AdS/CFT in the Mandelbrot Set

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Abstract

The AdS/CFT conjecture of Juan Maldacena is a cornerstone of Modern Physics, but why it works remains a mystery. The appearance of Cartan's rolling-ball analogy for G_2 symmetries in the Mandelbrot Set offers a window on correspondences between interactions in the higher-d precursor and 4-d spacetime implied by AdS/CFT. Recent theories suggest a literal embodiment of AdS/CFT in a black hole in 5-d $\rightarrow$ 4-d white hole/bubble on the holographic boundary. This transition is represented in the Mandelbrot Set at (-0.75,0*i*) where the boundary folds back on itself and we see a pseudo-symmetric mirroring of features in the cardioid, depicting 5-d evolution, with features in the circular region representing 4-d spacetime. This shows how the force of fermionic mass in the outward-pressing hypersurface of the early universe becomes the inward-facing pull of gravity toward the center of massive objects, in the present-day cosmos, which explains the weakness of gravity, and accelerating expansion.

Keywords: dimensional reduction, holographic universe, AdS/CFT, Lie group G_2, Mandelbrot Set, cosmic expansion, gravitation

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Introduction

The 1993 paper on dimensional reduction by Gerard 't Hooft [1], downloaded an unprecedented number of times when first posted on arXiv, launched a conversation on holography in Physics that continues today. A 1997 paper by Juan Maldacena expanding on that work [2] is still most downloaded [3]. The core idea of 't Hooft is that the 2-d surface of a black hole event horizon, where dimensions of spacetime are reduced, is like a hologram describing and encoding the features and Physics of our universe. Maldacena suggested this implies a conformal field theory (or CFT). He found the holographic principle could be applied upward as well, in a boundary of 4-d spacetime with a 5-d reality, and proposed that M-theory admits five non-compact dimensions. So we see a 2-d screen at infinity, where laws of 4-d Physics are written, perhaps during a 5-d $\rightarrow$ 4-d transition. This idea excites gravity theorists and cosmologists, who find many ways to put it to use, but it remains unclear what cosmological events created the context for AdS/CFT, which would explain why we observe this correspondence. The Mandelbrot Set may provide answers.

Theoretical Considerations

DGP gravity (for Dvali, Gabadadze, Porrati) [4] suggested a 5-d transition at extreme distances to explain accelerating expansion, in a braneworld scenario, by modifying gravity at long-range. Pourhasan, Afshordi, and Mann [5] asserted this could work cosmologically as a 5-d black hole $\rightarrow$ 4-d white hole transition from a prior universe. We know looking farther out into space means looking farther back in time, so the most distant horizon of our cosmos reveals an earlier cosmological epoch. What we see at the edge of our universe is the inside of a higher-dimensional cosmos – in this scenario – seen through the event horizon of a 5-d black hole, or poured through the wormhole throat in a 5-d black hole $\rightarrow$ 4-d spacetime transition. Our present-day universe is effectively inside out from the prior cosmos, or that universe is inside-out from ours. This is not surprising, if one knows about higher-d spheres. If the early cosmos was an expanding hypersphere; its surface has volume. This follows from a property observed in the circle or 1-sphere; its boundary is one dimension lower than what is contained. So with a conventional or 2-sphere; a 3-d volume is contained by its 2-d surface. Similarly; a hypersphere containing a 5-d volume has a 4-d surface.

Familiar particle varieties can exist in the 'skin' of the expanding fireball, therefore, and tend to be concentrated in this outer shell, if the early universe is a growing hypersphere. Most of the fermionic mass is found in this outward-pressing hypersurface rather than in the 5-d bulk, during this stage of cosmic evolution. From baryogenesis until the final decoupling; this mass is comprised of subatomic particles and nuclei which deform the fabric of space to expand the volume contained. But once decoupling gives way to recombination (named as though nuclei and electrons were once combined), there can be massive objects with many atoms and molecules. In the present day cosmos; there are objects which are an aggregate of molecular matter, and they generally gain mass as they increase in size. It is not quite so simple of course, because the density varies by the type or species we are talking about. Avogadro's number of Oxygen atoms weigh much less than the same number of Osmium atoms. In the current era though; objects made of atomic or molecular matter form centers the fabric of space puckers toward – making gravity wells where each sits in space. So fermionic mass was mainly outward-pressing in the early universe, but its gravitational pull is inward toward the center of massive objects in the current era.

What does the Mandelbrot Set show us?

Insights into this phenomenology, the 5-d $\rightarrow$ 4-d transition, and the AdS/CFT correspondence are found in the Mandelbrot Set. Cartan's rolling ball analogy for Lie group G_2 symmetries is in the overall shape of $\mathcal{M}$, where the ball's point of contact is at (-0.75,0*i*). The boundary folds back on itself, at that point, and form along the periphery changes character to create a pseudo-symmetric mirror across the boundary. This is highlighted by coloring in where iterand magnitude diminishes monotonically over 3 calculations, to show the Mandelbrot Butterfly. This reveals duality between weak interactions on the higher-d precursor side and EM forces on this side of the boundary, in our cosmos. To appreciate the significance, we need to see $\mathcal{M}$ in the complex domain as a shadow or projection of a figure in the quaternions and octonions. If we designate the octonion imaginaries (i, j, k, I, J, K, L) we can separate the octonions' 7 rotational axes into (I, J, K, L) for the 5-d early universe and (i, j, k) for the present-day cosmos to show the prior cosmos can be inside out from ours. [6] This model suggests nuclei of heavy and ultra-heavy elements were created in the 5-d precursor, but not all could pass through the wormhole. Some particle species created in the early universe were trapped there, as well, so extra particles theorized by supersymmetry or other considerations might exist, but still cannot be created in a lab.

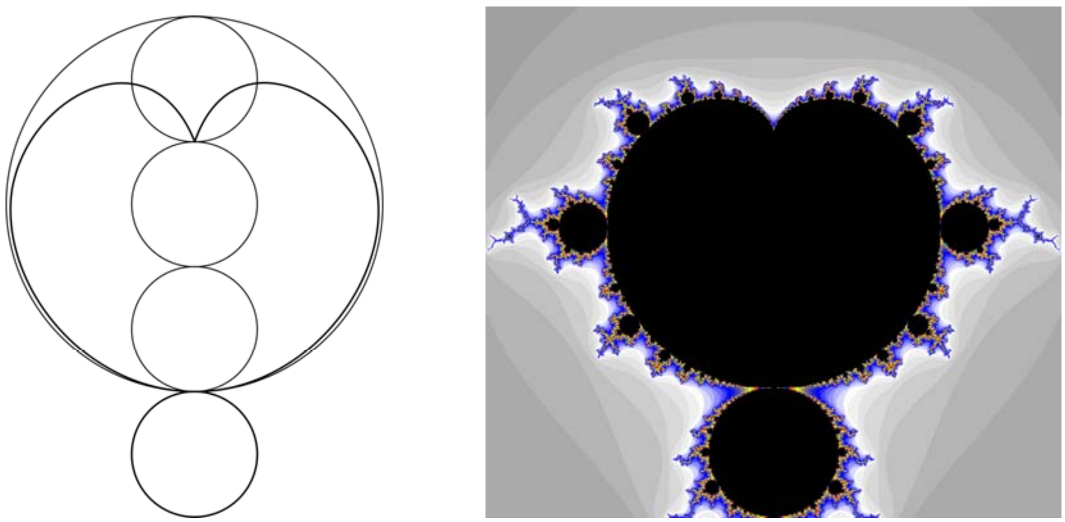

Fig. 1 – At left shows the construction of the cardioid and circular disc in $\mathcal{M}$ (on right) by rolling a circle of radius .25 on another, and then placing one at the extremum, reproducing the 3:1 ratio in Cartan's rolling ball G_2 analogy.

This is part of the appeal of using $\mathcal{M}$ to probe Physics. It allows us to see the dynamics we want to examine in an archetypal setting. The Mandelbrot Set displays local symmetries in every flavor and period balanced against global asymmetry. As Tan Lei first noted [7]; forms at the branching Misiurewicz points are perfectly symmetrical at the center, but conform to an asymmetric background at the edges, which is reflected in the corresponding Julia Sets. Symmetry is sharply shifted at boundary transitions, however, so symmetries preserved along the cardioid's boundary are broken on the other side, in the large circular region. Comparing forms on either side of the line at -0.75 informs us about electro-weak symmetry breaking, under the Mandelbrot mapping conjecture [8]. But this implies the 5-d $\rightarrow$ 4-d transition initiates both electro-weak symmetry breaking and decoupling. In Fig. 2 below of the Butterfly near (-0.75,0*i*); the pointed bands at the top depict matter-energy exchanges, as action paths of creation-annihilation operators. Below that is the 'skin' of the primordial hypersphere, or fabric of spacetime, where particles and nuclei are confined to move along or across the brane boundary, and time advances right to left. Below the fold, on our side of the brane, the grey discs are charged masses (in potential wells) in descending order, where time progresses from left to right, after decoupling.

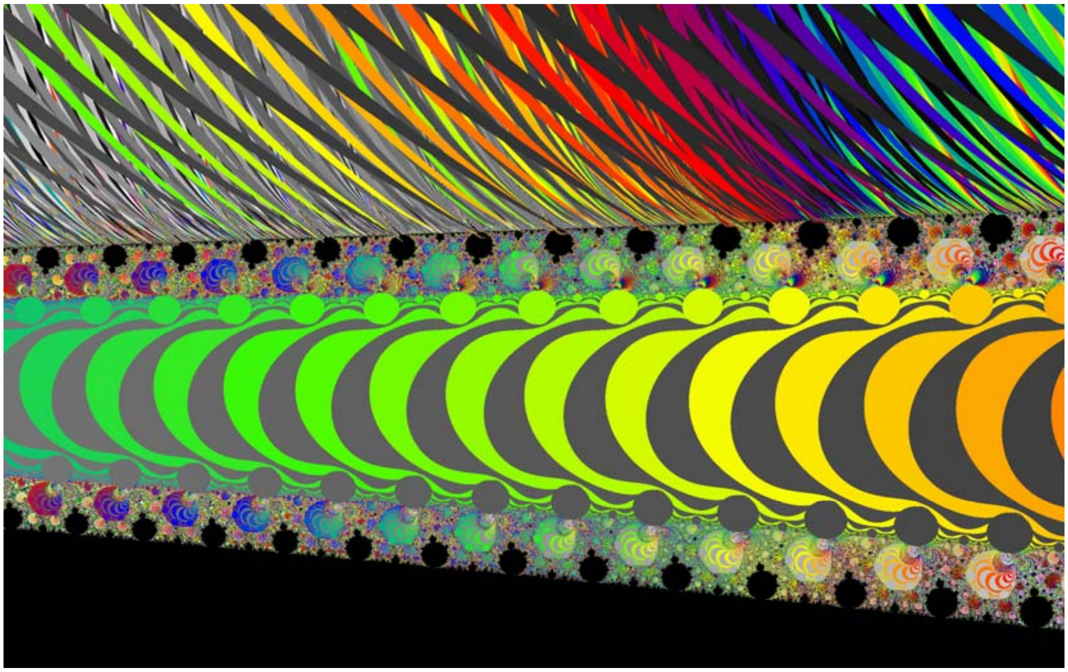

Fig. 2 – This diagram shows annihilation/creation operator paths in the early universe at top, nucleosynthesis of heavy elements from right to left (light orange to green discs), and macroscopic masses on bottom (grey discs) with decreasing charge from left to right, in the present-day cosmos.

In 4-d spacetime; macroscopic masses deform space to create a gravitational well of attraction. In the early universe, assuming a higher-d origin; fermionic particles familiar to our dimensional and energetic regime were squeezed onto the volumetric hypersurface, as discussed and as shown in Fig. 2 above, which pushed spacetime outward. We see element building in the prior cosmos, where when squeezed more tightly; ever denser nuclei deformed the fabric of space – stretching and expanding it – until a theoretical maximum was reached and a new era commenced. The reason is threefold. The contained hypervolume is greatest for a sphere in 5-d. [9] There was no further to go in density building, because the primordial energy was expended. And the action of Lie group G_2 forced the orientation of space to flip during the 5-d $\rightarrow$ 4-d transition. [10] This is all depicted in the Mandelbrot Set, the Butterfly figure, and the family of figures including the Julia Sets. The representation of $\mathcal{M}$ in Fig. 3 below is obtained by taking an array of concentric circles about (0,0*i*) then unrolling it and laying it flat. This shows how the cardioid region as seen from our space appears to be an inverted hyperbolic surface, mimicking Anti de Sitter space. So the Mandelbrot Set gives us a 2-d map of the 5-d $\rightarrow$ 4-d transition that gave birth to our cosmos.

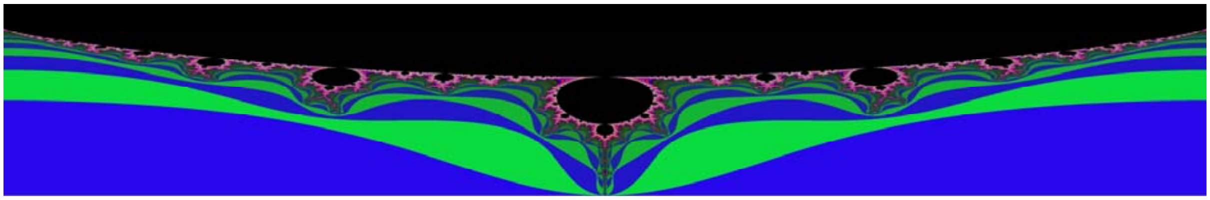

Fig. 3 – The Mandelbrot Set, mapping concentric circles about (0,0*i*) to rows of pixels, resembles Anti de Sitter space. The unfurled cardioid representing the 5-d precursor universe is at the top of screen, and the 4-d spacetime we inhabit is the bubble at center, using this model.

Conclusions

While what is presented above is still a toy model, in some areas, the author feels the correspondences indicated deserve further study. They show how AdS/CFT can arise in a unique cosmological setting, linking early-universe microphysics to present-day macroscopic astrophysics, and giving rise to familiar Physics laws.

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