Statement of Quantum Indeterminacy

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Abstract This article is a concise statement of the machinery of quantum indeterminacy — in response to the question: What is indeterminacy; is it something that can be written down?

Keywords foundations of quantum theory, quantum randomness, quantum indeterminacy, logical independence, self-reference, logical circularity, mathematical undecidability, Kurt Gödel.

1 Philosophy

The mathematics of quantum indeterminacy will be most unusual to physicists. Its principles are in *stability* and *non-prevention*, rather than *cause*. It has basis in Mathematical Logic, and reliance on the distinction between *true* and *provable* statements, made infamous through the work of Kurt Gödel.

The historical tradition in Physics has been to explain phenomena in terms of factors that cause them. This has meant looking for Postulates and Principles which imply physical consequences; along with mathematical framework that conveys those implications. In contrast, indeterminacy does not stem from rules that cause it, but from mathematical freedoms that do not prevent it. Indeterminacy is a matter of how rules of physics are carried out, not one of the rules themselves — a matter of information processing, not the substance of the information itself.

In 1930, Mathematics suffered a crisis of uncertainty after Kurt Gödel announced his First Incompleteness Theorem. Its consequence today is that there are statements within Applied Mathematics that are *true* but *not provable*. One such statement concerns existence of the square root of minus one. Knowing precisely what drives necessity for this number's presence in Quantum Theory resolves the question of quantum indeterminacy.

2 The Statement

Quantum indeterminacy is an association of *uncausedness* and *indefiniteness*. These peculiarities are inherent in transformations that fix *unitary* symmetries — without which *mixed states* cannot form.

Unitary symmetries are a consequence of self-reference propagating around *cyclic sequences* of transformations, where the requirement for self-consistency is satisfied by an imaginary scaling. The self-reference is an available process because it contradicts no rules of Applied Mathematics; it can be considered an uncaused, and unprevented, but stable process. Stability is met when the sequence of transformations coincidentally aligns to form symmetries. However, by definition, symmetries present *referential ambiguities* such as left|right handedness; and these are the source of indeterminacy's indefiniteness.

By way of the above, prepared mixed states convey information about orthogonality and handedness, as well as, information about imaginary scalings. The act of measurement involves conversion forth-and-back between this 'geometric' information and scalar information. But those conversions are not bijective; measurement retrieves the prepared orthogonal orientation, but reads handedness as ambiguous.

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3 Machinery

1. **Rules**
   (a) In an efficient set of rules or *axioms*, no one axiom can be proved or disproved from the others.
   (b) Rules are applied continually.
2. **Consequence and independence**
   (a) A definite set of mathematical axioms proves a definite set of theorems.
   (b) Alongside the theorems, there is a definite set of mathematical statements (in the same language) which those axioms neither prove nor disprove. These statements are known as *independent* (or *undecidable*).
   (c) In Applied Mathematics, that independent set is generally infinite.
3. **Outside information**
   (a) If a statement cannot be proved by the axioms, then independent statements can be brought in from *outside* the axiom system, to complete the proof.
   (b) It would seem intuitive that all implications of theorems are themselves theorems. However, when theorems are asserted *self-referentially*, a new independent statement is necessary in maintaining self-consistency.
4. **Net axiom content**
   (a) There is a *minimal* set of axioms — containing no superfluous information — that prove theorems that are statements capable of representing pure quantum states.
   (b) Mathematical statements capable of representing *mixed states* require an extra axiom, rule or statement — which can neither be proved nor disproved by the pure state axioms.
5. **Transformations**
   (a) Certain provable theorems of the minimal set of pure state axioms are transformations. Asserted self-referentially these *fix* new (orthogonal) unitary symmetries; all other outcomes being self-inconsistent and impossible.

Transformations can be regarded as machines acting on all vectors available to them in their environment. Machines whose output is identical to input, are by chance coincidence, capable of continual feedback; forming stable systems with their vectors. More complicated machines are possible involving chained sequences of transformations. Any composite transformation which accomplishes the feedback is an *identity mapping*; with its machine stability being reliant on self-reference or logical circularity. This self-reference *fixes* and sustains the persistent stability of entities like: eigenvalues, eigenvectors and (orthogonal) unitary symmetries [1, 2], but at a logical cost. Regrettably, the formalisms of Applied Mathematics does not express this logic. Examples are:

$$[\mathsf{A}/\lambda]\,\mathsf{x} \mapsto \mathsf{x} \qquad \mathsf{F}\mathsf{F}^{-1}\mathsf{x} \mapsto \mathsf{x} \qquad \mathsf{CABCAB}\mathsf{x} \mapsto \mathsf{x}$$

As the imaginary unit is independent, neither its existence nor non-existence is provable from axioms. But in orthogonal function spaces, the imaginary unit is implied in '3-way orthogonality'. So, the process of forming mixed state superpositions inadvertently enacts a new axiom, now denying non-existence of the imaginary unit. This is new information from outside the original axiom set. In the new orthogonality, there is new ambiguity in left|right handedness, which can be broken, only by distorting it, by way some outside agency — an electric field maybe.

Remark Physicists regularly agree on right handed frames for 3-space; but this is never expressed in the algebra; one physicist must show the other a 3d model.

References

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The detail is given in my book [pdf available on request]:

The Underlying Machinery of Quantum Indeterminacy — The Answer to a Century of Questions