SunQM-3s4: Using {N,n} QM structure and multiplier n' to analyze Saturn's (and other planets') ring structure

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Abstract

All Saturn rings do follow p{N,n} QM structure. By using planet's Earth-sized core as pCore{0,1} and r_1, Saturn's inner core, outer core, surface, B-ring, and moons of Mimas, Enceladus, Tethys, Dione, and Rhea, are almost perfectly at pCore{0,n=1..9//3} orbits or sizes. Also Uranus' major ring (ε ring), and its minor moons Portia, Puck, and its major moons Miranna, Arial, Umbriel, Titania, Oberon, are at orbit of pCore{0,n//2} with n ≈ 3, 3, 4, 5, 6,7, 9, and 10 approximately. All major rings of gas/ice planet are at Δn = +1 out of planet's surface (if using their Earth-sized core as r_1). For Neptune and Uranus, both of their surfaces are at pCore{0,2//2} size, and their major rings are at pCore{0,3//2} orbit. For Saturn and Jupiter, both of their surfaces are at around pCore{0,3//3} size, and their major rings are at pCore{0,4//3} orbit. By comparing between similar massed planets, the pCore{N,n} QM structure analysis suggests that Uranus' ring is younger than that of Neptune's, and Saturn's ring is younger than that of Jupiter's. Saturn's major ring (B-ring, base frequency n=4)'s inner edge can be described by a multiplier n' = 4*3^6 - 167 = 2749, its outer edge can be described by n' = 4*3^6 +225 = 3141, and its thickness can be described by n' =4*3^25 = 3.39E+12. Saturn's A, B, C, D rings may follow the radial probability density distribution curve. We can use p{N,n} QM and r^2 *|R(n,l)|^2 *|Y(l,m)|^2 to build Saturn major ring's true 3D structure probability function as shown in equation-4.

Introduction

A series of my previous papers has shown that the formation of Solar system (as well as each planet) was governed by its {N,n} QM [1] ~ [13]. In paper SunQM-3s6, -3s7, and -3s8, I showed that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial QM. In paper SunQM-3s3 and -3s9, I showed that the surface mass (atmosphere) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) θφ-2D dimension QM. In current paper, I'd like to further demonstrate that the rings of all planets are also governed by their p{N,n} QM. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see my paper SnQM-p1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = $3.4*10^{12}$, 5.6E-9 = $5.6*10^{-9}$. Note: The reading sequence for SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s5. Note: for all SunQM series papers, reader should check "SunQM-3s10: Updates and Q/A for SunQM series papers" for the most recent updates and corrections.

I. Using {N,n} QM to analyze Saturn's ring structure

Saturn ring data listed in Table 1 comes from wiki "Rings of Saturn". The middle r is calculated as the average of ring's inner edge r and out edge r. The major ring of Saturn is B-ring, with the middle r =1.05E+8 m.

From wiki "Rings of Saturn", all rings lie in Saturn's equatorial plane, except Phoebe-ring, which "*lies in the plane of Saturn's orbit, or roughly the ecliptic, and thus is tilted 27 degrees from Saturn's equatorial plane and the other rings*". Also see wiki "Rings of Saturn", figure "Side view of Saturn system, showing Enceladus in relation to the E Ring".

Table 1. Saturn's ring data obtained from wiki "Rings of Saturn".

Saturn's ring	mass	vertical thickness	major or minor	inner edge	outer edge	orbit, r_n calc. middle r	width
	kg	m		m	m	m	m
D-ring			major-4	6.69E+07	7.45E+07	7.07E+07	7.50E+06
C-ring	1.10E+18	5	major-3	7.47E+07	9.20E+07	8.34E+07	1.75E+07
B-ring	7~24E+18	5~15, edge up to 2500	major-1	9.20E+07	1.18E+08	1.05E+08	2.55E+07
A-ring	4~5E+18	10~30	major-2	1.22E+08	1.37E+08	1.30E+08	1.46E+07
F-ring			minor			1.41E+08	3~50E+4
G-ring			minor	1.66E+08	1.75E+08	1.71E+08	9.00E+06
E-ring			minor	1.80E+08	4.80E+08	3.30E+08	3.00E+08
Phoebe-ring	retrograde, 27° tilted		minor	4.00E+09	> 1.3E+10	> 4E+9	> 9E+9

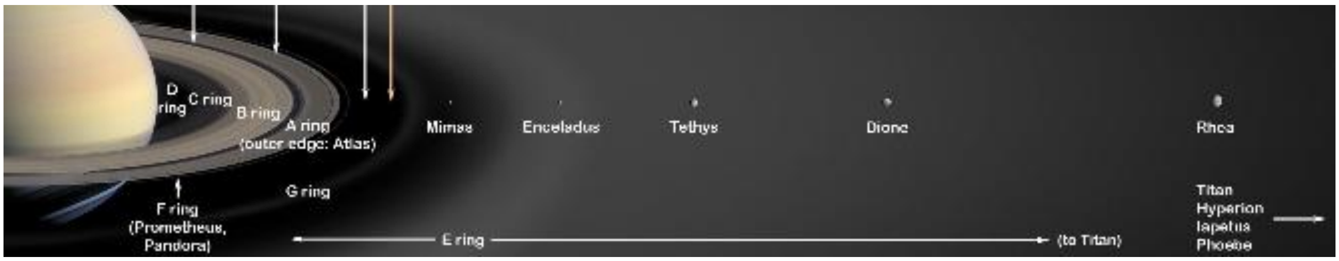


Figure 1. Saturn, its rings and major moons—from Mimas to Rhea (NASA Artist's Concept). Copied from wiki "Rings of Saturn", Author: https://photojournal.jpl.nasa.gov/catalog/PIA03550. Copy right: Public domain.

I-a. All Saturn rings do follow p{N,n} QM structure

Saturn's exterior p{N,n} QM structure (including major moons) has been analyzed in paper SunQM-1s3 Table 6a. Table 2 columns 4 through 7 of current paper copied part of the result. There, the Saturn surface was used to be p{0,1} and r_1 for the analysis, so we name it as pSurface{N,n}. Comparing to pSurface{N,n//3} (see Table 2 columns 6 &7), the pSurface{N,n//2 } analysis (see Table 2 columns 4 & 5) gives more accurate result for the outer major moons. Despite that the two most outer major moons (Titan, and Iapetus) occupy the approximate pSurface{2,1//2} and pSurface{3,1//2} orbits, the five inner major moons (Mimas, Enceladus, Tethys, Dione, Rhea) have no well defined pSurface{N,n} QM orbits, although all of them sit in the pSurface{1,1//2}o orbit space. Comparing to Mimas, Saturn's major ring B-ring is even closer to Saturn. So using pSurface{0,1} will not generate any useful result for Saturn's major ring analysis.

In paper SunQM-1s3, it was found that Saturn has a p{N,n//3} QM structure with an Earth-sized core at 5.82E+7 / 3^2 = 6.47E+6 meters. Also Saturn's atmosphere surface band pattern has n=3 character (see paper SunQM-3s3 section II). After many tries, now I find that if using Saturn's Earth-sized core as r_1 or p{0,1//3}, then we can have a nearly perfect n = 4, 5, 6, 7, 8, and 9 orbit sequence for B-ring, and moons of Mimas, Enceladus, Tethys, Dione, and Rhea, with Saturn's surface at n=3 (see columns 8 through 11 of Table 2). Now let us define this p{N,n} QM as Saturn's pCore{N,n} QM structure, meaning that it uses Saturn's Earth-sized core as r_1. The relationship between Saturn's pSurface{0,n} QM structure and Saturn's pCore{0,n} QM structure is that pSurface{0,n} = pCore{0,n*3}. Note: **at N = 0, in a {0,n//q} QM structure, n is same for any q quantum number**, so {0,n//2} = {0,n//3}. With the multiplier n' knowledge (see paper SunQM-2), it is obvious that pCore{0,n} QM structure is the high-frequency multiplier of the base-frequency pSurface{0,n} QM structure with pFactor = 3. Comparing to pCore{N,n//2} (see Table 2 columns 8 &9), the pCore{N,n//3 } analysis (see Table 2 columns 10 & 11) gives more accurate result for the inner major moons.

The analysis in Table 2 reveals that the current Saturn-moon system is organized by two different p{0,n} QM structures. In a large scale, Saturn-moon system is governed by Saturn's pSurface{0,n//2} QM structure, with Saturn's surface at pSurface{0,1//2}, with rings of A, B, C, D, in pSurface{0,1//2}o orbit space (but it occupies only half of the pSurface{0,1//2}o orbit space, ends at pSurface{0,1.5//2} not at pSurface{0,2//2}, see Figure 1); with E-ring and moons of Mimas, Enceladus, Tethys, Dione, and Rhea are all in the pSurface{1,1//2}o orbit space (but it also occupies only half of the pSurface{1,1//2}o orbit space, starts at pSurface{0,2//2} = pSurface{1,1//2}, ends at pSurface{0,3//2} = pSurface{1,1.5//2}, instead of ending at pSurface{0,4//2} = pSurface{2,1//2}, also see Figure 1); with Titan at orbit of pSurface{2,1//2}, and cleared all mass in the orbit space of pSurface{2,1//2}o (due to it has large mass); and with Lapetus at orbit of pSurface{3,1//2}, and cleared all mass in the orbit space of pSurface{3,1//2}o in Saturn's equatorial plane (even it has relative low mass). The Phoebe-ring in the pSurface{3,1//2}o orbit space (starts at pSurface{3,1//2}, and ends at pSurface{4,1//2}), but it is retrograde and out of Saturn's equatorial plane.

Within the size of pSurface{1,1//2}, Saturn's internal structure, rings, and moons are governed by the pCore{0,n//3} QM structure, with Saturn's inner core, outer core, surface, B-ring, Mimas, Enceladus, Tethys, Dione, and Rhea, are almost perfectly at pCore{0,n=1..9//3} orbits or sizes (see columns 10 & 11 of Table 2).

Using this result, we can guess how Saturn was formed and evolved (as the hypothesis-1):

1) Shortly (?) after the pre-Sun ball was collapsed to size of {2,1}, the original Saturn was formed at orbit {2,3} by accreting all mass in {2,3}o orbit space. It was formed in the same way as all other planets, with an Earth-sized core at r ≈ 6.47E+6 meters (or close to 6.32E+6 m as estimated in Table2 of SunQM-3s6) at size of pCore{0,1//2}, with an old surface r = 4x 6.47E+6 = 2.53E+7 meters at size of pCore{0,2//2} (which had ~20% of current mass, see paper SunQM-3s6 Table 2). So it had a pCore{0,n//2} QM structure initially. It might have a few old-rings and old-moons (the pre-moon of Mimas, Enceladus, Tethys, Dione, and Rhea, with mass probably < 1E+15 kg) on orbits of pCore{0,n=5..9//2}. It did not have moon Phoebe, and might or might not have Titan and Lapetus. An early collision made the original Saturn's equatorial plane 27˚ tilted from the Saturn orbit's plane.

2) Then the H-fusion was ignited at the pre-Sun's center. The heat expanded the ice-evap line to around {1,8}, and the out-flew mass (largely H, He, H_2O, CH_4, etc.) from inside the {2,1} pre-Sun ball was captured by Jupiter at orbit {2,2} and Saturn at orbit {2,3}. Then Saturn increased its mass from the original 20% to the current 100%, and also increased its size from the original pCore{0,n=2//2} to today's pCore{0,n=3//2} with r = 5.82E+7 m. At the end of this stage, Saturn changed its QM structure from pCore{0,n//2} to pCore{0,n//3}. During this time, a thick ring might be formed and it covered the whole orbit space of pCore{0,n=4..9//2}o, so that the pre-moons inside this thick ring grew up as today's Mimas, Enceladus, Tethys, Dione, and Rhea, and fine adjusted their orbit to the current pCore{0,n=5..9//2} orbits. Then at the end of this stage, we had a Saturn with today's size, five inner moons at today's orbits, and might or might not have p{0,4//2} ring, or Titan, or Lapetus.

3) Moons of Titan and Lapetus were captured by the full sized Saturn at the orbit of pSurface{2,1//2} and pSurface{3,1//2}. Alternatively, they might be formed in situ in step 1, and grew up in step 2.

4) An intruding celestial body (might be bigger than the retrograde moon Phoebe) collided with Saturn, excited the vaporized mass to the higher excited orbits including orbit spaces of pSurface{0,1//2}o, pSurface{1,1//2}o, pSurface{2,1//2}o, and pSurface{3,1//2}o. However, all seven major moons were intact, and they stayed in their original orbits. The fast self-spin of Saturn produced very strong QM nLL (disk-lyzing) effect in its SpnRefFrm (spin-reference-frame), and quickly disk-lyzed the mass in pSurface{N=0..3,1//2}o orbit spaces to its equatorial plane. Then Lapetus at pSurface{3,1} orbit cleared out all ring mass in pSurface{3,1}o orbit space in Saturn's equatorial plane. The very large massed Titan at pSurface{2,1} orbit quickly cleared out all mass in pSurface{2,1}o orbit space. The relatively small massed five inner moons in pSurface{1,1}o orbit space are still clearing out mass in this orbit space, and leaving residue part of very low density mass in pSurface{1,1}o orbit space as E-ring, waiting to be completely cleared by five inner moons.

The original E-ring covered from pSurface{1,1//2} = pSurface{0,2//2} to pSurface{2,1//2} = pSurface{0,4//2}. Since it is governed by pCore{N,n} QM structure, now it should be rewritten as started from pCore{0,6//2} = pCore{0,6//3}and ended at pCore{0,12//2} = pCore{0,12//3}. Because the mass occupancy was too low, E-ring quantum collapsed not only in θ-dimemsion (as disk-lyzation), but also in r-dimension. So E-ring's outer edge was quantum collapsed from the original pSurface{2,1//2} = pSurface{0,4//2} to today's pSurface{0,3//2} (see Table 2 column 5, = pSurface{1,1.5//2}), or equivalent to pCore{0,9//2} = pCore{0,9//3} = pCore{2,1//3} (see column 10 & 11). Finally, the ring mass in pSurface{0,1//2}o orbit space, started from pSurface{0,1//2} = pCore{0,3//3} and ends at pSurface{1,1//2} = pSurface{0,2//2} = pCore{0,6//2}, also quantum collapsed due to its mass occupancy was too low. the ring mass in pCore{0,5//2}o orbit space was collapsed as G-ring at orbit pCore{0,5//2}, the ring mass in pCore{0,4//2}o orbit space was collapsed as rings of A, B, C, D, with the middle radius of the major ring (B-ring) at orbit pCore{0,4//2}, and the ring mass in pCore{0,3//2}o orbit space was collapsed into Saturn with the surface at pCore{0,3//2}.

5) Before (or even during) the collision, moon Phoebe was captured by Saturn in a retrograde orbit at pSurface{4,1//2}, with the orbit plane out of Saturn's equatorial plane. After collision, the vaporized mass from Saturn excited to as far as the pSurface{3,1//2}o orbit space, therefore encounter moon Phoebe at pSurface{4,1//2} orbit. This caused Phoebe moon to release part of its surface mass to the pSurface{3,1//2}o orbit space, therefore formed retrograde Phoebe ring with the ring plane same as that of moon Phoebe's orbit plane. Why the Phoebe ring is in the pSurface{3,1//2}o orbit space, but not in the pSurface{4,1//2}o orbit space? Because the released Phoebe moon surface's (retrograde) mass was caused by the hitting of the Saturn's vaporized and excited prograde mass, so its retrograde speed was slowed, therefore it had to fly inward to pick up the speed, so the Phoebe ring's radius is smaller than Phoebe moon orbit's radius. After the vaporized mass from Saturn in the pSurface{3,1//2}o orbit space was disk-lyzed, and accreted by moon Lapetus, I believe that today's Phoebe moon should have stopped the releasing of its surface mass to the Phoebe ring. (Note: Pheobe ring's explanation is mostly based on other scientists' explanation shown in wiki "Rings of Saturn").

Table 2. p{N,n} QM structure analysis for Saturn's rings.

			use Saturn surface r as r_1				use Saturn's Earth-sized core as r_1			
			pSurface{N,n//2}		pSurface{N,n//3}		pCore{N,n//2}		pCore{N,n//3}	
pFactor =			2		3		2		3	
p{0,1}RF, r1=			**5.82E+07**		**5.82E+07**		**6.47E+06**		**6.47E+06**	
{1,1}, r=r1*pFactor^2			2.33E+08		5.24E+08		2.59E+07		5.82E+07	
{2,1}, r=r1*pFactor^4			9.32E+08		4.72E+09		1.04E+08		5.24E+08	
	mass, kg	orbit, r_n	n= sqrt(r_n/r_1)		n= sqrt(r_n/r_1)		n= sqrt(r_n/r_1)		n= sqrt(r_n/r_1)	
Saturn Earth-sized core, p{0,1//2}		6.47E+06					1.0	{0,1//2}	1.0	{0,1//3}
Saturn, p{0,2//2} core		2.59E+07					2.0	{0,2//2}	2.0	{0,2//3}
Saturn, surface p{0,3//2}	5.68E+26	5.82E+07	1.00	{0,1//2}	1.00	{0,1//3}	3.0	{0,3//2}	3.0	{0,3//3} ={1,1//3}
rings										
D-ring, inner edge		6.69E+07	1.1				3.22		3.22	
D-ring, outer edge		7.45E+07	1.1				3.39		3.39	
C-ring, inner edge		7.47E+07	1.1				3.40		3.40	
C-ring, outer edge		9.20E+07	1.3				3.77		3.77	
B-ring, inner edge		9.20E+07	1.3				3.77		3.77	
B-ring, outer edge		1.18E+08	1.4				4.27		4.27	
A-ring, inner edge		1.22E+08	1.4				4.34		4.34	
A-ring, outer edge		1.37E+08	1.5	{0,1.5//2}			4.60		4.60	
B-ring (major ring, middle r)	7 ~ 24E+18	1.05E+08	1.3		1.3		4.0	{0,4//2}	4.0	{0,4//3}
G-ring, middle r		1.71E+08					5.1	{0,5//2}	5.1	{0,5//3}
E-ring,in	minor	1.80E+08	1.8	{0,2//2}= {1,1//2}	1.8	{0,2//3}	5.3		5.3	{0,5.3//3} ≈{1,2//3}
E-ring,out		4.80E+08	2.9	{0,3//2}={1,1.5//2}	2.9	{0,3//3} ={1,1//3}	8.6		8.6	{0,9//3}={1,3//3}={2,1//3}
Phoebe-ring.in	minor	4.00E+09	8.3	{0,8//2}= {3,1//2}	8.3	{0,8.3//3} ≈{2,1//3}	24.9		24.9	
Phoebe-ring.out		1.30E+10	14.9	{0,15//2}= {4,1//2}	14.9	{0,15//3}= {1,5//2} ≈{2,2//3}	44.8		44.8	
moons										
Mimas	4E+19	1.86E+08	1.8	{0,2//2}	1.8	{0,2//3}	5.4	{0,5//2}	5.4	{0,5//3}
Enceladus	1.1E+20	2.38E+08	2.0	{0,2//2}	2.0	{0,2//3}	6.1	{0,6//2}	6.1	{0,6//3} ={1,2//3}
Tethys	6.2E+20	2.95E+08	2.2	{0,2//2}	2.2	{0,2//3}	6.7	{0,7//2}	6.7	{0,7//3}
Dione	1.1E+21	3.77E+08	2.5	{0,2.5//2}	2.5	{0,2.5//3}	7.6	{0,8//2}	7.6	{0,8//3}
Rhea	2.3E+21	5.27E+08	3.0	{0,3//2}	3.0	{0,3//3}	9.0	{0,9//2}	9.0	{0,9//3} ={2,1//3}
Titan	1.35E+23	1.22E+09	4.6	{0,4.6//2} ={2,1//2}	4.6	{0,4.6//3}	13.7	{0,14//2} ≈{0,16//2} ={4,1//2}	13.7	{0,14//3} ≈{1,5//3} ≈{2,2//3}
Lapetus	1.80E+21	3.56E+09	7.8	{0,7.8//2}= {3,1//2}	7.8	{0,7.8//3}	23.5		23.5	{0,24//3} ≈{1,8//3} ≈{2,3//3} ={3,1//3}
Phoebe-moon, retrograde	8.29E+18	1.29E+10	14.9	{0,15//2}= {4,1//2}	14.9	{0,15//3}	44.7		44.7	{0,45//3} ≈{2,5//3} ≈{3,2//3}

Note: Saturn's original Earth-sized core r =5.82E+7 /3^2 =6.47E+6 m. I did not use SunQM-3s6 Table 2's r=6.32E+6 m, because I believe the former one is more accurate. Note: Saturn ring's data and moon's data comes from wiki "Rings of Saturn" and wiki "Moons of Saturn".

A recent ALMA study (Vlahakis, et al.) [14] revealed that the disk-lyzation and ring formation of HL Tauri may happen within a period as short as 1 million years, (see wiki "HL Tauri"). So the whole process of step 4 in hypothesis-1 may also be in a short period (e.g., 1 million years). Because E-ring has not been cleared out by 5 inner moons yet, therefore Saturn's collision might have happened very recently.

There is a major alternative hypothesis (named as hypothesis-2) for the formation of Saturn's rings. The main difference between two hypotheses is that there was no (step 4) collision in hypothesis-2, the rings were directly formed when the original Saturn was capturing 4x of mass (in the step 2 of hypothesis-1). If hypothesis-2 is correct, then Saturn rings must have formed ~ 4 billion years ago. Then after ~ 4 billion years have passed, the E-ring still have not been accreted into 5 inner moons, it must have been prevented by some reason (like Jupiter's perturbation). So in hypothesis-2, Saturn's ring will last forever as long as Jupiter continues to be the big brother of Saturn.

Whatever it is, if only from {N,n} QM study, we are not able to tell between hypothesis-1 and hypothesis-2, which one is more correct. A recent study by a NASA group (O'Donoghue, J. et al.) [15] showed that Saturn's ring is diminishing, so this study result favors the hypothesis-1.

I-b. Using n=4 radial probability curve to explain how Saturn's ring shrinks (by quantum collapse) as the ring mass decreasing

Figure 2 shows the standard QM radial probability density distribution from n=1 to n=4 (plotted using data in Table 3). Let us first analyze that how a QM ball structure (with ~100% mass occupancy or at lease above ~30% mass occupancy) changes as the mass decreases. Since Saturn's main body has a p{0,3//2} (or a p{0,2//2}o orbit spaced) QM structure with ~50% mass occupancy, let us analyze a n=2's ball shaped QM structure. From Table 3, we can see that for n=2, r^2 *|R(n,l)|^2 has maximum value at r/r_1 =4 for |211> QM state, and the sum probability of |211>, |210>, and |200> states fills all r/r_1 range from 4 to ~9. So when having 100% mass occupancy, a celestial body at n=2 QM state will have its mass mainly occupy from r/r_1 ≈4 (or n=2) up to r/r_1 ≈9 (or n=3). This is exactly where the rule of "all mass between r_n to r_{n+1} belongs to orbit n" (see paper SunQM-3s2) come from. When mass occupancy getting decreased (by either removing some mass or decreasing the thermo pressure), the spinning QM's nLL effect (see paper SunQM-3s1) tells us that the |200> state will get its mass occupancy decreased first, because it has the highest state energy. |210> state will get its mass occupancy decreased the second, because it has the second highest state energy. |211> state will get its mass occupancy decreased the last, because it has the lowest state energy. This will cause a p{0,2//2} QM ball-shaped structure become more and more flatten. So from QM point of view, that "Saturn is more flatten than Jupiter" is not due to Saturn spins faster than Jupiter (actually it spins slower), it is because Saturn has too little mass to fill in |200> and |210> states, while Jupiter (due to it has close to 100% mass occupancy) has enough mass to fill in |200> and |210> states (if Jupiter is analyzed in pCore{N,n//3}, see section II below). When mass occupancy is further decreased to below a critical point (guessed to be from ~30% down to ~10%), then the whole n orbit spaced mass ball will quantum collapse into the n-1 orbit space, and leave only << 1% mass in the n orbit space to form ring. So governed by its QM-force, when the % mass occupancy further decreased by the heat dissipation, Saturn cannot gradually decrease its r/r_1 from =9 to =4. Instead, it will have to keep r/r_1 =9, while gradually increasing its flatten.

Now let us analyze how the QM ring structure (which must have mass occupancy << 1% for the whole n orbit space it located) change as ring mass decreases. Since Saturn's ABCD rings have a p{0,4//2} (or a p{0,4//2}o orbit spaced) QM structure, let us analyze the n=4's ring QM structure. From Table 3 we can see that for n=4, r^2 *|R(n,l)|^2 has maximum value at r/r_1 = 16 for |43m>, r/r_1 ≈ 21 for |42m>, r/r_1 ≈ 24 for |41m>, and r/r_1 ≈ 25 for |400> QM states. From the nLL effect theory (see paper SunQM-3s1), we know that (at ring stage) all above states will degenerate into |400>, |411>, |422>, and |433> states with the state energy decreasing from left to right. So when having high % mass occupancy, a ring at n=4 QM state will have its mass mainly occupy from r/r_1 = 16 (or n=4) up to r/r_1 = 25 (or n=5). This also fits the rule of "all mass between r_n to r_{n+1} belongs to orbit n" (see paper SunQM-3s2). Then, as the ring mass further decreasing, the ring width (Δr) will decrease within the same n=4 shell space. So while the inner edge of ring keeps at r/r_1 =16, the outer edge of the ring will quantum collapse from r/r_1 ≈ 25 of |400> state, to r/r_1 ≈ 24 of |411> state, then to r/r_1 ≈ 21 of |422> state, finally to r/r_1 ≈ 16 of |433> state. Hence the shrink of ring is still a quantum collapse, but it is based on the quantum number of *l*, not n.

Now it is clear that as we continue removing away mass from a ring, the ring's QM structure will quantum collapse from its outer edge towards its inner edge. This is why E-ring's out edge quantum collapsed from the previous pSurface{2,1//2} = pSurface{0,4//2} to today's pSurface{0,3//2}, because as time passed, E-ring got cooler (so its thermo pressure get lower), and more mass was either fell into the Saturn's surface, or accreted to the moon in the same n orbit shell, so that the mass occupancy in the E-ring's pSurface{1,1//2}o orbit space got lower. For the same reason, this caused an original disk (the continues ring mass distribution from n=4 pCore{0,4//2} to n=6 pCore{0,6//2}) shrank into two narrower and separated rings, one at n=4 orbit pCore{0,4//2} as ring-A, B, C, and D, and another at n=5 orbit pCore{0,5//2} as ring-G.

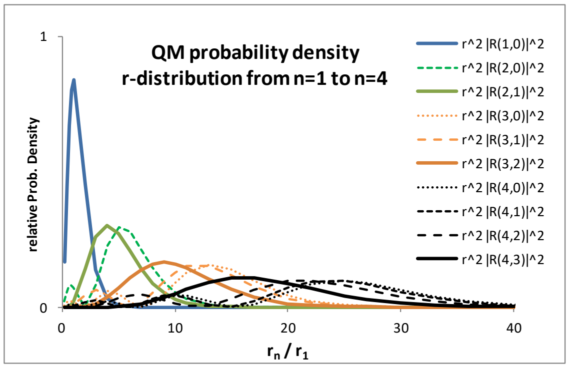


Figure 2. QM radial probability density distribution $r^2|R(n,l)|^2$ vs. r/r_1 from n=1 to n=4.

Table 3. $r^2|R(n,l)|^2$ vs. r/r_1 from n=1 to 4, with pCore{0,1} at Earth-sized core.

r_1= (a=)	6.47E+06	meter									Figure 4b		
factor	1.00E+07												
r/r_1=	r^2 |R(1,0)|^2	r^2 |R(2,0)|^2	r^2 |R(2,1)|^2	r^2 |R(3,0)|^2	r^2 |R(3,1)|^2	r^2 |R(3,2)|^2	r^2 |R(4,0)|^2	r^2 |R(4,1)|^2	r^2 |R(4,2)|^2	r^2 |R(4,3)|^2	Saturn, Prob.n=1..3	r/r_1 /4	Saturn's inner core, Prob.n=1..2
0.2	1.66E-01	2.05E-02	8.44E-05	6.07E-03	2.96E-05	7.04E-09	2.56E-03	1.32E-05	4.23E-09	1.74E-13	0.19	0.05	0.19
0.4	4.45E-01	5.31E-02	1.11E-03	1.56E-02	3.86E-04	3.94E-07	6.56E-03	1.72E-04	2.37E-07	4.02E-11	0.51	0.1	0.50
0.6	6.71E-01	7.49E-02	4.58E-03	2.17E-02	1.59E-03	3.93E-06	9.09E-03	7.06E-04	2.36E-06	9.32E-10	0.77	0.15	0.75
0.8	7.99E-01	8.00E-02	1.19E-02	2.27E-02	4.08E-03	1.93E-05	9.45E-03	1.80E-03	1.16E-05	8.42E-09	0.92	0.2	0.89
1	8.37E-01	7.11E-02	2.37E-02	1.95E-02	8.07E-03	6.45E-05	8.02E-03	3.55E-03	3.85E-05	4.54E-08	0.96	0.25	0.93
2	4.53E-01	0.00E+00	1.40E-01	3.31E-04	4.24E-02	2.12E-03	2.47E-04	1.79E-02	1.23E-03	7.05E-06	0.64	0.5	0.59
3	1.38E-01	8.66E-02	2.60E-01	3.10E-02	6.20E-02	1.24E-02	1.37E-02	2.39E-02	6.91E-03	1.10E-04	0.59	0.75	0.48
4	3.32E-02	2.27E-01	3.02E-01	5.90E-02	4.47E-02	3.58E-02	2.33E-02	1.40E-02	1.86E-02	6.64E-04	0.70	1	0.56
5	7.02E-03	2.93E-01	2.71E-01	4.74E-02	1.40E-02	7.01E-02	1.51E-02	2.02E-03	3.30E-02	2.40E-03	0.70	1.25	0.57
6	1.37E-03	2.76E-01	2.07E-01	1.68E-02	0.00E+00	1.07E-01	2.71E-03	1.62E-03	4.38E-02	6.26E-03	0.61	1.5	0.48
7	2.52E-04	2.16E-01	1.41E-01	1.45E-04	1.42E-02	1.39E-01	1.12E-03	1.38E-02	4.66E-02	1.30E-02	0.51	1.75	0.36
8	4.46E-05	1.49E-01	8.85E-02	1.17E-02	4.97E-02	1.59E-01	1.26E-02	3.02E-02	4.03E-02	2.30E-02	0.46	2	0.24
9	7.63E-06	9.47E-02	5.22E-02	4.60E-02	9.20E-02	1.66E-01	2.91E-02	4.14E-02	2.79E-02	3.58E-02	0.45	2.25	0.15
10	1.27E-06	5.62E-02	2.93E-02	8.83E-02	1.28E-01	1.60E-01	4.08E-02	4.24E-02	1.41E-02	5.05E-02	0.00	2.5	0.09
11	2.09E-07	3.16E-02	1.58E-02	1.25E-01	1.50E-01	1.46E-01	4.25E-02	3.40E-02	3.80E-03	6.56E-02	0.00	2.75	0.05
13	5.34E-09	8.93E-03	4.16E-03	1.57E-01	1.51E-01	1.05E-01	2.13E-02	8.17E-03	3.81E-03	9.19E-02	0.00	3.25	0.01
15	1.30E-10	2.25E-03	9.98E-04	1.38E-01	1.17E-01	6.50E-02	1.06E-03	1.10E-03	2.97E-02	1.06E-01	0.00	3.75	0.00
17	3.06E-12	5.20E-04	2.23E-04	9.72E-02	7.60E-02	3.63E-02	8.37E-03	2.25E-02	6.44E-02	1.06E-01	0.00	4.25	0.00
20	1.05E-14	5.16E-05	2.12E-05	4.44E-02	3.19E-02	1.30E-02	5.64E-02	7.31E-02	9.75E-02	8.71E-02	0.00	5	0.00
22	2.33E-16	1.04E-05	4.21E-06	2.33E-02	1.61E-02	6.08E-03	8.55E-02	9.46E-02	9.93E-02	6.87E-02	0.00	5.5	0.00
25	7.46E-19	8.88E-07	3.50E-07	7.76E-03	5.12E-03	1.77E-03	9.93E-02	9.62E-02	8.06E-02	4.26E-02	0.00	6.25	0.00
27	1.59E-20	1.66E-07	6.44E-08	3.48E-03	2.24E-03	7.42E-04	9.07E-02	8.29E-02	6.27E-02	2.90E-02	0.00	6.75	0.00
30	4.87E-23	1.28E-08	4.88E-09	9.66E-04	6.04E-04	1.89E-04	6.55E-02	5.63E-02	3.79E-02	1.50E-02	0.00	7.5	0.00
33	1.46E-25	9.42E-10	3.56E-10	2.48E-04	1.52E-04	4.53E-05	4.02E-02	3.30E-02	2.04E-02	7.19E-03	0.00	8.25	0.00
36	4.31E-28	6.72E-11	2.51E-11	5.97E-05	3.59E-05	1.03E-05	2.18E-02	1.73E-02	1.00E-02	3.22E-03	0.00	9	0.00
40	1.79E-31	1.90E-12	7.01E-13	8.29E-06	4.88E-06	1.35E-06	8.40E-03	6.43E-03	3.47E-03	1.01E-03	0.00	10	0.00

Note-1: if use r_1=0.647 (with unit of E+7 m), then the maximum Prob. =0.837. If use r_1=6.47E+6 m, then the maximum Prob. =0.837E-7. E-7 probability is due to that this R(nl) is normalized for the Bohr radius $r_1 = a_0$ =5.29E-11 m. So when using this probability, I need to scale it up to ~1E+7 times to make it around 1. We can avoid this trouble by deducing out the radial wave function R(nl) that specifically normalized to Saturn's r_1 =6.47E+6 m. But I am only a citizen scientist of QM, it is too much work for me to do it. Therefore, in Table 3, a factor of 1E+7 is multiplied to all r^2 *|R(n,l)|^2 to bring the values around one, not around 1E-7.

I-c. Saturn's A, B, C, D rings may follow the radial wave function guided probability density distribution

Section I-a showed that B-ring is a n=4 of Saturn's pCore{N,n} QM structure. Figure 3 shows that, if the brightness of the ring directly proportion to the total mass in θ-dimension, then we can see that the radial mass distribution of Saturn's D, C, B, A, F, G rings may follow the radial wave function's probability density curve r^2 *|R(4,3)|^2, with the main-ring (B-ring) at the maximum point r/r_1=4. In this figure, the r^2 *|R(4,3)|^2 curve data is shown in Table 3. The (A, B1, B2, C, D) intensity bars are estimated (by me) according to the brightness of rings. Notice that it is the total mass in θ-dimension per unit r (not per volume). From the transparency of rings (which is D> C> A> B), we know that it is the ring thickness (B1 > B2 > A > C > D) that directly correlates (or even proportion) to the |433> radial probability density. It is not clear that the real mass density per volume is same (or different) for all A, B, C, D rings.

If we choose Saturn's original atmosphere surface (r = 2.53E+7 m, see Table 2 in paper SunQM-3s6), then we find Saturn's B-ring (r = 1.05E+8 m) is exactly at r/r_1 =4, or n =2 orbit. So Saturn ring's θ-dimension total mass per unit r (∝ brightness) may also directly correlates to the |211> radial wave function's probability density r^2 *|R(2,1)|^2 (figure is not shown here). Readers can plot it out by yourself if you want to see the result.

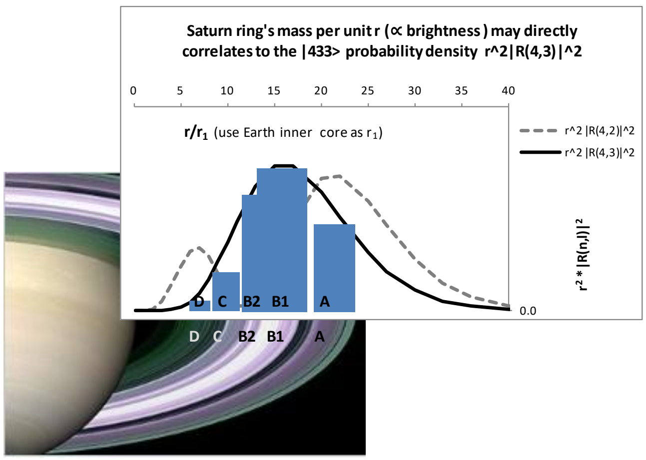


Figure 3. The radial mass distribution in Saturn A, B, C, D rings may follow radial probability density distribution r^2 *|R(n,l)|^2 at n=4 (or n=2, etc.). Saturn's picture is copied from wiki "Rings of Saturn". Original author: NASA / JPL - http://photojournal.jpl.nasa.gov/catalog/?IDNumber=PIA07873. Copy Right: Public Domain.

Notice that the origin of r^2 *|R(n,l)|^2 curve in Figure 3 is not at the center of Saturn body. Also the width of the curve is much narrower than the regular r^2 *|R(n,l)|^2 curve with its origin at the center of Saturn body, and with peak at n=4 or r/r_1 =16. So, to match the r^2 *|R(n,l)|^2 curve in Figure 3, We need to build a new probability function based on the regular one by 1) narrowing the peak by a factor of A, and 2) shifting the r by B. This means the r/r_1 in the original r^2 *|R(n,l)|^2 formula now is replaced by (r-B)/r_1*A = (r/r_1 - B/r_1)*A. The new formula for ABCD rings with n=4 is:

|R(4,3,shrunk & shifted)|^2 = 1/35/768^2 *[r_1^(-3)]* [((r-B)/r_1 *A)^6] * exp[-(r-B) /r_1 *A /2]

or, the shrunk & shifted

r^2 *|R(4,3)|^2 = [(r/r_1 - B/r_1) *A *r_1]^2 /35/768^2 *[r_1^(-3)] * [((r-B)/r_1 *A)^6] * exp[-(r - B) /r_1*A /2] [eq-1]

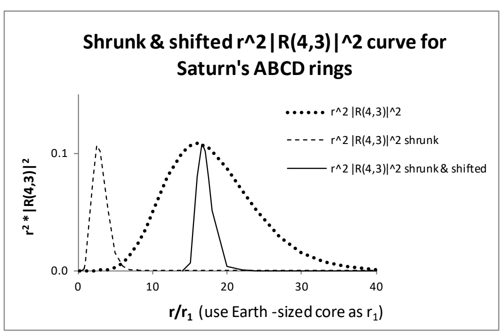
where r_1 = 6.47E+6 m, A =6, B/r_1 =14. Table 4 and Figure 4 show the result. Note: the original A = n^2 = 4^2 =16. The original B /r_1 = n^2 - 1 =15 (meaning to set r/r_1 =15 as R(nl) =0), so the original r/r_1 (maximum at r/r_1 = n^2) is replaced by (r/r_1 -(n^2 - 1)) *n^2, maximum still at = (n^2 - n^2 + 1) *n^2 = n^2 ! However, the bandwidth of this curve at A =16 and B /r_1 =15 is too narrow to cover ABDC rings (from pSurface{0,1//2} to {0,1.5//2}, which is equivalent to r/r_1 from =16 to ≈20). So in Table 4, I have to adjust to A =6, and B /r_1 = 14, to make the bandwidth of this curve to cover ABDC rings from pSurface{0,1//2} to {0,1.5//2}. Figure 4a shows the original r^2 *|R(4,3)|^2 curve, the shrunk-only, and the shrunk & shifted curves (data is shown in column 2, 3, and 4 of Table 4). Notice that to plot the shrunk & shifted curve, r/r_1 has to be >= 14 (see column 4 of Table 4).

Figure 4b shows the radial probability distribution r^2 *|R(n,l)|^2 for both Saturn's main body and its A, B, C, D rings. The dashed line is the Saturn main body's probability, which ends at r/r_1 =9, or covers from pCore{0,1//3} to pCore{0,3//3}. The dotted line is the Saturn inner core's probability. The solid line is the ABCD ring's probability, which covers from r/r_1 ≈16 to ≈20, or from pCore{0,4//3} to around pCore{0,4.5//3}.

Table 4. r^2 *|R(n,l)|^2 vs. r/r_1 for n=4, with pCore{0,1} at Earth-sized core.

r_1=	6.47E+06	A=	B/r_1 =
factor	1.00E+07	6	14
r/r_1=	r^2 *|R(4,3)|^2	r^2 *|R(4,3)|^2 shrunk	r^2 *|R(4,3)|^2 shrunk & shifted
0.2	1.74E-13	1.77E-07	
0.4	4.02E-11	2.48E-05	
0.6	9.32E-10	3.49E-04	
0.8	8.42E-09	1.91E-03	
1	4.54E-08	6.26E-03	
2	7.05E-06	7.98E-02	
2.5	3.27E-05	1.06E-01	
3	1.10E-04	1.02E-01	
4	6.64E-04	5.07E-02	
5	2.40E-03	1.50E-02	
6	6.26E-03	3.22E-03	
7	1.30E-02	5.50E-04	
8	2.30E-02	7.97E-05	
9	3.58E-02	1.02E-05	
10	5.05E-02	1.18E-06	
12	7.98E-02	1.25E-08	
14	1.01E-01	1.07E-10	0.00E+00
15	1.06E-01	9.23E-12	6.26E-03
16	1.08E-01	7.70E-13	7.98E-02
16.6	1.07E-01	1.71E-13	1.077E-01
16.7	1.07E-01	1.33E-13	1.079E-01
16.8	1.07E-01	1.03E-13	1.069E-01
17	1.06E-01	6.23E-14	1.02E-01
17.5	1.04E-01	1.75E-14	7.80E-02
18	1.02E-01	4.90E-15	5.07E-02
20	8.71E-02	2.82E-17	3.22E-03
22	6.87E-02	1.50E-19	7.97E-05
24	5.07E-02	7.45E-22	1.18E-06
27	2.90E-02	2.36E-25	1.19E-09
30	1.50E-02	6.76E-29	7.70E-13
33	7.19E-03	1.79E-32	3.76E-16
36	3.22E-03	4.43E-36	1.50E-19
40	1.01E-03	6.32E-41	3.50E-24

Note: in column 3, r/r_1 is replaced by r/r_1 *A. In column 4, r/r_1 is replaced by (r - B) $/r_1$ *A.



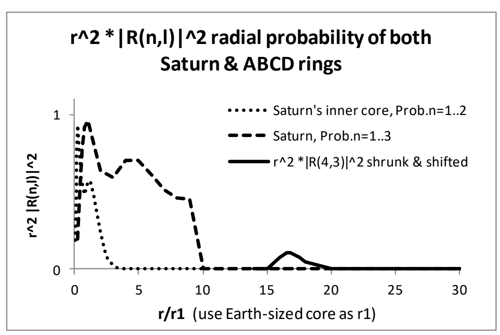


Figure 4a. The radial probability distribution for Saturn A, B, C, D rings can be described by a shrunk and shifted r^2 *|R(4,3)|^2 curve (the probability intensities are arbitrary).
Figure 4b. The radial probability distribution r^2 *|R(n,l)|^2 for both Saturn and its ABCD rings. Notice that the probability intensities are arbitrary.

I-d. Using the multiplier n' to describe Saturn major ring (B-ring)'s p{N,n} QM structure

According to wiki "Rings of Saturn", "*The B Ring is the largest, brightest, and most massive of the rings. Its thickness is estimated as 5 to 15 m ... are concentric, appearing as narrow ringlets, ... the outer edge of the B Ring contains vertical structures deviating up to 2.5 km from the main ring plane…The total mass of the B Ring was estimated to be somewhere in the range of 7 to 24E+18 kg. This compares to a mass for Mimas of 37.5E+18 kg*".

First, let us try to use the multiplier n' to describe the width (in r-dimension) of the B-ring. Table 1 shows that B-ring has the inner edge r = 9.2E+7 m and the outer edge r = 1.18E+8 m. Using Saturn's Earth-sized core as pCore{0,1//3} and r_1, B-ring can be described as n=4, or at the pCore{0,4//3} orbit (here we choose pFactor q=3). According to the multiplier n' theory (see paper SunQM-2, section I-h), a base-frequency n=4 can also be described by high-frequency (multiplier) n' as n' = 4*3^1 = 12, or n' = 4*3^2 = 36, etc. Table 5 columns 2 through 8 shows that how to use p{N,n} QM multiplier n' to describe the width of Saturn's B-ring. In the table, the n' multiplier level is represented by letter w, so that n' = n * q^w, or for n' = 4 * 3^1 = 12, w=1; for n' = 4 * 3^2 = 36, w=2; etc. Since n' = n * q^w, or n = n' / (q^w), and r_n = r_1 * n^2, therefore r_n = r_1 * (n')^2 / q^(2w). For n=4, r_n = 6.47E+6 * 4^2 = 1.04E+8 m for all different n'(s). Define x as the quantum number that deviates from the n' quantum number, then $r_{n'+x}$ is calculated as: $r_{n'+x}$ = r_1 * (n'+x)^2 / q^(2w). We can choose a smaller x (or a negative x) to let a $r_{n'+x}$ value matching B-ring's inner edge's r, and then this n'+x value is the multiplier quantum number for B-ring's inner edge. Similarly, we can choose (a relative larger) x to let $r_{n'+x}$ matches B-ring's outer edge, and get the multiplier quantum number for B-ring's outer edge. The result in Table 5 shows that when changing w from 0, to 5 (Table 5 columns 2 through 7), the calculated $r_{n'+x}$ does not match B-ring's both inner and outer edges well. But at w=6, the calculated $r_{n'+x}$ matches B-ring's both inner and outer edges well (see Table 5 column 8, the yellow colored cells). So Saturn B-ring (base frequency n=4)'s inner edge can be described by a multiplier n' = 4*3^6 - 167 = 2749, and its outer edge can be described by a multiplier n' = 4*3^6 +225 = 3141. Of cause, in this description, the higher the w (and n'), the higher the accuracy.

Table 5. Using p{N,n} QM multiplier n' to describe the width of Saturn's B-ring.

	B-ring							ringlet within B-ring	
pFactor q =	3	3	3	3	3	3	3		3
multiplier level w=	0	1	2	3	4	5	6		6
n' = n*q^w	4	12	36	108	324	972	2916		2916
r_1 =	6.47E+06	6.47E+06	6.47E+06	6.47E+06	6.47E+06	6.47E+06	6.47E+06		6.47E+06
r_n = r_1 *n'^2 / q^(2w)	1.04E+08	1.04E+08	1.04E+08	1.04E+08	1.04E+08	1.04E+08	1.04E+08		1.04E+08
B-ring, x=	0	-1	-2	-6	-18	-56	-167	ringlet, x=	0
$r_{n'+x}$ = r_1 * (n'+x)^2 / q^(2w) =9.2E+7 m	1.04E+08	8.70E+07	9.23E+07	9.23E+07	9.23E+07	9.19E+07	9.20E+07	$r_{n'+x}$ = r_1 * (n'+x)^2 / q^(2w)=	1.0352E+08
B-ring, x=	0	1	3	8	25	75	225	ringlet, x=	1
$r_{n'+x}$ = r_1 * (n'+x)^2 / q^(2w) = 1.18E+8 m	1.04E+08	1.21E+08	1.21E+08	1.19E+08	1.20E+08	1.20E+08	1.20E+08	$r_{n'+x}$ = r_1 * (n'+x)^2 / q^(2w)=	1.0359E+08
								ringlet width	7.10E+04

Multiplier n' can be used to describe not only the ring width, but also the ring thickness. Now let us try to determine the multiplier n' number for the thickness (in θ-dimension) of the B-ring. Let us first simplify the question by assuming the thickness of B-ring to be 200 meters. In a rθφ spherical coordinate system with the origin at the Saturn's center, define θ' as the angle between the top end of B-ring (100 m above equatorial plane) to equatorial plane, the θ' can be calculated as: 2 * r_n * sin(θ') = 200, where r_n = 1.05E+8 m is the middle r of B-ring (see table 1). Then sin(θ') = 200 /2 /1.05E+8 = 9.52E-7, or θ' = 9.52E-7 (sin(θ') ≈ θ' since it is small). According to paper SunQM-3s2 and SunQM-3s1, all rings of a spinning celestial body is formed by this celestial body QM's nLL effect (where L = n-1). From book "A Modern Approach to QM, pp339, eq-9.146", the YLL=

$$\langle \theta, \phi | l, l \rangle = Y_{l,l}(\theta, \phi) = \frac{(-1)^l}{2^l l!} \sqrt{\frac{(2l+1)!}{4\pi}} e^{il\phi} \sin^l \theta \qquad (9.146)$$

[eq-2]

where *l* is the maximum value (named L) and it equals n-1. Obviously the |YLL|^2 probability distribution in θ-dimension (near θ = π/2, or near θ' = 0) should directly correlate to the mass density θ-distribution near the equator. So the most straight forward method to determine the n' for a ring thickness is to do the integration ∫ |YLL|^2 * sin(θ) dθ dφ, or

A = C* ∫ [sin(θ)]^[2(n'-1)] * sin(θ) dθ dφ, [θ=0, π; φ=0, 2π],

where C is a constant, and the L' is the maximum value of *l* in multiplier value, L' = n'-1. And then also do the same integration but within much smaller θ range (near θ=π/2, or within +/- 9.52E-7 which is the thickness of the B-ring):

B = C* ∫ [sin(θ)]^[2(n'-1)] * sin(θ) dθ dφ, [θ=π/2 - 9.52E-7, π/2 + 9.52E-7; φ=0, 2π],

and then adjust the n' value to make B/A ration = 0.9, so that 90% of probability (= mass) is located within θ' = +/- 9.52E-7 (or within +/- 100 meter of equatorial plane). However, for the calculation of ∫ sin(θ')^y dθ', the integration software R does not allow y > 9E+7 (equivalents to n' < 4.5E+7). So this method is abandoned. Therefore I have to use the following method (which is an even more roughed approximation) to estimate the n' value.

In the integration of ∫ [sin(θ)]^[2(n'-1)] * sin(θ) dθ dφ, the mass distribution in θ-dimension completely depends on the function [sin(θ)]^[2(n'-1)] * sin(θ). When n' is very large (> 1E+12), this function can be simplified as sin(θ)^(2n'). We know that sin(θ)^(2n') function curve getting narrower near the center θ = π/2 as n' getting larger (see Figure 1a of paper SunQM-3s9). Due to θ = π/2 - θ', so sin(θ)^(2n') = sin(π/2 - θ')^(2n') = cos(θ')^(2n'). Also we know θ' = 9.52E-7. So we can simplify the integration analysis to be a sin(θ)^(2n') function analysis as: to find a n' that makes cos(9.52E-7)^(2n') = 0.1, or 90% down from 1. Using Microsoft Excel, we find 2n' ≈ 5E+12, or n' ≈ 2.5E+12.

According to the multiplier n' theory, a good n' has to satisfy the formula n' = n*q^w. Then for B-ring (where q=3, n=4, and n' ≈ 2.5E+12), what is w integer value for n' = n*q^w = 4*3^w ≈ 2.5E+12? Solving the simple equation we obtain w = [log(2.5E+12) - log4] / log3 ≈ 24.7. According to the multiplier n' theory, w has to be an integer, and the higher value of w, the more accurate the n' for QM description. Therefore we choose the closest higher value (of 24.7) as integer 25. So the final n' for B-ring' thickness is calculated to be =4*3^25 = 3.39E+12.

Using the final n', we can back calculate the θ' as, cos(θ')^(2n') = cos(θ')^(2* 3.39E+12) = 0.1, obtaining θ' = 8.20E-7. Then B-ring's half thickness (d) can be back calculated as: d / 1.05E+8 = 8.2E-7, obtaining d = 8.2E-7 * 1.05E+8 ≈ 86 m. Then at n' = 4 * 3^25 = 3.39E+12, with r_n = 1.05E+8 m, B-ring's thickness = 2d = 172 m. So the final answer is: with the known nLL orbit theory, known Saturn's Earth-core as r_1 = 6.47E+6 m, known r_n (=1.05E+8 m) for the B-ring, we calculated that the final n' = n*q^w = 4 * 3^25 = 3.39E+12, it makes 90% of B-ring's mass within θ' = +/- 8.2E-7, or within d = +/- 86 m (total thickness = 172 m).

With w=25, the width of the B-ring can be described as: the inner edge at n' = (4*3^6 - 167)*3^19 = 3.20E+12, and the outer edge at n' = (4*3^6 +225) * 3^19 = 3.65E+12.

B-ring can be further divided into many ringlets. Table 5 columns 9 & 10 show how to use the same method to describe the ringlets. A ~ 71 km wide ringlet in B-ring can be described with w=6, the inner edge at n' = 4*3^6 = 2916, and outer edge at n' = 4*3^6 +1 = 2917. A series of ~71 km wide (and consecutive numbered) ringlets in B-ring can be described with w=6, n'= 4*3^6 +0=2916, n'= 4*3^6 +1 =2917, n'= 4*3^6 +2 =2918, etc. The same calculation can be applied to the consecutive ringlets with any width, although the w value and n' value need to be re-adjusted.

I-e. Using p{N,n} QM and r^2 *|R(n,l)|^2 *|Y(l,m)|^2 to build Saturn major ring's true 3D structure probability function

Now we can build the true 3D structure of Saturn's major ring (ABDC rings) by using the true p{N,n} QM probability function |ψ|^2 = r^2 *|R(n,l)|^2 *|Y(l,m)|^2. The shrunk & shifted r^2 *|R(4,3)|^2 is shown in equation-1 (although its maximum probability has not been normalized to one). As explained in the section I-c, Saturn major ring is formed by QM's nLL effect, so it is at the |nLL> state, where L = n-1. So its |Y(*l*,m)|^2 = |Y(L,L)|^2 = |Y(n-1,n-1)|^2. Here the base frequency n needs to be replaced by its high frequency n'. According to equation-2, it is |Y(n'-1,n'-1)|^2 = C * sin(θ)^(2*(n'-1)), Since n' >> 1, so

|Y(L,L)|^2 ≈ C * sin(θ)^(2*n'), [eq-3]

where C is a constant, and n' = 4 * 3^25 = 3.39E+12 (which is obtained in section I-c). Multiply eq-1 to eq-3, we obtain

|ψ|^2 = r^2 *|R(4,3)|^2 *|Y(3,3)|^2 = [(r/r_1 - B/r_1) *A *r_1]^2 /35 / 768^2 *[r_1^(-3)] * [((r-B)/r_1 *A)^6] * exp[-(r-B) /r_1 *A /2] * C * sin(θ)^(2*n')

[eq-4]

where r_1 = 6.47E+6 m, A = 6, B/r_1 = 14, n' = 3.39E+12, C is a normalization factor need to be determined, and equation-4 is valid only in the r/r_1 range from 14 to infinity, or in the r range from 5.82E+7 /9 *14 = 9.05E+7 m to infinity. Equation-4 is the true 3D structure of Saturn's major ring represented in QM probability function |ψ|^2 = r^2|R(n,l)|^2 |Y(l,m)|^2. This probability function produces a ring with the middle r = 1.05E+8 m, the inner edge at r/r_1 ≈ 14, the outer edge at r/r_1 ≈ 20, the maximum probability at r/r_1 ≈ 16, and with 90% of mass within +/- 86 m of Saturn's equatorial plane.

We can also use QM probability integration formula to build 3D Saturn, plus its major ring (ABDC rings). The 3D probability integration formula for Saturn has been presented in paper SunQM-3s7, section V, and also shown in below:

5.68E+26 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2) *W * D * r^2 dr, [r=0, 5.82E+7 m]

where mass density D = 1.63E+5 / r^0.05 - 66672 (kg/m^3), and a adjustable factor W= 1E+7. The 3D probability integration formula for Saturn's ABCD ring can be written as:

Mass (r, θ, φ) = ∭ r^2 *|R(4,3)|^2 *|Y(*3,3*)|^2 *W *D *sin(θ) * r^2 dr dθ dφ, [r =5.82E+7 /9 *14 = 9.05E+7 m, ∞; θ=0, π; φ=0, 2π]

or using equation-4,

2.4E+19 kg = ∭ r^2 * [(r/r_1 - B/r_1) *A *r_1]^2 /35 / 768^2 *[r_1^(-3)] * [((r - B) /r_1 *A)^6] * exp[- (r - B) /r_1 *A /2] * C * sin(θ)^(2*n') *D *sin(θ) * r^2 dr dθ dφ, [r =9.05E+7 m, ∞; θ=0, π; φ=0, 2π]

where r_1 =6.47E+6 m, A=6, B/r_1 =14, n' = 3.39E+12, C is a normalization factor need to be determined using this integration equation, and D ≈ 3 kg/m^3, is the estimated mass density of ABCD rings (see the estimation below).

D is estimated as: Saturn's ABCD rings total mass M = 2.4E+19 kg. Assuming the averaged ring thickness = 2 * 86 = 172 m. The ring (simplified as a hollow cylinder)'s volume is V = (πR^2 - πr^2) *thickness = 3.14 * [(1.37E+8)^2 - (6.69E+7)^2] * 176 = 7.90E+18 m^3. Then, the mass density D = M / V = 2.4E+19 / 7.9E+18 = 3.04 kg/m^3.

II. Using p{N,n} QM to analyze Jupiter's ring structure

Jupiter's five major moons have been described by a pSurface{N,n//5} QM structure (see paper SunQM-1s3, Table 1a). Table 6 of current paper copied part of the result (see Table 6 columns 4 and 5). Although Jupiter's surface atmosphere band pattern has n=5 character (see paper SunQM-3s3 section I-a), and its current internal structure can also be analyzed with pFactor =5 (see paper SunQM-3s7 section I), n=5 does not give any meaningful result for Jupiter rings' pSurface{N,n} QM structure analysis. After many tries, I found that Jupiter's rings can be described by using pCore{N,n//2} QM structure with Jupiter's original Earth-sized core as r_1 and pCore{0,1//2}. Under this analysis (see Table 6 column 6 and 7), Jupiter's main ring (or ring-2, which is a narrow ring even it has the highest mass density) locates at exactly pCore{0,4//2} orbit. The Amalthea gossamer ring (or ring-3) occupies almost whole pCore{0,4//2}o orbit space, from pCore{0,4//2} to pCore{0,5//2}. The Thebe gossamer ring (or ring-4) occupies the whole pCore{0,4//2}o orbit space, plus the inner half of the pCore{0,5//2}o orbit space, from pCore{0,4//2} to pCore{0,5.5//2}. The Halo ring (or ring-1) occupies the outer half of the pCore{0,3//2}o orbit space, from pCore{0,3.5//2} to pCore{0,4//2}.

Table 6. p{N,n} QM structure analysis for Jupiter's rings.

			use Jupiter surface as r_1		use Earth-sized core as r_1		use Earth-sized core as r_1		use Jupiter surface as r_1	
			pSurface{N,n//5}		pCore{N,n//2}		pCore{N,n//3}		pSurface{N,n//3}	
pFactor =			5		2		3		3	
p{0,1}RF, r_1 =			**6.99E+07**		**7.59E+06**		**7.59E+06**		7.59E+06	
{1,1}, r=			1.75E+09		3.04E+07				**6.83E+07**	
	mass, kg	orbit, r_n	n= sqrt(r_n/r_1)		n=sqrt(r_n/r_1)		n=sqrt(r_n/r_1)		n=sqrt(r_n/r_1)	
original Earth-core		7.59E+06			1.00	{0,1//2}	1.00		1.00	
original p{0,2//3} core		3.04E+07			2.00	{0,2//2}	2.00		2.00	
Jupiter surface	1.898E+27	6.99E+07	1.00	{0,1//5}	3.03	{0,3//2}	3.03		3.03	
Rings										
ring-1, Halo ring, inner edge	E+7~E+9	9.20E+07	1.15		3.48	{0,3.5//2}	3.48		1.16	[0,1//3}
ring-1, Halo ring, outer edge		1.23E+08			4.02	{0,4//2}	4.02		1.34	
ring-2, Main ring, inner edge	E+11~E+16	1.23E+08			4.02	{0,4//2}	4.02		1.34	
ring-2, Main ring, outer edge		1.29E+08			4.12	{0,4//2}	4.12		1.37	
ring-3, Amalthea gossamer ring, inner edge	E+7~E+9	1.29E+08			4.12	{0,4//2}	4.12		1.37	
ring-3, Amalthea gossamer ring, outer edge		1.82E+08			4.90	{0,5//2}	4.90		1.63	
ring-4, Thebe gossamer ring, inner edge	E+7~E+9	1.29E+08			4.12	{0,4//2}	4.12		1.37	
ring-4, Thebe gossamer ring, outer edge		2.26E+08			5.46	{0,5.5//2}	5.46		1.82	{0,2//3}
(inner) minor moons										
Metis, 4-1 moon	3.60E+16	1.27E+08	1.35		4.09	{0,4//2}				
Adrastea, 4-2 moon	2.00E+15	1.28E+08			4.11	{0,4//2}				
Amalthea, 5-1 moon	2.08E+18	1.81E+08			4.88	{0,5//2}				
Thebe, 5-2 moon	4.30E+17	2.21E+08			5.40	{0,5.5//2}				
major moons										
Io	8.90E+22	4.22E+08	2.46	{0,2//5}	7.45	{0,7.5//2}	7.45	{1,2.5//3}	2.48	{0,2.5//3}
Europa	4.80E+22	6.71E+08	3.10	{0,3//5}	9.40	{0,9//2}	9.40	{1,3//3}	3.13	{0,3//3}
Ganymede	1.48E+23	1.07E+09	3.91	{0,4//5}	11.9	{0,12//2}	11.9	{1.4//3}	3.96	{0,4//3}
Callisto	1.08E+23	1.88E+09	5.19	{0,5//5}	15.7	{0,15//2}	15.7	{1,5//3}	5.25	{0,5//3}

Note: Jupiter's original Earth-sized core r =7.59E+6 m is obtained from paper SunQM-3s6 Table 2.
Note: Jupiter ring's (and moon's) data was obtained from wiki "Rings of Jupiter" and wiki "Moons of Jupiter".
Note: the result in columns 8 through 11 of Table 6 will be explained in section V.
Note: In column 8 through 11, Jupiter's interior structure is analyzed as p{N,n//3}, not p{N,n//5}, due to the superpoistion of q=5 and q=3 for Jupiter (see SunQM-3s7 for detailed discussion).

Moon Metis is renamed as "4-1 moon", due to it is the most inner moon in pCore{0,4//2}o orbit space. Moon Adrastea is renamed as "4-2 moon", due to it is the second (most inner) moon in pCore{0,4//2}o orbit space. From pCore{N,n//2} QM structure point of view, moon 4-1 and 4-2 will combine into one, so the new Metis/Adrastea moon will take pCore{0,4//2} orbit, similar as that moon 5-1 at orbit pCore{0,5//2} and moon 5-2 at orbit pCore{0,5.5/2}. According the rule "all mass between r_n and r_{n+1} belongs to orbit n", I believe that a ring covers r_n to r_{n+1} will start the accretion at its inner edge r_n (where the probability density, or mass density is the highest) to form the moonlet. As shown in Figure 2's explanation, as the moonlet accretes more mass, the leftover mass density in the ring will get lower, and then the ring width will get narrower as the outer edge of the ring starts to quantum collapse step by step. For example, for an n=4 ring, the outer edge will collapse from |400> to |411>, then to |422>, then to |433>. Finally, the moonlet at n=4 orbit will accrete all mass of the ring from r_n to r_{n+1}. So in most cases, **it is the moon locates at the inner edge (not the outer edge) of the ring will "eat" the whole ring**. Based on this analysis, the new Metis/Adrastea moon at the inner edge will "eat" both the main-ring (ring-2) and Amalthea gossamer ring (ring-3), and moon 5-1 at the inner edge will "eat" Thebe gossamer ring (ring-4).

Similar as that of Saturn's A, B, C, D rings, I believe that Jupiter's four rings (ring-1 through ring-4)'s r-dimension mass distribution also follows r^2 *|R(nl)|^2 curve with maximum mass density at n=4. These rings can be analyzed in the same way as that in Figure 3 and in section I-b through I-e.

There are two possible origins for Jupiter's ring.

The first one is: after Jupiter increased it s mass from the original 10% to 100%, there was one (or several) major collisions that excited some mass from Jupiter surface pCore{0,3//2} to the pCore{0,n=3..5//2}o orbit spaces. After de-excitation, all mass in the pCore{0,3//2}o orbit space fell back to Jupiter's surface. Most mass in the pCore{0,n=4..5//2}o orbit spaces also fell back to Jupiter's surface, only tiny amount of mass (which had the right orbit velocity vector) leftover in pCore{0,n=4..5//2}o orbit spaces. Under the Jupiter QM's nLL effect, this leftover mass disk-lyzed into a ring expanding from pCore{0,3.5//2} to pCore{0,5.5//2}, and with maximum mass density r-distribution at pCore{0,4//2}. So even all mass in pCore{0,3//2}o orbit space had already de-excited and fell back to Jupiter surface, the existence of Halo ring in the outer edge of pCore{0,3//2}o orbit space is only to make mass density r-distribution of the n=4 ring follows r^2 *|R(nl)|^2 curve with max at n=4. If this hypothesis is correct, then Jupiter's ring could be formed at any time after it gained 90% more mass (which is ~ 4 billion years ago). The fact that this ring is still exist today does favors the explanation that the ring was formed not too long ago, could be as recent as tens of millions years ago.

The second possible origin is that this ring was formed at the same time (or as the by-product) when Jupiter was capturing 90% more mass ~ 4 billion years ago. If so, then this ring must be as old as 4 billion years. Again, it is impossible to tell which origin is more correct only from {N,n} QM structural analysis.

III. Using p{N,n} QM to analyze Neptune's ring structure

In paper SunQM-1s3 Table 5a, using Neptune's surface r as r_1, a pSurface{N,n//2} QM structure was determined for Neptune's three major moons. Table 7 of current paper copied part of the result (see Table 7 columns 4 and 5). However, Neptune's rings do not fit into this pSurface{N,n//2} QM structure, although they do fit into the pCore{N,n//2} QM structure with Neptune's current Earth-sized core as r_1 and pCore{0,1//2}. Under this analysis (see Table 7 column 6 and 7), Neptune's main ring (Lassel ring) locates at exactly pCore{0,3//2} orbit. Notice that here I use Neptune's current Earth-sized r (not the original r). The reason is that I believe the current Neptune keeps the original size, so its current size is more accurate than the predicted one (in Table 2 of paper SunQM-3s6). Thus, Neptune is better to be described by a pCore{N,n//2} QM structure with r_1 at its Earth-sized core, and its surface at pCore{0,2//2} = pCore{1,1//2}, the major ring at pCore{0,3//2}, the inner most major moon at pCore{0,4//2} = pCore{2,1//2}, the major moon Triton at pCore{3,1//2}, the outer major moon Nerid at pCore{5,1//2} orbits (see Table 7 columns 6 and 7).

The origin of Neptune's rings may also have two possibilities: 1) A major collision excited Neptune surface mass to both pCore{0,2//2}o and pCore{0,3//2}o orbit spaces. After de-excitation, all mass in pCore{0,2//2}o and most mass in pCore{0,3//2}o orbit space fell back to Neptune surface, only tiny part of leftover mass in pCore{0,3//2}o orbit space was disk-lyzed and formed today's rings. 2) The catastrophic collision of Uranus excited some mass to such high orbit energy state so that it was captured by Neptune, and tiny part of the captured mass was stabilized at Neptune's pCore{0,3//2} orbit as rings.

Table 7. Determine Neptune ring's p{N,n} QM structure.

			use Neptune's surface as r_1		use Earth-sized core as r_1	
			pSurface{N,n//2}		pCore{N,n//2}	
pFactor =			2		2	
p{0,1}RF, r_1 =			**2.48E+07**		**6.19E+06**	
{1,1}, r=			9.91E+07		2.48E+07	
{2,1}, r=			3.96E+08		9.91E+07	
	mass, kg	orbit r_n	n=sqrt(r_n/r_1)		n=sqrt(r_n/r_1)	
current Earth-sized core		6.19E+06				
Neptune	1.02E+26	2.48E+07	1.00	{0,1//2}	2.00	{0,2//2} ={1,2//2}
rings						
Galle ring (minor-1), inner edge		4.09E+07	1.29		2.57	{0,2.5//2}
Galle ring (minor-1), inner edge		4.29E+07	1.32		2.63	{0,2.5//2}
Lassel ring (major ring), inner edge		5.32E+07	1.47		2.93	{0,3//2}
Lassel ring (major ring), outer edge		5.72E+07	1.52		3.04	{0,3//2}
Adams ring (minor-2)		6.29E+07	1.59		3.19	
moons						
Proteus	5.04E+19	1.18E+08	2.18	{0,2//2} ={1,1//2}	4.36	{0,4//2} ={2,1//2}
Triton	2.14E+22	3.55E+08	3.78	{0,4//2} ={2,1//2}	7.57	{0,8//2} ={3,1//2}
Nerid	2.70E+19	5.51E+09	14.9	{0,15//2} ={4,1//2}	29.8	{0,30//2} ={5,1//2}

Note: Neptune's original Earth-sized core r =2.48E+7 /2^2 = 6.19E+6 m. I did not use SunQM-3s6 Table 2's r =4.99E+6 m because I believe the former one is more accurate.

Note: Neptune ring's data and moon's data is obtained from wiki "Rings of Neptune" and wiki "Moons of Neptune".

IV. Using p{N,n} QM to analyze Uranus' ring structure

In paper SunQM-1s3 Table 6a, using Uranus' surface r as r_1, no meaningful pSurface{N,n//q} QM structure was obtained for Uranus' major moons. Table 8 (columns 4 and 5) of current paper copied part of the result. However, the same analysis shows that all Uranus' rings are perfectly located at the pSurface{0,1//2}o orbit space, with rings of ζ, 6, 5, 4, α, β, η, γ, δ, λ, ε and ν spread over from pSurface{0,1//2} to pSurface{0,2//2}, and μ ring sits at the pSurface{0,2//2} orbit (see column 4 and 5). Because rings of Neptune, Jupiter, and Saturn are all analyzed by using their Earth-sized core as r_1, here we also choose to use Uranus' current Earth-sized core as r_1 and pCore{0,1//2} for its ring analysis (see result in Table 8 columns 6 and 7). Under this analysis, the minor moons Portia, Puck, and major moons Miranna, Arial, Umbriel, Titania, Oberon, are sits at orbit of pCore{0,n//2} with n= 3, 4, 5, 6,7, 9, and 10 approximately. ζ ring sits in the inner half of pCore{0,2//2}o orbit space spread from pCore{0,2//2} to pCore{0,2.5//2}. Uranus' major ring (ε ring) locates at pCore{0,3//2} orbit. Uranus' outer most ring (μ ring) locates at pCore{0,4//2} orbit.

The origin of Uranus' rings can be explained as: a major collision excited Uranus surface mass from surface pSurface{0,1//2} to pSurface{0,1//2}o and pSurface{0,2//2}o orbit spaces. After de-excitation, most mass fell back to Uranus surface, and the tiny amount of leftover mass in pSurface{0,2//2}o orbit space was disk-lyzed and narrowed as μ ring at orbit pSurface{0,2//2} = pCore{0,4//2}. Also the tiny amount of leftover mass in pSurface{0,1//2}o orbit space was disk-lyzed, shrank and separated (as kind of ringlets) as 6, 5, 4, α, β, η, γ, δ, λ, ε and ν rings. According to the "ball-torus-7-11-gap effect" (see paper SunQM-1s1 section V), any mass near the surface of Uranus should be cleared out in the early stage of ring formation. So the ζ ring should not be there at all. The only explanation for the existence of ζ ring is that Uranus' rings must be very young, much younger than the age of Uranus' catastrophic collision (which might happened ~ 4 billion years ago and turned Uranus' spin axis by ~90 degrees). Wiki "Rings of Uranus" mentioned that "The rings of Uranus are thought to be relatively young, and not more than 600 million years old". My analysis using p{N,n} QM structure information favors this opinion.

According to pCore{N,n//2} QM structure, the (near) future of Uranus ring will be: 1) ζ ring will disappear, all of its mass will fall into Uranus; 2) all rings of 6, 5, 4, α, β, η, γ, δ, λ, and ν will be "eaten" by ε ring, and become a single ring at pCore{0,3//2} orbit; and 3) μ ring will stay at pCore{0,4//2} orbit and keep intact.

Table 8. Determine Uranus ring's p{N,n} QM structure.

			use Uranus' surface as r_1		use Earth-sized core as r_1	
			pSurface{N,n//2}		pCore{N,n//2}	
pFactor =			2		2	
p{0,1}RF, r_1 =			**2.56E+07**		**6.39E+06**	
{1,1}, r=			1.02E+08		2.56E+07	
{2,1}, r=			4.09E+08		1.02E+08	
	mass, kg	orbit rn	n=sqrt(r_n/r_1)		n=sqrt(r_n/r_1)	
Earth-sized core		6.39E+06				
Uranus	8.68E+25	2.56E+07	1.00	{0,1}	2.00	{0,1}
rings						
ζ ring, inner edge		2.68E+07	1.02	{0.1//2}	2.05	{0,2//2}
ζ rings, outer edge		4.14E+07	1.27		2.55	{0,2.5//2}
6 ring		4.18E+07	1.28		2.56	
5 ring		4.22E+07	1.28		2.57	
4 ring		4.26E+07	1.29		2.58	
α ring		4.47E+07	1.32		2.64	
β ring		4.57E+07	1.34		2.67	
η ring		4.72E+07	1.36		2.72	
γ ring		4.76E+07	1.36		2.73	
δ ring		4.83E+07	1.37		2.75	
λ ring		5.00E+07	1.40		2.80	
ε ring (major ring)		5.11E+07	1.41	{0,1.5//2}	2.83	{0,3//2}
ν ring, inner edge		6.61E+07	1.61		3.22	
ν ring, outer edge		6.99E+07	1.65		3.31	
μ ring, inner edge		8.60E+07	1.83	{0,2//2}	3.67	{0,4//2}
μ ring, outer edge		1.03E+08	2.01		4.01	{0,4//2}
moons						
Portia, minor moon	1.70E+18	6.61E+07	1.61		3.22	{0,3//2}
Puck, minor moon	2.90E+18	8.60E+07	1.83		3.67	{0,4//2}
Miranda	6.59E+19	1.29E+08	2.25		4.50	{0,5//2}
Arial	1.353E+21	1.91E+08	2.73		5.47	{0,6//2}
Umbriel	1.172E+21	2.66E+08	3.23		6.46	{0,7//2}
Titania	3.527E+21	4.36E+08	4.13		8.26	{0,9//2}
Oberon	3.014E+21	5.84E+08	4.78		9.56	{0,10//2}

Note: Uranus' original Earth-sized core r = 2.56E+7 /2^2 = 6.39E+6 m. I did not use SunQM-3s6 Table 2's r=5.55E+6 m, because I believe the former one is more accurate.
Note: Uranus ring's data and moon's data is obtained from wiki "Rings of Uranus" and wiki "Moons of Uranus".

V. The original Earth-sized core should be used as p{0,1} for all planets' exterior and interior p{N,n//q} QM structure analysis

In previous paper SunQM-1s3, all eight planets' exterior and interior p{N,n//q} QM structures were analyzed by using the planet surface as pSurface{0,1//q}. For their moons and rings, there were more unfitted than fitted. In current paper, by using the original Earth-sized core as pCore{0,1//q}, all major rings and major moons are found to be in the meaningful pCore{N,n//q} orbits. Plus, all major cores are fitted naturally because they are the Earth-sized core. By comparing to the old result in SunQM-1s3, here we summarize the new (and improved) results for each planet as shown below: for Earth and Venus, the results are the same because their surfaces are Earth-sized. For Neptune, similar result is obtained for the moons, although the major ring can now be fitted in (see Table 7). For Uranus, it was unfitted for all moons and rings, now the major ring and moons can be fitted in (see Table 8), so it is a big improvement. For Saturn, it was unfitted for all moons and rings, now the major ring and moons can be fitted in (see Table 2), so it is also a big improvement. For Jupiter, its moons (but not the rings) were well fitted under p{N,n//5}. Now under the new analysis, its major ring and inner (minor) moons can be fitted as pCore{N,n//2} QM structure (see Table 6 columns 6 & 7), and its major four moons can also be fitted as pCore{1,n=2..5//3} QM structure using Earth-sized core as pCore{0,1//3} (see Table 6 columns 8 through 11). This result may suggest that a Jupiter-massed planet should have a "global energy minimum" QM state with pCore{N,n//3} QM structure, and the current Jupiter is actually accidentally trapped in the p{N,n//5} QM structure. According to paper SunQM-3s7 section VIII, a p{N,n//q} QM state can be written as |qnlm>. And like that of n, q quantum number is also superpositional. So, among all possible superpositional q(s), q=5 may not be the ground state for a Jupiter-massed celestial body's |qnlm> state.

For Mars, we need to construct a new table (see Table 9) for illustration. Columns 4 & 5 of Table 9 shows that no meaningful result is obtained for its moons when using Mars current surface r as pSurface{0,1}. Columns 6 & 7 of Table 9 shows that when using Mars' original Earth-sized core as pCore{0,1//2} = pCore{0,1//4} = pCore{-1,4//4}, Mars core is at pCore{-1,1//2} = pCore{0,0.5//2} = pCore{-1,2//4}, Mars current surface is at pCore{0,0.75//2} = pCore{-1,3//4}, Phobos moon is at pCore{0,1.25//2} = pCore{-1,5//4} orbit, Deimos moon is at pCore{0,2//2} } = pCore{-1,8//4} orbit (see paper SunQM-3s6 for more discussion on the evolution of p{N,n} QM structure of Mars and Mercury). So the result clearly shows that the original Earth-sized core should be used as pCore{0,1} for all planets' exterior and interior p{N,n//q} QM structure analysis.

Table 9. Comparing Mars' p{N,n} QM structure analysis between using its original Earth-sized core as p{0,1}, or using its current surface as p{0,1}.

			use Mars' surface as r_1		use Earth-sized core as r_1	
			pSurface{N,n//2}		pCore{N,n//2}	
pFactor =			2		2	
p{0,1}RF, r_1 =			3.40E+06		6.19E+06	
	mass, kg	orbit r_n	$n=sqrt(r_n/r_1)$		$n=sqrt(r_n/r_1)$	
Mars core		1.80E+06	0.73		0.54	{-1,1//2}
Mars	6.417E+23	3.40E+06	1.00	{0,1}	0.74	{0,0.75//2}
Original Earth-sized core		6.19E+06			1.00	{0,1//2}
Phobos	1.08E+16	9.38E+06	1.66	{0,1.66/2}	1.23	{0,1.25//2}
Deimos	2.00E+15	2.35E+07	2.63	{0,2.63/2}	1.95	{0,2//2}

VI. Using p{N,n} QM structural analysis to compare the age of gas/ice planet's rings

It is interesting to see that all (major) rings of gas/ice planet are at Δn = +1 out of planet's surface (if using their Earth-sized core as r_1). In the case of Neptune and Uranus, both of their surfaces are at pCore{0,2//2}, and their major rings are at orbit pCore{0,3//2}. For Saturn and Jupiter, both of their surfaces are at around pCore{0,3//3}, and their major rings are at pCore{0,4//3} orbit. It is also interesting to see that those apparent random orbits of the inner moons of gas/ice planets can be described by a pCore{0,n//2} QM structure using their Earth-sized cores as r_1. This strongly suggests that the Earth-sized core does exists in all gas/ice planets, and it strongly affect the formation of pCore{N,n} QM structure in the near space of planets. Also the fact that all major rings are at Δn = +1 out of planet's surface suggests that all these major rings may have the similar origin. A single and most possible origin for all planets' rings is that a major collision caused mass excitation from planet surface to the Δn= 0, and +1 QM orbit space. After the de-excitation, most mass fell back to planet's surface, and the tiny leftover mass was disk-lyzed into a ring at the orbit of Δn = +1 out of planet's surface.

Our next question is: by comparing the pCore{N,n} QM structures of these rings, can we obtain some information about their (relative) ages? According to the "ball-torus-7-11-gap effect" (see SunQM-1s1), any mass near the surface of a planet should be cleared out in the early stage of ring formation. Suppose that the closeness of the inner ring to the surface of a planet directly correlates to the age of the ring, then we see Uranus' ζ ring (inner edge at pCore{0,2.05//2}) is relatively closest to Uranus' surface (at pCore{0,2//2}), followed by Saturn's D ring inner edge at pCore{0,3.22//2} relative to Saturn's surface at pCore{0,3//2}, then followed by Jupiter's Halo ring inner edge at pCore{0,3.48//2} relative to Jupiter's surface at pCore{0,3.03//2}, and the last one is Neptune's Galle ring inner edge at pCore{0,2.57//2} relative to Neptune's surface at pCore{0,2//2}. It suggests the ring ages from young to old are: Uranus', Saturn's, Jupiter's, and Neptune's. Actually, it is really hard to say between Uranus ring and Saturn's ring, which one is younger, because the much larger massed Saturn will clear out inner rings much faster than that of smaller massed Uranus. But by comparing between similar massed planets, the pCore{N,n} QM structure analysis do suggest that Uranus' ring is younger than that of Neptune's, and Saturn's ring is younger than that of Jupiter's.

VII. Using p{N,n} QM structural analysis to find the potential ring for Earth, Venus, etc.

The discussion in section VI shows that at least for gas/ice planets, there is a strong QM orbit at Δn = +1 out of planet's surface pCore{N,n//q}, where a (major) ring can be formed. I believe this rule is also valid for the rocky planets. If this is correct, then for our Earth, there could be a ring locates at 4x of Earth's radius, or r = 4x 6.38E+6 = 2.55E+7 meter. Similar idea has been proposed by other scientists before. According to the Giant Impact hypothesis (see wiki "Moon"), after the Theia collision, the Moon was first formed at orbit ~3x to 5x the radius of the Earth. See online "How close was the Moon to the Earth when it formed? (Intermediate)" [16]: "*simulations suggest is was about 3-5 times the radius of the Earth, or about 20 to 30 thousand kilometers*". Right before I publish this paper, I just learned that scientists already know that Earth has a geoconora: "*The geocorona is the luminous part of the outermost region of the Earth's atmosphere, the exosphere … It extends to at least 15.5 Earth radii and probably up to about 100 Earth radii* (see wiki "Geocorona", and paper [17] by Baliukin, I. I. et al.). If using Earth's r as pCore{0,1//2} and r_1, then r/r_1 = 15.5 equals to an almost perfect pCore{2,1//2} QM size, and r/r_1 = 64 (<100) equals to a pCore{3,1//2} QM size. From paper SunQM-1s3, we know that the current Moon's orbit is (almost perfectly) at p{3,1//2}. However, the fact that there is no ring of Earth has been observed (or probably too low mass density, or too short-lived to be observed) clearly shows that our Moon prevent the ring formation at this orbit.

Then what about other rocky planets? Mars' two moons will definitely prevent the ring formation at outside of the Mars. Mercury is not at a stable p{N,n} QM state due to it is too close to the rock-evaporation line (see paper SunQM-3s6), so it is unlikely to have any ring formation. Venus (due to it does not have any moon) may be the candidate for the ring formation. However, Venus has practically no spin (one Venus day = -5832.5 hours, in comparison to one Earth day = 24 hours). From paper SunQM-3s1, we know that a strong spin is the necessary condition for the disk-lyzation and ring formation at the outside of a celestial body. So Venus will not form ring structure even it should have the geoconora.

There is no ring has been observed for any planet does not mean that there is no extremely low mass density ring, or even a temporary ring. Just like the temporary and extreme low mass density Kordylewski dust cloud has been found recently at the Earth–Moon Lagrange point L5 [18], if we are lucky, we may still be able to find some temporary and extreme low density (ring-like) dust cloud at orbit about 4x of a rocky planet's radius. The mass of the ring may come either from the Solar wind, or from planet's own exosphere mass. Notice that the later one may also be the result (or the secondary effect) of a strong burst of Solar wind. This ring may exist temporarily, probably only observable after a strong burst of Solar wind.

Conclusion

Using planet's Earth-sized core as pCore{0,1} and r_1, Saturn's inner core, outer core, surface, B-ring, Mimas, Enceladus, Tethys, Dione, and Rhea, are almost perfectly at pCore{0,n=1..9//3} orbits or sizes. Also Uranus' major ring (ε ring), and its minor moons Portia, Puck, and its major moons Miranna, Arial, Umbriel, Titania, Oberon, are at orbit of pCore{0,n//2} with n ≈ 3, 3, 4, 5, 6,7, 9, and 10 approximately. All major rings of gas/ice planet are at Δn = +1 out of planet's surface (if using their Earth-sized core as r_1). Neptune and Uranus have their surfaces at pCore{0,2//2} size, and their major rings at pCore{0,3//2} orbit. Saturn and Jupiter have their surfaces at around pCore{0,3//3} size, and their major rings at pCore{0,4//3} orbit. By comparing between similar massed planets, the pCore{N,n} QM structure analysis suggests that Uranus' ring is younger than that of Neptune's, and Saturn's ring is younger than that of Jupiter's. Saturn major ring (B-ring, base frequency n=4)'s inner edge n' = 4*3^6 - 167 = 2749, outer edge n' = 4*3^6 +225 = 3141, and the thickness n' =4*3^25 = 3.39E+12. Saturn's A, B, C, D rings may follow the radial probability density distribution curve. Equation 4 shows how to build the Saturn major ring's true 3D structure probability function by using p{N,n} QM and r^2 *|R(n,l)|^2 *|Y(l,m)|^2.

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[19] A series of my papers that to be published (together with current paper):
SunQM-3s4: Using {N,n} QM structure and multiplier n' to analyze Saturn's (and other planets') ring structure.
SunQM-3s5: Using {N,n} QM structure and n*l*0 effect to analyze the bipolar outflow.
SunQM-4: Using {N,n} QM to explain how planets are formed through accretion.
SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe.
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal {N,n} QM.

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