 AD), Bhāskara II (1114-1185 AD) and others came up with a concept of zero. Brahamagupta (628), an outstanding Indian mathematician and astronomer of the 7th Century, in his Brahmasphula siddhanta was of the opinion that that 0/0 = 0 [58], while Bhāskara II defined [59] in Bijaganita that n / 0 = ∞.

The prohibition of division by zero is an artificially posted restriction and without any logical need or compelling reason. Mathematics as more or less a science created by human mind and consciousness does not exist outside of objective reality and absolutely independent of objective reality. Mathematics is one of human tools to depict objective reality. As soon as the touch with objective reality is lost, mathematics will change its own character to pure belief expressed by definitions, numbers and other mathematical constructs of human mind and consciousness.

And yet, despite a long history of debate going back to Aristotle himself, several problems of pure mathematics including the division zero by zero [30] are still not solved. In the present time Barukčić and Barukčić [60], supported by Paolilli [58], Mwangi [61], Czajko [62, 63] and Ufuoma [64], provided evidence that +0/+0=+1, while other authors [65–69] are not supporting this position. The position of these authors appears not to be compatible even with paraconsistent logic. On the very strong end of the spectrum of logic, paraconsistent logics is claiming that some contradictions are really true. Needless to say, all approaches to paraconsistency simply deny or are more or less in conflict with ex contradictione quodlibet principle. In other words, paraconsistent logic is a logical system which rejects the principle of explosion [70] or *ex contradictione sequitur quodlibet* (Latin, "from a contradiction, anything follows") while trying to deal with contradictions. The term 'paraconsistent' itself was coined by Francisco Miró Quesada at the Third Latin-American symposium on Mathematical Logic, Campinas, Brazil, July 11-17, 1976 [71]. Even if it is necessary to acknowledge the objective existence of contradictions in nature [72] this does not justifies the existence of logical or mathematical inconsistencies [57].

V. Conclusion

The division zero by zero is possible and defined. From the view of classical logic and Boolean algebra, it is $+0/+0 = +1$.

Financial support and sponsorship: Nil.

Conflict of interest statement:

The author declares that no conflict of interest exists according to the guidelines of the International Committee of Medical Journal Editors.

Acknowledgements

The open source, independent and non-profit Zotero Citation Manager has been used to create and manage references and bibliographies.

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