Events Simultaneity and Light Propagation in the context of the Galilean Principle of Relativity

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Abstract

The intent of this work is to present a discussion of the Galilean Principle of Relativity and of its implications for what concerns the nature of simultaneity of events and the characteristics of light propagation. It is shown that by using a clock synchronization procedure that makes use of isotropically propagating signals of generic nature, the simultaneity of distinct events can be established in a unique way by different observers, also when such observers are in relative motion between themselves. Such absolute nature of simultaneity is preserved in the passage from a stationary to a moving reference frame also when a set of generalized space-time coordinates is introduced. The corresponding transformations of coordinates between the two moving frames can be considered as a generalization of the Lorentz transformations to the case of synchronization signals having characteristic speed different from the speed of light in vacuum. The specific invariance properties of these coordinate transformations with respect to the characteristic speed of propagation of the synchronization signals and of the corresponding constitutive laws of the underlying physical phenomenon are also presented, leading to a different interpretation of their physical meaning with respect to the commonly accepted interpretation of the Lorentz transformations. On the basis of these results, the emission hypothesis of W. Ritz, that assumes that light is always emitted with the same relative speed with respect to its source and that is therefore fully consistent with the Galilean Principle of Relativity, is then applied to justify the outcomes of the Michelson-Morley and Fizeau interferometric experiments by introducing, for the latter case, an additional hypothesis regarding the possible influence of turbulence on the refractive index of the fluid. Finally, a test case to verify the validity of either the Galilean or the Relativistic velocity composition rule is presented. The test relies on the aberration of the light coming from celestial objects and on the analysis of the results obtained by applying the two different formulas for the resultant velocity vector to process the data of the observed positions, as measured by a moving observer, in order to determine the actual un-aberrated location of the source.

I. The Galilean Principle of Relativity

The Galilean Principle of Relativity has been originally formulated by Galileo Galilei in the following way[1]: *"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will* *be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted."*

The principle states the invariance of the physical phenomena with respect to uniform rectilinear motion, with constant velocity, of the entire physical system being considered. Though the original formulation by Galilei made explicit reference only to some specific physical phenomena, involving in particular mechanics and fluid-dynamics, the principle is considered valid also for all other physical phenomena, including electromagnetism and optics. This means that the results of any physical experiment shall not vary when the same test is repeated in a given laboratory and in another laboratory which is moving with uniform constant velocity W with respect to the first one.

In his formulation of the principle of relativity, Galilei remarked that any phenomenon which is characterized by having an isotropic propagation speed for a given state of motion of the ship will maintain this property also when the entire system, the ship, is moving with constant and uniform velocity with respect to its original condition. This invariance property is applicable both to wave-like phenomena that require a propagation medium to occur, like the circular propagation of the waves on the water surface, and to particle-like or corpuscular phenomena, involving the motion of physical objects, like the hand-launched balls mentioned in Galilei's example, or the motion of particles originating from a given source, provided that these corpuscular physical entities are emitted by the source with the same constant speed in all directions.

Let us now consider an observer S, stationary with the cabin deck, that is investigating the fall of the water drops from the bottle hanging from the ceiling. Let us suppose that this observer is equipped with a sheet of paper, rigidly connected to him, that is placed on the floor of the cabin with the aim of recording the point of impact of the water drops. Every water drop reaches the cabin floor plane in the same location, thus impressing on the sheet of paper a series of repeated coincident marks, all at the same point, that corresponds to the projection of the location of the bottle on the floor plane. For the observer S, therefore, the characteristic law of the fall of the drops from the bottle is that their motion occurs along a vertical line. According to the Galilean Principle of Relativity, these experimental results will be the same both when the ship is stationary and when it is moving with uniform rectilinear motion. The drops will always impress a series of coincident marks on the paper sheet, and the observer stationary with the cabin deck will judge their falling trajectory as vertical, independently from the state of motion of the ship, provided that it is rectilinear and uniform.

Let us now consider a second observer into the ship, and suppose that this second observer S' is moving horizontally with constant speed V with respect to the cabin structure, and therefore also with respect to the stationary observer S. Let us assume that also this observer is equipped with a sheet of paper that he puts on the floor and that rigidly follows his motion, thus sliding on the floor deck with speed V relative to the cabin structure. With this setup the water drops will not any longer hit the paper sheet of the moving observer S' all in the same position. Each drop will instead produce a separate mark on the paper sheet and the distance between consecutive marks will increase with the value of the relative speed V of the observer S'. Furthermore, let us suppose that a vertical rod has been fixed to the paper sheet in a position such that the top of the rod is aligned with the position of the bottom hole of the bottle at the time that a drop starts its fall. This drop will hit the paper sheet in a point that is not aligned with the base of the vertical rod. So, for the moving observer S', that is using the paper plane and the vertical rod as his reference frame to describe the motion of the drops, the fall of the water drops is not occurring along a vertical line, being not parallel to the orientation of the vertical rod, the z axis of S', and the deviation from the vertical direction depends on the value V of the speed of the observer. In this case therefore, the characteristic law of the same phenomenon has taken a different form for an observer that is in a state of uniform and rectilinear motion with respect to S thus showing that for the moving observer S' the law of fall of the drops is not invariant with respect to the velocity V.

According to the Galilean Principle of Relativity, also the moving observer S' will find again the same results of his experiment when the ship is moving with constant velocity W. The drops will continue to leave a series of separate marks on the paper sheet and both the distance between consecutive marks and the deviation from the vertical line of the fall trajectory of the drops will be the same independently from the state of rectilinear motion of the ship. Also for this observer, therefore, the law of the physical phenomenon being analyzed is invariant with respect to the velocity W of the physical system. Thus, whilst the physical phenomena are not affected by the state of uniform and rectilinear motion of the system, their description by means of mathematical laws depends on the choice of the reference frame of the observer and varies with its relative motion with respect to the physical system being investigated.

We can therefore say that the laws that describe the evolution of a physical system with respect to a given observer are not affected by the state of uniform and rectilinear motion of both the physical system and the observer, as a whole. Conversely, when we consider the characteristics laws of the same physical system with respect to two different observers that are in relative uniform motion with speed V between them, then such laws must change in the passage from one observer to the other one, and the way in which their mathematical form is transformed shall depend on the value V of the relative speed between the two observers. Anytime we are involved with the rules of transformations of the mathematical laws describing a given physical phenomenon in the passage from one reference frame to another one that is moving, we are therefore dealing with the case of two different observers, in relative uniform motion between themselves, observing and describing mathematically the behaviour of the same physical system.

For the observer S' that is moving with constant velocity V inside the cabin of the ship, we can also note that those phenomena which are characterized for the stationary observer S by a uniform speed in all directions, like the propagation of the wavefronts on the water surface of the bowl, or the motion of the hand-launched balls, will have a different propagation speed along different directions. Therefore, for a moving observer S' those phenomena are no longer characterized by an isotropic propagation speed, whilst this property is valid for an observer that is stationary with the ship's frame, i.e. for an observer which is stationary both with the source of the phenomenon and with its propagation medium, when the presence of a propagation medium is necessary for the specific phenomenon being investigated.

Assuming that the ship is in a state of uniform rectilinear motion, a reference frame stationary with the ship's deck is an inertial reference frame. If the observer moving inside the cabin is also translating with uniform constant speed with respect to the ship's deck, then also this moving frame is an inertial reference frame. The difference between these two inertial frames lies in their property of conserving, or not conserving, the isotropy of propagation of the phenomena. This characteristics therefore splits the class of the inertial frames into two groups, and the distinction is applicable also to phenomena that do not require a propagation medium, it is thus applicable also in vacuum. Thus we can conclude that the mathematical laws describing the evolution of the physical phenomena shall not be the same for *all* inertial reference frames, and for the associated observers. The mathematical laws of the same physical phenomenon must take, in general, a different form in an inertial frame for which the isotropy of propagation is conserved and into another inertial frame that is in relative motion with respect to the first one and for which the isotropy of propagation is not conserved.

An observer moving inside the ship's cabin will also notice variations of the frequency of periodic phenomena occurring into the system. The time separation between the peaks and valleys of the water waves appears different for an observer at rest with the propagation medium, the water inside the bowl, and for an observer moving on its surface. Similarly, the frequency of encounter of the water drops falling from the bottle will increase if the observer is moving upwards and decrease if the observer is moving downwards. The same variation of the observed frequency affects also other phenomena not mentioned by Galilei: the tone of a sound or the colors of the spectral lines emitted by an excited substance appear different for a moving observer with respect to a stationary one.

Finally, it can be noted that whilst the Galilean Principle of Relativity has been formulated for systems that are in a state of uniform rectilinear motion, the specific example used by Galilei in its original description, i.e. the ship and the physical entities contained in its cabin, is not actually representative of such a case, since the ship, whether at rest in the harbour or cruising on the sea, is transported by the Earth's motion along a non rectilinear path. Due to the curvature of the Earth and to its angular rotation, the state of motion of the ship contains a circular component and is characterized by a non-null angular velocity. Even if the amount of the deviation from uniform rectilinear motion is quite small and can be neglected, in first approximation, for many applications, the presence of the Earth's rotation has an influence of the physical phenomena being observed and it can indeed be detected by suitable physical experiences, for example by observing the variation of the plane of oscillation of a Foucault pendulum.

The same observation is applicable to any experiment performed into a Laboratory on the Earth, since the entire experimental setup is rigidly transported by the non-rectilinear motion of our planet. In general, this accelerated state of motion could have an influence on the results of the experiment and on the measurements being conducted. The actual extent and entity of the influence will depend on the phenomenon being investigated and on the specific experimental setup, being possibly not negligible for some very accurate experiments or measurements.

II. Simultaneity and Time Intervals

In order to describe the governing laws of physical phenomena by means of mathematical expressions it is necessary to define a set of space and time coordinates to associate each event being analyzed to a position in space and to a time of occurrence. The spatial position where the event occurs can be established by means of rigid rulers whilst the determination of time requires the use of clocks that must be synchronized in order to provide a consistent time basis. The synchronization of two clocks which are located in the same spatial position can be done by directly comparing their time readout at different instants. However, when we consider two non coincident clocks, the application of this synchronization method is not straightforward, because of the delay associated to the transport of the information from one location to the other one. It is therefore necessary to establish a method to determine when two events, the readout of the two clocks, occurring at different space locations are simultaneous. Such method will then allow to perform properly the synchronization of couple of distant clocks and will thus allow to synchronize any clock with a reference, or master, clock in such a way that a common time basis can be established and used to describe the time of occurrence of the physical events being investigated.

Let us consider two generic events occurring at two space locations A and B. Suppose that at the time of occurrence of each event some kind of synchronization signals are emitted from the two points of space where the events occur and that such synchronization signals travel with uniform and constant speed in all directions, i.e. that they propagate isotropically into a reference frame K that is stationary with respect to the two geometric locations A and B. Let v_c be the finite *characteristic speed* of propagation of such signals, which is assumed to be equal in all directions into the K frame. The locus of the points reached by each signal after a given interval of time from its emission is a sphere, having its center at the location of the corresponding originating event.

If the two events being considered are simultaneous, then the radii of the two spheres with centers in A and B are equal for every instant of time, since the two signals have the same propagation speed v_c. Therefore, the signal coming from A will encounter the other signal coming from B exactly at the midpoint M of segment AB.

Conversely, if two isotropically propagating signals having the same characteristic speed v_c meet each other at the midpoint M of segment AB, then, since the distance traveled by each one is the same by construction, being equal to half the length of AB, and since they both have the same propagation speed, the time elapsed from the emission to the encounter of each signal is the same for both, thus the two originating events A and B are simultaneous.

The same considerations are valid also for any other couple of isotropically propagating signals, emitted from A and B at the same time of the first two ones, but which are characterized by a different value v'_c of their finite propagation speed. Therefore, the criterion of simultaneity of events can be formulated in the following way:

Two events are simultaneous if and only if two isotropically propagating synchronization signals, emitted from the points A and B at the time of occurrence of the corresponding events, meet each other at the mid-point M of segment AB, **for any** *finite value of the characteristic speed v_c of the selected signals.* [1]

When this condition is verified we can say that, into the specified reference frame K being considered, the time t of the two events is the same, i.e. we can state $t_A = t_B$.

The simultaneity of events is therefore a characteristics that is invariant with respect to the speed of propagation v_c of the synchronization signals, thus resulting independent from the specific kind of signal being selected.

According to the above criterion, if an event A is simultaneous with a second event B and also with a third event C, then also the two events B and C are simultaneous. The synchronization signals selected to assess the mutual simultaneity between the three events can be different for each couple of events, the outcome of the process will be the same.

The physical nature of the specific signals being used for the synchronization is not relevant for the method, they could be particle-like or wave-like phenomena, nor the value of their characteristic propagation speed v_c, which is only assumed to be finite and equal in all directions. The only assumption required for the validity of the method is that the selected synchronization signals propagate isotropically with respect to the reference frame K. For example, in vacuum one could imagine to employ small particles, emitted in every direction with the same relative speed with

[1] In general, the location of the control point M need only to be selected in such a way that it is equidistant from A and B, i.e. it can be located at the center of a sphere having points A and B on its surface. The midpoint of the segment AB represents the minimum distance choice.

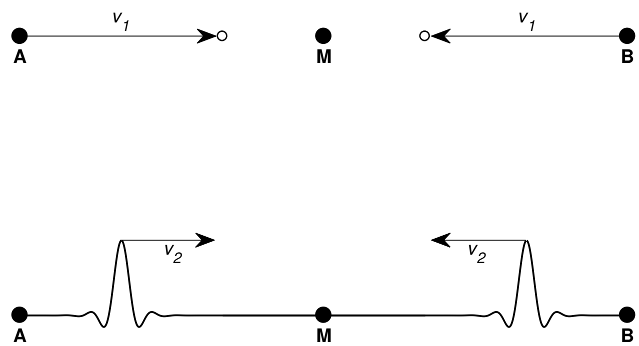

Figure 1: *Simultaneity assessment by means of particle-like (top) or wave-like (bottom) isotropic synchronization signals traveling with characteristic speed $v_c = v_1$ and $v_c = v_2$, respectively. M represents the midpoint of segment AB.*

respect to the source by a spring-loaded launching device, or one could consider to perturb an ideal string tensioned between its endpoints A and B and use the propagation of the resultant waveform as synchronization signal. In both cases we can ideally imagine of being able to tune the value of the signal speed to whatever finite value v_c, by properly adjusting the governing parameters of the selected physical phenomenon (string tension, spring and mass values). In presence of a homogeneous and isotropic medium, other kind of signals could also be employed like, for example, acoustic waves traveling in the air at the speed of sound.

In order to guarantee the isotropy of propagation of the synchronization signals, according to what stated by the Galilean Principle of Relativity, it is necessary that the frame of reference K identified to represent the coordinates of the two events A and B is stationary both with the source of the signals and with the propagation medium (for those phenomena that require a medium to propagate). In the above examples this means that the spring-loaded launcher of the particle-like objects, in one case, and the entire ideal string, in the other case, must be stationary with respect to the frame K.

The process can be applied to any pair of geometrical points in the space and to the corresponding couple of events. In such a way, it can be used to synchronize pairs of clocks placed at distinct space locations. Without losing generality we can assume that the origin of the reference frame K is coincident with one of the two points selected as the source of the synchronization signals. By using this method, therefore, it is possible to synchronize a *"master"* clock located in the origin of the reference frame K with a clock placed at any point of the entire space domain. This synchronization of the clocks guarantees also that the two clocks run at the same pace, spanning the same time intervals at the two different locations, i.e. it allows to state that $\Delta t_B = \Delta t_A$.

Repeating the same process for all points of the entire space domain it is possible to synchronize all the clocks located at the different geometrical locations of K with the reference time basis of the master clock located in the origin. All clocks will therefore beat in unison, spanning the same time intervals of the master clock. In this manner it is thus possible to associate, in a unique way and consistently with the Galilean Principle of Relativity, the space and time coordinates, expressed into the reference frame K, to any event occurring into the system being observed, and the process is not dependent neither on the type of physical signal used to perform the synchronization nor on its characteristics speed v_c, the only requirement for the validity of the synchronization method being that such signals are isotropically propagating with respect to the K frame.

Let us now consider a second reference frame, K′, that is in a state of uniform rectilinear motion with respect to the previous one, with a velocity having magnitude V, as measured in the reference frame K, and direction parallel to segment AB, oriented from A to B. Let A′ and B′ be the positions of the two geometrical points expressed in the reference frame K′ that coincide, respectively, with the positions of A and B at the time of occurrence of the corresponding events. Let A′B′ be the segment joining these two points and let M′ be the midpoint of this segment which is at rest into the frame K′. In order to assess the simultaneity of the two events being considered, an observer stationary with the K′ frame cannot use the same two synchronization signals that have been adopted by the observer of the K frame. In fact, due to the finite value of the characteristic signal speed v_c, the two K-based signals will meet together at M after some time from their emission from A and B. During this time period the midpoint M′ will have traveled a certain amount of distance from M and it will therefore occupy a different position in the space with respect to M. Since the two signals cannot meet both in M and in M′, it follows that the K′ observer would incorrectly judge the two events as being non-simultaneous.

In order to avoid this issue and to correctly evaluate the simultaneity of the two events also into the reference frame K′, it is necessary to make use of signals that propagate isotropically into this moving frame. This requires, according to the Galilean Principle of Relativity, that the sources of the signals and the propagation medium (for example, the ball launchers or the tensioned string) are both stationary with respect to the reference frame of the observer. A moving observer K′ can therefore assess the simultaneity of events A and B by using other two synchronization signals, distinct from the ones used by the observer of the K frame, emitted from the space locations A′ and B′, that are coincident with A and B at the time of occurrence of the corresponding events, provided that such signals travel isotropically with respect to his reference frame K′. The nature of these two *"primed"* signals and the corresponding characteristic speed v'_c, could be the same of the ones used for synchronization in frame K, or it could be different, provided that it is isotropic in K′. For example, one could imagine to use the traveling balls in frame K and the waveform propagating on the string in frame K′, or viceversa. In this way the two events A and B will be declared simultaneous also into the K′ reference frame, since the two *"primed"* signals, propagating with the same speed v'_c from A′ to M′ and from B′ to M′, will meet each other at the midpoint M′ of segment A′B′ [2]. In this way, whenever two events are declared simultaneous in one reference frame K, they result simultaneous also in the moving frame K′, and this conclusion regarding the coincidence in time of the two events is independent from the specific nature and the corresponding characteristic speed v_c, or v'_c, of the synchronization signals being used in the two reference frames.

When the specific physical signal chosen for the synchronization procedure needs some form of medium to propagate in the surrounding space, the requirement of isotropic propagation can be guaranteed only for a reference frame that is stationary with the specific propagation medium being considered. For example, in case of acoustic signals traveling in the atmosphere, only the clocks of those reference frames which are stationary with the air can be synchronized using such

[2] As previously noted, since the segment A′B′ is at rest into frame K′, but is moving with velocity V with respect to frame K, the location in space of its midpoint M′ at the time of detection of the primed signals will not be coincident with the position of M

acoustic signals. The clocks referred to any other reference frame in relative motion with respect to the previous ones, and therefore in motion with respect to the air, cannot be synchronized by means of those acoustic signals, since for such a moving frame the speed of sound would no longer be same in all directions, i.e. it would not be isotropic.

The synchronization procedure described above, and the related considerations, are valid also when light signals are used to establish the simultaneity of events, provided that the light sources being considered and the transparent light propagation medium, if present, are both stationary with respect to the reference frame of the observer and with the clocks that are being synchronized. The emission hyphothesis formulated by W. Ritz[3], that assumes that light is emitted in all directions with the same relative speed with respect to its source, being fully consistent with the Galilean Principle of Relativity and therefore also compliant with the above requirements of the synchronization procedure, justifies the usage of light signals to synchronize the clocks also into the moving reference frame K'.

The assessment of simultaneity of the events with respect to the moving reference frame can also be implemented in the following, more direct way. At each instant of time a generic geometrical point P' belonging to the moving frame K' happens to be coincident with one geometrical point P of the reference frame K. When the two geometrical points are coincident, $P' \equiv P$, the time indicated in that moment by the clock located at P can be readily extended also to P' since, being the two points coincident, there is no delay associated with the transfer of the information regarding the time readout between two different space locations. It is therefore possible to associate to P' the same time indicated by the clock associated to P. Since the clocks of the entire space K are all synchronized between them, they all indicate the same time. This same time stamp can therefore be assigned also to all geometrical points of K' because each point of the K' space domain will be coincident with one and only one location of the K space and will therefore take from it the corresponding time indication. In other terms, it is possible to assign to all geometrical points of the moving frame K', the same time indicated by the *"stationary"* clocks synchronized into reference frame K. This conclusion is valid for any geometrical location belonging to the reference frame K', thus allowing to establish, also for the observers of this *"moving"* reference frame, the same time basis of the *"stationary"* one, i.e. it is possible to set $t' = t$, from which it also follows $\Delta t' = \Delta t$.

III. Transformations of coordinates between moving frames

In this paragraph it will be shown that the absolute nature of simultaneity can also be consistently assessed by a moving observer through the use of a class of coordinate transformations similar to the Lorentz transformations.

Let us consider two events occurring at two distinct locations A and B of the space and be K a reference frame stationary with respect to the points A and B. Let v_c be the characteristic propagation speed of the isotropic signals that have been selected to synchronize the clocks into this reference frame. According to the previously described synchronization method, the two events are simultaneous if the synchronization signals emitted from A and B at the time of occurrence of the corresponding events meet each other at the midpoint of segment AB.

For any couple of events we can now introduce, into the reference frame K, the *characteristic interval*, s_c, that is a scalar quantity dependent from the space and time coordinates of the two events A and B and that is defined by the following relation containing the value of the signal propagation speed v_c as a constant parameter:

$$s_c^2 = v_c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \tag{1}$$

where $\Delta t = (t_B - t_A)$, and $\Delta x = (x_B - x_A)$, $\Delta y = (y_B - y_A)$, $\Delta z = (z_B - z_A)$.

Let us now consider a second reference frame K′, having its axes parallel to those of K, and let K′ be translating with constant speed V along the positive direction of the x axes with respect to frame K. We will call K the stationary frame and K′ the moving frame. Let us also introduce, into the moving frame K′, a new set of four *generalized* space-time coordinates, that will be indicated with $(\varepsilon', \eta', \zeta', \tau')$, and that are functions of the (x, y, z, t) coordinates of the stationary reference frame K:

$$(\varepsilon', \eta', \zeta', \tau') = f(x, y, z, t) \tag{2}$$

It is possible to select the functions f that defines the primed generalized space-time coordinates $(\varepsilon', \eta', \zeta', \tau')$ in such a way that the characteristic interval between two events results invariant in the passage from K to K′, and viceversa, i.e. to select f in such a way that it results:

$$\left[s'_c(\varepsilon', \eta', \zeta', \tau')\right]^2 = \left[s_c(x, y, z, t)\right]^2 \tag{3}$$

Since the y and z axes of the two reference frames are parallel by construction and are not mutually traslating along the respective directions, the corresponding coordinates of the two frames can be set equal to each other: $\eta' = y$ and $\zeta' = z$, from which it follows: $\Delta\eta' = \Delta y$ and $\Delta\zeta' = \Delta z$.

With this choice, the problem reduces to that of finding the relations between the (ε', τ') and (x, t) coordinates. We are therefore looking for the specific form of the transformations of coordinates that gives:

$$(v_c \Delta\tau')^2 - (\Delta\varepsilon')^2 = (v_c \Delta t)^2 - (\Delta x)^2 \tag{4}$$

This relation can be satisfied by putting:

$$\tau' = \gamma_c\big(t - \frac{V}{v_c^2}x\big) \quad ; \quad \varepsilon' = \gamma_c\big(x - Vt\big) \qquad \textit{with} \qquad \gamma_c = 1/\sqrt{(1 - V^2/v_c^2)} \tag{5}$$

as it can be verified by substituting these expressions into eq. (4):

$$\big(v_c\Delta\tau'\big)^2 - \big(\Delta\varepsilon'\big)^2 = \Big[v_c\gamma_c\Big(\Delta t - \frac{V}{v_c^2}\Delta x\Big)\Big]^2 - \Big[\gamma_c\Big(\Delta x - V\Delta t\Big)\Big]^2 =$$

$$= \gamma_c^2\Big(v_c^2\Delta t^2 + \frac{V^2}{v_c^2}\Delta x^2 - 2V\Delta x\Delta t\Big) - \gamma_c^2\Big(\Delta x^2 + V^2\Delta t^2 - 2V\Delta x\Delta t\Big) =$$

$$= \frac{v_c^2}{v_c^2 - V^2}\Big[(v_c^2 - V^2)\Delta t^2 - \frac{v_c^2 - V^2}{v_c^2}\Delta x^2\Big] = \big(v_c\Delta t\big)^2 - \big(\Delta x\big)^2$$

Therefore, the coordinate transformations that satisfy the invariance property (3) of the characteristic interval are:

$$\varepsilon' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad \eta' = y; \qquad \zeta' = z; \qquad \tau' = \frac{t - \frac{V}{v_c^2}x}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{6}$$

and the inverse transformations, from the generalized space-time coordinates of K′ to K, are:

$$x = \frac{\varepsilon' + V\tau'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad y = \eta'; \qquad z = \zeta'; \qquad t = \frac{\tau' + \frac{V}{v_c^2}\varepsilon'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{7}$$

When light signals propagating in vacuum are chosen as synchronization signals, the characteristic speed is equal to the speed of light in vacuum, $v_c = c$, and the above transformations of coordinates coincide with the Lorentz transformations.

It can be noted that the transformations (6), and the corresponding inverse (7), are not defined for $V = v_c$, whereas for $V > v_c$ the two generalized coordinates ε' and τ' become complex, having a non null imaginary part. Even in this case, these complex coordinates still preserve the invariance of the characteristic interval s_c, as it can be verified by direct substitution of (6) into equation (4). The invariance of the characteristic interval is therefore verified for all values of $V \neq v_c$ and it holds true for any finite value of the characteristic speed v_c of the selected isotropic signal used to synchronize the clocks into the stationary frame K. When the relative speed V of the moving reference frame is very small compared to the characteristic speed v_c of the selected synchronization signals, the speed ratio V/v_c tends to zero and the transformation of coordinates of eqs. (6) tends, in the limit $V/v_c \to 0$, to the Galilean one:

$$\varepsilon' = x - Vt; \qquad \eta' = y; \qquad \zeta' = z; \qquad \tau' = t; \tag{8}$$

Let us now consider, into frame K, two simultaneous events A and B occurring at two generic points of the space and let (x_A, y_A, z_A) and (x_B, y_B, z_B) be the coordinates of the geometrical locations of the two events and $t_A = t_B$ the corresponding time of occurrence. According to the definition of simultaneity given before, the synchronization signals emitted by A and B will both reach at the same time $t_M > t_A$ the midpoint M of segment AB, with M having coordinates:

$$(x_M, y_M, z_M) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2}\right)$$

The two characteristic intervals, into frame K, between the two simultaneous events A and B being considered and the event O of detection of the arrival of their synchronization signals at the midpoint M are given by:

$$s_{OA}^2 = (v_c \Delta t_{AM})^2 - L_{AM}^2 \qquad \textit{and} \qquad s_{OB}^2 = (v_c \Delta t_{BM})^2 - L_{BM}^2 \tag{9}$$

where:

$$\begin{aligned} \Delta t_{AM} &= (t_M - t_A) \\ \Delta t_{BM} &= (t_M - t_B) \\ L_{AM}^2 &= (x_A - x_M)^2 + (y_A - y_M)^2 + (z_A - z_M)^2 \\ L_{BM}^2 &= (x_B - x_M)^2 + (y_B - y_M)^2 + (z_B - z_M)^2 \end{aligned}$$

Since in the stationary frame K the two points A and B are located symmetrically with respect to the midpoint of the segment, it is $L_{BM} = L_{AM} = L/2$, where L is the length of segment AB, and since the time of emission of the signals is the same, $t_A = t_B$, it follows that $\Delta t_A = \Delta t_B$ and therefore it results:

$$s_{OA}^2 = s_{OB}^2 \tag{10}$$

This shows that, in the stationary frame K, two distinct events A and B are simultaneous when they are separated by the same characteristic interval from the event of the arrival of their synchronization signals at the midpoint of segment AB. Expression (10) can thus be considered as the mathematical formulation of the criterion for the simultaneity between two events described in the previous section and based on isotropic signals propagating with characteristic speed v_c into frame K.

In order to further generalize the formulation of the mathematical criterion of events simultaneity, let us consider a reference event Q, occurring at a given time t_Q at a point of space belonging to the midplane of segment AB, i.e. at a point belonging to the plane orthogonal to the line joining A and B and passing thru the midpoint M of segment AB. Since Q belongs to the midplane of AB, it is equidistant from A and from B: $L_{AQ} = L_{BQ}$, and since the two events A and B are simultaneous, it is $t_A = t_B$, and therefore we have also $(\Delta t_{AQ})^2 = (t_A - t_Q)^2 = (t_B - t_Q)^2 = (\Delta t_{BQ})^2$ for every time of occurrence of the event Q. Therefore, also in this case it results:

$$s_{OA}^2 = (v_c \Delta t_{AQ})^2 - L_{AQ}^2 = (v_c \Delta t_{BQ})^2 - L_{BQ}^2 = s_{OB}^2 \tag{11}$$

and this shows that the simultaneity of two distinct events can be established by evaluating and comparing the two characteristic intervals that separate events A and B from a generic reference event Q occurring at a location that is equidistant from the two events being considered. The two events A and B are simultaneous when the two intervals are equal.

The above considerations can be repeated for any other finite value of the parameter v_c, leading always to the same result expressed by relations (10) and (11), thus showing that simultaneity is invariant with respect to the propagation speed v_c of the selected synchronization signals.

Since the characteristic interval s_c is invariant under the generalized coordinate transformations (6) defined above, we have $(s'_{OA})^2 = (s_{OA})^2$ and $(s'_{OB})^2 = (s_{OB})^2$. If we now consider two simultaneous events into the stationary frame K, for these two events it is $s_{OA}^2 = s_{OB}^2$, and therefore it will also be:

$$(s'_{OA})^2 = (s'_{OB})^2 \tag{12}$$

Thus, according to the same criterion established before, based on the equality of the characteristic intervals, two events that are simultaneous in the stationary frame K are simultaneous also in the moving frame K', and viceversa. This invariance of simultaneity holds true for any value of the relative speed V between the two moving frames and for any kind of physical signals selected to synchronize the clocks, so it holds true for any finite value of their characteristic speed v_c and is thus consistent with the definition of simultaneity given in the previous section. It appears therefore that simultaneity is an absolute characteristic of the events, that can be defined and assessed univocally by different observers that are in a state of uniform relative motion one with respect to the other, by applying the same general criterion of equality of the characteristic intervals with respect to an equidistant reference event.

Let us now calculate, in the moving frame K', the generalized time coordinate τ' of two simultaneous events A and B. According to (6), we have:

$$\tau'_A = \frac{t_A - \frac{V}{v_c^2} x_A}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{13}$$

and

$$\tau'_B = \frac{t_B - \frac{V}{v_c^2} x_B}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{14}$$

Taking into account that $t_B = t_A$ it is possible to rewrite τ'_B as follows:

$$\tau'_B = \frac{t_A - \frac{V}{v_c^2} x_A - \frac{V}{v_c^2}(x_B - x_A)}{\sqrt{1 - \frac{V^2}{v_c^2}}} = \tau'_A - \frac{V}{v_c^2} \frac{(x_B - x_A)}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{15}$$

This relation shows that the generalized coordinate τ' of two simultaneous events A and B, evaluated in the moving frame K', has not the same value for the two events, being, in general:

$$\tau'_B \neq \tau'_A \tag{16}$$

In other words, two simultaneous events A and B turn out as being characterized by a different value of the corresponding generalized coordinate τ'.[3] Therefore the generalized coordinate τ' cannot be used by the moving observer to represent the time of occurrence of the events, i.e. τ' is not time.

We can now consider the governing laws that describe the isotropic propagation of the signals selected to synchronize the clocks. In particular let us consider the case of the tensioned ideal string. It is known that for small amplitudes, the transverse displacement u of the points of the string is determined by the solution of the d'Alembert equation:

$$v_s^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0 \qquad with \qquad u = u(x,t) \tag{17}$$

where $v_s = \sqrt{N/\lambda}$ gives the speed of propagation of the perturbations along the string as a function of the applied axial tension N and linear mass density λ of the string.

As discussed in the first paragraph related to the phenomenological description of the Galilean Principle of Relativity, any experimental determination of the string properties and of its response will give identical results when the same characterization tests are repeated into two different laboratories that are uniformly translating one with respect to the other. Therefore, the string behaviour will be represented by the same governing laws in both cases, i.e. the same equation (17) will be determined both by the observer of the stationary laboratory and by the observer of the moving one, and the propagation of the perturbations along the string will remain isotropic and will have the same characteristic speed v_s in both reference frames.

The situation is different if we consider, into a given laboratory, a moving observer with its associated moving reference frame. Let K be a reference frame stationary with the laboratory, and stationary also with respect to the string, and let K' be another reference frame translating with velocity V parallel to the string axis. For this frame, which is in relative motion with respect to the string, the perturbations on the string will be no more propagating isotropically, their speed being greater than v_s along one direction and lower than v_s in the opposite direction. Correspondingly, also the governing laws of the string will change when expressed into the moving frame K'. In this case therefore the governing law of the string, expressed by equation (17) for the stationary observer, should not be invariant in the transformation from the stationary frame K to the moving frame K'.

Let us now see how the wave equation (17) transforms in the moving frame K' when the generalized coordinates with characteristic speed v_c, as defined by (6), are used. The d'Alembert equation (17) can be reformulated as:

$$\left(v_s \frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\left(v_s \frac{\partial}{\partial x} - \frac{\partial}{\partial t}\right) u = 0 \tag{18}$$

and the two generalized coordinates (ε', τ') can be written in a more compact form as:

$$\varepsilon' = \gamma_c (x - Vt), \quad \tau' = \gamma_c (t - Vx/v_c^2) \tag{19}$$

[3] The only particular case for which $\tau'_B = \tau'_A$ occurs when $x_B = x_A$, i.e. when the two simultaneous events A and B are located in a plane orthogonal to the direction of the velocity vector V of frame K', and therefore in a plane orthogonal to the x axis of the K frame, being it parallel to V by construction.

where $\gamma_c = 1/\sqrt{1 - V^2/v_c^2}$. This change of variables can be applied to the d'Alembert equation by taking into account that:

$$\frac{\partial}{\partial x} = \frac{\partial \varepsilon'}{\partial x}\frac{\partial}{\partial \varepsilon'} + \frac{\partial \tau'}{\partial x}\frac{\partial}{\partial \tau'} = \gamma_c\Big(\frac{\partial}{\partial \varepsilon'} - \frac{V}{v_c^2}\frac{\partial}{\partial \tau'}\Big)$$

$$\frac{\partial}{\partial t} = \frac{\partial \varepsilon'}{\partial t}\frac{\partial}{\partial \varepsilon'} + \frac{\partial \tau'}{\partial t}\frac{\partial}{\partial \tau'} = \gamma_c\Big(\frac{\partial}{\partial \tau'} - V\frac{\partial}{\partial \varepsilon'}\Big)$$

In this way, the wave equation (18) takes the form:

$$\gamma_c^2\Big(A\,\frac{\partial^2 u}{\partial \tau'^2} + B\,\frac{\partial u}{\partial \varepsilon'}\frac{\partial u}{\partial \tau'} - C\,\frac{\partial^2 u}{\partial \varepsilon'^2}\Big) = 0 \tag{20}$$

where the three terms A, B and C are given by:

$$A = \Big(1 - \frac{V^2 v_s^2}{v_c^4}\Big) \qquad B = 2V\Big(\frac{v_s^2}{v_c^2} - 1\Big) \qquad C = (v_s^2 - V^2)$$

From these expressions it can be noted that when $v_s = v_c$ it results $B = 0$, and $A = 1/\gamma_c^2$, $C = v_c^2 - V^2$. Substituting these terms into (20), gives:

$$v_c^2\frac{\partial^2 u}{\partial \varepsilon'^2} - \frac{\partial^2 u}{\partial \tau'^2} = 0 \qquad with \qquad u = u(\varepsilon', \tau') \tag{21}$$

Therefore, when the parameter v_c contained into the generalized coordinate transformation (6) is equal to the speed of propagation v_s of the specific phenomenon being described, the corresponding equation governing the evolution of the perturbations along the string is invariant in the passage from the stationary frame K to the moving frame K'. This invariance property, however, is no longer verified when the characteristic speed v_c used in the coordinate transformation is different from v_s. In this case, in fact, the term B is not null, and the equation resulting from the change of coordinates has no longer the same form of the original wave equation.

This peculiar invariance property of the generalized coordinates is valid not only for the monodimensional case of the string equation that has been considered here, but also for the tridimensional case of the wave equation that has the same form of (17). Also in this more general case, the invariance of the governing equations is satisfied only when the parameter v_c that appears in the definition of the coordinate transformation has the same value of the characteristic speed of the isotropically propagating phenomenon being represented. If some of the physical properties characterizing the phenomenon change, thereby changing the corresponding physical speed of propagation, then the wave equation will take a different form in the passage from the stationary to the moving observer and its solutions will be different along different directions.

Let us now analyze the relationship between the generalized velocity $\mathbf{w}'$, evaluated into frame K' on the basis of the coordinate transformation defined by (6), and the expression of the velocity into frame K. This can be done by evaluating, from eqs. (7), the differentials:

$$dx = \frac{d\varepsilon' + V d\tau'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad dy = d\eta'; \qquad dz = d\zeta'; \qquad dt = \frac{d\tau' + \frac{V}{v_c^2}d\varepsilon'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{22}$$

Through the definition of the velocity in the stationary frame:

$$\mathbf{v} = \Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big)$$

and, by analogy, of the generalized velocity in the moving frame:

$$\mathbf{w}' = \left(\frac{d\varepsilon'}{d\tau'}, \frac{d\eta'}{d\tau'}, \frac{d\zeta'}{d\tau'}\right)$$

it follows that

$$v_x = \frac{w'_x + V}{1 + \frac{Vw'_x}{v_c^2}}; \qquad v_y = \frac{w'_y\sqrt{1-(V^2/v_c^2)}}{1 + \frac{Vw'_x}{v_c^2}}; \qquad v_z = \frac{w'_z\sqrt{1-(V^2/v_c^2)}}{1 + \frac{Vw'_x}{v_c^2}}; \tag{23}$$

The expression of the generalized velocity $\mathbf{w}'$ into frame K′ is found by inverting the above relations, obtaining:

$$w'_x = \frac{v_x - V}{1 - \frac{Vv_x}{v_c^2}}; \qquad w'_y = \frac{v_y\sqrt{1-(V^2/v_c^2)}}{1 - \frac{Vv_x}{v_c^2}}; \qquad w'_z = \frac{v_z\sqrt{1-(V^2/v_c^2)}}{1 - \frac{Vv_x}{v_c^2}}; \tag{24}$$

From these expressions it turns out that when the magnitude of the velocity in the stationary frame K is equal to the characteristic speed, i.e. when $|\mathbf{v}| = v_c$, then also the magnitude of the generalized velocity in the moving frame K′ has the same value: $|\mathbf{w}'| = v_c$. In fact, considering the case $v^2 = v_x^2 + v_y^2 + v_z^2 = v_c^2$ and evaluating the magnitude of $\mathbf{w}'$ from equations (24), it results:

$$|\mathbf{w}'|^2 = \frac{(v_x^2+v_y^2+v_z^2) - 2v_xV + V^2 - \frac{V^2}{v_c^2}(v_y^2+v_z^2)}{(1 - v_xV/v_c^2)^2} = \frac{[v_c^2 - 2v_xV + \frac{V^2v_x^2}{v_c^2}]}{(1 - v_xV/v_c^2)^2} = v_c^2\frac{[1 - 2\frac{v_xV}{v_c^2} + \frac{v_x^2V^2}{v_c^4}]}{(1 - v_xV/v_c^2)^2} = v_c^2$$

The transformations of coordinates defined by equations (6) represent therefore a change of variables that leaves invariant the characteristic speed. Considering the case of particle-like signals used to synchronize the clocks (for example the spring-loaded launcher device considered in the previous section), and applying the characteristic coordinate transformation (6) to calculate the generalized speed of the particles into the moving frame K′, it turns out that also these particle-like signals, that propagate isotropically with speed v_c in the stationary frame K, will have the same value v_c of the generalized speed w' along every direction, also in the moving frame K′. This invariance of the characteristic speed is valid for any finite value of the speed of the specific synchronization signal being considered. When the value of characteristic speed is equal to the speed of light in vacuum, $v_c = c$, the above result corresponds to the invariance of the speed of light under the Lorentz coordinate transformations.

It can be noted that this kind of coordinate transformations do not preserve the invariance of the relative velocity between two physical objects in the passage from a given reference frame K to another one, K′, that is in a state of uniform rectilinear motion with respect to the first one. For this reason, the Lorentz transformations, that are a particular case of eqs. (6) characterized by having characteristic speed of the synchronization signals equal to the speed of light in vacuum, $v_c = c$, appear incompatible with the Galilean Principle of Relativity, since any physical phenomenon whose constitutive laws depend from the relative velocities of the involved entities will have a different mathematical expression for two different observers, in contrast with the invariance postulated by Galilei in his formulation of the principle.

Let us now consider the case of a signal, particle-like or wave-like, traveling along the x axis of the stationary frame K with constant speed v, that is: $v_x = v$ and $v_y = v_z = 0$. Let K′ be a moving reference frame which is also traveling along the direction of the x axis of K with uniform constant speed equal to v. According to the Galilean rule of speed composition, the velocity of the signal with respect to such moving frame K′ is null, since it is given by $v' = v - V = v - v = 0$ for any value of the common speed v of both the signal and the reference frame. We want now to evaluate the generalized velocity of this signal into the moving frame K' according to the formulas (24) previously established. By putting $v_x = V = v$ and $v_y = v_z = 0$ it results:

$$w'_y = w'_z = 0$$

$$w'_x = \frac{v_x - V}{1 - \frac{V v_x}{v_c^2}} = \frac{v - v}{1 - \frac{v^2}{v_c^2}} = v_c \frac{\beta - \beta}{1 - \beta^2}$$

where $\beta = v/v_c$. Therefore it results $w'_x = 0 \quad \forall\, v \neq v_c$ whilst in the case $v = v_c$, for which $\beta = 1$, the previous expression gives an undetermined form of the type $0/0$, expression that can however be evaluated by applying the rule of de l'Hopital. Putting $f(\beta) = \beta - \beta$ and $g(\beta) = 1 - \beta^2$ it gives:

$$\lim_{\beta \to 1} w'_x = \lim_{\beta \to 1} v_c \frac{f}{g} = \lim_{\beta \to 1} v_c \frac{f'}{g'} = v_c \frac{0}{-2} = 0$$

Thus, also for the case $v = v_c$ the generalized speed of a signal traveling with speed v, as evaluated by an observer comoving with it at the same speed, $V = v$, is zero. When applied to the case of a light signal, or a photon, traveling in vacuum with velocity $v = c$, this result shows that the generalized speed of a light signal evaluated by a luminal observer, i.e. the speed of light evaluated by a reference frame moving at the same speed of light in vacuum, is null. This result appears in contrast with the postulate of invariance of the speed of light that is at the basis of the Special Theory of Relativity[2], since it shows that there is at least one observer, the luminal observer, for which the speed of light, calculated according to the rules determined by the theory itself, is zero instead of being equal to c as required by the postulate.

In summary, the generalized coordinate transformations defined by (6) are characterized by the following peculiar properties in the passage from a reference frame K to another frame K' that is translating with constant velocity V:

1. they make invariant the characteristic interval s_c defined by equation (1);
2. they leave invariant the constitutive laws representing isotropic propagation of a phenomenon having characteristic speed v_c in the stationary frame K;
3. they maintain, for the generalized speed of propagation in the moving frame K', the same value of the characteristic speed v_c that such phenomenon has in the stationary frame K.

The above properties are verified for any finite value of the characteristic speed v_c and correspond, for the case $v_c = c$, to the same properties of the Lorentz transformations that are valid for the propagation of light in vacuum and for the corresponding governing laws as described by the Maxwell equations of electromagnetism. [4]

Being valid for any value of the selected characteristic propagation speed v_c, these invariance properties of the generalized coordinate transformation (6) can be considered as a peculiar mathematical characteristics of this type of coordinate transformations, rather than a dependency of the properties of space and time from the motion of the observer.

It has been shown above by relation (16) that for a moving reference frame K', two simultaneous events do not have, in general, the same value of the generalized coordinate τ' which therefore cannot be used as a time identification of the events. This coordinate, instead, can be interpreted

[4] The above described invariance properties of the Lorentz transformations are true only for the vacuum case, for which the speed of light is equal to c. In any other transparent medium, for which the speed of light is lower than c, the Lorentz transformations do not verify any more these invariance properties.

in a different way as follows. Let us consider, in frame K, a generic event P occurring at a given time t and at a given point (x, y, z) of the space, and let us consider a second reference frame K', moving with uniform velocity V along the x axis, and having its origin O' coincident with the origin O of frame K at time $t = 0$. The amount of time needed by the synchronization signal emitted from P in order to reach the $x = 0$ plane of frame K is $\Delta t = x/v_c$. In this same amount of time, the origin of frame K' will have traveled a distance, along the x axis of frame K, equal to $\Delta x = V\,\Delta t = Vx/v_c$. A synchronization signal, traveling with characteristic speed v_c in frame K, would take a time interval equal to $\Delta t_c = \Delta x/v_c = Vx/v_c^2$ in order to cover such distance. It is therefore possible to write the expression of the generalized coordinate τ' in the following way:

$$\tau' = \gamma_c\,(t - \Delta t_c) \tag{25}$$

where $\Delta t_c = Vx/v_c^2$ and $\gamma_c = 1/\sqrt{1 - V^2/v_c^2}$. Equation (25) shows that τ' represents a *retarded* (or *advanced*, depending on the sign of the characteristic time delay Δt_c) and *scaled* generalized time coordinate which is a function of the position x of the event along the direction of motion of the moving frame K', of its velocity V, and of the characteristic speed v_c of the specific synchronization signal that has been considered. Looking now to the definition of the generalized coordinate ε' associated to the moving frame K', it turns out that it can be interpreted as a *scaled* version of the position $x' = (x - Vt)$ of the event P along the x axis of frame K', that uses the same value of the non-dimensional scaling factor γ_c that enters into the definition of the generalized coordinate τ', that is:

$$\varepsilon' = \gamma_c\,x' \tag{26}$$

For low values of the speed ratio V/v_c the characteristic delay Δt_c tends to zero and the characteristic scaling factor γ_c tends to one, thus the generalized coordinate transformation reduces to the Galilean one in the limit of low values of the speed V of the moving frame K' with respect to the characteristic speed v_c.

IV. Example of application to a specific physical system

Let us consider a circular water basin of radius R, with a flat bottom surface such that the water has constant depth across the entire area of the basin. Let us suppose to have, at the center of the basin, a vertical thin rod in contact with the water surface. By oscillating vertically this rod it is possible to generate waves that propagate from the center to the edge of the basin, traveling with the same speed in all directions by virtue of the intrinsic symmetry of the physical system, as shown in Figure 2. At every instant of time the wavefronts of these waves describe concentric circles and, thanks to the symmetry properties of the system, every wavefront reach simultaneously all the points of the external perimeter of the basin, as it is also known by experience. Indicating with w the value of the radial speed of propagation of the waves, the time elapsed from the start of a given wavefront from the center of the basin to its arrival at the border is given by $\Delta t = R/w$. Because of the symmetry of the system this value is the same for every direction and for every point of arrival on the external circumference.

Let us now consider an observer K that is stationary with the basin, and let us put the origin of the corresponding reference frame at the center of the basin. For this reference frame the mathematical law that characterize the physical phenomenon being considered, i.e. the propagation of the wavefronts from the center of the basin, is given by:

$$v_x = w\,cos(\theta); \qquad v_y = w\,sin(\theta) \tag{27}$$

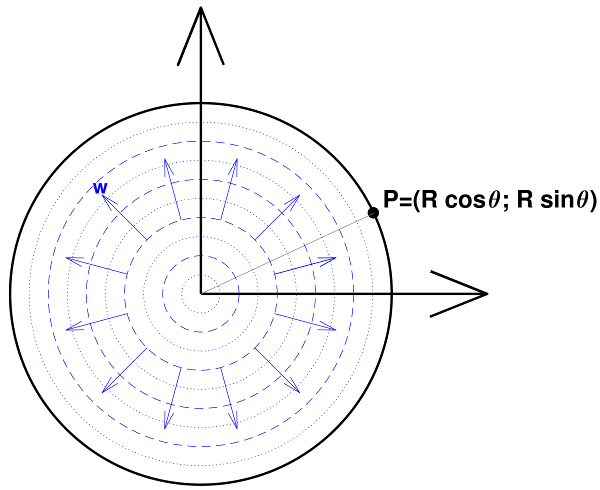

Figure 2: *Circular basin with constant depth water. The dashed and dotted blue circles indicate the position of the wave crests and troughs at a given instant of time.*

where $\theta = atg(y/x)$ and $(x;y)$ are the coordinates of the point. From eq. (27) the magnitude of the speed results

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{w^2\left(cos^2(\theta) + sin^2(\theta)\right)} = w$$

for every value of θ, which reflects the isotropy of propagation of the waves with respect to the observer K.

Let us now consider a moving observer K', with its corresponding reference frame, and let V be speed of this K' frame relative to the observer K. Let us orient the two reference frames such that the x axis of both is directed parallel to V. We want now to transform the physical law of the system, given by eq. (27) for the observer K, into the moving frame K'. We will do this by applying both the Galilean transformations and the Lorentz ones, using the pedices G (Galilean) for the first and R (Relativistic) for the second, obtaining:

$$v'_{Gx} = v_x - V = w\,cos(\theta) - V; \qquad v'_{Gy} = v_y = w\,sin(\theta) \tag{28}$$

$$v'_{Rx} = \frac{v_x - V}{1 - \frac{Vv_x}{c^2}} = \frac{w\,cos(\theta) - V}{1 - \frac{Vw\,cos(\theta)}{c^2}}; \qquad v'_{Ry} = \frac{v_y\sqrt{1-(V^2/c^2)}}{1 - \frac{Vv_x}{c^2}} = \frac{w\,sin(\theta)\sqrt{1-(V^2/c^2)}}{1 - \frac{Vw\,cos(\theta)}{c^2}} \tag{29}$$

where c is the speed of light in vacuum. Thus, when expressed into the moving frame K', the law of the phenomenon being observed has taken a new mathematical form, expressed respectively by eqs. (28) and (29), and the new form of this law now contains also the value of the relative speed V between the two reference frames. As it can be noted by calculating the magnitude of the transformed vector $\mathbf{v}'$, the new mathematical law does not correspond anymore to an isotropic speed of propagation of the wavefronts with respect to K', since the magnitude of both $\mathbf{v}'_{\mathbf{G}}$ and $\mathbf{v}'_{\mathbf{R}}$ is no longer the same for every direction considered, but becomes a function of θ. In fact, it results:

$$|\mathbf{v}'_{\mathbf{G}}|^2 = (v'_{GX})^2 + (v'_{Gy})^2 = w^2 + V^2 - 2wV cos(\theta)$$

$$|\mathbf{v'_R}|^2 = (v'_{Rx})^2 + (v'_{Ry})^2 = \frac{w^2 + V^2 - 2wVcos(\theta) - w^2V^2sin^2(\theta)/c^2}{1 - wVcos(\theta)/c^2}$$

This shows, consistently with what discussed into the first section, that a phenomenon which is characterized by an isotropic propagation speed for a stationary observer, does not appear isotropic to an observer which is translating with velocity V with respect to the first one, and this has been verified by applying both the Galilean transformations of coordinates and the relativistic ones expressed by the Lorentz transformations [5].

Let us now consider the two events that correspond to the start of a given wavefront from the center of the basin and to its arrival at a generic point P on the perimeter. Let us indicate with E_O the first event and E_P the second one. Setting the origin of time in correspondence of the first event, the in-plane and time coordinates of these two events into the reference frame K are:

$$E_O = (x_O; y_O; t_O) = (0; 0; 0) \qquad \textit{and} \qquad E_P = (x_P; y_P; t_P) = \Big(Rcos(\theta); Rsin(\theta); \frac{R}{w}\Big)$$

Let us calculate, using the Lorentz transformations, the space-time coordinates of these two events into the K' moving frame. Putting $\gamma = 1/\sqrt{1 - V^2/c^2}$, this gives, respectively:

$$E'_O = (x'_O; y'_O; t'_O) = (0; 0; 0)$$

$$E'_P = (x'_P; y'_P; t'_P) = \Big(\gamma\big(Rcos(\theta) - V\frac{R}{w}\big); \quad Rsin(\theta); \quad \gamma\big(\frac{R}{w} - \frac{V}{c^2}Rcos(\theta)\big)\Big)$$

The primed time variable of the second event E_P, calculated from the Lorentz transformations is given by

$$t'_P = \gamma R\big(\frac{1}{w} - \frac{V}{c^2}cos(\theta)\big)$$

Interpreting the variable t'_P as the time of the event for the moving observer would thus lead to conclude that the arrival of the wavefront at the perimeter of the basin is no longer simultaneous for all points along the edge, since it would depend on the angular position of point P thru the $cos(\theta)$ function. However, this conclusion is not in agreement with the experience, and it would constitute a violation of the symmetry of the physical system being considered, since it would mean that there are some points on the external circumference, some directions, for which the wavefront arrives earlier than other positions or directions, conclusion which is not in agreement with the symmetry characteristics of the water basin analyzed.

However it is still possible to properly assess the simultaneity of the wavefront arrival events into the moving reference frame K' by using the method that has been presented into Section III, based on the use of the space-time intervals. Let us in fact consider two events corresponding to the arrival of the waterfont at two different points A and B located on the edge of the circular basin. In the stationary frame K, these two events have coordinates:

$$E_A = (x_A; y_A; t_A) = \Big(Rcos(\alpha); Rsin(\alpha); \frac{R}{w}\Big) \quad \textit{and} \quad E_B = (x_B; y_B; t_B) = \Big(Rcos(\theta); Rsin(\theta); \frac{R}{w}\Big)$$

with α and θ representing the angular position of the two selected locations. We can calculate, in the stationary frame K, the relativistic intervals separating these two events from the event E_O that corresponds to the start of the wave from the center:

$$s^2_{OA} = c^2(t_A - t_O)^2 - (x_A - x_O)^2 - (y_A - y_O)^2 = c^2\frac{R^2}{w^2} - R^2cos^2(\alpha) - R^2sin^2(\alpha) = R^2\Big(\frac{c^2}{w^2} - 1\Big)$$

[5] As shown in section III, the only way to maintain the isotropy of propagation of the waves also for the moving observer would be that of using the transformations of coordinates defined by (6), with characteristic speed equal to the speed of propagation of the wave: $v_c = w$

$$s_{OB}^2 = c^2(t_B - t_O)^2 - (x_B - x_O)^2 - (y_B - y_O)^2 = c^2\frac{R^2}{w^2} - R^2cos^2(\theta) - R^2sin^2(\theta) = R^2\Big(\frac{c^2}{w^2} - 1\Big)$$

Thus, the relativistic interval that separates the start of the wavefront from the center of the basin from its arrival on the perimeter is the same for all the locations on the external circumference, it being independent from the angular location of the point of arrival, it results therefore $s_{OA}^2 = s_{OB}^2$ for every couple of points on the external perimeter.

Let us now calculate the relativistic space-time coordinates of the two arrival events with respect to the moving frame K' by applying the Lorentz transformations, thus obtaining:

$$E'_A = (x'_A; y'_A; t'_A) = \Big(\gamma R\big(cos(\alpha) - \frac{V}{w}\big); \quad Rsin(\alpha); \quad \gamma R\big(\frac{1}{w} - \frac{V}{c^2}cos(\alpha)\big)\Big)$$

$$E'_B = (x'_B; y'_B; t'_B) = \Big(\gamma R\big(cos(\theta) - \frac{V}{w}\big); \quad Rsin(\theta); \quad \gamma R\big(\frac{1}{w} - \frac{V}{c^2}cos(\theta)\big)\Big)$$

It is now possible to calculate the relativistic interval between E'_A and E'_O:

$$(s'_{OA})^2 = c^2(t'_A - t'_O)^2 - (x'_A - x'_O)^2 - (y'_A - y'_O)^2 =$$

$$= c^2\gamma^2R^2\Big(\frac{1}{w} - \frac{V}{c^2}cos(\alpha)\Big)^2 - \gamma^2R^2\Big(cos(\alpha) - \frac{V}{w}\Big)^2 - R^2sin^2(\alpha) =$$

$$= \gamma^2R^2\Big[c^2\Big(\frac{1}{w} - \frac{V}{c^2}cos(\alpha)\Big)^2 - \Big(cos(\alpha) - \frac{V}{w}\Big)^2 - \frac{1}{\gamma^2}sin^2(\alpha)\Big]$$

Inserting the expression of γ, expanding the various terms and taking into account that $1/\gamma^2 = (1 - V^2/c^2)$ finally gives:

$$(s'_{OA})^2 = \gamma^2R^2\Big(\frac{c^2}{w^2} + \frac{V^2}{c^2} - 1 - \frac{V^2}{w^2}\Big) = R^2\Big(\frac{c^2}{w^2} - 1\Big) \tag{30}$$

and similarly:

$$(s'_{OB})^2 = \gamma^2R^2\Big(\frac{c^2}{w^2} + \frac{V^2}{c^2} - 1 - \frac{V^2}{w^2}\Big) = R^2\Big(\frac{c^2}{w^2} - 1\Big) \tag{31}$$

which shows that also in this case it results $(s'_{OA})^2 = (s'_{OB})^2$. We have therefore found that also for the moving observer K', the relativistic interval that separates the start of the wavefront from the center of the basin from its arrival on the perimeter, is the same for all the locations on the external circumference, it being independent from the angular location of the point of arrival. Therefore, on the basis of criterion (11) presented into Section III, the simultaneity of the arrival of the wavefronts on the edge of the basin is consistently maintained also for the moving observer and its reference frame, provided that the assessment is based on the criterion of equality of the relativistic intervals.

The same considerations and results would be found also if we consider a different kind of physical phenomenon, like for example an acoustic sonar pulse or a light flash emitted into the water at the center of the basin. Thanks to the symmetry properties of the system being considered, also in these cases the sound, or light, wavefronts will reach simultaneously all the points on the external circumference. Repeating the same previous analysis for this other physical phenomena requires just to replace the value of the characteristic radial speed of the surface water waves w with the speed of sound or with the speed of light into the medium, water in this case. In this way we will therefore find again that the propagation of the sound or light waves will not be isotropical for a moving observer K', and it will be possible to assess the simultaneity of the arrivals of the wavefront at the basin edge by using the criterion based on the equality of the intervals.

V. Physical experiences on the speed of propagation of light

In this section two interferometric experiments on light propagation will be examined, comparing the experimental measurement results with the expected outcomes deriving from the Ritz emission theory that is, as already mentioned, fully in agreement with the Galilean Principle of Relativity and with the associated rule of vector sum of the velocities.

The first, well-known, experience being considered is the Michelson-Morley interferometer that typically has two orthogonal arms. This experiment has been conceived to investigate the possible effects induced on the propagation of light by the motion of the Earth along its orbit through the so-called luminiferous aether, or simply aether, that was supposed being the propagation medium of light. In this experiment all the optical components of the setup - the beam splitter, the mirrors, the target plane where the fringes can be observed - and any transparent optical medium, when present, are rigidly transported by the motion of the Earth along its trajectory. Because of its geometrical layout, shown schematically in Figure 3, the area of the optical path is null, therefore the angular motion of the Earth does not produce any shift of the fringes due to the Sagnac effect which is proportional to the product ΩA, where Ω is the component of the angular speed orthogonal to the plane of the optical path and A the corresponding area.

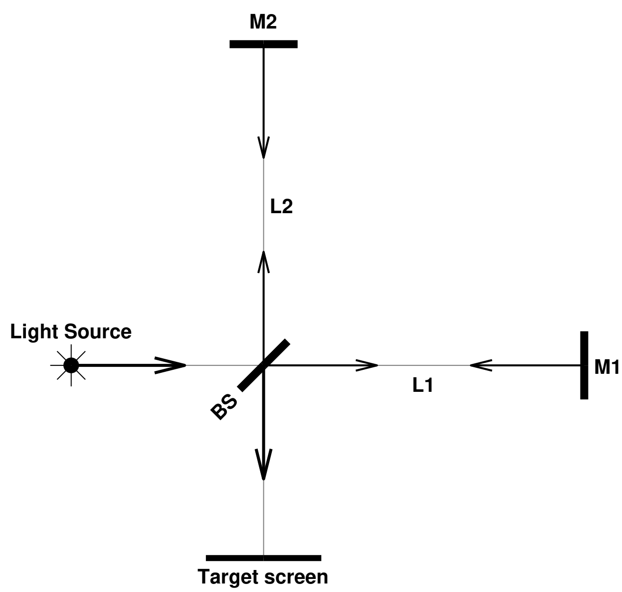

Figure 3: *Schematic layout of the Michelson-Morley interferometer with two orthogonal arms of length L1 and L2, where M1, M2 indicate the mirrors and BS the beam-splitter*

The results of the experiment, performed under a variety of conditions and in different geographical locations and times of the year, have always revealed no effect on the interference pattern deriving from either the speed or the orientation of the interferometer. The same null results have been obtained both in vacuum and in presence of a transparent medium having an index of refraction greater than one, for which the speed of light is less than c. These null outcomes of the test are fully consistent with the Galilean Principle of Relativity and the isotropy of propagation of light. In the hypothesis of the aether, instead, there should be a variation into the observed fringe pattern - unless also such hypothetical medium is rigidly following the motion of the Earth - as a result of a change of the orientation of the interferometer that has been purposely induced experimentally by mounting the entire optical setup on a heavy concrete platform floating over mercury and making it rotate around its vertical axis. However, no effect on the fringe pattern due to the rotation of the interferometer has been experimentally observed.

The null results of the experiment can be explained immediately by assuming, as in the emission hypothesis of W. Ritz, that light is always emitted and propagated with the same relative speed, equal to c/n, with respect to the optical components of the test setup, along both arms of the interferometer. This conclusion is valid both in vacuum and in presence of a transparent medium, the only difference between the two cases being the actual value of the relative speed of light. For an observer that is at rest with respect to the test apparatus, the light propagation is isotropic, having the same speed along the two orthogonal arms of the interferometer, and it will remain isotropic also when the test setup is moving with a non-null constant velocity V. On the contrary, an observer that is in a state of uniform relative motion with respect to the test apparatus would notice a non-isotropic propagation of light, with different values of the speed along different directions, in agreement with the Galilean vector sum of the velocity vectors. However, since also the optical components of the test setup would have different velocities with respect to this observer, determined by the same vector sum rule, the calculation of the time taken by light to travel the optical path along the two arms of the interferometer will give the same results obtained by the observer at rest, and therefore no alteration of the fringe patterns has to be expected from such calculation, in agreement with the experience.

In the Fizeau experience[4], the propagation of light into a stream of water flowing within pipes has been investigated by analyzing the fringe patterns generated at the recombination of two light beams that are counter-propagating in the fluid stream. The analysis of the test results obtained by Fizeau with its original apparatus, shown schematically in Figure 4, led to the following expression for the relative velocity W_F of the light with respect to the stationary system of the laboratory:

$$W_F = w + v(1 - \frac{1}{n^2}); \tag{32}$$

where v is the velocity of the fluid flowing into the pipes having circular cross-section, and $w = c/n$ is the speed of light into the fluid being utilized for the test, characterized by an index of refraction equal to n when such fluid is stationary.

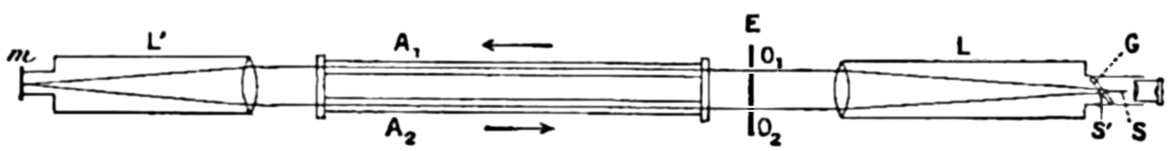

Figure 4: *Original layout of Fizeau's experiment*

We want now to investigate if the experimental result expressed by eq. (32) that was derived by Fizeau, can be obtained by applying the Ritz emission hypothesis and the associated Galilean vector sum of the velocities of light into the propagation medium and the velocity of the fluid. This requires the determination of the actual value of the speed v of the fluid flow to be used in the vector sum formula, since in this kind of experiment the propagation medium used, typically water, is not characterized by a common and uniform state of motion of all its particles inside the volume occupied. The motion of the fluid in fact cannot be represented as a pure rigid body translation with constant speed, therefore there is not just a single value of the velocity for the entire fluid, but a rather complex velocity field with a distribution that varies from point to point inside the volume of the pipes.

In addition, the specific geometrical layout of the Fizeau test setup introduces several factors that can affect the characteristics of the interference fringes formed at the recombination of the two counter propagating beams, in particular:

1. the non null area of the optical path, because of which interference fringes can arise also in absence of fluid motion (and actually also without any fluid) due to the Sagnac effect associated to the Earth's rotation;
2. the radial shape of the velocity profile of the fluid motion at the various sections of the pipes;
3. the axial speed of the fluid flow which varies along the pipe length and therefore along the optical path;
4. the non-axial components of the fluid velocity associated to a turbulent regime of the flow.

The Sagnac effect can be considered as a constant bias, since both the angular velocity of the laboratory where the experiment is performed and the area of the optical path, determined by the geometrical layout of the test setup are not varied during the execution of the measures, therefore the product ΩA remains constant.

The radial distribution of the axial velocity profile of the fluid flow can cause a distortion of the shape of the incident wavefront of the light beam. For the beam propagating in the same direction of the fluid stream an incident planar wavefront could be deformed in a way similar to that of a plane-concave lens, since the equivalent optical length of the light rays closer to the centerline of the pipes will be shortened by the dragging effect due to the fluid flow more than that of the rays travelling farther from the pipe centerline. Conversely, the wavefront deformation associated to the light beam traveling against the fluid stream should be similar to the one generated by a plane-convex lens. On the recombining plane, the interference pattern generated by the two counter-propagating beams would be affected by the actual shape of the two distorted wavefront and in particular this effect could generate a variation of the fringe spacing if the shape of the radial velocity profile changes as a result of changes of the average flow rate. This effect should be more pronounced comparing the distortion associated with a laminar flow regime, characterized by a parabolic velocity profile, with respect to that of a turbulent flow regime, where the velocity profile in the central portion of the pipes is more flat. The two different shapes of the velocity profiles for laminar and turbulent regimes are shown qualitatively in Figure 5

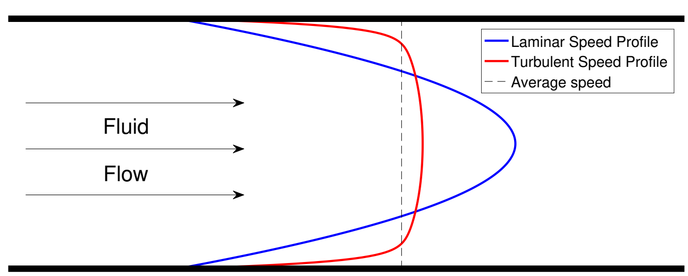

Figure 5: *Laminar vs turbulent velocity profiles inside circular pipes*

The axial velocity of the flow is also not constant along the length of the pipes, and therefore along the optical path traveled by the light beams into the moving fluid. The total amount of dragging effect to be added, or subtrated, to the speed of propagation of the light would be dependent from the integral, across the entire length of the optical path, of all the local values of the axial fluid velocity on every section of the pipes. This calculation is not straightforward, being the actual velocity field quite complex, especially in the transition regions close to the end of the tubes. Using longer tubes could help in reducing the sensitivity of the results to the local effects concentrated at the ends of the tubes, but even along the central straight portion of the pipes the flow regime would reach a stable, fully developed state with a constant radial velocity profile distribution, only after some distance along the pipe. Overall, this variation of the speed profile along the length of the pipes will have the effect of reducing the average axial speed seen by the light beam at the center of the pipes, with respect to the value determined at a specific section, typically located towards the exit end of the pipes, where the actual velocity profile measurement is performed and the value of the velocity at the center of the flow is determined.

In the case of turbulent flow regime, which is the actual flow regime used for the measures in the original Fizeau experiment, the velocity field of the fluid flow is characterized by having also radial components of the fluid velocities in addition to the axial ones. These radial components are associated to the presence of fluid vortices, typical of the turbulent flow regime, having different scales, and which can be randomic and non-stationary. In this regime, the motion of the fluid particles with respect to the stationary frame of the laboratory does not corresponds to a pure axial motion along the axis of the pipes, but contains also circular components, due to the vorticity of the flow, with the associated accelerations. Under these conditions the invariance of the physical phenomena asserted by the Galilean Principle of Relativity is no longer applicable, therefore it may be possible that the physical properties of the entities involved in the test are somehow affected by the accelerated state of motion of the particles that constitute the system being observed, thereby changing to some extent the values of their physical characteristics with respect to the corresponding values determined under stationary conditions. In particular, in the case of the Fizeau's experiment, the specific state of motion associated to turbulence could have an impact on the light propagation inside the transparent medium flowing into the pipes. On average it could introduce an additional *"dragging"* term, generated by the circular motion of the fluid into the turbulent vortices, that creates an additional delay of the axial propagation of the light beam. In other terms, the turbulent motion of the fluid, with the associated vortices, can have the effect of reducing the average equivalent propagation speed of the light beam inside the fluid which therefore would have a greater index of refraction in turbulent conditions with respect to the stationary case.

In order to separate this term from the ones associated to the variation of the axial components of the fluid velocity, it can be taken into account by including into the equation an *"equivalent"* index of refraction n^*, which would be dependent on the level of turbulence of the fluid, and would take values greater than the one corresponding to the stationary fluid, i.e. $n^* \geq n$. Being associated to the presence of a turbulent flow regime, such equivalent index of refraction can be expressed as a function of the Reynolds number Re that is used to characterize the level of turbulence of the flow: $n^* = n^*(Re)$. For low values of the Reynolds number, within the laminar range, n^* would be equal to the index of refraction of the stationary fluid, whereas for Reynolds number values greater than the threshold corresponding to the onset of turbulent flow, an increase of n^* with the Reynolds number could be expected.

Taking into account of these effects, the expression of the relative speed of light W with respect to the stationary observer, calculated on the basis of the classical Galilean rule of vector sum, can be written in the following form:

$$W = w^* + \bar{v} = \frac{c}{n^*} + \frac{1}{L}\int_0^L v(x)\,dx; \tag{33}$$

where the first term accounts for the effect of variation of the refraction index, and the integral of the second term is extended to the entire length L of the optical path of each light beam. It appears therefore, in particular on the basis of items 3) and 4) above, that the actual value of the speed of light measured with respect to a stationary observer should be lower than the value predicted by the Galilean formula of speed composition, when that formula is evaluated using the peak value of the fluid speed inside the pipes and the nominal value of the refraction index of the stationary fluid, and this result is consistent with the outcome of the experiment. The actual amount of deviation would depend on the specific characteristics of the experimental setup being considered, in particular for what concerns its hydraulic characteristics and parameters.

Recent repetitions of the Fizeau experiment have highlighted that the effects due to turbulence could be the major contributor to the fringe shift observed as a result of the variation of the fluid flow rate and average velocity. In particular Lahaye *et al.* [5] explicitly mention that for low value of the fluid speed, $\bar{v} < 1$ *m/s*, it has not been possible to acquire any valid test point because of the difficulties in getting stable pictures on the digital sensor used to detect the fringes and their variation. Also in a similar work from Maers *et al.*[6] the experimental data of fringe shift versus flow velocity-difference have been measured only for water velocities in the range $0.5 < \bar{v} < 3.6\ m/s$ for which the flow is fully in the turbulent regime, having Reynolds number in the range $12.700 < Re < 91.400$.

The expression of the Reynolds number for the flow into circular pipes is:

$$Re = \frac{\rho D}{\mu} v = \frac{\nu}{D} \bar{v} \tag{34}$$

where ρ is the density of the fluid, μ and ν represent its dynamic and kinematic viscosity, D is the pipe diameter and $\bar{v}$ the macroscopic velocity of the fluid flow. According to the previously described assumption we could write the dependency of the equivalent refraction index from the Reynolds number as follows:

$$n^* = \begin{cases} n & if \quad Re < Re_L \\ n + \alpha(Re - Re_T) & if \quad Re > Re_T \end{cases} \tag{35}$$

where α is a constant to be determined, and Re_L and Re_T represent, respectively, the Reynolds numbers corresponding to the end of the laminar flow regime and to the onset of the turbulent one. Considering velocities of the fluid flow in the turbulent range, $\bar{v} > v_T$, it is therefore possible to put the expression of the equivalent index of refraction of the turbulent fluid in the form:

$n^* = n(1 + \delta)$ where $\delta = \delta(Re) = \frac{\alpha(Re - Re_T)}{n} \ll 1$.

The evaluation of the resultant speed of light into the moving turbulent flow using expression (33) would require calculating the integral of all the local axial velocities of the fluid along the optical path, but this in turn would require the precise knowledge of the flow field in each point into the pipes, which is not available. Due to the lack of detailed knowledge of the flow velocity field at every position inside the pipes, it will be assumed, as stated also in [6], that the velocities of the fluid are constant in the straight sections of the tubes through which the light beams travels, and have a radial profile typical of a turbulent regime. In this way the expression of the resultant speed of light becomes:

$$W = \frac{c}{n(1 + \delta)} + \bar{v}; \tag{36}$$

Taking into account that $\delta \ll 1$, it is possible to expand the first term of (36) into powers of δ. Making then use of the definition of δ and truncating the expansion to first order it results:

$$W = \frac{c}{n}(1 - \delta + \delta^2 ...) + \bar{v} \simeq \frac{c}{n}(1 - \alpha\frac{\Delta Re}{n}) + \bar{v} = \frac{c}{n} - \alpha\frac{c}{n^2}\Delta Re + \bar{v}$$

where $\Delta Re = (Re - Re_T) = \frac{\nu}{D}(\bar{v} - v_T)$. Substituting this into the previous equation gives:

$$W = \frac{c}{n} + \bar{v} - \alpha \frac{c}{n^2} \frac{\nu}{D}(\bar{v} - v_T) \tag{37}$$

Putting now

$$\alpha = \frac{D}{\nu c} \tag{38}$$

the expression of the light speed with respect to the stationary observer finally becomes:

$$W = \frac{c}{n} + \bar{v}\left(1 - \frac{1}{n^2}\right) + \frac{1}{n^2} v_T \tag{39}$$

that corresponds to the expression obtained by Fizeau by taking into account that the term v_T/n^2 can be neglected, being much smaller, by several orders of magnitude, than the other constant term c/n.

The above derivation shows that it is possible to provide an interpretation based on the Galilean vector sum of velocities of the experimental results obtained by Fizeau, without the need to invoke any space-time distortion. The proposed approach is based on the hypothesis that the index of refraction of the fluid is altered by the turbulent flow regime. This hypothesis could be verified by further experimental investigations of the optical properties of fluids under turbulent flow regimes, or by a theoretical analysis that would however require to have a very detailed model representing the complex velocity field of the fluid under such flow regime.

VI. A test case for the velocity composition rule

In this section a test case is proposed to investigate the validity of the Galilean velocity vector addition rule versus the relativistic one that derives from the Lorentz transformations. The test is based on the analysis of the phenomenon of stellar aberration, i.e. on the observed variation of the position of the celestial objects as a function of the motion of the observer and of its velocity, motion that coincide with that of the Earth along its orbit in the case of a terrestrial telescope. Since the two formulas for the composition of the velocity of the light with the velocity of the observer are different, the expected variation of the position of the star evaluated by means of the relativistic rule is different from that obtained with the classical vector sum rule, and the amount of the difference depends on the value of the ratio of the speed of the observer with respect to the speed of light. Being the orbital velocity of the Earth about 10^4 times smaller than c, such differences are very small and their analysis therefore requires very accurate measurements of the observed position of the celestial objects in order to resolve the differences between the two cases.

Let us consider the light coming from a very far celestial source, such that the corresponding wavefront can be considered planar over the entire area of the Earth's orbit. For an observer at rest into the center of mass of the Solar system the position of this source is fully characterized by two angles which can be expressed as the in-plane azimuth angle and the out-of-plane elevation angle with respect to the plane of the Earth's orbit.

Let $\mathbf{V}$ be the velocity vector describing the motion of an observer that is moving into the ecliptical plane. Let $\mathbf{c}$ be the vector defining the velocity of propagation of the incoming light with respect to the stationary frame, and let us consider a moving reference frame having its x axis aligned with the direction of the velocity vector $\mathbf{V}$ of the observer and the y axis lying into the plane formed by the direction of the incoming light and $\mathbf{V}$. The resultant vector $\mathbf{c}'$ that defines the apparent position of the light source for the moving observer, will also lie into the xy plane according both to the Galilean vector-sum rule and to the relativistic velocity-composition rule.

However, the observed variation of the angle of incidence, i.e. the amount of *aberration*, is different in the two cases. It can be calculated by applying the two velocity composition rules and focusing the analysis on the x and y components of the vectors.

Let us define, in the reference frame of the Sun, the direction of the light source by the angle θ that the incoming light vector makes with the direction of the velocity of the observer. Let v by the speed of the observer, which is assumed to be directed along the positive direction of the x axis of the observer's reference frame, and $\beta = v/c$ be the ratio of the observer speed with respect to the speed of light. Let us indicate with θ' the aberrated direction of the source as seen by the moving observer. The relationship between the angles θ and θ', derived respectively from the classical vector sum and from the relativistic velocity composition rule, is given by the following two exact trigonometric expressions, indicated by the pedices G and R, respectively:

$$sin(\theta - \theta'_G) = \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \tag{40}$$

and

$$sin(\theta - \theta'_R) = \beta \frac{sin(\theta)}{1+\beta cos(\theta)} + \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)(1+\sqrt{1-\beta^2})} \tag{41}$$

For very small values of the observer speed, compared to the speed of light, the difference between the two angles θ and θ' is also very small, therefore it is possible to determine the solution of the above expressions by approximating the sine function with its argument, $sin(\theta - \theta') \simeq (\theta - \theta')$, thus giving:

$$\theta'_G = \theta - \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \tag{42}$$

and

$$\theta'_R = \theta - \beta \frac{sin(\theta)}{1+\beta cos(\theta)} - \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)(1+\sqrt{1-\beta^2})} \tag{43}$$

The two expressions (42) and (43) allow to calculate the expected apparent position θ' of the light source for the moving observer when the corresponding position θ of the celestial object into the stationary frame is known.

Conversely, in order to perform the calculation of the *un-aberrated* position of the source starting from the one observed into the moving frame, it is necessary to use the inverse relationships between θ and θ' that are given by:

$$\theta_G = \theta' + \beta sin(\theta') \tag{44}$$

and

$$\theta_R = \theta' + \frac{\beta sin(\theta')}{1-\beta cos(\theta')} + \frac{sin(2\theta')}{2} \frac{\sqrt{1-\beta^2} - 1}{(1 - \beta cos(\theta'))}$$

When $\beta \ll 1$ this last expression can be rewritten as a power series of β truncated to the term of second order, giving:

$$\theta_R \simeq \theta' + \beta sin(\theta') + \frac{1}{4}\beta^2 sin(2\theta') \tag{45}$$

The comparison of equations (44) and (45) shows that the reconstructed position of the light source calculated using the relativistic formula differs from the one obtained from the Galilean vector sum by a term which is quadratic into β. For a given value of the velocity v of the observer, the amplitude of this term depends on the angle between the incident light and the direction of the velocity vector of the observer, being maximum when $|sin(2\theta')| = 1$, therefore when $\theta' = \pi/4 + k\pi$, and being null when the observer velocity is either parallel or it forms a right angle with respect to the direction of the incident light.

Let us now consider the case of an observer moving around the Sun with constant angular velocity Ω on a circular orbit having radius R, and of a distant light source located into the same plane of this orbit and stationary with respect to the Sun, as shown in Figure 6. The vector of the observer velocity always lies into the plane of the orbit, therefore in this case the aberration of the incoming light produces, for such moving observer, an apparent motion of the source which is also always lying into the same plane of the orbit. For this orbiting observer the stationary light source thus shows an apparent oscillation of its position along an horizontal line parallel to the plane of the orbit and characterized by the same time period of the orbit.

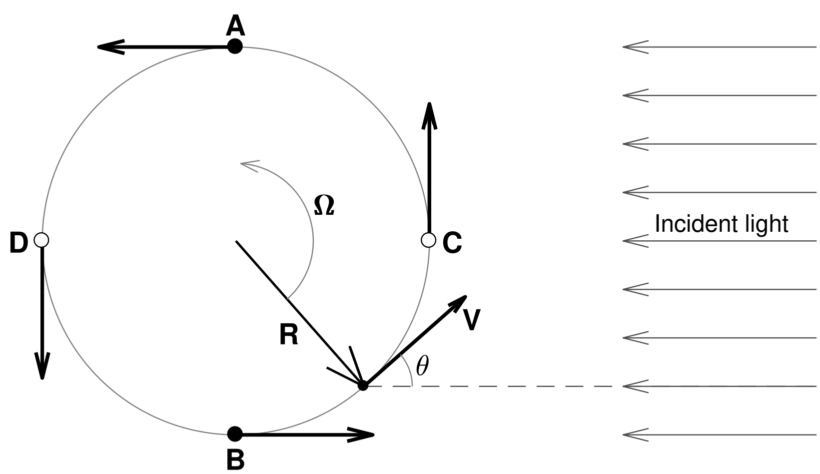

Figure 6: *Orbiting observer with complanar light source*

It is possible to identify four notable locations along the orbit which are significant because of their peculiar properties with respect to the aberration of the source. In the two positions labeled **A** and **B** the velocity of the observer is parallel to the incident light, therefore when the moving observer is in these points of the orbit there is no aberration of the incoming light and the observed position of the star coincides with the one observed into the stationary frame of the Sun. The position of the celestial object observed in these two points can therefore be taken as a reference position, since it requires no calculation in order to remove the aberration term.

Conversely, when the moving observer is in the two locations labeled **C** and **D**, its velocity is orthogonal to the direction of the incident light. In these two locations there is the maximum aberration of the apparent position of the star. However, the value of the aberration term is the same for both the classical and the relativistic rule. Therefore, the calculation of the un-aberrated position of the light source, by means of equations (44) or (45) leads to the same result for both

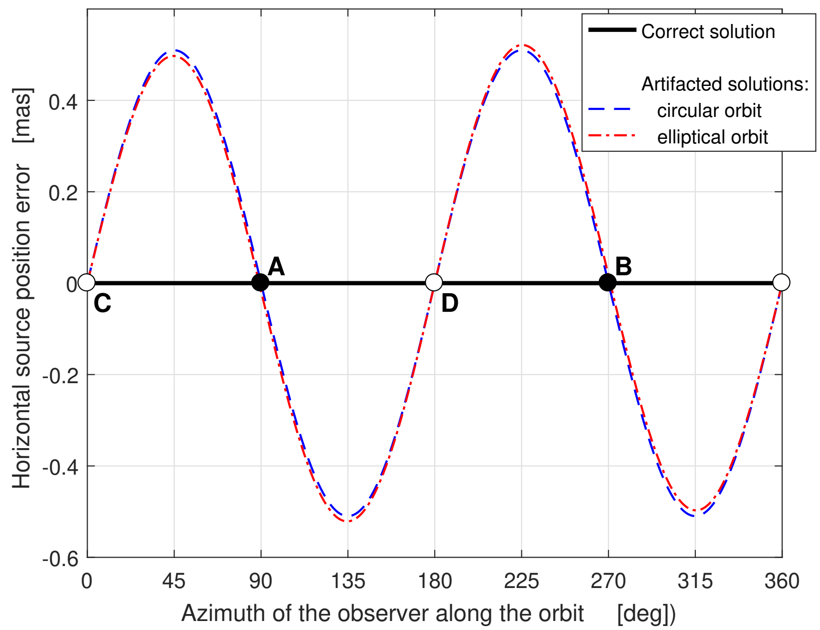

Figure 7: *Comparison of the un-aberrated position of the light source calculated by means of the two different velocity composition rules for an Earth based observer*

the classical and the relativistic rule. In the particular case of a stationary source considered here, the position of the source calculated by the moving observer located in these two points results coincident with the position observed at locations **A**, **B**.

For any other point of the orbit, the un-aberrated position of the source calculated by means of the classical rule will be different from the one obtained from the relativistic formula, and the maximum difference between the two results will occur when the moving observer is at the midpoints between **A**,**B** and **C**,**D**, i.e. at an azimuth angle along the orbit of $\psi = \pi/4 + k\pi/2$. Assuming a stationary source, since the angle between the light direction and the velocity of the observer is $\theta = \Omega t$, one of the two computed results, either the Galilean or the Relativistic one, will contain an harmonic oscillation of the horizontal position of the celestial object, having amplitude equal to $\beta^2/4$, and with period equal to one half the period of the observer's orbit. Such peculiar behaviour of the reconstructed position of the distant light source, characterized by a twice per revolution oscillation that constitutes its specific signature, represents an artifact of the calculated solution, artifact which is due to the inconsistency of the analytical formula used with respect to the actual rule followed by the physical phenomenon.

Let us now consider the case of a terrestrial observer and let's approximate the Earth's orbit with a circle of radius $R = 150x10^6\,km$, and period T equal to one year. In this case the orbital speed is constant and its value is $v \simeq 30\,km/s$, which gives $\beta \simeq 10^{-4}$.

With these values of the orbital parameters the two resulting curves of the calculated horizontal position of the source, deriving from the application of equations (44) or (45), are shown in Figure 7. In this figure, also the resulting artifacted solution calculated taking into account the elliptical shape of the Earth's orbit is presented. Due to the small eccentricity of the actual orbit, the deviations of these results from the reference case of a circular trajectory are very small, as shown in the graph that has been calculated considering a celestial object aligned to the major axis of the ecliptic.

The values of the un-aberrated position of the source corresponding to the four notable orbital locations **A**,**B** and **C**,**D** are indicated in Figure 7 with the same markers used in the previous Figure 6. Both the correct and the artifacted curves pass through points **A** and **B**, since for these locations there is no aberration at all and the value of the horizontal position of the celestial object is given directly by the observed position. Both curves also give the same results for locations **C** and **D** where the velocity of the observer is orthogonal to the incoming light direction. [6]

The above described artifact, characterized by its twice per revolution frequency content, must be present in either the classical or the relativistic computed results, and has the same specific signature characteristics for any observed stationary source lying into the orbital plane, with almost the same amplitude of oscillation and with the same frequency content, independently from the specific celestial object or the specific region of the electromagnetic spectrum being observed.

When the celestial object being analyzed does not lie into the orbital plane there will be a contribution due to aberration also in the out-of-plane position of the source. Considerations similar to those discussed for an in-plane source apply also to this more general case: the vertical component of the calculated position of the source will contain a twice per revolution spurious term in either the classical or the relativistic results. The amplitude of the artifacted vertical component is null when the celestial object is located in the orbital plane, it then increases with the out-of-plane elevation of the source, reaching a maximum for an elevation angle of $\pi/4$, for which the term $\beta^2 sin(2\theta')$ is maximum. For elevations greater than $\pi/4$ the amplitude of the vertical spurious term will then decrease again and will become zero for circumpolar objects, for which also the in-plane component vanishes.

The presence of a twice per revolution frequency term into the computed results of the un-aberrated position of stationary celestial objects is therefore a general characteristics, a specific signature, that allows to identify the incorrect velocity composition rule between the two that have been analyzed.

VII. Conclusions

In the previous sections it has been shown that simultaneity of events can be assessed in a unique and consistent way by using a general method of clock synchronization that does not necessarily require the use of light signals. By using this method two events that are simultaneous for one observer result simultaneous also for another observer that is moving with respect to the first one. This shows that the concept of simultaneity is independent from the state of motion of the observer and from the specific clock synchronization signal that has been selected, and such absolute nature of simultaneity allows to introduce a definition of time which is common for all observers.

The above considerations have led to an alternative physical interpretation of the Lorentz transformation of coordinates and suggest that some interferometric experiments on light propagation can be explained without invoking the space-time deformation assumed by the Theory of Relativity, and by applying, instead, the Ritz emission theory[3] which assumes that light is always emitted with the same relative speed, equal to c in vacuum, with respect to its source.

Finally, a test case to discriminate between the Relativistic and the Galilean velocity composition rules has been proposed. The test is based on the analysis of the aberration of the light coming from stationary celestial objects as perceived by an orbiting observer, and on the different results

[6] In the general case of a non stationary source, the corresponding computed value of the horizontal position of the object evaluated at **C** and **D** could differ from the one corresponding to the reference locations **A** and **B**.

obtained by using the two different velocity composition formulas to remove the aberration term from the observed position of the various light sources of the sky. In order to be applied to measured data, this comparison requires that the observed position of the sources is determined with high accuracy, since the differences that have to be investigated are of the order of milliarcseconds, a level of accuracy that should be achievable by the most advanced large ground telescopes or space based astrometric instruments like, for example, the Gaia scientific satellite.

Should the outcome of the test be in favour of the classical Galilean velocity vector sum, this could constitute a supporting element to reconsider the validity of Ritz emission theory in place of the Special Theory of Relativity. Despite having radical differences in their fundamental assumptions, the two theories share some important aspects that marked a sharp distinction from the approach previously adopted for the analysis of electromagnetic phenomena and for classical mechanics. Regarding the propagation of light both theories negate the existence of the aether, whilst for what concerns mechanics and the dynamics of motion of bodies, in both theories the interactions between non-coincident physical entities are not instantaneous as it was assumed in the Newtonian approach. Because of the assumption of instantaneous action at distance, the equations of motion of classical Newtonian mechanics contain, as stated by L. Landau[7], "a certain degree of imprecision". The removal of the hypothesis of instantaneous action at distance, which is inherent into the action-reaction principle when applied to physical entities having a non-null geometrical separation between them, allows both theories to provide the correct predictions of the precession of the motion of the perihelion of Mercury. It may be possible, therefore, that also other experimental observations that have been considered as being in agreement with the Theory of Relativity could find an alternative interpretation not based on the concept of space-time deformation.

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