Standard Model fermions and Higgs scalar field from pCCR operator algebra

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Abstract

A fundamental theory of fermions is proposed by constructing the Dirac gamma matrices using pCCR operators. The result is the fermions of the Standard Model, 3 generations of leptons and quarks and the chiral nature of su(2) is revealed. In addition the pCCR operators generates a Higgs complex scalar doublet and that space-time is found to be 4d.

1 Introduction

At present there is no universally accepted explanation for the group structure and particle content of the Standard Model (SM) [1]. Understanding the nature of the elementary particles seems to be the key to understanding the structure of the SM. Quantum Field Theory QFT assumes a field and its associated creation and annihilation operators for each type of particle. Here CCR operators are used which acts on a single field. It is assumed the CCR operators are space-time dependent.

2 Primary CCR algebra - pCCR

Assume a CCR algebra which has the following commuting relations [2]:

$$[f_{ab}, f^{\dagger}_{cd}] = \delta_{abcd} \tag{1}$$

The CCR operators are unitary, generalise to

$$f_{ab}e^{iPx}_{ab} \quad f^{\dagger}_{ab}e^{-iPx}_{ab} \tag{2}$$

Let $g \in \{f_{ab}e^{iPx}_{ab}\}$ and let $g^{\dagger} \in \{f^{\dagger}_{ab}e^{-iPx}_{ab}\}$ A general function is the product of pCCR creation operators

$$S_f = \prod_{j,k} g^{\dagger j} g^k S_i \tag{3}$$

S_f and S_i are in general matrices, but could be vectors. A general matrix generated from unit creation operators is

$$M_{ab} = e^{ikx}_{ab} = cos(kx)_{ab} + isin(kx)_{ab} \tag{4}$$

3 Dirac Gamma matrices

Special matrices arise when $kx \in [-2\pi, 2\pi]$ has the following values:

$$e^0 = +1_0 \quad e^{\pm i2\pi} = +1_{\pm 2} \quad e^{\pm i\pi} = -1_{\pm 1} \quad e^{\pm \frac{\pi}{2}} = \pm i_{\pm \frac{1}{2}} \quad e^{\pm \frac{3\pi}{2}} = \mp i_{\pm \frac{1}{2}}$$

The subscripts indicates the value of $\frac{kx}{\pi}$
The 3 1's form a set:

$$1_\phi = \{1_{-2}, 0, 1_2\}$$

1_ϕ has 3 pairs of 1's $\{1_{02} = (1_0, 1_2), 1_{0-2} = (1_0, 1_{-2}), 1_{22} = (1_2, 1_{-2})\}$
There is only 1 pair of -1's $\{-1_1, -1_{-1}\}$
The imaginary units, fixes the size of the Dirac gamma matrices to 4x4 and therefore space-time is 4d. (see appendix for the Dirac gamma matrices). Form the matrices $\{\gamma^0, \gamma^1, \gamma^3\}$ from the 3 pairs of 1's and the 1 pair of -1's.

$$\{\gamma^0_{02}, \gamma^0_{0-2}, \gamma^0_{22}\}, \{\gamma^1_{02}, \gamma^1_{0-2}, \gamma^1_{22}\}, \{\gamma^3_{02}, \gamma^3_{0-2}, \gamma^3_{22}\}$$

With only 1 γ^2, 6 sets of 4d Dirac Gamma matrices are:

$$\gamma^\mu_k = \begin{cases} \{\gamma^0_{02}, \gamma^1_{0-2}, \gamma^2, \gamma^3_{22}\} \\ \{\gamma^0_{02}, \gamma^1_{22}, \gamma^2, \gamma^3_{0-2}\} \\ \{\gamma^0_{0-2}, \gamma^1_{02}, \gamma^2, \gamma^3_{22}\} \\ \{\gamma^0_{0-2}, \gamma^1_{22}, \gamma^2, \gamma^3_{02}\} \\ \{\gamma^0_{22}, \gamma^1_{02}, \gamma^2, \gamma^3_{0-2}\} \\ \{\gamma^0_{22}, \gamma^1_{0-2}, \gamma^2, \gamma^3_{02}\} \end{cases}$$

The 4d Dirac Gamma matrices act on 4d complex field, generating the fermions.

4 Chiral su(2)

Only the 3 1_ϕ elements transform under su(2) and therefore is chiral. There are 3 su(2) chiral doublets -

$$\begin{pmatrix} \gamma^\mu_1 \\ \gamma^\mu_3 \end{pmatrix}, \quad \begin{pmatrix} \gamma^\mu_2 \\ \gamma^\mu_5 \end{pmatrix}, \quad \begin{pmatrix} \gamma^\mu_4 \\ \gamma^\mu_6 \end{pmatrix}$$

Generalise the weak-hyper charge Y [3]

$$Y = \sum_c 2(Q - T_3)_c \tag{5}$$

where c is the number of colors, Q is the electric charge and T_3 is the su(2) normalised diagonal matrix (see appendix). The weak hypercharge can be defined to be $Y = \pm 1$.

5 Leptons

The chiral doublets have $c = 1$ and for $Y = -1$, the electric charges are $Q = (0, -1)$ thus 3 chiral lepton doublets. $Y = 1$ results in 3 chiral anti-lepton doublets.

6 Quarks

Another set of Dirac gamma matrices can be formed

$$\gamma_k^\mu(1_\phi) \otimes 1_\phi$$

$$\begin{pmatrix} \gamma_1^\mu \\ \gamma_3^\mu \end{pmatrix}_h, \quad \begin{pmatrix} \gamma_2^\mu \\ \gamma_5^\mu \end{pmatrix}_h, \quad \begin{pmatrix} \gamma_4^\mu \\ \gamma_6^\mu \end{pmatrix}_h$$

The chiral doublets have $c = 3$, and for $Y = 1$ the electric charges are $Q = (2/3, -1/3)$ - 3 chiral quark doublets. $Y = -1$ results in 3 chiral anti-quark doublets.

7 Higgs doublet

Complex scalar doublet $\Phi = (1_{-2}, 1_2)$
The doublet has $c = 1, Y = +1$, $Q = (0, +1)$ complex scalar doublet which matches the Higgs complex scalar doublet [4].

8 Summary

The pCCR operators generates sets of gamma matrices which act on 4d complex vectors. Each set corresponds to the 6 leptons and 6 quarks in 3 colors. In addition the pCCR operators generate a Higgs complex scalar doublet and that space-time is 4d.

9 Appendix

For reference the Dirac Gamma matrices [5] are:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The su(2) normalised diagonal matrix is [6]

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

References

[1] Daniel V. Schroeder Michael E. Peskin. *An Introduction to Quantum Field Theory*. Westview Press, 1985.

[2] Daniel V. Schroeder Michael E. Peskin. *An Introduction to Quantum Field Theory*, chapter 2, page 19. Westview Press, 1985.

[3] Pham Xuan Yem Quang Ho-Kim. *Elementary Particles and Their Interactions*, chapter 9, pages 310,311. Springer-Verlag, 1998.

[4] Pham Xuan Yem Quang Ho-Kim. *Elementary Particles and Their Interactions*, chapter 9, page 313. Springer-Verlag, 1998.

[5] Pham Xuan Yem Quang Ho-Kim. *Elementary Particles and Their Interactions*, chapter 3, pages 57,59. Springer-Verlag, 1998.

[6] Pham Xuan Yem Quang Ho-Kim. *Elementary Particles and Their Interactions*, chapter 3, page 59. Springer-Verlag, 1998.