Observations of structure of a possible unification algebra

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Abstract. A C-loop algebra, designated $\mathbb{U}$ is assembled as the product: $M_4(C) \otimes \mathbb{T}$. When $M_4(C)$ is assigned to represent $Cl_{1,3}(R) \otimes \mathbb{C}$ and the principle of spatial equivalence is invoked, a sub-algebra designated $\mathbb{W}$ is found to have features that suggest it could provide an underlying basis for the standard model of fundamental particles. $\mathbb{U}$ is of the same order as $Cl_{0,10}(R)$, but has a "natural" partition into $Cl_{1,3}(R) \otimes \mathbb{C} \otimes \mathbb{W}$, suggesting that its use in string/M theories in the place of $Cl_{0,10}(R)$ may generate a description of reality.

1. Introduction

This paper describes algebraic structures using labels for unit elements which highlight features related to spatial equivalence, as set out in section 2.

Sections 3 documents the algebraic structures. They have been investigated by using the multiplication tables for unit elements combined with random coefficients to generate random products, which are then used to check the properties of the algebras, testing for distributivity, associativity, flexibility, alternativity and power associativity. The Loops package[1] for GAP4[2] has also been used.

In sections 4 and 5 observations and postulates are presented relating the algebraic structures to physics. The author has limited understanding of subjects such as torsion, manifolds and fiber bundles, so the postulates are speculative, but they demonstrate the potential for the algebraic structures to provide a basis for the unification of general relativity with quantum mechanics.

Sections 6 to 9 present details relating to the assembly of the algebraic structures and their properties.

Contents

2. Notation for algebras $\mathbb{T}$, $\mathbb{M}$, $\mathbb{U}$, $\mathbb{W}$ and $\mathbb{D}$

Labels for algebras and their unit elements are based on patterns related to spatial equivalence when a sub-algebra is used to represent the space-time Clifford algebra.

Unit elements for $\mathbb{M} \cong M_4(C)$ are represented by:

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
S	L	M	N	V	D	E	F	iU	iX	iY	iZ	iT	iP	iQ	iR
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
iS	iL	iM	iN	iV	iD	iE	iF	U	X	Y	Z	T	P	Q	R

Unit elements for the trigintaduonion algebra, $\mathbb{T}$, are represented by:

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ

Cayley tables for these algebras are shown in section 7. They have been arranged so that, if the signs of products are ignored, they form the same latin square. This is referred to as "alignment" of the algebras. Note that the subscripts $_\iota, _\jmath, _\kappa$ identify orientation with respect to iX, iY, iZ for the alignment.

An algebra labeled $\mathbb{U}$ is generated as the tensor product $\mathbb{T} \otimes \mathbb{M}$. For $\mathbb{U}$, all unit elements of its sub-algebra, $\mathbb{T}$, commute with all unit elements of its sub-algebra, $\mathbb{M}$. Unit elements of $\mathbb{U}$ are labeled using combinations of the labels assigned to $\mathbb{T}$ and $\mathbb{M}$, such as $\nu_\kappa iR$. The labels are listed in Section 9.

A further sub-algebra of $\mathbb{U}$, labeled $\mathbb{W}$, is identified which has a Cayley table, shown in section 3.4.2, which is aligned with those of $\mathbb{T}$ and $\mathbb{M}$. It is a "resonant" subalgebra of $\mathbb{U}$, where "resonance" is defined as meeting a requirement for spatial equivalence for all sub-algebras when three unit elements of $\mathbb{M}$ are used to represent unit spatial vector elements for a Clifford algebra.

Unit elements for $\mathbb{W}$ are represented by:

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
$\sigma_o S$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_o V$	$\lambda_\iota D$	$\lambda_\jmath E$	$\lambda_\kappa F$	$\mu_o iU$	$\mu_\iota iX$	$\mu_\jmath iY$	$\mu_\kappa iZ$	$\nu_o iT$	$\nu_\iota iP$	$\nu_\jmath iQ$	$\nu_\kappa iR$
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
$\alpha_o iS$	$\alpha_\iota iL$	$\alpha_\jmath iM$	$\alpha_\kappa iN$	$\beta_o iV$	$\beta_\iota iD$	$\beta_\jmath iE$	$\beta_\kappa iF$	$\gamma_o U$	$\gamma_\iota X$	$\gamma_\jmath Y$	$\gamma_\kappa Z$	$\delta_o T$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$

A label, $\mathbb{D}$ is used for algebras isomorphic to the algebra of real 4×4 diagonal matrices.

3. Algebraic structures

3.1. Structure of $\mathbb{M} \cong M_4(C)$

The structure of $\mathbb{M}$ is well known.

3.1.1. Embedded group and automorphism group. $\mathbb{M}$ contains an embedded group, the Dirac matrix group, of order 64, generated by its 32 basis elements. The automorphism group for the Dirac matrix group, as determined using the Loops package for GAP4, has 42 conjugacy classes, and its structure description is:
((C2 x C2 x C2 x C2) : A6) : (C2 x C2)

3.1.2. Sub-groups of order 32. The Dirac matrix group has 31 sub-groups of order 32, which can be sorted into 3 classes, as shown in table 10 in section 6.

3.1.3. Sub-groups of order 16. The Dirac matrix group has 155 sub-groups of order 16. There are five types of these sub-groups which differ in the signature of their unit components or in their commutation properties. Once unit matrices for $\mathbb{M}$ are assigned to represent complexified unit multivector elements for a space-time Clifford algebra, further distinctions are found which identify sub-groups based on their relationship to space-like and time-like vectors. These sub-groups have been labeled as shown in table 2 in section 6.

3.1.4. Sub-groups of order 8. The Dirac matrix group has 155 sub-groups of order 8. There are 80 non-abelian sub-groups. There are 60 abelian sub-groups which exclude the unit imaginary, and 15 abelian sub-groups which include the unit imaginary. Once unit matrices for $\mathbb{M}$ are assigned to represent complexified unit multivector elements for a space-time Clifford algebra, further distinctions are found which identify sub-groups based on their relationship to space-like and time-like vectors. These sub-groups are shown in tables 4-6 in section 6.

3.1.5. Graded algebras. $\mathbb{M}$ can be used to represent multivectors for graded polar vector algebras such as the Clifford algebras $Cl_4(C)$, $Cl_{0,5}(R)$, $Cl_{2,3}(R)$, $Cl_{1,4}(R)$.

3.2. Structure of $\mathbb{T}$

The structure of the trigintaduonion algebra, $\mathbb{T}$, has been described by Cawagas et al[3], but that description does not detail the differing ways in which lower order subalgebras participate in sedenion-type subalgebras. These details are shown in tables 3-9 in section 6.

3.2.1. Embedded loop and automorphism group. $\mathbb{T}$ contains an embedded loop T_L of order 64 generated by its 32 basis elements. The automorphism group for T_L, as determined using the Loops package[1] for GAP4[2], has 42 conjugacy classes, and its structure description is:
C2 x C2 x ((C2 x C2 x C2) . PSL(3,2))

3.2.2. Sub-loops of order 32. T_L has 31 sedenion-type subloops of $\mathbb{T}$ of order 32, falling into four isomorphism classes, which Cawagas et al. designated $S_L, S_L^{\alpha}, S_L^{\beta}, S_L^{\gamma}$. In this paper these have been numbered and referred to as $S_{15}^{0}..S_{15}^{0}$, $S_1^{\alpha}..S_7^{\alpha}$, $S_8^{\beta}..S_{14}^{\beta}$, S_0^{γ}, as shown in table 10 in section 6.

3.2.3. Sub-loops of order 16. T_L has 155 octonion-type sub-loops of order 16, falling into two isomorphism classes: octonion loops which Cawagas et al. designated O_L and quasi-octonion loops which they designated $\widetilde{O}_L$. These octonion-type sub-loops of T_L participate in sedenion-type subloops of T_L in a variety of ways, as shown in table 3 in section 6.

3.2.4. Sub-loops of order 8. T_L has 155 quaternionic subloops of order 8, falling into one isomorphism class which Cawagas et al. designated Q_8. These quaternionic subloops of T_L participate in sedenion-type subloops of T_L in a variety of ways, as shown in tables 4-6 in section 6.

3.2.5. Graded algebras. It is postulated that $\mathbb{T}$ can be used to represent graded multivectors for axial vector algebras, which can be associated with graded multivectors for polar vector algebras represented by Clifford algebras such as $Cl_4(C)$, $Cl_{0,5}(R)$, $Cl_{2,3}(R)$, $Cl_{1,4}(R)$.

3.3. Structure of $\mathbb{U} \cong \mathbb{M} \otimes \mathbb{T}$

3.3.1. Embedded loop and automorphism group. $\mathbb{U}$ contains an embedded loop U_L of order 2048 generated by its 1024 basis elements. The automorphism group for U_L, as determined using the Loops package for GAP4, is:
C2xC2x((((C2xC2xC2xC2):A6):(C2xC2))x((C2xC2xC2).PSL(3,2))).

3.3.2. Associative subalgebras of $\mathbb{U}$. $\mathbb{M}$ is associative. $\mathbb{T}$ is di-associative, pairs of its imaginary unit elements generate sub-algebras isomorphic to $\mathbb{H}$. As noted by Cawagas et al, there are 155 of these sub-algebras. So, for $\mathbb{U}$, there are 155 associative sub-algebras isomorphic to $\mathbb{H} \otimes \mathbb{M}$.

3.3.3. Aligned sub-algebras of $\mathbb{U}$. The Cayley tables for $\mathbb{M}$ and $\mathbb{T}$, as presented in tables 12 and 13 in section 7, have been arranged so that, if the signs of products are ignored, they form the same latin square. Referring to this as "alignment", there are many possible ways of aligning unit elements of the two algebras. For sub-algebras of $\mathbb{M}$ and $\mathbb{T}$, paired combinations of unit elements from this alignment generate "aligned" sub-algebras of $\mathbb{U}$.

3.3.4. Resonant subalgebras of $\mathbb{U}$. For the alignment of table 12 with 13, unit elements of aligned sub-algebras are listed in tables 4-10 in section 6. It can be seen that all $\mathbb{M}$ subgroups that are related by spatial rotation are aligned with $\mathbb{T}$ subloops which have related patterns of participation in sedenion type subloops. This property is designated as a "resonance". A sub-algebra generated using paired products for a resonant alignment, such as $\mathbb{W}$, is designated a "resonant" sub-algebra.

3.3.5. Partition of $\mathbb{U}$. For $\mathbb{U}$, all unit elements from $\mathbb{W}$ together with the unit imaginary element from $\mathbb{M}$ all commute with all unit elements from a subalgebra of $\mathbb{M}$ isomorphic to $Cl_{1,3}(R)$. If $\mathbb{U}$ is used as a basis for a total space for a manifold with fiber bundles, this suggests the choice of a $Cl_{1,3}(R)$ base manifold. For a complexified unit element of $\mathbb{W}$ such as $\mu_\iota X + \mu_\iota iX$, the partial derivative with respect to a coordinate, x, associated with the unit vector, would be a complex function of the associated unit element, μ_ι, of $\mathbb{T}$. This suggests association of $\mathbb{C} \otimes \mathbb{W}$ with fiber bundles, as covariant derivatives of solder forms for tangent vectors define torsion on tangent frame bundles[4].

3.4. Structure of $\mathbb{W}$ and $\mathbb{C} \otimes \mathbb{W}$

Each unit element of $\mathbb{W}$ is a product of a unit element of $\mathbb{M}$ with a unit element of $\mathbb{T}$. As all imaginary unit elements of $\mathbb{T}$ square to -1 and anti-commute, unit elements of $\mathbb{W}$ other than the identity have opposite signature and opposite commutation properties to the corresponding unit elements of $\mathbb{M}$. This relates the Lie bracket of products for $\mathbb{W}$ to the Jordan brace of products for $\mathbb{M}$ and vice-versa, a form of supersymmetry.

3.4.1. Embedded loop and automorphism group.

$\mathbb{W}$ contains an embedded loop W_L of order 64 generated by its 32 basis elements. The automorphism group for W_L, as determined using the Loops package for GAP4, has 40 conjugacy classes, and its structure description is:
$(C2) \times (C2) \times (C2) \times (S4)$.
The automorphism group of its complexification has 80 conjugacy classes and its structure description is:
$(C2) \times (C2) \times (C2) \times (C2) \times (S4)$.
GAP4 also reports:
Its smallgroups ID is (384,20162).
It is a pc group of size 384 with 8 generators, with a trivial Frattini subgroup, and a subgroup lattice of 5127 classes, 18480 subgroups.

3.4.2. Cayley table for $\mathbb{W}$ basis elements.

The Cayley table for $\mathbb{W}$ basis elements (excluding negative elements) is shown in table 1.

Table 1. Cayley table for $\mathbb{W}$ basis elements

Ref.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	$\sigma_o S$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_o V$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\delta_o iU$	$\delta_\iota iX$	$\delta_\jmath iY$	$\delta_\kappa iZ$	$\mu_o iT$	$\mu_\iota iP$	$\mu_\jmath iQ$	$\mu_\kappa iR$	$\alpha_o iS$	$\alpha_\iota iL$	$\alpha_\jmath iM$	$\alpha_\kappa iN$	$\lambda_o iV$	$\lambda_\iota iD$	$\lambda_\jmath iE$	$\lambda_\kappa iF$	$\nu_o U$	$\nu_\iota X$	$\nu_\jmath Y$	$\nu_\kappa Z$	$\gamma_o T$	$\gamma_\iota P$	$\gamma_\jmath Q$	$\gamma_\kappa R$
$\sigma_o S$	$+\sigma_o S$	$+\sigma_\iota L$	$+\sigma_\jmath M$	$+\sigma_\kappa N$	$+\beta_o V$	$+\beta_\iota D$	$+\beta_\jmath E$	$+\beta_\kappa F$	$+\delta_o iU$	$+\delta_\iota iX$	$+\delta_\jmath iY$	$+\delta_\kappa iZ$	$+\mu_o iT$	$+\mu_\iota iP$	$+\mu_\jmath iQ$	$+\mu_\kappa iR$	$+\alpha_o iS$	$+\alpha_\iota iL$	$+\alpha_\jmath iM$	$+\alpha_\kappa iN$	$+\lambda_o iV$	$+\lambda_\iota iD$	$+\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\nu_o U$	$+\nu_\iota X$	$+\nu_\jmath Y$	$+\nu_\kappa Z$	$+\gamma_o T$	$+\gamma_\iota P$	$+\gamma_\jmath Q$	$+\gamma_\kappa R$
$\sigma_\iota L$	$+\sigma_\iota L$	$+\sigma_o S$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\beta_\iota D$	$-\beta_o V$	$+\beta_\kappa F$	$+\beta_\jmath E$	$+\delta_\iota iX$	$+\delta_o iU$	$-\delta_\kappa iZ$	$-\delta_\jmath iY$	$+\mu_\iota iP$	$+\mu_o iT$	$-\mu_\kappa iR$	$-\mu_\jmath iQ$	$+\alpha_\iota iL$	$+\alpha_o iS$	$-\alpha_\kappa iN$	$-\alpha_\jmath iM$	$+\lambda_\iota iD$	$+\lambda_o iV$	$-\lambda_\kappa iF$	$-\lambda_\jmath iE$	$-\nu_\iota X$	$-\nu_o U$	$+\nu_\kappa Z$	$+\nu_\jmath Y$	$-\gamma_\iota P$	$-\gamma_o T$	$+\gamma_\kappa R$	$+\gamma_\jmath Q$
$\sigma_\jmath M$	$+\sigma_\jmath M$	$+\sigma_\kappa N$	$+\sigma_o S$	$+\sigma_\iota L$	$+\beta_\jmath E$	$-\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$	$-\delta_o iU$	$+\delta_\iota iX$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$+\mu_o iT$	$-\mu_\iota iP$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$	$+\alpha_o iS$	$-\alpha_\iota iL$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$-\lambda_o iV$	$+\lambda_\iota iD$	$+\nu_\jmath Y$	$-\nu_\kappa Z$	$+\nu_o U$	$-\nu_\iota X$	$-\gamma_\jmath Q$	$+\gamma_\kappa R$	$-\gamma_o T$	$+\gamma_\iota P$
$\sigma_\kappa N$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$+\sigma_\iota L$	$+\sigma_o S$	$-\beta_\kappa F$	$+\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$	$+\delta_\iota iX$	$-\delta_o iU$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$+\mu_\iota iP$	$-\mu_o iT$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\alpha_\iota iL$	$+\alpha_o iS$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$-\lambda_\iota iD$	$+\lambda_o iV$	$+\nu_\kappa Z$	$-\nu_\jmath Y$	$-\nu_\iota X$	$+\nu_o U$	$+\gamma_\kappa R$	$-\gamma_\jmath Q$	$-\gamma_\iota P$	$+\gamma_o T$
$\beta_o V$	$+\beta_o V$	$+\beta_\iota D$	$-\beta_\jmath E$	$+\beta_\kappa F$	$+\sigma_o S$	$+\sigma_\iota L$	$-\sigma_\jmath M$	$+\sigma_\kappa N$	$+\mu_o iT$	$-\mu_\iota iP$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$+\delta_o iU$	$-\delta_\iota iX$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$+\lambda_o iV$	$-\lambda_\iota iD$	$+\lambda_\jmath iE$	$-\lambda_\kappa iF$	$+\alpha_o iS$	$-\alpha_\iota iL$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\gamma_o T$	$+\gamma_\iota P$	$-\gamma_\jmath Q$	$+\gamma_\kappa R$	$-\nu_o U$	$+\nu_\iota X$	$-\nu_\jmath Y$	$+\nu_\kappa Z$
$\beta_\iota D$	$+\beta_\iota D$	$+\beta_o V$	$-\beta_\kappa F$	$+\beta_\jmath E$	$-\sigma_\iota L$	$-\sigma_o S$	$-\sigma_\kappa N$	$+\sigma_\jmath M$	$+\mu_\iota iP$	$-\mu_o iT$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$+\delta_\iota iX$	$-\delta_o iU$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\alpha_\iota iL$	$-\alpha_o iS$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\gamma_\iota P$	$+\gamma_o T$	$-\gamma_\kappa R$	$+\gamma_\jmath Q$	$-\nu_\iota X$	$+\nu_o U$	$-\nu_\kappa Z$	$+\nu_\jmath Y$
$\beta_\jmath E$	$+\beta_\jmath E$	$+\beta_\kappa F$	$-\beta_o V$	$+\beta_\iota D$	$+\sigma_\jmath M$	$-\sigma_\kappa N$	$-\sigma_o S$	$-\sigma_\iota L$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$-\mu_o iT$	$+\mu_\iota iP$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$+\delta_o iU$	$-\delta_\iota iX$	$+\lambda_\jmath iE$	$-\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$	$-\alpha_\jmath iM$	$+\alpha_\kappa iN$	$-\alpha_o iS$	$+\alpha_\iota iL$	$+\gamma_\jmath Q$	$-\gamma_\kappa R$	$+\gamma_o T$	$-\gamma_\iota P$	$-\nu_\jmath Y$	$+\nu_\kappa Z$	$-\nu_o U$	$+\nu_\iota X$
$\beta_\kappa F$	$+\beta_\kappa F$	$+\beta_\jmath E$	$-\beta_\iota D$	$+\beta_o V$	$-\sigma_\kappa N$	$+\sigma_\jmath M$	$-\sigma_\iota L$	$-\sigma_o S$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$-\mu_\iota iP$	$+\mu_o iT$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$	$-\delta_\iota iX$	$+\delta_o iU$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$+\alpha_\iota iL$	$-\alpha_o iS$	$+\gamma_\kappa R$	$-\gamma_\jmath Q$	$+\gamma_\iota P$	$-\gamma_o T$	$+\nu_\kappa Z$	$-\nu_\jmath Y$	$+\nu_\iota X$	$-\nu_o U$
$\delta_o iU$	$+\delta_o iU$	$-\delta_\iota iX$	$+\delta_\jmath iY$	$+\delta_\kappa iZ$	$+\mu_o iT$	$+\mu_\iota iP$	$-\mu_\jmath iQ$	$-\mu_\kappa iR$	$-\sigma_o S$	$+\sigma_\iota L$	$-\sigma_\jmath M$	$-\sigma_\kappa N$	$-\beta_o V$	$-\beta_\iota D$	$+\beta_\jmath E$	$+\beta_\kappa F$	$-\nu_o U$	$-\nu_\iota X$	$+\nu_\jmath Y$	$+\nu_\kappa Z$	$+\gamma_o T$	$+\gamma_\iota P$	$-\gamma_\jmath Q$	$-\gamma_\kappa R$	$+\alpha_o iS$	$+\alpha_\iota iL$	$-\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\lambda_o iV$	$-\lambda_\iota iD$	$+\lambda_\jmath iE$	$+\lambda_\kappa iF$
$\delta_\iota iX$	$+\delta_\iota iX$	$-\delta_o iU$	$+\delta_\kappa iZ$	$+\delta_\jmath iY$	$-\mu_\iota iP$	$-\mu_o iT$	$-\mu_\kappa iR$	$-\mu_\jmath iQ$	$-\sigma_\iota L$	$+\sigma_o S$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\beta_\iota D$	$-\beta_o V$	$-\beta_\kappa F$	$-\beta_\jmath E$	$-\nu_\iota X$	$-\nu_o U$	$+\nu_\kappa Z$	$+\nu_\jmath Y$	$-\gamma_\iota P$	$-\gamma_o T$	$-\gamma_\kappa R$	$-\gamma_\jmath Q$	$-\alpha_\iota iL$	$-\alpha_o iS$	$+\alpha_\kappa iN$	$+\alpha_\jmath iM$	$-\lambda_\iota iD$	$-\lambda_o iV$	$-\lambda_\kappa iF$	$-\lambda_\jmath iE$
$\delta_\jmath iY$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$+\delta_o iU$	$+\delta_\iota iX$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$-\mu_o iT$	$+\mu_\iota iP$	$+\sigma_\jmath M$	$+\sigma_\kappa N$	$+\sigma_o S$	$-\sigma_\iota L$	$-\beta_\jmath E$	$+\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$	$-\nu_\jmath Y$	$-\nu_\kappa Z$	$+\nu_o U$	$+\nu_\iota X$	$+\gamma_\jmath Q$	$-\gamma_\kappa R$	$-\gamma_o T$	$+\gamma_\iota P$	$+\alpha_\jmath iM$	$+\alpha_\kappa iN$	$-\alpha_o iS$	$-\alpha_\iota iL$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$
$\delta_\kappa iZ$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$+\delta_\iota iX$	$+\delta_o iU$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$-\mu_\iota iP$	$+\mu_o iT$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\sigma_\iota L$	$+\sigma_o S$	$+\beta_\kappa F$	$-\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$	$-\nu_\kappa Z$	$-\nu_\jmath Y$	$+\nu_\iota X$	$+\nu_o U$	$-\gamma_\kappa R$	$+\gamma_\jmath Q$	$-\gamma_\iota P$	$+\gamma_o T$	$+\alpha_\kappa iN$	$+\alpha_\jmath iM$	$-\alpha_\iota iL$	$-\alpha_o iS$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$
$\mu_o iT$	$+\mu_o iT$	$-\mu_\iota iP$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$+\delta_o iU$	$+\delta_\iota iX$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$-\beta_o V$	$-\beta_\iota D$	$-\beta_\jmath E$	$+\beta_\kappa F$	$-\sigma_o S$	$+\sigma_\iota L$	$+\sigma_\jmath M$	$-\sigma_\kappa N$	$-\gamma_o T$	$-\gamma_\iota P$	$-\gamma_\jmath Q$	$+\gamma_\kappa R$	$+\nu_o U$	$+\nu_\iota X$	$+\nu_\jmath Y$	$-\nu_\kappa Z$	$-\lambda_o iV$	$-\lambda_\iota iD$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\alpha_o iS$	$+\alpha_\iota iL$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$
$\mu_\iota iP$	$+\mu_\iota iP$	$-\mu_o iT$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$-\delta_\iota iX$	$-\delta_o iU$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$-\beta_\iota D$	$-\beta_o V$	$-\beta_\kappa F$	$+\beta_\jmath E$	$-\sigma_\iota L$	$+\sigma_o S$	$+\sigma_\kappa N$	$-\sigma_\jmath M$	$-\gamma_\iota P$	$-\gamma_o T$	$-\gamma_\kappa R$	$+\gamma_\jmath Q$	$-\nu_\iota X$	$-\nu_o U$	$+\nu_\kappa Z$	$-\nu_\jmath Y$	$-\lambda_\iota iD$	$-\lambda_o iV$	$-\lambda_\kappa iF$	$+\lambda_\jmath iE$	$-\alpha_\iota iL$	$-\alpha_o iS$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$
$\mu_\jmath iQ$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$-\mu_o iT$	$+\mu_\iota iP$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$+\delta_o iU$	$+\delta_\iota iX$	$+\beta_\jmath E$	$+\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$	$-\sigma_\jmath M$	$+\sigma_\kappa N$	$+\sigma_o S$	$-\sigma_\iota L$	$-\gamma_\jmath Q$	$-\gamma_\kappa R$	$-\gamma_o T$	$+\gamma_\iota P$	$+\nu_\jmath Y$	$-\nu_\kappa Z$	$+\nu_o U$	$+\nu_\iota X$	$+\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$	$-\alpha_\jmath iM$	$+\alpha_\kappa iN$	$-\alpha_o iS$	$-\alpha_\iota iL$
$\mu_\kappa iR$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$-\mu_\iota iP$	$+\mu_o iT$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$	$+\delta_\iota iX$	$+\delta_o iU$	$+\beta_\kappa F$	$+\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$	$+\sigma_\kappa N$	$-\sigma_\jmath M$	$-\sigma_\iota L$	$+\sigma_o S$	$-\gamma_\kappa R$	$-\gamma_\jmath Q$	$-\gamma_\iota P$	$+\gamma_o T$	$-\nu_\kappa Z$	$+\nu_\jmath Y$	$+\nu_\iota X$	$+\nu_o U$	$+\lambda_\kappa iF$	$+\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\alpha_\iota iL$	$-\alpha_o iS$
$\alpha_o iS$	$+\alpha_o iS$	$-\alpha_\iota iL$	$-\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\lambda_o iV$	$-\lambda_\iota iD$	$-\lambda_\jmath iE$	$-\lambda_\kappa iF$	$+\nu_o U$	$+\nu_\iota X$	$+\nu_\jmath Y$	$+\nu_\kappa Z$	$+\gamma_o T$	$+\gamma_\iota P$	$+\gamma_\jmath Q$	$+\gamma_\kappa R$	$+\sigma_o S$	$-\sigma_\iota L$	$-\sigma_\jmath M$	$-\sigma_\kappa N$	$-\beta_o V$	$-\beta_\iota D$	$-\beta_\jmath E$	$-\beta_\kappa F$	$+\delta_o iU$	$+\delta_\iota iX$	$+\delta_\jmath iY$	$+\delta_\kappa iZ$	$+\mu_o iT$	$+\mu_\iota iP$	$+\mu_\jmath iQ$	$+\mu_\kappa iR$
$\alpha_\iota iL$	$+\alpha_\iota iL$	$-\alpha_o iS$	$-\alpha_\kappa iN$	$-\alpha_\jmath iM$	$+\lambda_\iota iD$	$+\lambda_o iV$	$-\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\nu_\iota X$	$+\nu_o U$	$-\nu_\kappa Z$	$-\nu_\jmath Y$	$+\gamma_\iota P$	$+\gamma_o T$	$-\gamma_\kappa R$	$-\gamma_\jmath Q$	$+\sigma_\iota L$	$-\sigma_o S$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\beta_\iota D$	$-\beta_o V$	$+\beta_\kappa F$	$+\beta_\jmath E$	$-\delta_\iota iX$	$-\delta_o iU$	$+\delta_\kappa iZ$	$+\delta_\jmath iY$	$-\mu_\iota iP$	$-\mu_o iT$	$+\mu_\kappa iR$	$+\mu_\jmath iQ$
$\alpha_\jmath iM$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\alpha_o iS$	$-\alpha_\iota iL$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$-\lambda_o iV$	$+\lambda_\iota iD$	$-\nu_\jmath Y$	$+\nu_\kappa Z$	$-\nu_o U$	$+\nu_\iota X$	$+\gamma_\jmath Q$	$-\gamma_\kappa R$	$+\gamma_o T$	$-\gamma_\iota P$	$+\sigma_\jmath M$	$+\sigma_\kappa N$	$-\sigma_o S$	$+\sigma_\iota L$	$+\beta_\jmath E$	$-\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$	$+\delta_\jmath iY$	$-\delta_\kappa iZ$	$+\delta_o iU$	$-\delta_\iota iX$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$-\mu_o iT$	$+\mu_\iota iP$
$\alpha_\kappa iN$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\alpha_\iota iL$	$-\alpha_o iS$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$-\lambda_\iota iD$	$+\lambda_o iV$	$-\nu_\kappa Z$	$+\nu_\jmath Y$	$+\nu_\iota X$	$-\nu_o U$	$-\gamma_\kappa R$	$+\gamma_\jmath Q$	$+\gamma_\iota P$	$-\gamma_o T$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$+\sigma_\iota L$	$-\sigma_o S$	$-\beta_\kappa F$	$+\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$-\delta_\iota iX$	$+\delta_o iU$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$-\mu_\iota iP$	$+\mu_o iT$
$\lambda_o iV$	$+\lambda_o iV$	$-\lambda_\iota iD$	$+\lambda_\jmath iE$	$-\lambda_\kappa iF$	$-\alpha_o iS$	$-\alpha_\iota iL$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$	$+\gamma_o T$	$-\gamma_\iota P$	$+\gamma_\jmath Q$	$-\gamma_\kappa R$	$+\nu_o U$	$-\nu_\iota X$	$+\nu_\jmath Y$	$-\nu_\kappa Z$	$+\beta_o V$	$+\beta_\iota D$	$-\beta_\jmath E$	$+\beta_\kappa F$	$-\sigma_o S$	$+\sigma_\iota L$	$-\sigma_\jmath M$	$+\sigma_\kappa N$	$-\mu_o iT$	$+\mu_\iota iP$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$-\delta_o iU$	$+\delta_\iota iX$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$
$\lambda_\iota iD$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\alpha_\iota iL$	$+\alpha_o iS$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$+\gamma_\iota P$	$-\gamma_o T$	$+\gamma_\kappa R$	$-\gamma_\jmath Q$	$+\nu_\iota X$	$-\nu_o U$	$+\nu_\kappa Z$	$-\nu_\jmath Y$	$+\beta_\iota D$	$+\beta_o V$	$-\beta_\kappa F$	$+\beta_\jmath E$	$-\sigma_\iota L$	$+\sigma_o S$	$-\sigma_\kappa N$	$+\sigma_\jmath M$	$-\mu_\iota iP$	$+\mu_o iT$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$-\delta_\iota iX$	$+\delta_o iU$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$
$\lambda_\jmath iE$	$+\lambda_\jmath iE$	$-\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$	$-\alpha_\jmath iM$	$+\alpha_\kappa iN$	$+\alpha_o iS$	$+\alpha_\iota iL$	$-\gamma_\jmath Q$	$+\gamma_\kappa R$	$-\gamma_o T$	$+\gamma_\iota P$	$+\nu_\jmath Y$	$-\nu_\kappa Z$	$+\nu_o U$	$-\nu_\iota X$	$+\beta_\jmath E$	$+\beta_\kappa F$	$-\beta_o V$	$+\beta_\iota D$	$+\sigma_\jmath M$	$-\sigma_\kappa N$	$+\sigma_o S$	$-\sigma_\iota L$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$+\mu_o iT$	$-\mu_\iota iP$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$	$-\delta_o iU$	$+\delta_\iota iX$
$\lambda_\kappa iF$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$+\alpha_\iota iL$	$+\alpha_o iS$	$-\gamma_\kappa R$	$+\gamma_\jmath Q$	$-\gamma_\iota P$	$+\gamma_o T$	$-\nu_\kappa Z$	$+\nu_\jmath Y$	$-\nu_\iota X$	$+\nu_o U$	$+\beta_\kappa F$	$+\beta_\jmath E$	$-\beta_\iota D$	$+\beta_o V$	$-\sigma_\kappa N$	$+\sigma_\jmath M$	$-\sigma_\iota L$	$+\sigma_o S$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$+\mu_\iota iP$	$-\mu_o iT$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$+\delta_\iota iX$	$-\delta_o iU$
$\nu_o U$	$+\nu_o U$	$+\nu_\iota X$	$-\nu_\jmath Y$	$-\nu_\kappa Z$	$-\gamma_o T$	$-\gamma_\iota P$	$+\gamma_\jmath Q$	$+\gamma_\kappa R$	$-\alpha_o iS$	$+\alpha_\iota iL$	$-\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\lambda_o iV$	$-\lambda_\iota iD$	$+\lambda_\jmath iE$	$+\lambda_\kappa iF$	$-\delta_o iU$	$+\delta_\iota iX$	$-\delta_\jmath iY$	$-\delta_\kappa iZ$	$-\mu_o iT$	$-\mu_\iota iP$	$+\mu_\jmath iQ$	$+\mu_\kappa iR$	$+\sigma_o S$	$+\sigma_\iota L$	$-\sigma_\jmath M$	$-\sigma_\kappa N$	$-\beta_o V$	$-\beta_\iota D$	$+\beta_\jmath E$	$+\beta_\kappa F$
$\nu_\iota X$	$+\nu_\iota X$	$+\nu_o U$	$-\nu_\kappa Z$	$-\nu_\jmath Y$	$+\gamma_\iota P$	$+\gamma_o T$	$+\gamma_\kappa R$	$+\gamma_\jmath Q$	$-\alpha_\iota iL$	$+\alpha_o iS$	$+\alpha_\kappa iN$	$+\alpha_\jmath iM$	$-\lambda_\iota iD$	$-\lambda_o iV$	$-\lambda_\kappa iF$	$-\lambda_\jmath iE$	$-\delta_\iota iX$	$+\delta_o iU$	$-\delta_\kappa iZ$	$-\delta_\jmath iY$	$+\mu_\iota iP$	$+\mu_o iT$	$+\mu_\kappa iR$	$+\mu_\jmath iQ$	$-\sigma_\iota L$	$-\sigma_o S$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\beta_\iota D$	$-\beta_o V$	$-\beta_\kappa F$	$-\beta_\jmath E$
$\nu_\jmath Y$	$+\nu_\jmath Y$	$+\nu_\kappa Z$	$-\nu_o U$	$-\nu_\iota X$	$-\gamma_\jmath Q$	$+\gamma_\kappa R$	$+\gamma_o T$	$-\gamma_\iota P$	$+\alpha_\jmath iM$	$+\alpha_\kappa iN$	$+\alpha_o iS$	$-\alpha_\iota iL$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$	$-\delta_o iU$	$-\delta_\iota iX$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$+\mu_o iT$	$-\mu_\iota iP$	$+\sigma_\jmath M$	$+\sigma_\kappa N$	$-\sigma_o S$	$-\sigma_\iota L$	$-\beta_\jmath E$	$+\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$
$\nu_\kappa Z$	$+\nu_\kappa Z$	$+\nu_\jmath Y$	$-\nu_\iota X$	$-\nu_o U$	$+\gamma_\kappa R$	$-\gamma_\jmath Q$	$+\gamma_\iota P$	$-\gamma_o T$	$+\alpha_\kappa iN$	$+\alpha_\jmath iM$	$-\alpha_\iota iL$	$+\alpha_o iS$	$+\lambda_\kappa iF$	$-\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$	$-\delta_\iota iX$	$-\delta_o iU$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$+\mu_\iota iP$	$-\mu_o iT$	$+\sigma_\kappa N$	$+\sigma_\jmath M$	$-\sigma_\iota L$	$-\sigma_o S$	$+\beta_\kappa F$	$-\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$
$\gamma_o T$	$+\gamma_o T$	$+\gamma_\iota P$	$+\gamma_\jmath Q$	$-\gamma_\kappa R$	$-\nu_o U$	$-\nu_\iota X$	$-\nu_\jmath Y$	$+\nu_\kappa Z$	$-\lambda_o iV$	$-\lambda_\iota iD$	$-\lambda_\jmath iE$	$+\lambda_\kappa iF$	$-\alpha_o iS$	$+\alpha_\iota iL$	$+\alpha_\jmath iM$	$-\alpha_\kappa iN$	$-\mu_o iT$	$+\mu_\iota iP$	$+\mu_\jmath iQ$	$-\mu_\kappa iR$	$-\delta_o iU$	$-\delta_\iota iX$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$	$-\beta_o V$	$-\beta_\iota D$	$-\beta_\jmath E$	$+\beta_\kappa F$	$+\sigma_o S$	$+\sigma_\iota L$	$+\sigma_\jmath M$	$-\sigma_\kappa N$
$\gamma_\iota P$	$+\gamma_\iota P$	$+\gamma_o T$	$+\gamma_\kappa R$	$-\gamma_\jmath Q$	$+\nu_\iota X$	$+\nu_o U$	$-\nu_\kappa Z$	$+\nu_\jmath Y$	$-\lambda_\iota iD$	$-\lambda_o iV$	$-\lambda_\kappa iF$	$+\lambda_\jmath iE$	$-\alpha_\iota iL$	$+\alpha_o iS$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\mu_\iota iP$	$+\mu_o iT$	$+\mu_\kappa iR$	$-\mu_\jmath iQ$	$+\delta_\iota iX$	$+\delta_o iU$	$-\delta_\kappa iZ$	$+\delta_\jmath iY$	$-\beta_\iota D$	$-\beta_o V$	$-\beta_\kappa F$	$+\beta_\jmath E$	$-\sigma_\iota L$	$-\sigma_o S$	$+\sigma_\kappa N$	$-\sigma_\jmath M$
$\gamma_\jmath Q$	$+\gamma_\jmath Q$	$+\gamma_\kappa R$	$+\gamma_o T$	$-\gamma_\iota P$	$-\nu_\jmath Y$	$+\nu_\kappa Z$	$-\nu_o U$	$-\nu_\iota X$	$+\lambda_\jmath iE$	$+\lambda_\kappa iF$	$+\lambda_o iV$	$-\lambda_\iota iD$	$-\alpha_\jmath iM$	$+\alpha_\kappa iN$	$+\alpha_o iS$	$-\alpha_\iota iL$	$-\mu_\jmath iQ$	$+\mu_\kappa iR$	$+\mu_o iT$	$-\mu_\iota iP$	$-\delta_\jmath iY$	$+\delta_\kappa iZ$	$-\delta_o iU$	$-\delta_\iota iX$	$+\beta_\jmath E$	$+\beta_\kappa F$	$+\beta_o V$	$-\beta_\iota D$	$-\sigma_\jmath M$	$+\sigma_\kappa N$	$-\sigma_o S$	$-\sigma_\iota L$
$\gamma_\kappa R$	$+\gamma_\kappa R$	$+\gamma_\jmath Q$	$+\gamma_\iota P$	$-\gamma_o T$	$+\nu_\kappa Z$	$-\nu_\jmath Y$	$-\nu_\iota X$	$-\nu_o U$	$+\lambda_\kappa iF$	$+\lambda_\jmath iE$	$+\lambda_\iota iD$	$-\lambda_o iV$	$+\alpha_\kappa iN$	$-\alpha_\jmath iM$	$-\alpha_\iota iL$	$+\alpha_o iS$	$-\mu_\kappa iR$	$+\mu_\jmath iQ$	$+\mu_\iota iP$	$-\mu_o iT$	$+\delta_\kappa iZ$	$-\delta_\jmath iY$	$-\delta_\iota iX$	$-\delta_o iU$	$+\beta_\kappa F$	$+\beta_\jmath E$	$+\beta_\iota D$	$-\beta_o V$	$+\sigma_\kappa N$	$-\sigma_\jmath M$	$-\sigma_\iota L$	$-\sigma_o S$

3.4.3. Sub-algebras of $\mathbb{W}$ and $\mathbb{C}\otimes\mathbb{W}$. The Cayley tables of $\mathbb{M}$, $\mathbb{T}$ and $\mathbb{W}$ have been configured in a resonant alignment, their sub-algebras are also aligned, and are configured for spatial equivalence when $\mathbb{M}$ is used to represent $Cl_{1,3}(R)\otimes\mathbb{C}$. The sub-algebras of $\mathbb{W}$ are in one-to-one correspondence with sub-algebras of $\mathbb{M}$ and $\mathbb{T}$.

The 155 sub-algebras of $\mathbb{W}$ with four unit elements, all of which are associative, are listed in tables 4-6 in section 6. 80 of them relate to non-abelian sub-algebras of $\mathbb{M}$ and 75 to abelian sub-algebras of $\mathbb{M}$. The sub-algebras of $\mathbb{W}$ with eight unit elements, none of which are associative, are listed in tables 7-9 in section 6.

Analysis of the 155 sub-algebras of $\mathbb{W}$ with 8 unit elements reveals that 15 of them are isomorphic to the split octonions. The other 140 are not power associative. Its 15 sub-algebras isomorphic to the split octonions generate, when complexified, 15 sub-algebras of $\mathbb{C}\otimes\mathbb{W}$ isomorphic to $\mathbb{C}\otimes\mathbb{O}$. Cohl Furey has postulated that minimal left ideals of a $Cl_6(C)$ algebra extracted from $\mathbb{C}\otimes\mathbb{O}$ correspond to one family of fundamental particles[5][6], and refers to others who have advocated the existence of a connection between non-associative algebras and particle theory[7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25]. $\mathbb{U}\cong[\mathbb{C}\otimes\mathbb{W}\otimes Cl_{1,3}(R)]$ has, as sub-algebras, the algebras on which all of these approaches are based, suggesting that it has the potential to provide a basis for the standard model of fundamental particles.

It is postulated that these 15 $\mathbb{C}\otimes\mathbb{O}$ sub-algebras, correspond to three spatial orientations for five families of particles - three families of standard model fermions and two families of dark matter particles. These 15 sub-algebras have, as sub-algebras, all the sub-algebras generated by complexification of the 75 four element subalgebras of $\mathbb{W}$ related to abelian sub-algebras of $\mathbb{M}$.

The other 80 sub-algebras of $\mathbb{W}$ with four unit elements, when complexified, generate sub-algebras isomorphic to $\mathbb{C}\otimes\mathbb{D}$, the algebra of 4×4 complex diagonal matrices. These are commuting and associative. It is postulated that they are associated with vector bosons.

It is postulated that the sub-algebra of $\mathbb{W}$ with unit elements $[\sigma_o S, \sigma_o iS, \alpha_o S, \alpha_o iS]$, a complex doublet, is associated with the Higgs mechanism[26].

3.4.4. Grading distinctions. For sub-algebras of $\mathbb{C}\otimes\mathbb{W}$ with fewer than 32 unit elements, there are distinctions between otherwise isomorphic sub-algebras arising from the patterns of participation of quaternionic and octonion-type sub-loops of T_L in $S_L, S_L^\alpha, S_L^\beta, S_L^\gamma$ sub-loops of T_L. Once $\mathbb{M}$ is assigned to represent $Cl_{1,3}(R)\otimes\mathbb{C}$, further distinctions arise between otherwise isomorphic sub-algebras of $\mathbb{C}\otimes\mathbb{W}$. It is postulated that these distinctions generate the complexity of the standard model.

3.4.5. Matrix representation of $\mathbb{C} \otimes \mathbb{W}$. The notation used to generate $\mathbb{T}$ features dis-association operators, symbols that define the products of unit elements of $\mathbb{T}$ using a modified form of the usual Moufang loop construction for octonions, as setout in table 14 in section 8.2.

A general element of $\mathbb{C} \otimes \mathbb{W}$ can be represented using a complex 4×4 matrix with entries with dis-association operators and using coefficients $a_{-}, b_{-}i$, $c_{-}, d_{-}i$ with subscripts which are lower case versions of the associated unit element from $\mathbb{M}$.

$$\begin{pmatrix}
\begin{matrix} +a_s+b_si+c_s\sigma_o+d_si\alpha_o \\ +a_q+b_qi+c_q\delta_\jmath+d_qi\nu_\jmath \\ +a_d+b_di+c_d\lambda_\iota+d_di\beta_\iota \\ +a_z+b_zi+c_z\gamma_\kappa+d_zi\mu_\kappa \end{matrix} &
\begin{matrix} +a_v+b_vi+c_v\lambda_o+d_vi\beta_o \\ +a_y+b_yi+c_y\gamma_\jmath+d_yi\mu_\jmath \\ +a_l+b_li+c_l\sigma_\iota+d_li\alpha_\iota \\ +a_r+b_ri+c_r\delta_\kappa+d_ri\nu_\kappa \end{matrix} &
\begin{matrix} +a_m+b_mi+c_m\sigma_\jmath+d_mi\alpha_\jmath \\ +a_t+b_ti+c_t\delta_o+d_ti\nu_o \\ +a_f+b_fi+c_f\lambda_\kappa+d_fi\beta_\kappa \\ +a_x+b_xi+c_x\gamma_\iota+d_xi\mu_\iota \end{matrix} &
\begin{matrix} +a_e+b_ei+c_e\lambda_\jmath+d_ei\beta_\jmath \\ +a_u+b_ui+c_u\gamma_o+d_ui\mu_o \\ +a_n+b_ni+c_n\sigma_\kappa+d_ni\alpha_\kappa \\ +a_p+b_pi+c_p\delta_\iota+d_pi\nu_\iota \end{matrix} \\
\begin{matrix} -a_v-b_vi-c_v\lambda_o-d_vi\beta_o \\ +a_y+b_yi+c_y\gamma_\jmath+d_yi\mu_\jmath \\ -a_l-b_li-c_l\sigma_\iota-d_li\alpha_\iota \\ +a_r+b_ri+c_r\delta_\kappa+d_ri\nu_\kappa \end{matrix} &
\begin{matrix} +a_s+b_si+c_s\sigma_o+d_si\alpha_o \\ -a_q-b_qi-c_q\delta_\jmath-d_qi\nu_\jmath \\ +a_d+b_di+c_d\lambda_\iota+d_di\beta_\iota \\ -a_z-b_zi-c_z\gamma_\kappa-d_zi\mu_\kappa \end{matrix} &
\begin{matrix} -a_e-b_ei-c_e\lambda_\jmath-d_ei\beta_\jmath \\ +a_u+b_ui+c_u\gamma_o+d_ui\mu_o \\ -a_n-b_ni-c_n\sigma_\kappa-d_ni\alpha_\kappa \\ +a_p+b_pi+c_p\delta_\iota+d_pi\nu_\iota \end{matrix} &
\begin{matrix} +a_m+b_mi+c_m\sigma_\jmath+d_mi\alpha_\jmath \\ -a_t-b_ti-c_t\delta_o-d_ti\nu_o \\ +a_f+b_fi+c_f\lambda_\kappa+d_fi\beta_\kappa \\ -a_x-b_xi-c_x\gamma_\iota-d_xi\mu_\iota \end{matrix} \\
\begin{matrix} -a_m-b_mi-c_m\sigma_\jmath-d_mi\alpha_\jmath \\ -a_t-b_ti-c_t\delta_o-d_ti\nu_o \\ +a_f+b_fi+c_f\lambda_\kappa+d_fi\beta_\kappa \\ +a_x+b_xi+c_x\gamma_\iota+d_xi\mu_\iota \end{matrix} &
\begin{matrix} -a_e-b_ei-c_e\lambda_\jmath-d_ei\beta_\jmath \\ -a_u-b_ui-c_u\gamma_o-d_ui\mu_o \\ +a_n+b_ni+c_n\sigma_\kappa+d_ni\alpha_\kappa \\ +a_p+b_pi+c_p\delta_\iota+d_pi\nu_\iota \end{matrix} &
\begin{matrix} +a_s+b_si+c_s\sigma_o+d_si\alpha_o \\ +a_q+b_qi+c_q\delta_\jmath+d_qi\nu_\jmath \\ -a_d-b_di-c_d\lambda_\iota-d_di\beta_\iota \\ -a_z-b_zi-c_z\gamma_\kappa-d_zi\mu_\kappa \end{matrix} &
\begin{matrix} +a_v+b_vi+c_v\lambda_o+d_vi\beta_o \\ +a_y+b_yi+c_y\gamma_\jmath+d_yi\mu_\jmath \\ -a_l-b_li-c_l\sigma_\iota-d_li\alpha_\iota \\ -a_r-b_ri-c_r\delta_\kappa-d_ri\nu_\kappa \end{matrix} \\
\begin{matrix} +a_e+b_ei+c_e\lambda_\jmath+d_ei\beta_\jmath \\ -a_u-b_ui-c_u\gamma_o-d_ui\mu_o \\ -a_n-b_ni-c_n\sigma_\kappa-d_ni\alpha_\kappa \\ +a_x+b_xi+c_x\gamma_\iota+d_xi\mu_\iota \end{matrix} &
\begin{matrix} -a_m-b_mi-c_m\sigma_\jmath-d_mi\alpha_\jmath \\ +a_t+b_ti+c_t\delta_o+d_ti\nu_o \\ +a_f+b_fi+c_f\lambda_\kappa+d_fi\beta_\kappa \\ +a_p+b_pi+c_p\delta_\iota+d_pi\nu_\iota \end{matrix} &
\begin{matrix} -a_v-b_vi-c_v\lambda_o-d_vi\beta_o \\ +a_y+b_yi+c_y\gamma_\jmath+d_yi\mu_\jmath \\ +a_l+b_li+c_l\sigma_\iota+d_li\alpha_\iota \\ -a_z-b_zi-c_z\gamma_\kappa-d_zi\mu_\kappa \end{matrix} &
\begin{matrix} +a_s+b_si+c_s\sigma_o+d_si\alpha_o \\ -a_q-b_qi-c_q\delta_\jmath-d_qi\nu_\jmath \\ -a_d-b_di-c_d\lambda_\iota-d_di\beta_\iota \\ -a_r-b_ri-c_r\delta_\kappa-d_ri\nu_\kappa \end{matrix}
\end{pmatrix}$$

4. Dimensionality

4.1. Number of dimensions

The quest for a theory of everything has generated models with various physical dimensionalities - five dimensions for the original Kaluza-Klein theory[27], ten for string theories[28], eleven for M-theory[29]. For nature to be embedded in dimensionalities such as four, five, ten or eleven, whether real or complex, appears arbitrary.

We observe three spatial dimensions and one temporal dimension, which sets a minimum for the number of physical dimensions. The original Kaluza-Klein theory added a further physical dimension and extended general relativity unifying it with classical electromagnetism, suggesting the existence of five physical dimensions.

The Big Bang comprises a transition from a singularity to a manifold expanding through time. The singularity can be regarded as a one dimensional amplitude for zero dimensionality. If that was an unstable configuration, so that a transition to finite amplitudes for more than one dimension was favoured, this raises the question - why a transition to three, four, five, ten or eleven dimensions?

If physical n-space is regarded as composed of an assembly of distorted n-spherical quanta, and if the degree of distortion required is related to the densest possible packing ratio, those densest possible packing ratios vary with dimensionality (n). Suppose that a singularity corresponds to stacking those quanta in an n-needle, one “above” the other. That stack also has a packing ratio.

The packing ratios for assemblies of spheres for continua of different dimensionalities are:

The area of a 2 dimensional circle is $\pi R^2 = 3.142R^2$.
The volume of a 3 dimensional 3-ball is $4\pi/3.R^3 = 4.189R^3$.
The volume of a 4 dimensional 4-ball is $\pi^2/2.R^4 = 4.935R^4$.
The volume of a 5 dimensional 5-ball is $8\pi^2/15.R^5 = 5.264R^5$.
The volume of a 6 dimensional 6-ball is $\pi^3/6.R^6 = 5.168R^6$.
The volume of a 7 dimensional 7-ball is $16\pi^3/105.R^7 = 4.725R^7$.
The volume of an 8 dimensional 8-ball is $\pi^4/24.R^8 = 4.059R^8$.
The volume of a 9 dimensional 9-ball is $32\pi^4/945.R^9 = 3.299R^9$.
The volume of a 10 dimensional 10-ball is $\pi^5/120.R^8 = 2.550R^{10}$.

For an assembly of n-balls stacked into an n-needle of unit quantum radius, the packing fractions are:
For a stack of 2 dimensional unit circles on edge: $(\pi R^2)/(2R \times 2R) \rightarrow$ 78.6%
For a 3-needle of unit 3-balls: $(4\pi^3/3.R^3)/(\pi R^2 \times 2R) \rightarrow$ 66.6%
For a 4-needle of unit 4-balls: $(\pi^2/2.R^4)/(4\pi^3/3.R^3 \times 2R) \rightarrow$ 58.9%
For a 5-needle of unit 5-balls: $(8\pi^2/15.R^5)/((\pi^2/2.R^4) \times 2R) \rightarrow$ 53.3%
For a 6-needle of unit 6-balls: $(\pi^3/6.R^6)/(8\pi^2/15.R^5 \times 2R) \rightarrow$ 49.1%
For a 7-needle of unit 7-balls: $(16\pi^3/105.R^7)/(\pi^3/6.R^6) \times 2R) \rightarrow$ 45.7%
For an 8-needle of unit 8-balls: $(\pi^4/24.R^8)/(16\pi^3/105.R^7) \times 2R) \rightarrow$ 45.9%
For a 9-needle of unit 9-balls: $(32\pi^4/945.R^9)/(\pi^4/24.R^8) \times 2R) \rightarrow$ 34.9%
For a 10-needle of unit 10-balls: $(\pi^5/120.R^{10})/(32\pi^4/945.R^9) \times 2R) \rightarrow$ 38.6%

For euclidean n-space the densest packing fractions, as listed by Cohn and Elkies[30], are:
For 2 dimensional 2-balls (circles) of equal radius: 91%
For 3 dimensional 3-balls of equal radius: 74%
For 4 dimensional 4-balls of equal radius: in the range 61.7 to 64.8%
For 5 dimensional 5-balls of equal radius: in the range 46.5 to 52.5%
For 6 dimensional 6-balls of equal radius: in the range 37.3 to 41.8%
For 7 dimensional 7-balls of equal radius: in the range: 29.5 to 32.8%
For 8 dimensional 8-balls of equal radius: 25.4%
For 9 dimensional 9-balls of equal radius: in the range: 14.6 to 19.5%
For 10 dimensional 10-balls of equal radius: in the range: 10.0 to 14.9%

For n-balls arranged in a n-disc, that is an euclidean n-space extended in $n-1$ dimensions and limited to unit quantum diameter in the nth dimension, the densest packing fractions are product of the densest packing fractions for the euclidean $(n-1)$ space and the packing fraction for the n-needle of the same dimensionality. For instance, for three dimensions spheres would be packed into the disc in the densest packing of parallel cylinders in a plane.

For euclidean space the densest packing fractions for n-discs are:
For 2 dimensional 2-balls (circles) of equal radius: 71%
For 3 dimensional 3-balls of equal radius: 49%
For 4 dimensional 4-balls of equal radius: in the range 36 to 38%
For 5 dimensional 5-balls of equal radius: in the range 25 to 28%
For 6 dimensional 6-balls of equal radius: in the range 18 to 21%
For 7 dimensional 7-balls of equal radius: in the range: 13 to 15%
For 8 dimensional 8-balls of equal radius: 11.7%
For 9 dimensional 9-balls of equal radius: in the range: 5 to 7%
For 10 dimensional 10-balls of equal radius: in the range: 4 to 6%

In dimensionalities lower than 5 an euclidean n-space has a higher densest possible packing fraction than that of an n-needle and that of an n-disc. This suggests the hypothesis that, for a four dimensional manifold, a singularity would be unstable, tending to expand into a 4 sphere. If that expansion were to overshoot, becoming disc like, there would be a tendency for it to contract again.

However, space is not necessarily euclidean. Packing fractions for hyperbolic space are higher than for euclidean space, but are difficult to calculate. For 3 dimensions and 4 dimensions, densest packing fractions have been calculated as:
For 3 dimensional 3-balls of equal radius: 85.3%
For 4 dimensional 4-balls of equal radius: 71.6%
Compared to euclidean n-space, these figures are 15% and 13% higher respectively. Extrapolating to higher dimensions, for dimensionalities higher than 5, euclidean n-space would still have a lower packing fraction than an n-needle. However, in 5 dimensions, it is possible that for hyperbolic 5-space there could be a denser packing than for a 5-needle.

This analysis suggets that expansion of a singularity into four dimensions for euclidean space or five dimensions for hyperbolic space could be favoured.

4.2. Types of dimensions

The original Kaluza-Klein theory[27] accounted for electro-magnetism by introducing an additional dimension. This dimension is not observed, suggesting that it would differ from the observed spatial dimensions. Time is observed, but also differs from spatial dimensions. $\mathbb{M}$ is isomorphic to the Clifford algebra $Cl_{0,5}(R)$ and to $Cl_{1,3}(R) \otimes \mathbb{C}$. $Cl_{0,5}(R)$ is generated using five polar vector unit elements with negative signature. It is postulated that four dimensions corresponds to the dimensions of space and imaginary time and the fifth dimension to the extra dimension for 5D Kaluza-Klein theory, and that conventional time is emergent. This suggests the concept of reality as a 3-dimensional wavefront distorted into a fourth dimension propagating in a fifth dimension.

String and M-theories[28][29] also postulate additional dimensions. $\mathbb{U}$ is of the same order as $Cl_{0,10}(R)$, but has a "natural" partition into $Cl_{1,3}(R)\otimes\mathbb{C}\otimes\mathbb{W}$, suggesting that it could be used in string/M theories in the place of $Cl_{0,10}(R)$. $\mathbb{U}$ may constitute a representation of a combination of a manifold with 5 polar vectors and 5 axial vectors. This suggests assigning the polar vectors to a five-dimensional space and the axial vectors to a form of torsion for each dimension. General relativity[31] is usually formulated using the assumption that affine connection has a vanishing torsion tensor, but non-vanishing torsion has been proposed for Einstein-Cartan-Sciama-Kibble and other theories. In an overview[32], Tejinder Singh comments:

"Thus on the one hand we have the torsion-dominated limit, which are the Dirac equations, and on the other hand we have the gravity dominated limit, which are the Einstein equations. In the former case, gravity is absent (Minkowski space-time) and matter behaviour is quantum. In the latter case matter behaviour is classical, and gravity dominates over torsion. Thus we may conclude that there must be a more general underlying theory in which the torsion-free part and the torsion part of the spin-connection are both present, and to which GTR and quantum theory are both approximations."

The number of degrees of freedom for torsion for a given dimensionality are limited. For a manifold of dimension d = 5 with a maximally symmetric submanifold of dimension n = 4, there are up to $1+4+6=11$ allowed torsion components which are in general functions of the fifth coordinate[33]. The Einstein field equations (for four dimensions) have 10 degrees of freedom, four of which are unphysical. This suggests a correlation between the $4+6$ allowed torsion components and the degrees of freedom for physical space-time with imaginary time substituted for time to make the four dimensions symmetric.

In this paper the algebra $\mathbb{U}$ has been assembled as the tensor product $\mathbb{M}\otimes\mathbb{T}$, for which all unit elements of $\mathbb{M}$ commute with all unit elements of $\mathbb{T}$. An alternative form of torsion could be generated by assembling an algebra for which this is not the case. One such algebra will be described in a further paper by this author.

5. The Higgs mechanism

A grand unification algebra should provide a basis for the Higgs mechanism[26]. The mexican hat potential is unusual. This section describes algebraic elements that could account for that potential.

The Higgs mechanism acts on a complex doublet and involves scalar fields. For $\mathbb{U}$ a scalar subalgebra can be assembled as the product:
$[\sigma_o S, \sigma_o iS, \alpha_o S, \alpha_o iS] \otimes [\sigma_o S, \sigma_o T, \sigma_o V, \sigma_o U] \otimes [\sigma_o S, \lambda_o S, \mu_o S, \nu_o S]$.

$[\sigma_o S, \sigma_o T, \sigma_o V, \sigma_o U] \otimes [\sigma_o S, \lambda_o S, \mu_o S, \nu_o S]$ is isomorphic to $\mathbb{H} \otimes \mathbb{H}$ and to $M_4(R)$. Its unit elements can be assigned unit matrices from table 1 as follows:
$[\sigma_o S] = [S]$, $[\sigma_o T, \sigma_o V, \sigma_o U] = [TVU]$, $[\lambda_o S, \mu_o S, \nu_o S] = [LMN]$
$[\lambda_o T, \mu_o T, \nu_o T] = [PQR]$, $[\lambda_o V, \mu_o V, \nu_o V] = [DEF]$, $[\lambda_o U, \mu_o U, \nu_o U] = [XYZ]$

Subalgebras of $M_4(R)$ for which the scalar component is associated with a mexican hat potential can be found by considering unitary abelian subgroups of $M_4(R)$. Unitary abelian subgroups of $M_4(R)$ can be represented by diagonal 4×4 matrices.

$$\begin{bmatrix} e^{i\theta_1} & 0 & 0 & 0 \\ 0 & e^{i\theta_2} & 0 & 0 \\ 0 & 0 & e^{i\theta_3} & 0 \\ 0 & 0 & 0 & e^{i\theta_4} \end{bmatrix}$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$, allowing it to be rewritten:

$$\begin{bmatrix} e^{ia} & 0 & 0 & 0 \\ 0 & e^{ib} & 0 & 0 \\ 0 & 0 & e^{ic} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The product of two elements of this type with parameters a, b, c and a', b', c' has parameters $a + a', b + b', c + c'$. A subgroup of the Heisenberg group H(5) shares this property:

$$\begin{bmatrix} 1 & a & b & c + ab \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix has determinant = 1, and the commuting products of the form:

$$\begin{bmatrix} 1 & a + a' & b + b' & c + c' + (a + a') \times (b + b') \\ 0 & 1 & 0 & b + b' \\ 0 & 0 & 1 & a + a' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix can be written in terms of unit elements of $M_4(R)$ as:
$[S] + a/2[V + Y] + b/2[M + F] + (c + ab)/4[E + U + N + P]$.

There are other combinations of unit elements of $M_4(C)$ with similar properties. To find these combinations, it is useful to arrange the matrices in an array with anticommuting basis matrices and the identity in each row/column, forming a 6×6 array:

$$\begin{bmatrix} S & V & T & X & Y & Z \\ V & S & U & P & Q & R \\ T & U & S & D & E & F \\ X & P & D & S & N & M \\ Y & Q & E & N & S & L \\ Z & R & F & M & L & S \end{bmatrix}$$

Interchanging rows and matching columns preserves commutation/anticommutation relationships and group properties with respect to position in the array. For example, rows and columns 1 and 2 can be interchanged to make the array:

$$\begin{bmatrix} S & V & U & P & Q & R \\ V & S & T & X & Y & Z \\ U & T & S & D & E & F \\ P & X & D & S & N & M \\ Q & Y & E & N & S & L \\ R & Z & F & M & L & S \end{bmatrix}$$

Inspecting this array to assign unit matrices for an equivalent H5 subgroup group, they would be:
$[S] + a/2[V + Q] + b/2[M + F] + (c + ab)/4[E + N + T + X]$
This combination has the same properties. Interchanging rows and columns 1 and 2 has not changed the signatures of the matrices allocated to each position.

If a further interchange is made that does affect the signatures, e.g interchanging rows and columns 1 and 4, to generate:

$$\begin{bmatrix} S & Q & U & P & V & R \\ Q & S & E & N & Y & L \\ U & E & S & D & T & F \\ P & N & D & S & X & M \\ V & Y & E & N & S & L \\ R & L & F & M & Z & S \end{bmatrix}$$

For the combination:
$[S] + a/2[Y + Q] + b/2[M + F] + (c + ab)/4[P + U + T + X]$
The determinant is no longer 1. To make this combination generate a unitary matrix, a factor has to be applied to $[S]$. That factor is $\sqrt{(\pm 1 \pm 2(a/2)^2)}$, provided that the factor is real and not imaginary.

For the resulting matrix, there are four plus/minus permutations, for which the possible matrices for [S] are:

$$\begin{bmatrix} \sqrt{(1+a^2/2)} & 0 & 0 & 0 \\ 0 & \sqrt{(1+a^2/2)} & 0 & 0 \\ 0 & 0 & \sqrt{(1+a^2/2)} & 0 \\ 0 & 0 & 0 & \sqrt{(1+a^2/2)} \end{bmatrix}$$

Which always has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt{(-1-a^2/2)} & 0 & 0 & 0 \\ 0 & \sqrt{(-1-a^2/2)} & 0 & 0 \\ 0 & 0 & \sqrt{(-1-a^2/2)} & 0 \\ 0 & 0 & 0 & \sqrt{(-1-a^2/2)} \end{bmatrix}$$

Which never has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt{(1-a^2/2)} & 0 & 0 & 0 \\ 0 & \sqrt{(1-a^2/2)} & 0 & 0 \\ 0 & 0 & \sqrt{(1-a^2/2)} & 0 \\ 0 & 0 & 0 & \sqrt{(1-a^2/2)} \end{bmatrix}$$

Which has real entries for $a^2/2 \leq 1$, and determinant $= 1 - a^2 + a^4/4$

$$\begin{bmatrix} \sqrt{(-1+a^2/2)} & 0 & 0 & 0 \\ 0 & \sqrt{(-1+a^2/2)} & 0 & 0 \\ 0 & 0 & \sqrt{(-1+a^2/2)} & 0 \\ 0 & 0 & 0 & \sqrt{(-1+a^2/2)} \end{bmatrix}$$

Which has real entries for $a^2/2 \geq 1$, and determinant $= 1 - a^2 + a^4/4$

The function $f(a) = 1 - a^2 + a^4/4$ has the form of a mexican hat potential.

For the assignment of unit elements of $\mathbb{U}$ to matrices:
$[\sigma_o S] = [S]$, $[\sigma_o T, \sigma_o V, \sigma_o U] = [TVU]$, $[\lambda_o S, \mu_o S, \nu_o S] = [LMN]$
$[\lambda_o T, \mu_o T, \nu_o T] = [PQR]$, $[\lambda_o V, \mu_o V, \nu_o V] = [DEF]$, $[\lambda_o U, \mu_o U, \nu_o U] = [XYZ]$

The group represented by a plus/minus choice for:
$\sqrt{(\pm 1 \pm a^2/2)}[S] + a/2[Y + Q] + b/2[M + F] + (c + ab)/4[P + U + T + X]$
is isomorphic to that for the same plus/minus choice for:
$\sqrt{(\pm 1 \pm a^2/2)}[\sigma_o S] + a/2[\mu_o U + \mu_o T] + b/2[\mu_o S + \nu_o V]$
$+ (c + ab)/4[\lambda_o T + \sigma_o U + \sigma_o T + \lambda_o U]$
which can be rearranged into:
$\sqrt{(\pm 1 \pm a^2/2)}[\sigma_o S] + (U + T) \times [a/2\mu_o + (c + ab)/4\sigma + (c + ab)/4\lambda] + b/2[\mu_o S + \nu_o V]$
for which $[TVU]$ symmetry is broken.

6. Subalgebra tables

Table 2. Classification of sub-algebras with 8 unit elements of $\mathbb{W} \cong M_4(C)$ with respect to unit elements of a $Cl_{1,3}$ multivector

Type 1 subgroups having [+ - - - + + + -] signature							
Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements
1a	SLMNDEFV	1a	SLiMiNDiEiFV	1a	SiLMiNiDEiFV	1a	SiLiMNiDiEFV
1b	SLMNPQRT	1b	SLiMiNPiQiRT	1b	SiLMiNiPQiRT	1b	SiLiMNiPiQRT
1c	SLMNXYZU	1c	SLiMiNXiYiZU	1c	SiLMiNiXYiZU	1c	SiLiMNiXiYZU
1d	SVTUDPXL	1d	SViTiUDiPiXL	1d	SiVTiUiDPiXL	1d	SiViTUiDiPXL
1d	SVTUEQYM	1d	SViTiUEiQiYM	1d	SiVTiUiEQiYM	1d	SiViTUiEiQYM
1d	SVTUFRZN	1d	SViTiUFiRiZN	1d	SiVTiUiFRiZN	1d	SiViTUiFiRZN
1e	SLiEiFiURQiX	1e	SLEFiUiRiQiX	1e	SiLiEFURiQiX	1e	SiLEiFUiRQiX
1e	SMiFiDiUPRiY	1e	SMFDiUiPiRiY	1e	SiMiFDUPiRiY	1e	SiMFiDUiPRiY
1e	SNiDiEiUQPiZ	1e	SNDEiUiQiPiZ	1e	SiNiDEUQiPiZ	1e	SiNDiEUiQPiZ
1f	SLiQiRiVZYiD	1f	SLQRiViZiYiD	1f	SiLiQRVZiYiD	1f	SiLQiRViZYiD
1f	SMiRiPiVXZiE	1f	SMRPiViXiZiE	1f	SiMiRPVXiZiE	1f	SiMRiPViXZiE
1f	SNiPiQiVYXiF	1f	SNPQiViYiXiF	1f	SiNiPQVYiXiF	1f	SiNPiQViYXiF
1g	SLiYiZiTFEiP	1g	SLYZiTiFiEiP	1g	SiLiYZTFiEiP	1g	SiLYiZTiFEiP
1g	SMiZiXiTDFiQ	1g	SMZXiTiDiFiQ	1g	SiMiZXTDiFiQ	1g	SiMZiXTiDFiQ
1g	SNiXiYiTEDiR	1g	SNXYiTiEiDiR	1g	SiNiXYTEiDiR	1g	SiNXiYTiEDiR
Type 2 subgroups having [+ - - - - - - +] signature							
Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements		
2a	SLMNiDiEiFiV						
2b	SLMNiPiQiRiT						
2c	SLMNiXiYiZiU						
2d	SVTUiDiPiXiL						
2d	SVTUiEiQiYiM						
2d	SVTUiFiRiZiN						
2e	SLiEiFUiRiQX						
Type 3 subgroups having [+ - + + - + + +] signature							
Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements		
3a	SLiMiNiDEFiV	3a	SiLMiNDiEFiV	3a	SiLiMNDEiFiV		
3b	SLiMiNiPQRiT	3b	SiLMiNPiQRiT	3b	SiLiMNPQiRiT		
3c	SLiMiNiXYZiU	3c	SiLMiNXiYZiU	3c	SiLiMNXYiZiU		
3d	SViTiUiDPXiL	3d	SiVTiUDiPXiL	3d	SiViTUDPiXiL		
3d	SViTiUiEQYiM	3d	SiVTiUEiQYiM	3d	SiViTUEQiYiM		
3d	SViTiUiFRZiN	3d	SiVTiUFiRZiN	3d	SiViTUFRiZiN		
3e	SLEFURQX	3e	SiLiEFiUiRQX	3e	SiLEiFiURiQX		
3e	SMFDUPRY	3e	SiMiFDiUiPRY	3e	SiMFiDiUPiRY		
3e	SNDEUQPZ	3e	SiNiDEiUiQPZ	3e	SiNDiEiUQiPZ		
3f	SLQRVZYD	3f	SiLiQRiViZYD	3f	SiLQiRiVZiYD		
3f	SMRPVXZE	3f	SiMiRPiViXZE	3f	SiMRiPiVXiZE		
3f	SNPQVYXF	3f	SiNiPQiViYXF	3f	SiNPiQiVYiXF		
3g	SLYZTEFP	3g	SiLYiZiTiEFP	3g	SiLiYZiTEiFP		
3g	SMZXTFDQ	3g	SiMZiXiTiFDQ	3g	SiMiZXiTFiDQ		
3g	SNXYTDER	3g	SiNXiYiTiDER	3g	SiNiXYiTDiER		
Type 4 subgroups having [+ + + + - - - -] signature							
Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements		
4a	SiVDiLViDLiS	4a	SiVEiMViEMiS	4a	SiVFiNViFNiS		
4b	SiTPiLTiPLiS	4b	SiTQiMTiQMiS	4b	SiTRiNTiRNiS		
4c	SXiUiLiXULiS	4c	SYiUiMiYUMiS	4c	SZiUiNiZUNiS		
4d	SXERiXiEiRiS	4d	SYPFiYiPiFiS	4d	SZDQiZiDiQiS		
4d	SXQFiXiQiFiS	4d	SYDRiYiDiRiS	4d	SZPEiZiPiEiS		
Type 5 subgroups having [+ - - - + + + -] signature							
Subtype	Unit elements	Subtype	Unit elements	Subtype	Unit elements		
5a	SLMNiLiMiNiS						
5b	SLiEiFiLEFiS	5b	SMiFiDiMFDiS	5b	SNiDiEiNDEiS		
5c	SLiQiRiLQRiS	5c	SMiRiPiMRPiS	5c	SNiPiQiNPQiS		
5d	SLiYiZiLYZiS	5d	SMiZiXiMZXiS	5d	SNiXiYiNXYiS		
5e	SVTUiViTiUiS						
5f	SViPiXiVPXiS	5f	SViQiYiVQYiS	5f	SViRiZiVRZiS		
5g	STiXiDiTXDiS	5g	STiYiEiTYEiS	5g	STiZiFiTZFiS		
5h	SUiDiPiUDPiS	5h	SUiEiQiUEQiS	5h	SUiFiRiUFRiS		

TABLE 3. Classification of sub-algebras with 8 unit elements of $\mathbb{T}$ with respect to participation in sedenion-type sub-algebras

Octonion/Quasi-Octonion unit elements, Type ($\mathbb{O}$ or $\widetilde{O}$ and sedenion participation)		
Note: automorphism group for $\mathbb{O}$ has structure description (C2 x C2 x C2).PSL(3,2), small groups ID [1344,814]		
Note: automorphism group for $\widetilde{O}$ has structure description (((C2 x C2 x C2):(C2 x C2)):C3) :C2, small groups ID [192,1494]		
Type a: $\mathbb{O}$, sedenion participation: $1 \times S_0$, $1 \times S^\alpha$,$1 \times S^\gamma$		
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \lambda_\kappa\ \lambda_o\ \mathbb{O}^a\ S^0_{18}S^\alpha_1S^\gamma_0$		
$\sigma_o\ \sigma_\iota\ \delta_\jmath\ \delta_\kappa\ \lambda_o\ \gamma_\kappa\ \gamma_\jmath\ \lambda_\iota\ \mathbb{O}^a\ S^0_{28}S^\alpha_5S^\gamma_0$	$\sigma_o\ \sigma_\jmath\ \delta_\kappa\ \delta_\iota\ \lambda_o\ \gamma_\iota\ \gamma_\kappa\ \lambda_\jmath\ \mathbb{O}^a\ S^0_{29}S^\alpha_6S^\gamma_0$	$\sigma_o\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \lambda_o\ \gamma_\jmath\ \gamma_\iota\ \lambda_\kappa\ \mathbb{O}^a\ S^0_{30}S^\alpha_7S^\gamma_0$
$\sigma_o\ \sigma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \delta_o\ \lambda_\jmath\ \lambda_\kappa\ \delta_\iota\ \mathbb{O}^a\ S^0_{25}S^\alpha_2S^\gamma_0$	$\sigma_o\ \sigma_\jmath\ \gamma_\kappa\ \gamma_\iota\ \delta_o\ \lambda_\kappa\ \lambda_\iota\ \delta_\jmath\ \mathbb{O}^a\ S^0_{26}S^\alpha_3S^\gamma_0$	$\sigma_o\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \delta_o\ \lambda_\iota\ \lambda_\jmath\ \delta_\kappa\ \mathbb{O}^a\ S^0_{27}S^\alpha_4S^\gamma_0$
Type b: $\widetilde{O}$, sedenion participation: $1 \times S_0$, $1 \times S^\beta$,$1 \times S^\gamma$		
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \gamma_o\ \widetilde{O}^b\ S^0_{17}S^\beta_8S^\gamma_0$		
$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\iota\ \delta_\iota\ \gamma_\iota\ \sigma_\iota\ \widetilde{O}^b\ S^0_{19}S^\beta_9S^\gamma_0$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\jmath\ \delta_\jmath\ \gamma_\jmath\ \sigma_\jmath\ \widetilde{O}^b\ S^0_{20}S^\beta_{10}S^\gamma_0$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\kappa\ \delta_\kappa\ \gamma_\kappa\ \sigma_\kappa\ \widetilde{O}^b\ S^0_{21}S^\beta_{11}S^\gamma_0$
$\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa\ \gamma_o\ \delta_\kappa\ \delta_\jmath\ \gamma_\iota\ \widetilde{O}^b\ S^22_0S^\beta_{12}S^\gamma_0$	$\sigma_o\ \sigma_\jmath\ \lambda_\kappa\ \lambda_\iota\ \gamma_o\ \delta_\iota\ \delta_\kappa\ \gamma_\jmath\ \widetilde{O}^b\ S^0_{23}S^\beta_{13}S^\gamma_0$	$\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \gamma_o\ \delta_\jmath\ \delta_\iota\ \gamma_\kappa\ \widetilde{O}^b\ S^0_{24}S^\beta_{14}S^\gamma_0$
Type c: $\mathbb{O}$, sedenion participation: $2 \times S_0$, $1 \times S^\gamma$		
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \delta_\kappa\ \delta_o\ \mathbb{O}^c\ S^0_{15}S^0_{16}S^\gamma_0$		
Type d: $\widetilde{O}$, sedenion participation: $2 \times S^0$,$1 \times S^\alpha$		
$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\iota\ \delta_\iota\ \mu_\iota\ \sigma_\iota\ \widetilde{O}^d\ S^0_{15}S^0_{19}S^\alpha_2$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\jmath\ \delta_\jmath\ \mu_\jmath\ \sigma_\jmath\ \widetilde{O}^d\ S^0_{15}S^0_{20}S^\alpha_3$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\kappa\ \delta_\kappa\ \mu_\kappa\ \sigma_\kappa\ \widetilde{O}^d\ S^0_{15}S^0_{21}S^\alpha_4$
$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \mu_o\ \delta_\kappa\ \delta_\jmath\ \mu_\iota\ \widetilde{O}^d\ S^0_{15}S^0_{22}S^\alpha_5$	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \mu_o\ \delta_\iota\ \delta_\kappa\ \mu_\jmath\ \widetilde{O}^d\ S^0_{15}S^0_{23}S^\alpha_6$	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \mu_o\ \delta_\jmath\ \delta_\iota\ \mu_\kappa\ \widetilde{O}^d\ S^0_{15}S^0_{24}S^\alpha_7$
Type e: $\mathbb{O}$, sedenion participation: $2 \times S^0$, $1 \times S^\alpha$		
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \mu_\kappa\ \mu_o\ \mathbb{O}^e\ S^0_{15}S^0_{17}\ S^\alpha_1$		
Type f: $\widetilde{O}$, sedenion participation: $2 \times S^0$,$1 \times S^\beta$		
$\sigma_o\ \sigma_\iota\ \delta_\jmath\ \delta_\kappa\ \beta_o\ \mu_\kappa\ \mu_\jmath\ \beta_\iota\ \widetilde{O}^f\ S^0_{12}S^0_{15}\ S^\beta_{28}$	$\sigma_o\ \sigma_\jmath\ \delta_\kappa\ \delta_\iota\ \beta_o\ \mu_\iota\ \mu_\kappa\ \beta_\jmath\ \widetilde{O}^f\ S^0_{13}S^0_{15}\ S^\beta_{29}$	$\sigma_o\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \beta_o\ \mu_\jmath\ \mu_\iota\ \beta_\kappa\ \widetilde{O}^f\ S^0_{14}S^0_{15}\ S^\beta_{30}$
$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \delta_o\ \beta_\kappa\ \beta_\jmath\ \delta_\iota\ \widetilde{O}^f\ S^0_9S^0_{15}\ S^\beta_{25}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \delta_o\ \beta_\iota\ \beta_\kappa\ \delta_\jmath\ \widetilde{O}^f\ S^0_{10}S^0_{15}\ S^\beta_{26}$	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \delta_o\ \beta_\jmath\ \beta_\iota\ \delta_\kappa\ \widetilde{O}^f\ S^0_{11}S^0_{15}\ S^\beta_{27}$
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \beta_\kappa\ \beta_o\ \widetilde{O}^f\ S^0_8S^0_{15}\ S^\beta_{18}$		
Type g: $\mathbb{O}$, sedenion participation: $1 \times S^0$, $2 \times S^\alpha$		
$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\iota\ \nu_\iota\ \mu_\iota\ \sigma_\iota\ \mathbb{O}^g\ S^0_{19}S^\alpha_1\ S^\alpha_5$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\jmath\ \nu_\jmath\ \mu_\jmath\ \sigma_\jmath\ \mathbb{O}^g\ S^0_{20}S^\alpha_1\ S^\alpha_6$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\kappa\ \nu_\kappa\ \mu_\kappa\ \sigma_\kappa\ \mathbb{O}^g\ S^0_{21}S^\alpha_1\ S^\alpha_7$
$\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa\ \mu_o\ \nu_\kappa\ \nu_\jmath\ \mu_\iota\ \mathbb{O}^g\ S^0_{22}S^\alpha_1\ S^\alpha_2$	$\sigma_o\ \sigma_\jmath\ \lambda_\kappa\ \lambda_\iota\ \mu_o\ \nu_\iota\ \nu_\kappa\ \mu_\jmath\ \mathbb{O}^g\ S^0_{23}S^\alpha_1\ S^\alpha_3$	$\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \mu_o\ \nu_\jmath\ \nu_\iota\ \mu_\kappa\ \mathbb{O}^g\ S^0_{24}S^\alpha_1\ S^\alpha_4$
Type h: $\widetilde{O}$, sedenion participation: $1 \times S^0$,$2 \times S^\alpha$		
$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\iota\ \nu_\iota\ \gamma_\iota\ \alpha_\iota\ \widetilde{O}^h\ S^0_{19}S^\alpha_3\ S^\alpha_4$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\iota\ \delta_\iota\ \gamma_\iota\ \alpha_\iota\ \widetilde{O}^h\ S^0_{19}S^\alpha_6\ S^\alpha_7$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\jmath\ \nu_\jmath\ \gamma_\jmath\ \alpha_\jmath\ \widetilde{O}^h\ S^0_{20}S^\alpha_2\ S^\alpha_4$
$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\jmath\ \delta_\jmath\ \gamma_\jmath\ \alpha_\jmath\ \widetilde{O}^h\ S^0_{20}S^\alpha_5\ S^\alpha_7$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\kappa\ \delta_\kappa\ \gamma_\kappa\ \alpha_\kappa\ \widetilde{O}^h\ S^0_{21}S^\alpha_5\ S^\alpha_6$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\kappa\ \nu_\kappa\ \gamma_\kappa\ \alpha_\kappa\ \widetilde{O}^h\ S^0_{21}S^\alpha_2\ S^\alpha_3$
$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \gamma_\kappa\ \mu_o\ \widetilde{O}^h\ S^0_{17}S^\alpha_3\ S^\alpha_6$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \gamma_\kappa\ \mu_o\ \widetilde{O}^h\ S^0_{17}S^\alpha_2\ S^\alpha_5$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \mu_\kappa\ \mu_o\ \widetilde{O}^h\ S^0_{17}S^\alpha_4\ S^\alpha_7$
$\sigma_o\ \alpha_\iota\ \beta_\jmath\ \lambda_\kappa\ \mu_o\ \nu_\kappa\ \delta_\jmath\ \gamma_\iota\ \widetilde{O}^h\ S^0_{22}S^\alpha_3\ S^\alpha_7$	$\sigma_o\ \alpha_\iota\ \lambda_\jmath\ \beta_\kappa\ \mu_o\ \delta_\kappa\ \nu_\jmath\ \gamma_\iota\ \widetilde{O}^h\ S^0_{22}S^\alpha_4\ S^\alpha_6$	$\sigma_o\ \alpha_\jmath\ \beta_\kappa\ \lambda_\iota\ \mu_o\ \nu_\iota\ \delta_\kappa\ \gamma_\jmath\ \widetilde{O}^h\ S^0_{23}S^\alpha_4\ S^\alpha_5$
$\sigma_o\ \alpha_\jmath\ \lambda_\kappa\ \beta_\iota\ \mu_o\ \delta_\iota\ \nu_\kappa\ \gamma_\jmath\ \widetilde{O}^h\ S^0_{23}S^\alpha_2\ S^\alpha_7$	$\sigma_o\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \mu_o\ \delta_\jmath\ \nu_\iota\ \gamma_\kappa\ \widetilde{O}^h\ S^0_{24}S^\alpha_3\ S^\alpha_5$	$\sigma_o\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \mu_o\ \nu_\jmath\ \delta_\iota\ \gamma_\kappa\ \widetilde{O}^h\ S^0_{24}S^\alpha_2\ S^\alpha_6$
Type i: $\widetilde{O}$, sedenion participation: $1 \times S^0$,$2 \times S^\beta$		
$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \mu_\kappa\ \gamma_o\ \widetilde{O}^i\ S^0_{17}S^\beta_{10}\ S^\beta_{13}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \mu_\kappa\ \gamma_o\ \widetilde{O}^i\ S^0_{17}S^\beta_9\ S^\beta_{12}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \gamma_\kappa\ \gamma_o\ \widetilde{O}^i\ S^0_{17}S^\beta_{11}\ S^\beta_{14}$
$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \gamma_\iota\ \sigma_\iota\ \widetilde{O}^i\ S^0_{19}S^\beta_8\ S^\beta_{12}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \gamma_\jmath\ \sigma_\jmath\ \widetilde{O}^i\ S^0_{20}S^\beta_8\ S^\beta_{13}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \gamma_\kappa\ \sigma_\kappa\ \widetilde{O}^i\ S^0_{21}S^\beta_8\ S^\beta_{14}$
$\sigma_o\ \alpha_\iota\ \beta_\jmath\ \lambda_\kappa\ \gamma_o\ \delta_\kappa\ \nu_\jmath\ \mu_\iota\ \widetilde{O}^i\ S^0_{22}S^\beta_{11}\ S^\beta_{13}$	$\sigma_o\ \alpha_\iota\ \lambda_\jmath\ \beta_\kappa\ \gamma_o\ \nu_\kappa\ \delta_\jmath\ \mu_\iota\ \widetilde{O}^i\ S^0_{22}S^\beta_{10}\ S^\beta_{14}$	$\sigma_o\ \alpha_\jmath\ \lambda_\kappa\ \beta_\iota\ \gamma_o\ \nu_\iota\ \delta_\kappa\ \mu_\jmath\ \widetilde{O}^i\ S^0_{23}S^\beta_{11}\ S^\beta_{12}$
$\sigma_o\ \alpha_\jmath\ \beta_\kappa\ \lambda_\iota\ \gamma_o\ \delta_\iota\ \nu_\kappa\ \mu_\jmath\ \widetilde{O}^i\ S^0_{23}S^\beta_9\ S^\beta_{14}$	$\sigma_o\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \gamma_o\ \nu_\jmath\ \delta_\iota\ \mu_\kappa\ \widetilde{O}^i\ S^0_{24}S^\beta_9\ S^\beta_{13}$	$\sigma_o\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \gamma_o\ \delta_\jmath\ \nu_\iota\ \mu_\kappa\ \widetilde{O}^i\ S^0_{24}S^\beta_{10}\ S^\beta_{12}$
$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \mu_\iota\ \alpha_\iota\ \widetilde{O}^i\ S^0_{19}S^\beta_{10}\ S^\beta_{11}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \mu_\jmath\ \alpha_\jmath\ \widetilde{O}^i\ S^0_{20}S^\beta_9\ S^\beta_{11}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \mu_\kappa\ \alpha_\kappa\ \widetilde{O}^i\ S^0_{21}S^\beta_9\ S^\beta_{10}$
$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \gamma_o\ \nu_\kappa\ \nu_\jmath\ \gamma_\iota\ \widetilde{O}^i\ S^0_{22}S^\beta_8\ S^\beta_9$	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \gamma_o\ \nu_\iota\ \nu_\kappa\ \gamma_\jmath\ \widetilde{O}^i\ S^0_{23}S^\beta_8\ S^\beta_{10}$	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \gamma_o\ \nu_\jmath\ \nu_\iota\ \gamma_\kappa\ \widetilde{O}^i\ S^0_{24}S^\beta_8\ S^\beta_{11}$
$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\iota\ \delta_\iota\ \mu_\iota\ \alpha_\iota\ \widetilde{O}^i\ S^0_{19}S^\beta_{131}\ S^\beta_{14}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\jmath\ \delta_\jmath\ \mu_\jmath\ \alpha_\jmath\ \widetilde{O}^i\ S^0_{20}S^\beta_{12}\ S^\beta_{14}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\kappa\ \delta_\kappa\ \mu_\kappa\ \alpha_\kappa\ \widetilde{O}^i\ S^0_{21}S^\beta_{12}\ S^\beta_{13}$
Type j: $\widetilde{O}$, sedenion participation: $1 \times S^0$,$1 \times S^\alpha$,$1 \times S^\beta$		
$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \beta_\kappa\ \lambda_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_6S^\beta_{10}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \lambda_\kappa\ \lambda_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_7S^\beta_{11}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \beta_\kappa\ \lambda_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_5S^\beta_9$
$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \nu_\kappa\ \delta_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_3S^\beta_{10}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \nu_\kappa\ \delta_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_2S^\beta_9$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \delta_\kappa\ \delta_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_4S^\beta_{11}$
$\sigma_o\ \alpha_\iota\ \nu_\jmath\ \delta_\kappa\ \lambda_o\ \gamma_\kappa\ \mu_\jmath\ \beta_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_6S^\beta_{11}$	$\sigma_o\ \alpha_\iota\ \delta_\jmath\ \nu_\kappa\ \lambda_o\ \mu_\kappa\ \gamma_\jmath\ \beta_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_7S^\beta_{10}$	$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \beta_o\ \gamma_\kappa\ \gamma_\jmath\ \beta_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_2S^\beta_8$
$\sigma_o\ \alpha_\jmath\ \nu_\kappa\ \delta_\iota\ \lambda_o\ \gamma_\iota\ \mu_\kappa\ \beta_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_7S^\beta_9$	$\sigma_o\ \alpha_\jmath\ \delta_\kappa\ \nu_\iota\ \lambda_o\ \mu_\iota\ \gamma_\kappa\ \beta_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_5S^\beta_{11}$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \beta_o\ \gamma_\iota\ \gamma_\kappa\ \beta_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_3S^\beta_8$
$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \beta_o\ \gamma_\jmath\ \gamma_\iota\ \beta_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_4S^\beta_8$	$\sigma_o\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \lambda_o\ \mu_\jmath\ \gamma_\iota\ \beta_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_6S^\beta_9$	$\sigma_o\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \lambda_o\ \gamma_\jmath\ \mu_\iota\ \beta_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_5S^\beta_{10}$
$\sigma_o\ \alpha_\iota\ \gamma_\jmath\ \mu_\kappa\ \delta_o\ \beta_\kappa\ \lambda_\jmath\ \nu_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_4S^\beta_{10}$	$\sigma_o\ \sigma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \nu_o\ \beta_\kappa\ \beta_\jmath\ \nu_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_5S^\beta_8$	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \nu_o\ \lambda_\kappa\ \lambda_\jmath\ \nu_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_1S^\beta_{12}$
$\sigma_o\ \alpha_\iota\ \mu_\jmath\ \gamma_\kappa\ \delta_o\ \lambda_\kappa\ \beta_\jmath\ \nu_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_3S^\beta_{11}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \nu_o\ \lambda_\iota\ \lambda_\kappa\ \nu_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_1S^\beta_{13}$	$\sigma_o\ \alpha_\jmath\ \mu_\kappa\ \gamma_\iota\ \delta_o\ \lambda_\iota\ \beta_\kappa\ \nu_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_4S^\beta_9$
$\sigma_o\ \alpha_\jmath\ \gamma_\kappa\ \mu_\iota\ \delta_o\ \beta_\iota\ \lambda_\kappa\ \nu_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_2S^\beta_{11}$	$\sigma_o\ \sigma_\jmath\ \gamma_\kappa\ \gamma_\iota\ \nu_o\ \beta_\iota\ \beta_\kappa\ \nu_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_6S^\beta_8$	$\sigma_o\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \delta_o\ \beta_\jmath\ \lambda_\iota\ \nu_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_3S^\beta_9$
$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \nu_o\ \lambda_\jmath\ \lambda_\iota\ \nu_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_1S^\beta_{14}$	$\sigma_o\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \nu_o\ \beta_\jmath\ \beta_\iota\ \nu_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_7S^\beta_8$	$\sigma_o\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \delta_o\ \lambda_\jmath\ \beta_\iota\ \nu_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_2S^\beta_{10}$
$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \lambda_o\ \mu_\kappa\ \mu_\jmath\ \lambda_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_1S^\beta_9$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \lambda_o\ \mu_\iota\ \mu_\kappa\ \lambda_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_1S^\beta_{10}$	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \lambda_o\ \mu_\jmath\ \mu_\iota\ \lambda_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_1S^\beta_{11}$
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \nu_\kappa\ \nu_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_1S^\beta_8$	$\sigma_o\ \alpha_\iota\ \delta_\jmath\ \nu_\kappa\ \beta_o\ \gamma_\kappa\ \mu_\jmath\ \lambda_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_3S^\beta_{14}$	$\sigma_o\ \alpha_\iota\ \nu_\jmath\ \delta_\kappa\ \beta_o\ \mu_\kappa\ \gamma_\jmath\ \lambda_\iota\ \widetilde{O}^j\ S^0_{28}S^\alpha_4S^\beta_{13}$
$\sigma_o\ \alpha_\jmath\ \delta_\kappa\ \nu_\iota\ \beta_o\ \gamma_\iota\ \mu_\kappa\ \lambda_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_4S^\beta_{12}$	$\sigma_o\ \alpha_\jmath\ \nu_\kappa\ \delta_\iota\ \beta_o\ \mu_\iota\ \gamma_\kappa\ \lambda_\jmath\ \widetilde{O}^j\ S^0_{29}S^\alpha_2S^\beta_{14}$	$\sigma_o\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \beta_o\ \mu_\jmath\ \gamma_\iota\ \lambda_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_3S^\beta_{12}$
$\sigma_o\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \beta_o\ \gamma_\jmath\ \mu_\iota\ \lambda_\kappa\ \widetilde{O}^j\ S^0_{30}S^\alpha_2S^\beta_{13}$	$\sigma_o\ \alpha_\iota\ \mu_\jmath\ \gamma_\kappa\ \nu_o\ \lambda_\jmath\ \beta_\kappa\ \delta_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_6S^\beta_{14}$	$\sigma_o\ \alpha_\iota\ \gamma_\jmath\ \mu_\kappa\ \nu_o\ \beta_\jmath\ \lambda_\kappa\ \delta_\iota\ \widetilde{O}^j\ S^0_{25}S^\alpha_7S^\beta_{13}$
$\sigma_o\ \alpha_\jmath\ \mu_\kappa\ \gamma_\iota\ \nu_o\ \lambda_\kappa\ \beta_\iota\ \delta_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_7S^\beta_{12}$	$\sigma_o\ \alpha_\jmath\ \gamma_\kappa\ \mu_\iota\ \nu_o\ \beta_\kappa\ \lambda_\iota\ \delta_\jmath\ \widetilde{O}^j\ S^0_{26}S^\alpha_5S^\beta_{14}$	$\sigma_o\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \nu_o\ \beta_\iota\ \lambda_\jmath\ \delta_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_6S^\beta_{12}$
$\sigma_o\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \nu_o\ \lambda_\iota\ \beta_\jmath\ \delta_\kappa\ \widetilde{O}^j\ S^0_{27}S^\alpha_5S^\beta_{13}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \nu_\kappa\ \nu_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_7S^\beta_{14}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \delta_\kappa\ \nu_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_6S^\beta_{13}$
$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \delta_\kappa\ \nu_o\ \widetilde{O}^j\ S^0_{16}S^\alpha_5S^\beta_{12}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \beta_\kappa\ \beta_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_4S^\beta_{14}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \lambda_\kappa\ \beta_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_2S^\beta_{12}$
$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \lambda_\kappa\ \beta_o\ \widetilde{O}^j\ S^0_{18}S^\alpha_3S^\beta_{13}$		
Type k: $\mathbb{O}$, sedenion participation: $3 \times S_0$		
$\sigma_o\ \gamma_\kappa\ \delta_\iota\ \lambda_\jmath\ \mu_\kappa\ \nu_\iota\ \beta_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{24}S^0_{25}S^0_{29}$	$\sigma_o\ \gamma_\jmath\ \delta_\iota\ \lambda_\kappa\ \mu_\jmath\ \nu_\iota\ \beta_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{23}S^0_{25}S^0_{30}$	$\sigma_o\ \gamma_\iota\ \delta_\jmath\ \lambda_\kappa\ \mu_\iota\ \nu_\jmath\ \beta_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{22}S^0_{26}S^0_{30}$
$\sigma_o\ \gamma_\iota\ \mu_o\ \alpha_\iota\ \mu_\iota\ \gamma_o\ \sigma_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{19}S^0_{22}$	$\sigma_o\ \beta_o\ \lambda_\iota\ \alpha_\iota\ \lambda_o\ \beta_\iota\ \sigma_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{19}S^0_{28}$	$\sigma_o\ \nu_o\ \delta_\iota\ \alpha_\iota\ \delta_o\ \nu_\iota\ \sigma_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{19}S^0_{25}$
$\sigma_o\ \gamma_\jmath\ \mu_o\ \alpha_\jmath\ \mu_\jmath\ \gamma_o\ \sigma_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{20}S^0_{23}$	$\sigma_o\ \nu_o\ \delta_\jmath\ \alpha_\jmath\ \delta_o\ \nu_\jmath\ \sigma_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{20}S^0_{26}$	$\sigma_o\ \beta_o\ \lambda_\jmath\ \alpha_\jmath\ \lambda_o\ \beta_\jmath\ \sigma_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{20}S^0_{29}$
$\sigma_o\ \gamma_\kappa\ \mu_o\ \alpha_\kappa\ \mu_\kappa\ \gamma_o\ \sigma_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{21}S^0_{24}$	$\sigma_o\ \nu_o\ \delta_\kappa\ \alpha_\kappa\ \delta_o\ \nu_\kappa\ \sigma_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{21}S^0_{27}$	$\sigma_o\ \beta_o\ \lambda_\kappa\ \alpha_\kappa\ \lambda_o\ \beta_\kappa\ \sigma_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{21}S^0_{30}$
$\sigma_o\ \gamma_\kappa\ \lambda_\iota\ \delta_\jmath\ \mu_\kappa\ \beta_\iota\ \nu_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{24}S^0_{26}S^0_{28}$	$\sigma_o\ \gamma_\jmath\ \lambda_\iota\ \delta_\kappa\ \mu_\jmath\ \beta_\iota\ \nu_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{23}S^0_{27}S^0_{28}$	$\sigma_o\ \gamma_\iota\ \lambda_\jmath\ \delta_\kappa\ \mu_\iota\ \beta_\jmath\ \nu_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{22}S^0_{27}S^0_{29}$
$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \alpha_\jmath\ \lambda_\kappa\ \lambda_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{23}S^0_{26}$	$\sigma_o\ \delta_o\ \mu_\iota\ \beta_\iota\ \nu_o\ \gamma_\iota\ \lambda_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{19}S^0_{26}S^0_{27}$	$\sigma_o\ \delta_o\ \mu_\jmath\ \beta_\jmath\ \nu_o\ \gamma_\jmath\ \lambda_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{20}S^0_{25}S^0_{27}$
$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \alpha_\kappa\ \lambda_\iota\ \lambda_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{24}S^0_{27}$	$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \alpha_\iota\ \lambda_\jmath\ \lambda_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{18}S^0_{22}S^0_{25}$	$\sigma_o\ \delta_o\ \mu_\kappa\ \beta_\kappa\ \nu_o\ \gamma_\kappa\ \lambda_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{21}S^0_{25}S^0_{26}$
$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \alpha_\iota\ \alpha_\jmath\ \alpha_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{17}S^0_{18}$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \alpha_\jmath\ \delta_\kappa\ \delta_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{23}S^0_{29}$	$\sigma_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \mu_o\ \lambda_\iota\ \delta_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{19}S^0_{23}S^0_{24}$
$\sigma_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \mu_o\ \lambda_\jmath\ \delta_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{20}S^0_{22}S^0_{24}$	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \alpha_\kappa\ \delta_\iota\ \delta_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{24}S^0_{30}$	$\sigma_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \mu_o\ \lambda_\kappa\ \delta_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{21}S^0_{22}S^0_{23}$
$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \alpha_\iota\ \delta_\jmath\ \delta_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{16}S^0_{22}S^0_{28}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_o\ \nu_o\ \mu_o\ \alpha_o\ \mathbb{O}^k\ S^0_{19}S^0_{20}S^0_{21}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \alpha_\jmath\ \gamma_\kappa\ \gamma_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{26}S^0_{29}$
$\sigma_o\ \lambda_o\ \nu_\iota\ \mu_\iota\ \beta_o\ \delta_\iota\ \gamma_\iota\ \alpha_o\ \mathbb{O}^k\ S^0_{19}S^0_{29}S^0_{30}$	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \alpha_\kappa\ \gamma_\iota\ \gamma_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{27}S^0_{30}$	$\sigma_o\ \lambda_o\ \nu_\jmath\ \mu_\jmath\ \beta_o\ \delta_\jmath\ \gamma_\jmath\ \alpha_o\ \mathbb{O}^k\ S^0_{20}S^0_{28}S^0_{30}$
$\sigma_o\ \lambda_o\ \nu_\kappa\ \mu_\kappa\ \beta_o\ \delta_\kappa\ \gamma_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{21}S^0_{28}S^0_{29}$	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \alpha_\iota\ \gamma_\jmath\ \gamma_\kappa\ \alpha_o\ \mathbb{O}^k\ S^0_{17}S^0_{25}S^0_{28}$	

Table 4. Unit elements for aligned $\mathbb{M}$ and $\mathbb{T}$ subalgebras with 4 unit elements and complexified aligned $\mathbb{C}\otimes\mathbb{W}$ sub-algebras

Unit elements of $\mathbb{M}$		Unit elements of $\mathbb{T}$ and sedenion-type loop participation						Unit elements of $\mathbb{C}\otimes\mathbb{W}$	
Sub-group	Type	Sub-loop	Type	S^0_L	S^α_L	S^β_L	S^γ_L	Complexified aligned sub-loop	Type
SLMN	Non-abelian	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa$	Non-Abelian	$S^0_{15}S^0_{16}S^0_{17}S^0_{18}$	S^α_1	S^β_8	S^γ_0	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \sigma_oiS\ \sigma_\iota iL\ \sigma_\jmath iM\ \sigma_\kappa iN$	Abelian
SLiMiN	Non-abelian	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa$	Non-Abelian	$S^0_{16}S^0_{17}S^0_{18}$	$S^\alpha_2S^\alpha_5$	$S^\beta_9S^\beta_{12}$	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \sigma_oiS\ \sigma_\iota iL\ \alpha_\jmath M\ \alpha_\kappa N$	Abelian
SiLMiN	Non-abelian	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa$	Non-Abelian	$S^0_{16}S^0_{17}S^0_{18}$	$S^\alpha_3S^\alpha_6$	$S^\beta_{10}S^\beta_{13}$	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \sigma_oiS\ \alpha_\iota L\ \sigma_\jmath iM\ \alpha_\kappa N$	Abelian
SiLiMN	Non-abelian	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa$	Non-Abelian	$S^0_{16}S^0_{17}S^0_{18}$	$S^\alpha_4S^\alpha_7$	$S^\beta_{11}S^\beta_{14}$	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \sigma_oiS\ \alpha_\iota L\ \alpha_\jmath M\ \sigma_\kappa iN$	Abelian
SLEF	Non-abelian	$\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa$	Non-Abelian	$S^0_{15}S^0_{16}S^0_{22}S^0_{28}$	S^α_5	S^β_{12}	S^γ_0	$\sigma_oS\ \sigma_\iota L\ \lambda_\jmath E\ \lambda_\kappa F\ \sigma_oiS\ \sigma_\iota iL\ \lambda_\jmath iE\ \lambda_\kappa iF$	Abelian
SLiEiF	Non-abelian	$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa$	Non-Abelian	$S^0_{16}S^0_{22}S^0_{28}$	$S^\alpha_1S^\alpha_2$	$S^\beta_8S^\beta_9$	–	$\sigma_oS\ \sigma_\iota L\ \beta_\jmath iE\ \beta_\kappa iF\ \sigma_oiS\ \sigma_\iota iL\ \beta_\jmath E\ \beta_\kappa F$	Abelian
SiLiEF	Non-abelian	$\sigma_o\ \alpha_\iota\ \beta_\jmath\ \lambda_\kappa$	Non-Abelian	$S^0_{16}S^0_{22}S^0_{28}$	$S^\alpha_4S^\alpha_6$	$S^\beta_{11}S^\beta_{13}$	–	$\sigma_oS\ \alpha_\iota iL\ \beta_\jmath iE\ \lambda_\kappa F\ \sigma_oiS\ \alpha_\iota L\ \beta_\jmath E\ \lambda_\kappa iF$	Abelian
SiLEiF	Non-abelian	$\sigma_o\ \alpha_\iota\ \lambda_\jmath\ \beta_\kappa$	Non-Abelian	$S^0_{16}S^0_{22}S^0_{28}$	$S^\alpha_3S^\alpha_7$	$S^\beta_{10}S^\beta_{14}$	–	$\sigma_oS\ \alpha_\iota iL\ \lambda_\jmath E\ \beta_\kappa iF\ \sigma_oiS\ \alpha_\iota L\ \lambda_\jmath iE\ \beta_\kappa F$	Abelian
SMFD	Non-abelian	$\sigma_o\ \sigma_\jmath\ \lambda_\kappa\ \lambda_\iota$	Non-Abelian	$S^0_{15}S^0_{16}S^0_{23}S^0_{29}$	S^α_6	S^β_{13}	S^γ_0	$\sigma_oS\ \sigma_\jmath M\ \lambda_\kappa F\ \lambda_\iota D\ \sigma_oiS\ \sigma_\jmath iM\ \lambda_\kappa iF\ \lambda_\iota iD$	Abelian
SMiFiD	Non-abelian	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota$	Non-Abelian	$S^0_{16}S^0_{23}S^0_{29}$	$S^\alpha_1S^\alpha_3$	$S^\beta_8S^\beta_{10}$	–	$\sigma_oS\ \sigma_\jmath M\ \beta_\kappa iF\ \beta_\iota iD\ \sigma_oiS\ \sigma_\jmath iM\ \beta_\kappa F\ \beta_\iota D$	Abelian
SiMiFD	Non-abelian	$\sigma_o\ \alpha_\jmath\ \beta_\kappa\ \lambda_\iota$	Non-Abelian	$S^0_{16}S^0_{23}S^0_{29}$	$S^\alpha_2S^\alpha_7$	$S^\beta_9S^\beta_{14}$	–	$\sigma_oS\ \alpha_\jmath iM\ \beta_\kappa iF\ \lambda_\iota D\ \sigma_oiS\ \alpha_\jmath M\ \beta_\kappa F\ \lambda_\iota iD$	Abelian
SiMFiD	Non-abelian	$\sigma_o\ \alpha_\jmath\ \lambda_\kappa\ \beta_\iota$	Non-Abelian	$S^0_{16}S^0_{23}S^0_{29}$	$S^\alpha_4S^\alpha_5$	$S^\beta_{11}S^\beta_{12}$	–	$\sigma_oS\ \alpha_\jmath iM\ \lambda_\kappa F\ \beta_\iota iD\ \sigma_oiS\ \alpha_\jmath M\ \lambda_\kappa iF\ \beta_\iota D$	Abelian
SNDE	Non-abelian	$\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath$	Non-Abelian	$S^0_{15}S^0_{16}S^0_{24}S^0_{30}$	S^α_7	S^β_{14}	S^γ_0	$\sigma_oS\ \sigma_\kappa N\ \lambda_\iota D\ \lambda_\jmath E\ \sigma_oiS\ \sigma_\kappa iN\ \lambda_\iota iD\ \lambda_\jmath iE$	Abelian
SNiDiE	Non-abelian	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath$	Non-Abelian	$S^0_{16}S^0_{24}S^0_{30}$	$S^\alpha_1S^\alpha_4$	$S^\beta_8S^\beta_{11}$	–	$\sigma_oS\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \sigma_oiS\ \sigma_\kappa iN\ \beta_\iota D\ \beta_\jmath E$	Abelian
SiNiDE	Non-abelian	$\sigma_o\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath$	Non-Abelian	$S^0_{16}S^0_{24}S^0_{30}$	$S^\alpha_3S^\alpha_5$	$S^\beta_{10}S^\beta_{12}$	–	$\sigma_oS\ \alpha_\kappa iN\ \beta_\iota iD\ \lambda_\jmath E\ \sigma_oiS\ \alpha_\kappa N\ \beta_\iota D\ \lambda_\jmath iE$	Abelian
SiNDiE	Non-abelian	$\sigma_o\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath$	Non-Abelian	$S^0_{16}S^0_{24}S^0_{30}$	$S^\alpha_2S^\alpha_6$	$S^\beta_9S^\beta_{13}$	–	$\sigma_oS\ \alpha_\kappa iN\ \lambda_\iota D\ \beta_\jmath iE\ \sigma_oiS\ \alpha_\kappa N\ \lambda_\iota iD\ \beta_\jmath E$	Abelian
SLYZ	Non-abelian	$\sigma_o\ \sigma_\iota\ \gamma_\jmath\ \gamma_\kappa$	Non-Abelian	$S^0_{17}S^0_{25}S^0_{28}$	$S^\alpha_2S^\alpha_5$	S^β_8	S^γ_0	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \sigma_oiS\ \sigma_\iota iL\ \gamma_\jmath iY\ \gamma_\kappa iZ$	Abelian
SLiYiZ	Non-abelian	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa$	Non-Abelian	$S^0_{15}S^0_{17}S^0_{25}S^0_{28}$	S^α_1	$S^\beta_9S^\beta_{12}$	–	$\sigma_oS\ \sigma_\iota L\ \mu_\jmath iY\ \mu_\kappa iZ\ \sigma_oiS\ \sigma_\iota iL\ \mu_\jmath Y\ \mu_\kappa Z$	Abelian
SiLiYZ	Non-abelian	$\sigma_o\ \alpha_\iota\ \mu_\jmath\ \gamma_\kappa$	Non-Abelian	$S^0_{17}S^0_{25}S^0_{28}$	$S^\alpha_3S^\alpha_6$	$S^\beta_{11}S^\beta_{14}$	–	$\sigma_oS\ \alpha_\iota iL\ \mu_\jmath iY\ \gamma_\kappa Z\ \sigma_oiS\ \alpha_\iota L\ \mu_\jmath Y\ \gamma_\kappa iZ$	Abelian
SiLYiZ	Non-abelian	$\sigma_o\ \alpha_\iota\ \gamma_\jmath\ \mu_\kappa$	Non-Abelian	$S^0_{17}S^0_{25}S^0_{28}$	$S^\alpha_4S^\alpha_7$	$S^\beta_{10}S^\beta_{13}$	–	$\sigma_oS\ \alpha_\iota iL\ \gamma_\jmath Y\ \mu_\kappa iZ\ \sigma_oiS\ \alpha_\iota L\ \gamma_\jmath iY\ \mu_\kappa Z$	Abelian
SMZX	Non-abelian	$\sigma_o\ \sigma_\jmath\ \gamma_\kappa\ \gamma_\iota$	Non-Abelian	$S^0_{17}S^0_{26}S^0_{29}$	$S^\alpha_3S^\alpha_6$	S^β_8	S^γ_0	$\sigma_oS\ \sigma_\jmath M\ \gamma_\kappa Z\ \gamma_\iota X\ \sigma_oiS\ \sigma_\jmath iM\ \gamma_\kappa iZ\ \gamma_\iota iX$	Abelian
SMiZiX	Non-abelian	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota$	Non-Abelian	$S^0_{15}S^0_{17}S^0_{26}S^0_{29}$	S^α_1	$S^\beta_{10}S^\beta_{13}$	–	$\sigma_oS\ \sigma_\jmath M\ \mu_\kappa iZ\ \mu_\iota iX\ \sigma_oiS\ \sigma_\jmath iM\ \mu_\kappa Z\ \mu_\iota X$	Abelian
SiMiZX	Non-abelian	$\sigma_o\ \alpha_\jmath\ \mu_\kappa\ \gamma_\iota$	Non-Abelian	$S^0_{17}S^0_{26}S^0_{29}$	$S^\alpha_4S^\alpha_7$	$S^\beta_9S^\beta_{12}$	–	$\sigma_oS\ \alpha_\jmath iM\ \mu_\kappa iZ\ \gamma_\iota X\ \sigma_oiS\ \alpha_\jmath M\ \mu_\kappa Z\ \gamma_\iota iX$	Abelian
SiMZiX	Non-abelian	$\sigma_o\ \alpha_\jmath\ \gamma_\kappa\ \mu_\iota$	Non-Abelian	$S^0_{17}S^0_{26}S^0_{29}$	$S^\alpha_2S^\alpha_5$	$S^\beta_{11}S^\beta_{14}$	–	$\sigma_oS\ \alpha_\jmath iM\ \gamma_\kappa Z\ \mu_\iota iX\ \sigma_oiS\ \alpha_\jmath M\ \gamma_\kappa iZ\ \mu_\iota X$	Abelian
SNXY	Non-abelian	$\sigma_o\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath$	Non-Abelian	$S^0_{17}S^0_{27}S^0_{30}$	$S^\alpha_4S^\alpha_7$	S^β_8	S^γ_0	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \sigma_oiS\ \sigma_\kappa iN\ \gamma_\iota iX\ \gamma_\jmath iY$	Abelian
SNiXiY	Non-abelian	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath$	Non-Abelian	$S^0_{15}S^0_{17}S^0_{27}S^0_{30}$	S^α_1	$S^\beta_{11}S^\beta_{14}$	–	$\sigma_oS\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \sigma_oiS\ \sigma_\kappa iN\ \mu_\iota X\ \mu_\jmath Y$	Abelian
SiNiXY	Non-abelian	$\sigma_o\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath$	Non-Abelian	$S^0_{17}S^0_{27}S^0_{30}$	$S^\alpha_2S^\alpha_5$	$S^\beta_{10}S^\beta_{13}$	–	$\sigma_oS\ \alpha_\kappa iN\ \mu_\iota iX\ \gamma_\jmath Y\ \sigma_oiS\ \alpha_\kappa N\ \mu_\iota X\ \gamma_\jmath iY$	Abelian
SiNXiY	Non-abelian	$\sigma_o\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath$	Non-Abelian	$S^0_{17}S^0_{27}S^0_{30}$	$S^\alpha_3S^\alpha_6$	$S^\beta_9S^\beta_{12}$	–	$\sigma_oS\ \alpha_\kappa iN\ \gamma_\iota X\ \mu_\jmath iY\ \sigma_oiS\ \alpha_\kappa N\ \gamma_\iota iX\ \mu_\jmath Y$	Abelian
SLQR	Non-abelian	$\sigma_o\ \sigma_\iota\ \delta_\jmath\ \delta_\kappa$	Non-Abelian	$S^0_{18}S^0_{22}S^0_{25}$	$S^\alpha_1S^\alpha_2$	S^β_{12}	S^γ_0	$\sigma_oS\ \sigma_\iota L\ \delta_\jmath Q\ \delta_\kappa R\ \sigma_oiS\ \sigma_\iota iL\ \delta_\jmath iQ\ \delta_\kappa iR$	Abelian
SLiQiR	Non-abelian	$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa$	Non-Abelian	$S^0_{15}S^0_{18}S^0_{22}S^0_{25}$	S^α_5	$S^\beta_8S^\beta_9$	–	$\sigma_oS\ \sigma_\iota L\ \nu_\jmath iQ\ \nu_\kappa iR\ \sigma_oiS\ \sigma_\iota iL\ \nu_\jmath Q\ \nu_\kappa R$	Abelian
SiLiQR	Non-abelian	$\sigma_o\ \alpha_\iota\ \nu_\jmath\ \delta_\kappa$	Non-Abelian	$S^0_{18}S^0_{22}S^0_{25}$	$S^\alpha_3S^\alpha_7$	$S^\beta_{11}S^\beta_{13}$	–	$\sigma_oS\ \alpha_\iota iL\ \nu_\jmath iQ\ \delta_\kappa R\ \sigma_oiS\ \alpha_\iota L\ \nu_\jmath Q\ \delta_\kappa iR$	Abelian
SiLQiR	Non-abelian	$\sigma_o\ \alpha_\iota\ \delta_\jmath\ \nu_\kappa$	Non-Abelian	$S^0_{18}S^0_{22}S^0_{25}$	$S^\alpha_4S^\alpha_6$	$S^\beta_{10}S^\beta_{14}$	–	$\sigma_oS\ \alpha_\iota iL\ \delta_\jmath Q\ \nu_\kappa iR\ \sigma_oiS\ \alpha_\iota L\ \delta_\jmath iQ\ \nu_\kappa R$	Abelian
SMRP	Non-abelian	$\sigma_o\ \sigma_\jmath\ \delta_\kappa\ \delta_\iota$	Non-Abelian	$S^0_{18}S^0_{23}S^0_{26}$	$S^\alpha_1S^\alpha_3$	S^β_{13}	S^γ_0	$\sigma_oS\ \sigma_\jmath M\ \delta_\kappa R\ \delta_\iota P\ \sigma_oiS\ \sigma_\jmath iM\ \delta_\kappa iR\ \delta_\iota iP$	Abelian
SMiRiP	Non-abelian	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota$	Non-Abelian	$S^0_{15}S^0_{18}S^0_{23}S^0_{26}$	S^α_6	$S^\beta_8S^\beta_{10}$	–	$\sigma_oS\ \sigma_\jmath M\ \nu_\kappa iR\ \nu_\iota iP\ \sigma_oiS\ \sigma_\jmath iM\ \nu_\kappa R\ \nu_\iota P$	Abelian
SiMiRP	Non-abelian	$\sigma_o\ \alpha_\jmath\ \nu_\kappa\ \delta_\iota$	Non-Abelian	$S^0_{18}S^0_{23}S^0_{26}$	$S^\alpha_4S^\alpha_5$	$S^\beta_9S^\beta_{14}$	–	$\sigma_oS\ \alpha_\jmath iM\ \nu_\kappa iR\ \delta_\iota P\ \sigma_oiS\ \alpha_\jmath M\ \nu_\kappa R\ \delta_\iota iP$	Abelian
SiMRiP	Non-abelian	$\sigma_o\ \alpha_\jmath\ \delta_\kappa\ \nu_\iota$	Non-Abelian	$S^0_{18}S^0_{23}S^0_{26}$	$S^\alpha_2S^\alpha_7$	$S^\beta_{11}S^\beta_{12}$	–	$\sigma_oS\ \alpha_\jmath iM\ \delta_\kappa R\ \nu_\iota iP\ \sigma_oiS\ \alpha_\jmath M\ \delta_\kappa iR\ \nu_\iota P$	Abelian
SNPQ	Non-abelian	$\sigma_o\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath$	Non-Abelian	$S^0_{18}S^0_{24}S^0_{27}$	$S^\alpha_1S^\alpha_4$	S^β_{14}	S^γ_0	$\sigma_oS\ \sigma_\kappa N\ \delta_\iota P\ \delta_\jmath Q\ \sigma_oiS\ \sigma_\kappa iN\ \delta_\iota iP\ \delta_\jmath iQ$	Abelian
SNiPiQ	Non-abelian	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath$	Non-Abelian	$S^0_{15}S^0_{18}S^0_{24}S^0_{27}$	S^α_7	$S^\beta_8S^\beta_{11}$	–	$\sigma_oS\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \sigma_oiS\ \sigma_\kappa iN\ \nu_\iota P\ \nu_\jmath Q$	Abelian
SiNiPQ	Non-abelian	$\sigma_o\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath$	Non-Abelian	$S^0_{18}S^0_{24}S^0_{27}$	$S^\alpha_2S^\alpha_6$	$S^\beta_{10}S^\beta_{12}$	–	$\sigma_oS\ \alpha_\kappa iN\ \nu_\iota iP\ \delta_\jmath Q\ \sigma_oiS\ \alpha_\kappa N\ \nu_\iota P\ \delta_\jmath iQ$	Abelian
SiNPiQ	Non-abelian	$\sigma_o\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath$	Non-Abelian	$S^0_{18}S^0_{24}S^0_{27}$	$S^\alpha_3S^\alpha_5$	$S^\beta_9S^\beta_{13}$	–	$\sigma_oS\ \alpha_\kappa iN\ \delta_\iota P\ \nu_\jmath iQ\ \sigma_oiS\ \alpha_\kappa N\ \delta_\iota iP\ \nu_\jmath Q$	Abelian
SVTU	Non-abelian	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o$	Non-Abelian	$S^0_{19}S^0_{20}S^0_{21}$	–	$S^\beta_9S^\beta_{10}S^\beta_{11}$	S^γ_0	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \sigma_oiS\ \lambda_oiV\ \delta_oiT\ \gamma_oiU$	Abelian
SiVTiU	Non-abelian	$\sigma_o\ \beta_o\ \delta_o\ \mu_o$	Non-Abelian	$S^0_{19}S^0_{20}S^0_{21}$	$S^\alpha_1S^\alpha_5S^\alpha_6S^\alpha_7$	–	–	$\sigma_oS\ \beta_oiV\ \delta_oT\ \mu_oiU\ \sigma_oiS\ \beta_oV\ \delta_oiT\ \mu_oU$	Abelian
SViTiU	Non-abelian	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o$	Non-Abelian	$S^0_{15}S^0_{19}S^0_{20}S^0_{21}$	$S^\alpha_2S^\alpha_3S^\alpha_4$	–	–	$\sigma_oS\ \lambda_oV\ \nu_oiT\ \mu_oiU\ \sigma_oiS\ \lambda_oiV\ \nu_oT\ \mu_oU$	Abelian
SiViTU	Non-abelian	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o$	Non-Abelian	$S^0_{19}S^0_{20}S^0_{21}$	–	$S^\beta_8S^\beta_{12}S^\beta_{13}S^\beta_{14}$	–	$\sigma_oS\ \beta_oiV\ \nu_oiT\ \gamma_oU\ \sigma_oiS\ \beta_oV\ \nu_oT\ \gamma_oiU$	Abelian
SUDP	Non-abelian	$\sigma_o\ \gamma_o\ \lambda_\iota\ \delta_\iota$	Non-Abelian	$S^0_{19}S^0_{23}S^0_{24}$	–	$S^\beta_9S^\beta_{13}S^\beta_{14}$	S^γ_0	$\sigma_oS\ \gamma_oU\ \lambda_\iota D\ \delta_\iota P\ \sigma_oiS\ \gamma_oiU\ \lambda_\iota iD\ \delta_\iota iP$	Abelian
SiUiDP	Non-abelian	$\sigma_o\ \mu_o\ \beta_\iota\ \delta_\iota$	Non-Abelian	$S^0_{19}S^0_{23}S^0_{24}$	$S^\alpha_1S^\alpha_3S^\alpha_4S^\alpha_5$	–	–	$\sigma_oS\ \mu_oiU\ \beta_\iota iD\ \delta_\iota P\ \sigma_oiS\ \mu_oU\ \beta_\iota D\ \delta_\iota iP$	Abelian
SiUDiP	Non-abelian	$\sigma_o\ \mu_o\ \lambda_\iota\ \nu_\iota$	Non-Abelian	$S^0_{15}S^0_{19}S^0_{23}S^0_{24}$	$S^\alpha_2S^\alpha_6S^\alpha_7$	–	–	$\sigma_oS\ \mu_oiU\ \lambda_\iota D\ \nu_\iota iP\ \sigma_oiS\ \mu_oU\ \lambda_\iota iD\ \nu_\iota P$	Abelian
SUiDiP	Non-abelian	$\sigma_o\ \gamma_o\ \beta_\iota\ \nu_\iota$	Non-Abelian	$S^0_{19}S^0_{23}S^0_{24}$	–	$S^\beta_8S^\beta_{10}S^\beta_{11}S^\beta_{12}$	–	$\sigma_oS\ \gamma_oU\ \beta_\iota iD\ \nu_\iota iP\ \sigma_oiS\ \gamma_oiU\ \beta_\iota D\ \nu_\iota P$	Abelian
SUEQ	Non-abelian	$\sigma_o\ \gamma_o\ \lambda_\jmath\ \delta_\jmath$	Non-Abelian	$S^0_{20}S^0_{22}S^0_{24}$	–	$S^\beta_{10}S^\beta_{12}S^\beta_{14}$	S^γ_0	$\sigma_oS\ \gamma_oU\ \lambda_\jmath E\ \delta_\jmath Q\ \sigma_oiS\ \gamma_oiU\ \lambda_\jmath iE\ \delta_\jmath iQ$	Abelian
SiUiEQ	Non-abelian	$\sigma_o\ \mu_o\ \beta_\jmath\ \delta_\jmath$	Non-Abelian	$S^0_{20}S^0_{22}S^0_{24}$	$S^\alpha_1S^\alpha_2S^\alpha_3S^\alpha_6$	–	–	$\sigma_oS\ \mu_oiU\ \beta_\jmath iE\ \delta_\jmath Q\ \sigma_oiS\ \mu_oU\ \beta_\jmath E\ \delta_\jmath iQ$	Abelian
SiUEiQ	Non-abelian	$\sigma_o\ \mu_o\ \lambda_\jmath\ \nu_\jmath$	Non-Abelian	$S^0_{15}S^0_{20}S^0_{22}S^0_{24}$	$S^\alpha_3S^\alpha_5S^\alpha_7$	–	–	$\sigma_oS\ \mu_oiU\ \lambda_\jmath E\ \nu_\jmath iQ\ \sigma_oiS\ \mu_oU\ \lambda_\jmath iE\ \nu_\jmath Q$	Abelian
SUiEiQ	Non-abelian	$\sigma_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath$	Non-Abelian	$S^0_{20}S^0_{22}S^0_{24}$	–	$S^\beta_8S^\beta_9S^\beta_{11}S^\beta_{13}$	–	$\sigma_oS\ \gamma_oU\ \beta_\jmath iE\ \nu_\jmath iQ\ \sigma_oiS\ \gamma_oiU\ \beta_\jmath E\ \nu_\jmath Q$	Abelian
SUFR	Non-abelian	$\sigma_o\ \gamma_o\ \lambda_\kappa\ \delta_\kappa$	Non-Abelian	$S^0_{21}S^0_{22}S^0_{23}$	–	$S^\beta_{11}S^\beta_{12}S^\beta_{13}$	S^γ_0	$\sigma_oS\ \gamma_oU\ \lambda_\kappa F\ \delta_\kappa R\ \sigma_oiS\ \gamma_oiU\ \lambda_\kappa iF\ \delta_\kappa iR$	Abelian
SiUiFR	Non-abelian	$\sigma_o\ \mu_o\ \beta_\kappa\ \delta_\kappa$	Non-Abelian	$S^0_{21}S^0_{22}S^0_{23}$	$S^\alpha_1S^\alpha_2S^\alpha_3S^\alpha_7$	–	–	$\sigma_oS\ \mu_oiU\ \beta_\kappa iF\ \delta_\kappa R\ \sigma_oiS\ \mu_oU\ \beta_\kappa F\ \delta_\kappa iR$	Abelian
SiUFiR	Non-abelian	$\sigma_o\ \mu_o\ \lambda_\kappa\ \nu_\kappa$	Non-Abelian	$S^0_{15}S^0_{21}S^0_{22}S^0_{23}$	$S^\alpha_4S^\alpha_5S^\alpha_6$	–	–	$\sigma_oS\ \mu_oiU\ \lambda_\kappa F\ \nu_\kappa iR\ \sigma_oiS\ \mu_oU\ \lambda_\kappa iF\ \nu_\kappa R$	Abelian
SUiFiR	Non-abelian	$\sigma_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa$	Non-Abelian	$S^0_{21}S^0_{22}S^0_{23}$	–	$S^\beta_8S^\beta_9S^\beta_{10}S^\beta_{14}$	–	$\sigma_oS\ \gamma_oU\ \beta_\kappa iF\ \nu_\kappa iR\ \sigma_oiS\ \gamma_oiU\ \beta_\kappa F\ \nu_\kappa R$	Abelian
STXD	Non-abelian	$\sigma_o\ \delta_o\ \gamma_\iota\ \lambda_\iota$	Non-Abelian	$S^0_{19}S^0_{29}S^0_{30}$	$S^\alpha_6S^\alpha_7$	S^β_9	S^γ_0	$\sigma_oS\ \delta_oT\ \gamma_\iota X\ \lambda_\iota D\ \sigma_oiS\ \delta_oiT\ \gamma_\iota iX\ \lambda_\iota iD$	Abelian
SiTiXD	Non-abelian	$\sigma_o\ \nu_o\ \mu_\iota\ \lambda_\iota$	Non-Abelian	$S^0_{15}S^0_{19}S^0_{29}S^0_{30}$	S^α_2	$S^\beta_{13}S^\beta_{14}$	–	$\sigma_oS\ \nu_oiT\ \mu_\iota iX\ \lambda_\iota D\ \sigma_oiS\ \nu_oT\ \mu_\iota X\ \lambda_\iota iD$	Abelian
STiXiD	Non-abelian	$\sigma_o\ \delta_o\ \mu_\iota\ \beta_\iota$	Non-Abelian	$S^0_{19}S^0_{29}S^0_{30}$	$S^\alpha_1S^\alpha_5$	$S^\beta_{10}S^\beta_{11}$	–	$\sigma_oS\ \delta_oT\ \mu_\iota iX\ \beta_\iota iD\ \sigma_oiS\ \delta_oiT\ \mu_\iota X\ \beta_\iota D$	Abelian
SiTXiD	Non-abelian	$\sigma_o\ \nu_o\ \gamma_\iota\ \beta_\iota$	Non-Abelian	$S^0_{19}S^0_{29}S^0_{30}$	$S^\alpha_3S^\alpha_4$	$S^\beta_8S^\beta_{12}$	–	$\sigma_oS\ \nu_oiT\ \gamma_\iota X\ \beta_\iota iD\ \sigma_oiS\ \nu_oT\ \gamma_\iota iX\ \beta_\iota D$	Abelian
STYE	Non-abelian	$\sigma_o\ \delta_o\ \gamma_\jmath\ \lambda_\jmath$	Non-Abelian	$S^0_{20}S^0_{28}S^0_{30}$	$S^\alpha_5S^\alpha_7$	S^β_{10}	S^γ_0	$\sigma_oS\ \delta_oT\ \gamma_\jmath Y\ \lambda_\jmath E\ \sigma_oiS\ \delta_oiT\ \gamma_\jmath iY\ \lambda_\jmath iE$	Abelian
SiTiYE	Non-abelian	$\sigma_o\ \nu_o\ \mu_\jmath\ \lambda_\jmath$	Non-Abelian	$S^0_{15}S^0_{20}S^0_{28}S^0_{30}$	S^α_3	$S^\beta_{12}S^\beta_{14}$	–	$\sigma_oS\ \nu_oiT\ \mu_\jmath iY\ \lambda_\jmath E\ \sigma_oiS\ \nu_oT\ \mu_\jmath Y\ \lambda_\jmath iE$	Abelian
STiYiE	Non-abelian	$\sigma_o\ \delta_o\ \mu_\jmath\ \beta_\jmath$	Non-Abelian	$S^0_{20}S^0_{28}S^0_{30}$	$S^\alpha_1S^\alpha_6$	$S^\beta_9S^\beta_{11}$	–	$\sigma_oS\ \delta_oT\ \mu_\jmath iY\ \beta_\jmath iE\ \sigma_oiS\ \delta_oiT\ \mu_\jmath Y\ \beta_\jmath E$	Abelian
SiTYiE	Non-abelian	$\sigma_o\ \nu_o\ \gamma_\jmath\ \beta_\jmath$	Non-Abelian	$S^0_{20}S^0_{28}S^0_{30}$	$S^\alpha_2S^\alpha_4$	$S^\beta_8S^\beta_{13}$	–	$\sigma_oS\ \nu_oiT\ \gamma_\jmath Y\ \beta_\jmath iE\ \sigma_oiS\ \nu_oT\ \gamma_\jmath iY\ \beta_\jmath E$	Abelian
STZF	Non-abelian	$\sigma_o\ \delta_o\ \gamma_\kappa\ \lambda_\kappa$	Non-Abelian	$S^0_{21}S^0_{28}S^0_{29}$	$S^\alpha_5S^\alpha_6$	S^β_{11}	S^γ_0	$\sigma_oS\ \delta_oT\ \gamma_\kappa Z\ \lambda_\kappa F\ \sigma_oiS\ \delta_oiT\ \gamma_\kappa iZ\ \lambda_\kappa iF$	Abelian
SiTiZF	Non-abelian	$\sigma_o\ \nu_o\ \mu_\kappa\ \lambda_\kappa$	Non-Abelian	$S^0_{15}S^0_{21}S^0_{28}S^0_{29}$	S^α_4	$S^\beta_{12}S^\beta_{13}$	–	$\sigma_oS\ \nu_oiT\ \mu_\kappa iZ\ \lambda_\kappa F\ \sigma_oiS\ \nu_oT\ \mu_\kappa Z\ \lambda_\kappa iF$	Abelian
STiZiF	Non-abelian	$\sigma_o\ \delta_o\ \mu_\kappa\ \beta_\kappa$	Non-Abelian	$S^0_{21}S^0_{28}S^0_{29}$	$S^\alpha_1S^\alpha_7$	$S^\beta_9S^\beta_{10}$	–	$\sigma_oS\ \delta_oT\ \mu_\kappa iZ\ \beta_\kappa iF\ \sigma_oiS\ \delta_oiT\ \mu_\kappa Z\ \beta_\kappa F$	Abelian
SiTZiF	Non-abelian	$\sigma_o\ \nu_o\ \gamma_\kappa\ \beta_\kappa$	Non-Abelian	$S^0_{21}S^0_{28}S^0_{29}$	$S^\alpha_2S^\alpha_3$	$S^\beta_8S^\beta_{14}$	–	$\sigma_oS\ \nu_oiT\ \gamma_\kappa Z\ \beta_\kappa iF\ \sigma_oiS\ \nu_oT\ \gamma_\kappa iZ\ \beta_\kappa F$	Abelian
SVPX	Non-abelian	$\sigma_o\ \lambda_o\ \delta_\iota\ \gamma_\iota$	Non-Abelian	$S^0_{19}S^0_{26}S^0_{27}$	$S^\alpha_3S^\alpha_4$	S^β_9	S^γ_0	$\sigma_oS\ \lambda_oV\ \delta_\iota P\ \gamma_\iota X\ \sigma_oiS\ \lambda_oiV\ \delta_\iota iP\ \gamma_\iota iX$	Abelian
SiViPX	Non-abelian	$\sigma_o\ \beta_o\ \nu_\iota\ \gamma_\iota$	Non-Abelian	$S^0_{19}S^0_{26}S^0_{27}$	$S^\alpha_6S^\alpha_7$	$S^\beta_8S^\beta_{12}$	–	$\sigma_oS\ \beta_oiV\ \nu_\iota iP\ \gamma_\iota X\ \sigma_oiS\ \beta_oV\ \nu_\iota P\ \gamma_\iota iX$	Abelian
SViPiX	Non-abelian	$\sigma_o\ \lambda_o\ \nu_\iota\ \mu_\iota$	Non-Abelian	$S^0_{15}S^0_{19}S^0_{26}S^0_{27}$	S^α_2	$S^\beta_{10}S^\beta_{11}$	–	$\sigma_oS\ \lambda_oV\ \nu_\iota iP\ \mu_\iota iX\ \sigma_oiS\ \lambda_oiV\ \nu_\iota P\ \mu_\iota X$	Abelian
SiVPiX	Non-abelian	$\sigma_o\ \beta_o\ \delta_\iota\ \mu_\iota$	Non-Abelian	$S^0_{19}S^0_{26}S^0_{27}$	$S^\alpha_1S^\alpha_5$	$S^\beta_{13}S^\beta_{14}$	–	$\sigma_oS\ \beta_oiV\ \delta_\iota P\ \mu_\iota iX\ \sigma_oiS\ \beta_oV\ \delta_\iota iP\ \mu_\iota X$	Abelian
SVQY	Non-abelian	$\sigma_o\ \lambda_o\ \delta_\jmath\ \gamma_\jmath$	Non-Abelian	$S^0_{20}S^0_{25}S^0_{27}$	$S^\alpha_2S^\alpha_4$	S^β_{10}	S^γ_0	$\sigma_oS\ \lambda_oV\ \delta_\jmath Q\ \gamma_\jmath Y\ \sigma_oiS\ \lambda_oiV\ \delta_\jmath iQ\ \gamma_\jmath iY$	Abelian
SiViQY	Non-abelian	$\sigma_o\ \beta_o\ \nu_\jmath\ \gamma_\jmath$	Non-Abelian	$S^0_{20}S^0_{25}S^0_{27}$	$S^\alpha_5S^\alpha_7$	$S^\beta_8S^\beta_{13}$	–	$\sigma_oS\ \beta_oiV\ \nu_\jmath iQ\ \gamma_\jmath Y\ \sigma_oiS\ \beta_oV\ \nu_\jmath Q\ \gamma_\jmath iY$	Abelian
SViQiY	Non-abelian	$\sigma_o\ \lambda_o\ \nu_\jmath\ \mu_\jmath$	Non-Abelian	$S^0_{15}S^0_{20}S^0_{25}S^0_{27}$	S^α_3	$S^\beta_9S^\beta_{11}$	–	$\sigma_oS\ \lambda_oV\ \nu_\jmath iQ\ \mu_\jmath iY\ \sigma_oiS\ \lambda_oiV\ \nu_\jmath Q\ \mu_\jmath Y$	Abelian
SiVQiY	Non-abelian	$\sigma_o\ \beta_o\ \delta_\jmath\ \mu_\jmath$	Non-Abelian	$S^0_{20}S^0_{25}S^0_{27}$	$S^\alpha_1S^\alpha_6$	$S^\beta_{12}S^\beta_{14}$	–	$\sigma_oS\ \beta_oiV\ \delta_\jmath Q\ \mu_\jmath iY\ \sigma_oiS\ \beta_oV\ \delta_\jmath iQ\ \mu_\jmath Y$	Abelian
SVRZ	Non-abelian	$\sigma_o\ \lambda_o\ \delta_\kappa\ \gamma_\kappa$	Non-Abelian	$S^0_{21}S^0_{25}S^0_{26}$	$S^\alpha_2S^\alpha_3$	S^β_{11}	S^γ_0	$\sigma_oS\ \lambda_oV\ \delta_\kappa R\ \gamma_\kappa Z\ \sigma_oiS\ \lambda_oiV\ \delta_\kappa iR\ \gamma_\kappa iZ$	Abelian
SiViRZ	Non-abelian	$\sigma_o\ \beta_o\ \nu_\kappa\ \gamma_\kappa$	Non-Abelian	$S^0_{21}S^0_{25}S^0_{26}$	$S^\alpha_5S^\alpha_6$	$S^\beta_8S^\beta_{14}$	–	$\sigma_oS\ \beta_oiV\ \nu_\kappa iR\ \gamma_\kappa Z\ \sigma_oiS\ \beta_oV\ \nu_\kappa R\ \gamma_\kappa iZ$	Abelian
SViRiZ	Non-abelian	$\sigma_o\ \lambda_o\ \nu_\kappa\ \mu_\kappa$	Non-Abelian	$S^0_{15}S^0_{21}S^0_{25}S^0_{26}$	S^α_4	$S^\beta_9S^\beta_{10}$	–	$\sigma_oS\ \lambda_oV\ \nu_\kappa iR\ \mu_\kappa iZ\ \sigma_oiS\ \lambda_oiV\ \nu_\kappa R\ \mu_\kappa Z$	Abelian
SiVRiZ	Non-abelian	$\sigma_o\ \beta_o\ \delta_\kappa\ \mu_\kappa$	Non-Abelian	$S^0_{21}S^0_{25}S^0_{26}$	$S^\alpha_1S^\alpha_7$	$S^\beta_{12}S^\beta_{13}$	–	$\sigma_oS\ \beta_oiV\ \delta_\kappa R\ \mu_\kappa iZ\ \sigma_oiS\ \beta_oV\ \delta_\kappa iR\ \mu_\kappa Z$	Abelian

Table 5. Unit elements for aligned $\mathbb{M}$ and $\mathbb{T}$ subalgebras with 4 unit elements and complexified aligned $\mathbb{C} \otimes \mathbb{W}$ sub-algebras

Unit elements of $\mathbb{M}$		Unit elements of $\mathbb{T}$ and sedenion-type loop participation						Unit elements of $\mathbb{C} \otimes \mathbb{W}$	
Sub-group	Type	Sub-loop	Type	S_L^0	S_L^α	S_L^β	S_L^γ	Complexified aligned sub-loop	Type
SVDL	Abelian	$\sigma_o\ \lambda_o\ \lambda_\iota\ \sigma_\iota$	Non-Abelian	$S_{15}^0 S_{16}^0 S_{19}^0 S_{25}^0$	S_2^α	S_9^β	S_0^γ	$\sigma_o S\ \lambda_o V\ \lambda_\iota D\ \sigma_\iota L\ \sigma_o iS\ \lambda_o iV\ \lambda_\iota iD\ \sigma_\iota iL$	Non-Abelian
SiViDL	Abelian	$\sigma_o\ \beta_o\ \beta_\iota\ \sigma_\iota$	Non-Abelian	$S_{16}^0 S_{19}^0 S_{25}^0$	$S_1^\alpha S_5^\alpha$	$S_8^\beta S_{12}^\beta$	–	$\sigma_o S\ \beta_o iV\ \beta_\iota iD\ \sigma_\iota L\ \sigma_o iS\ \beta_o V\ \beta_\iota D\ \sigma_\iota iL$	Non-Abelian
SViDiL	Abelian	$\sigma_o\ \lambda_o\ \beta_\iota\ \alpha_\iota$	Non-Abelian	$S_{16}^0 S_{19}^0 S_{25}^0$	$S_3^\alpha S_4^\alpha$	$S_{10}^\beta S_{11}^\beta$	–	$\sigma_o S\ \lambda_o V\ \beta_\iota iD\ \alpha_\iota iL\ \sigma_o iS\ \lambda_o iV\ \beta_\iota D\ \alpha_\iota L$	Non-Abelian
SiVDiL	Abelian	$\sigma_o\ \beta_o\ \lambda_\iota\ \alpha_\iota$	Non-Abelian	$S_{16}^0 S_{19}^0 S_{25}^0$	$S_6^\alpha S_7^\alpha$	$S_{13}^\beta S_{14}^\beta$	–	$\sigma_o S\ \beta_o iV\ \lambda_\iota D\ \alpha_\iota iL\ \sigma_o iS\ \beta_o V\ \lambda_\iota iD\ \alpha_\iota L$	Non-Abelian
SVEM	Abelian	$\sigma_o\ \lambda_o\ \lambda_\jmath\ \sigma_\jmath$	Non-Abelian	$S_{15}^0 S_{16}^0 S_{20}^0 S_{26}^0$	S_3^α	S_{10}^β	S_0^γ	$\sigma_o S\ \lambda_o V\ \lambda_\jmath E\ \sigma_\jmath M\ \sigma_o iS\ \lambda_o iV\ \lambda_\jmath iE\ \sigma_\jmath iM$	Non-Abelian
SiViEM	Abelian	$\sigma_o\ \beta_o\ \beta_\jmath\ \sigma_\jmath$	Non-Abelian	$S_{16}^0 S_{20}^0 S_{26}^0$	$S_1^\alpha S_6^\alpha$	$S_8^\beta S_{13}^\beta$	–	$\sigma_o S\ \beta_o iV\ \beta_\jmath iE\ \sigma_\jmath M\ \sigma_o iS\ \beta_o V\ \beta_\jmath E\ \sigma_\jmath iM$	Non-Abelian
SViEiM	Abelian	$\sigma_o\ \lambda_o\ \beta_\jmath\ \alpha_\jmath$	Non-Abelian	$S_{16}^0 S_{20}^0 S_{26}^0$	$S_2^\alpha S_4^\alpha$	$S_9^\beta S_{11}^\beta$	–	$\sigma_o S\ \lambda_o V\ \beta_\jmath iE\ \alpha_\jmath iM\ \sigma_o iS\ \lambda_o iV\ \beta_\jmath E\ \alpha_\jmath M$	Non-Abelian
SiVEiM	Abelian	$\sigma_o\ \beta_o\ \lambda_\jmath\ \alpha_\jmath$	Non-Abelian	$S_{16}^0 S_{20}^0 S_{26}^0$	$S_5^\alpha S_7^\alpha$	$S_{12}^\beta S_{14}^\beta$	–	$\sigma_o S\ \beta_o iV\ \lambda_\jmath E\ \alpha_\jmath iM\ \sigma_o iS\ \beta_o V\ \lambda_\jmath iE\ \alpha_\jmath M$	Non-Abelian
SVFN	Abelian	$\sigma_o\ \lambda_o\ \lambda_\kappa\ \sigma_\kappa$	Non-Abelian	$S_{15}^0 S_{16}^0 S_{21}^0 S_{27}^0$	S_4^α	S_{11}^β	S_0^γ	$\sigma_o S\ \lambda_o V\ \lambda_\kappa F\ \sigma_\kappa N\ \sigma_o iS\ \lambda_o iV\ \lambda_\kappa iF\ \sigma_\kappa iN$	Non-Abelian
SiViFN	Abelian	$\sigma_o\ \beta_o\ \beta_\kappa\ \sigma_\kappa$	Non-Abelian	$S_{16}^0 S_{21}^0 S_{27}^0$	$S_1^\alpha S_7^\alpha$	$S_8^\beta S_{14}^\beta$	–	$\sigma_o S\ \beta_o iV\ \beta_\kappa iF\ \sigma_\kappa N\ \sigma_o iS\ \beta_o V\ \beta_\kappa F\ \sigma_\kappa iN$	Non-Abelian
SViFiN	Abelian	$\sigma_o\ \lambda_o\ \beta_\kappa\ \alpha_\kappa$	Non-Abelian	$S_{16}^0 S_{21}^0 S_{27}^0$	$S_2^\alpha S_3^\alpha$	$S_9^\beta S_{10}^\beta$	–	$\sigma_o S\ \lambda_o V\ \beta_\kappa iF\ \alpha_\kappa iN\ \sigma_o iS\ \lambda_o iV\ \beta_\kappa F\ \alpha_\kappa N$	Non-Abelian
SiVFiN	Abelian	$\sigma_o\ \beta_o\ \lambda_\kappa\ \alpha_\kappa$	Non-Abelian	$S_{16}^0 S_{21}^0 S_{27}^0$	$S_5^\alpha S_6^\alpha$	$S_{12}^\beta S_{13}^\beta$	–	$\sigma_o S\ \beta_o iV\ \lambda_\kappa F\ \alpha_\kappa iN\ \sigma_o iS\ \beta_o V\ \lambda_\kappa iF\ \alpha_\kappa N$	Non-Abelian
STPL	Abelian	$\sigma_o\ \delta_o\ \delta_\iota\ \sigma_\iota$	Non-Abelian	$S_{18}^0 S_{19}^0 S_{28}^0$	$S_1^\alpha S_5^\alpha$	S_9^β	S_0^γ	$\sigma_o S\ \delta_o T\ \delta_\iota P\ \sigma_\iota L\ \sigma_o iS\ \delta_o iT\ \delta_\iota iP\ \sigma_\iota iL$	Non-Abelian
SiTiPL	Abelian	$\sigma_o\ \nu_o\ \nu_\iota\ \sigma_\iota$	Non-Abelian	$S_{15}^0 S_{18}^0 S_{19}^0 S_{28}^0$	S_2^α	$S_8^\beta S_{12}^\beta$	–	$\sigma_o S\ \nu_o iT\ \nu_\iota iP\ \sigma_\iota L\ \sigma_o iS\ \nu_o T\ \nu_\iota P\ \sigma_\iota iL$	Non-Abelian
STiPiL	Abelian	$\sigma_o\ \delta_o\ \nu_\iota\ \alpha_\iota$	Non-Abelian	$S_{18}^0 S_{19}^0 S_{28}^0$	$S_6^\alpha S_7^\alpha$	$S_{10}^\beta S_{11}^\beta$	–	$\sigma_o S\ \delta_o T\ \nu_\iota iP\ \alpha_\iota iL\ \sigma_o iS\ \delta_o iT\ \nu_\iota P\ \alpha_\iota L$	Non-Abelian
SiTPiL	Abelian	$\sigma_o\ \nu_o\ \delta_\iota\ \alpha_\iota$	Non-Abelian	$S_{18}^0 S_{19}^0 S_{28}^0$	$S_3^\alpha S_4^\alpha$	$S_{13}^\beta S_{14}^\beta$	–	$\sigma_o S\ \nu_o iT\ \delta_\iota P\ \alpha_\iota iL\ \sigma_o iS\ \nu_o T\ \delta_\iota iP\ \alpha_\iota L$	Non-Abelian
STQM	Abelian	$\sigma_o\ \delta_o\ \delta_\jmath\ \sigma_\jmath$	Non-Abelian	$S_{18}^0 S_{20}^0 S_{29}^0$	$S_1^\alpha S_6^\alpha$	S_{10}^β	S_0^γ	$\sigma_o S\ \delta_o T\ \delta_\jmath Q\ \sigma_\jmath M\ \sigma_o iS\ \delta_o iT\ \delta_\jmath iQ\ \sigma_\jmath iM$	Non-Abelian
SiTiQM	Abelian	$\sigma_o\ \nu_o\ \nu_\jmath\ \sigma_\jmath$	Non-Abelian	$S_{15}^0 S_{18}^0 S_{20}^0 S_{29}^0$	S_3^α	$S_8^\beta S_{13}^\beta$	–	$\sigma_o S\ \nu_o iT\ \nu_\jmath iQ\ \sigma_\jmath M\ \sigma_o iS\ \nu_o T\ \nu_\jmath Q\ \sigma_\jmath iM$	Non-Abelian
STiQiM	Abelian	$\sigma_o\ \delta_o\ \nu_\jmath\ \alpha_\jmath$	Non-Abelian	$S_{18}^0 S_{20}^0 S_{29}^0$	$S_5^\alpha S_7^\alpha$	$S_9^\beta S_{11}^\beta$	–	$\sigma_o S\ \delta_o T\ \nu_\jmath iQ\ \alpha_\jmath iM\ \sigma_o iS\ \delta_o iT\ \nu_\jmath Q\ \alpha_\jmath M$	Non-Abelian
SiTQiM	Abelian	$\sigma_o\ \nu_o\ \delta_\jmath\ \alpha_\jmath$	Non-Abelian	$S_{18}^0 S_{20}^0 S_{29}^0$	$S_2^\alpha S_4^\alpha$	$S_{12}^\beta S_{14}^\beta$	–	$\sigma_o S\ \nu_o iT\ \delta_\jmath Q\ \alpha_\jmath iM\ \sigma_o iS\ \nu_o T\ \delta_\jmath iQ\ \alpha_\jmath M$	Non-Abelian
STRN	Abelian	$\sigma_o\ \delta_o\ \delta_\kappa\ \sigma_\kappa$	Non-Abelian	$S_{18}^0 S_{21}^0 S_{30}^0$	$S_1^\alpha S_7^\alpha$	S_{11}^β	S_0^γ	$\sigma_o S\ \delta_o T\ \delta_\kappa R\ \sigma_\kappa N\ \sigma_o iS\ \delta_o iT\ \delta_\kappa iR\ \sigma_\kappa iN$	Non-Abelian
SiTiRN	Abelian	$\sigma_o\ \nu_o\ \nu_\kappa\ \sigma_\kappa$	Non-Abelian	$S_{15}^0 S_{18}^0 S_{21}^0 S_{30}^0$	S_4^α	$S_8^\beta S_{14}^\beta$	–	$\sigma_o S\ \nu_o iT\ \nu_\kappa iR\ \sigma_\kappa N\ \sigma_o iS\ \nu_o T\ \nu_\kappa R\ \sigma_\kappa iN$	Non-Abelian
STiRiN	Abelian	$\sigma_o\ \delta_o\ \nu_\kappa\ \alpha_\kappa$	Non-Abelian	$S_{18}^0 S_{21}^0 S_{30}^0$	$S_5^\alpha S_6^\alpha$	$S_9^\beta S_{10}^\beta$	–	$\sigma_o S\ \delta_o T\ \nu_\kappa iR\ \alpha_\kappa iN\ \sigma_o iS\ \delta_o iT\ \nu_\kappa R\ \alpha_\kappa N$	Non-Abelian
SiTRiN	Abelian	$\sigma_o\ \nu_o\ \delta_\kappa\ \alpha_\kappa$	Non-Abelian	$S_{18}^0 S_{21}^0 S_{30}^0$	$S_2^\alpha S_3^\alpha$	$S_{12}^\beta S_{13}^\beta$	–	$\sigma_o S\ \nu_o iT\ \delta_\kappa R\ \alpha_\kappa iN\ \sigma_o iS\ \nu_o T\ \delta_\kappa iR\ \alpha_\kappa N$	Non-Abelian
SXUL	Abelian	$\sigma_o\ \gamma_\iota\ \gamma_o\ \sigma_\iota$	Non-Abelian	$S_{17}^0 S_{19}^0 S_{22}^0$	–	$S_8^\beta S_9^\beta S_{12}^\beta$	S_0^γ	$\sigma_o S\ \gamma_\iota X\ \gamma_o U\ \sigma_\iota L\ \sigma_o iS\ \gamma_\iota iX\ \gamma_o iU\ \sigma_\iota iL$	Non-Abelian
SiXiUL	Abelian	$\sigma_o\ \mu_\iota\ \mu_o\ \sigma_\iota$	Non-Abelian	$S_{15}^0 S_{17}^0 S_{19}^0 S_{22}^0$	$S_1^\alpha S_2^\alpha S_5^\alpha$	–	–	$\sigma_o S\ \mu_\iota iX\ \mu_o iU\ \sigma_\iota L\ \sigma_o iS\ \mu_\iota X\ \mu_o U\ \sigma_\iota iL$	Non-Abelian
SXiUiL	Abelian	$\sigma_o\ \gamma_\iota\ \mu_o\ \alpha_\iota$	Non-Abelian	$S_{17}^0 S_{19}^0 S_{22}^0$	$S_3^\alpha S_4^\alpha S_6^\alpha S_7^\alpha$	–	–	$\sigma_o S\ \gamma_\iota X\ \mu_o iU\ \alpha_\iota iL\ \sigma_o iS\ \gamma_\iota iX\ \mu_o U\ \alpha_\iota L$	Non-Abelian
SiXUiL	Abelian	$\sigma_o\ \mu_\iota\ \gamma_o\ \alpha_\iota$	Non-Abelian	$S_{17}^0 S_{19}^0 S_{22}^0$	–	$S_{10}^\beta S_{11}^\beta S_{13}^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\iota iX\ \gamma_o U\ \alpha_\iota iL\ \sigma_o iS\ \mu_\iota X\ \gamma_o iU\ \alpha_\iota L$	Non-Abelian
SYUM	Abelian	$\sigma_o\ \gamma_\jmath\ \gamma_o\ \sigma_\jmath$	Non-Abelian	$S_{17}^0 S_{20}^0 S_{23}^0$	–	$S_8^\beta S_{10}^\beta S_{13}^\beta$	S_0^γ	$\sigma_o S\ \gamma_\jmath Y\ \gamma_o U\ \sigma_\jmath M\ \sigma_o iS\ \gamma_\jmath iY\ \gamma_o iU\ \sigma_\jmath iM$	Non-Abelian
SiYiUM	Abelian	$\sigma_o\ \mu_\jmath\ \mu_o\ \sigma_\jmath$	Non-Abelian	$S_{15}^0 S_{17}^0 S_{20}^0 S_{23}^0$	$S_1^\alpha S_3^\alpha S_6^\alpha$	–	–	$\sigma_o S\ \mu_\jmath iY\ \mu_o iU\ \sigma_\jmath M\ \sigma_o iS\ \mu_\jmath Y\ \mu_o U\ \sigma_\jmath iM$	Non-Abelian
SYiUiM	Abelian	$\sigma_o\ \gamma_\jmath\ \mu_o\ \alpha_\jmath$	Non-Abelian	$S_{17}^0 S_{20}^0 S_{23}^0$	$S_2^\alpha S_4^\alpha S_5^\alpha S_7^\alpha$	–	–	$\sigma_o S\ \gamma_\jmath Y\ \mu_o iU\ \alpha_\jmath iM\ \sigma_o iS\ \gamma_\jmath iY\ \mu_o U\ \alpha_\jmath M$	Non-Abelian
SiYUiM	Abelian	$\sigma_o\ \mu_\jmath\ \gamma_o\ \alpha_\jmath$	Non-Abelian	$S_{17}^0 S_{20}^0 S_{23}^0$	–	$S_9^\beta S_{11}^\beta S_{12}^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\jmath iY\ \gamma_o U\ \alpha_\jmath iM\ \sigma_o iS\ \mu_\jmath Y\ \gamma_o iU\ \alpha_\jmath M$	Non-Abelian
SZUN	Abelian	$\sigma_o\ \gamma_\kappa\ \gamma_o\ \sigma_\kappa$	Non-Abelian	$S_{17}^0 S_{21}^0 S_{24}^0$	–	$S_8^\beta S_{11}^\beta S_{14}^\beta$	S_0^γ	$\sigma_o S\ \gamma_\kappa Z\ \gamma_o U\ \sigma_\kappa N\ \sigma_o iS\ \gamma_\kappa iZ\ \gamma_o iU\ \sigma_\kappa iN$	Non-Abelian
SiZiUN	Abelian	$\sigma_o\ \mu_\kappa\ \mu_o\ \sigma_\kappa$	Non-Abelian	$S_{15}^0 S_{17}^0 S_{21}^0 S_{24}^0$	$S_1^\alpha S_4^\alpha S_7^\alpha$	–	–	$\sigma_o S\ \mu_\kappa iZ\ \mu_o iU\ \sigma_\kappa N\ \sigma_o iS\ \mu_\kappa Z\ \mu_o U\ \sigma_\kappa iN$	Non-Abelian
SZiUiN	Abelian	$\sigma_o\ \gamma_\kappa\ \mu_o\ \alpha_\kappa$	Non-Abelian	$S_{17}^0 S_{21}^0 S_{24}^0$	$S_2^\alpha S_3^\alpha S_5^\alpha S_6^\alpha$	–	–	$\sigma_o S\ \gamma_\kappa Z\ \mu_o iU\ \alpha_\kappa iN\ \sigma_o iS\ \gamma_\kappa iZ\ \mu_o U\ \alpha_\kappa N$	Non-Abelian
SiZUiN	Abelian	$\sigma_o\ \mu_\kappa\ \gamma_o\ \alpha_\kappa$	Non-Abelian	$S_{17}^0 S_{21}^0 S_{24}^0$	–	$S_9^\beta S_{10}^\beta S_{12}^\beta S_{13}^\beta$	–	$\sigma_o S\ \mu_\kappa iZ\ \gamma_o U\ \alpha_\kappa iN\ \sigma_o iS\ \mu_\kappa Z\ \gamma_o iU\ \alpha_\kappa N$	Non-Abelian
SXER	Abelian	$\sigma_o\ \gamma_\iota\ \lambda_\jmath\ \delta_\kappa$	Non-Abelian	$S_{22}^0 S_{26}^0 S_{30}^0$	$S_3^\alpha S_7^\alpha$	S_{12}^β	S_0^γ	$\sigma_o S\ \gamma_\iota X\ \lambda_\jmath E\ \delta_\kappa R\ \sigma_o iS\ \gamma_\iota iX\ \lambda_\jmath iE\ \delta_\kappa iR$	Non-Abelian
SiXEiR	Abelian	$\sigma_o\ \mu_\iota\ \lambda_\jmath\ \nu_\kappa$	Non-Abelian	$S_{15}^0 S_{22}^0 S_{26}^0 S_{30}^0$	S_5^α	$S_{10}^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\iota iX\ \lambda_\jmath E\ \nu_\kappa iR\ \sigma_o iS\ \mu_\iota X\ \lambda_\jmath iE\ \nu_\kappa R$	Non-Abelian
SXiEiR	Abelian	$\sigma_o\ \gamma_\iota\ \beta_\jmath\ \nu_\kappa$	Non-Abelian	$S_{22}^0 S_{26}^0 S_{30}^0$	$S_4^\alpha S_6^\alpha$	$S_8^\beta S_9^\beta$	–	$\sigma_o S\ \gamma_\iota X\ \beta_\jmath iE\ \nu_\kappa iR\ \sigma_o iS\ \gamma_\iota iX\ \beta_\jmath E\ \nu_\kappa R$	Non-Abelian
SiXiER	Abelian	$\sigma_o\ \mu_\iota\ \beta_\jmath\ \delta_\kappa$	Non-Abelian	$S_{22}^0 S_{26}^0 S_{30}^0$	$S_1^\alpha S_2^\alpha$	$S_{11}^\beta S_{13}^\beta$	–	$\sigma_o S\ \mu_\iota iX\ \beta_\jmath iE\ \delta_\kappa R\ \sigma_o iS\ \mu_\iota X\ \beta_\jmath E\ \delta_\kappa iR$	Non-Abelian
SXQF	Abelian	$\sigma_o\ \gamma_\iota\ \delta_\jmath\ \lambda_\kappa$	Non-Abelian	$S_{22}^0 S_{27}^0 S_{29}^0$	$S_4^\alpha S_6^\alpha$	S_{12}^β	S_0^γ	$\sigma_o S\ \gamma_\iota X\ \delta_\jmath Q\ \lambda_\kappa F\ \sigma_o iS\ \gamma_\iota iX\ \delta_\jmath iQ\ \lambda_\kappa iF$	Non-Abelian
SiXiQF	Abelian	$\sigma_o\ \mu_\iota\ \nu_\jmath\ \lambda_\kappa$	Non-Abelian	$S_{15}^0 S_{22}^0 S_{27}^0 S_{29}^0$	S_5^α	$S_{11}^\beta S_{13}^\beta$	–	$\sigma_o S\ \mu_\iota iX\ \nu_\jmath iQ\ \lambda_\kappa F\ \sigma_o iS\ \mu_\iota X\ \nu_\jmath Q\ \lambda_\kappa iF$	Non-Abelian
SXiQiF	Abelian	$\sigma_o\ \gamma_\iota\ \nu_\jmath\ \beta_\kappa$	Non-Abelian	$S_{22}^0 S_{27}^0 S_{29}^0$	$S_3^\alpha S_7^\alpha$	$S_8^\beta S_9^\beta$	–	$\sigma_o S\ \gamma_\iota X\ \nu_\jmath iQ\ \beta_\kappa iF\ \sigma_o iS\ \gamma_\iota iX\ \nu_\jmath Q\ \beta_\kappa F$	Non-Abelian
SiXQiF	Abelian	$\sigma_o\ \mu_\iota\ \delta_\jmath\ \beta_\kappa$	Non-Abelian	$S_{22}^0 S_{27}^0 S_{29}^0$	$S_1^\alpha S_2^\alpha$	$S_{10}^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\iota iX\ \delta_\jmath Q\ \beta_\kappa iF\ \sigma_o iS\ \mu_\iota X\ \delta_\jmath iQ\ \beta_\kappa F$	Non-Abelian
SYPF	Abelian	$\sigma_o\ \gamma_\jmath\ \delta_\iota\ \lambda_\kappa$	Non-Abelian	$S_{23}^0 S_{27}^0 S_{28}^0$	$S_4^\alpha S_5^\alpha$	S_{13}^β	S_0^γ	$\sigma_o S\ \gamma_\jmath Y\ \delta_\iota P\ \lambda_\kappa F\ \sigma_o iS\ \gamma_\jmath iY\ \delta_\iota iP\ \lambda_\kappa iF$	Non-Abelian
SiYiPF	Abelian	$\sigma_o\ \mu_\jmath\ \nu_\iota\ \lambda_\kappa$	Non-Abelian	$S_{15}^0 S_{23}^0 S_{27}^0 S_{28}^0$	S_6^α	$S_{11}^\beta S_{12}^\beta$	–	$\sigma_o S\ \mu_\jmath iY\ \nu_\iota iP\ \lambda_\kappa F\ \sigma_o iS\ \mu_\jmath Y\ \nu_\iota P\ \lambda_\kappa iF$	Non-Abelian
SYiPiF	Abelian	$\sigma_o\ \gamma_\jmath\ \nu_\iota\ \beta_\kappa$	Non-Abelian	$S_{23}^0 S_{27}^0 S_{28}^0$	$S_2^\alpha S_7^\alpha$	$S_8^\beta S_{10}^\beta$	–	$\sigma_o S\ \gamma_\jmath Y\ \nu_\iota iP\ \beta_\kappa iF\ \sigma_o iS\ \gamma_\jmath iY\ \nu_\iota P\ \beta_\kappa F$	Non-Abelian
SiYPiF	Abelian	$\sigma_o\ \mu_\jmath\ \delta_\iota\ \beta_\kappa$	Non-Abelian	$S_{23}^0 S_{27}^0 S_{28}^0$	$S_1^\alpha S_3^\alpha$	$S_9^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\jmath iY\ \delta_\iota P\ \beta_\kappa iF\ \sigma_o iS\ \mu_\jmath Y\ \delta_\iota iP\ \beta_\kappa F$	Non-Abelian
SYDR	Abelian	$\sigma_o\ \gamma_\jmath\ \lambda_\iota\ \delta_\kappa$	Non-Abelian	$S_{23}^0 S_{25}^0 S_{30}^0$	$S_2^\alpha S_7^\alpha$	S_{13}^β	S_0^γ	$\sigma_o S\ \gamma_\jmath Y\ \lambda_\iota D\ \delta_\kappa R\ \sigma_o iS\ \gamma_\jmath iY\ \lambda_\iota iD\ \delta_\kappa iR$	Non-Abelian
SiYDiR	Abelian	$\sigma_o\ \mu_\jmath\ \lambda_\iota\ \nu_\kappa$	Non-Abelian	$S_{15}^0 S_{23}^0 S_{25}^0 S_{30}^0$	S_6^α	$S_9^\beta S_{14}^\beta$	–	$\sigma_o S\ \mu_\jmath iY\ \lambda_\iota D\ \nu_\kappa iR\ \sigma_o iS\ \mu_\jmath Y\ \lambda_\iota iD\ \nu_\kappa R$	Non-Abelian
SYiDiR	Abelian	$\sigma_o\ \gamma_\jmath\ \beta_\iota\ \nu_\kappa$	Non-Abelian	$S_{23}^0 S_{25}^0 S_{30}^0$	$S_4^\alpha S_5^\alpha$	$S_8^\beta S_{10}^\beta$	–	$\sigma_o S\ \gamma_\jmath Y\ \beta_\iota iD\ \nu_\kappa iR\ \sigma_o iS\ \gamma_\jmath iY\ \beta_\iota D\ \nu_\kappa R$	Non-Abelian
SiYiDR	Abelian	$\sigma_o\ \mu_\jmath\ \beta_\iota\ \delta_\kappa$	Non-Abelian	$S_{23}^0 S_{25}^0 S_{30}^0$	$S_1^\alpha S_3^\alpha$	$S_{11}^\beta S_{12}^\beta$	–	$\sigma_o S\ \mu_\jmath iY\ \beta_\iota iD\ \delta_\kappa R\ \sigma_o iS\ \mu_\jmath Y\ \beta_\iota D\ \delta_\kappa iR$	Non-Abelian
SZDQ	Abelian	$\sigma_o\ \gamma_\kappa\ \lambda_\iota\ \delta_\jmath$	Non-Abelian	$S_{24}^0 S_{25}^0 S_{29}^0$	$S_2^\alpha S_6^\alpha$	S_{14}^β	S_0^γ	$\sigma_o S\ \gamma_\kappa Z\ \lambda_\iota D\ \delta_\jmath Q\ \sigma_o iS\ \gamma_\kappa iZ\ \lambda_\iota iD\ \delta_\jmath iQ$	Non-Abelian
SiZDiQ	Abelian	$\sigma_o\ \mu_\kappa\ \lambda_\iota\ \nu_\jmath$	Non-Abelian	$S_{15}^0 S_{24}^0 S_{25}^0 S_{29}^0$	S_7^α	$S_9^\beta S_{13}^\beta$	–	$\sigma_o S\ \mu_\kappa iZ\ \lambda_\iota D\ \nu_\jmath iQ\ \sigma_o iS\ \mu_\kappa Z\ \lambda_\iota iD\ \nu_\jmath Q$	Non-Abelian
SZiDiQ	Abelian	$\sigma_o\ \gamma_\kappa\ \beta_\iota\ \nu_\jmath$	Non-Abelian	$S_{24}^0 S_{25}^0 S_{29}^0$	$S_3^\alpha S_5^\alpha$	$S_8^\beta S_{11}^\beta$	–	$\sigma_o S\ \gamma_\kappa Z\ \beta_\iota iD\ \nu_\jmath iQ\ \sigma_o iS\ \gamma_\kappa iZ\ \beta_\iota D\ \nu_\jmath Q$	Non-Abelian
SiZiDQ	Abelian	$\sigma_o\ \mu_\kappa\ \beta_\iota\ \delta_\jmath$	Non-Abelian	$S_{24}^0 S_{25}^0 S_{29}^0$	$S_1^\alpha S_4^\alpha$	$S_{10}^\beta S_{12}^\beta$	–	$\sigma_o S\ \mu_\kappa iZ\ \beta_\iota iD\ \delta_\jmath Q\ \sigma_o iS\ \mu_\kappa Z\ \beta_\iota D\ \delta_\jmath iQ$	Non-Abelian
SZPE	Abelian	$\sigma_o\ \gamma_\kappa\ \delta_\iota\ \lambda_\jmath$	Non-Abelian	$S_{24}^0 S_{26}^0 S_{28}^0$	$S_3^\alpha S_5^\alpha$	S_{14}^β	S_0^γ	$\sigma_o S\ \gamma_\kappa Z\ \delta_\iota P\ \lambda_\jmath E\ \sigma_o iS\ \gamma_\kappa iZ\ \delta_\iota iP\ \lambda_\jmath iE$	Non-Abelian
SiZiPE	Abelian	$\sigma_o\ \mu_\kappa\ \nu_\iota\ \lambda_\jmath$	Non-Abelian	$S_{15}^0 S_{24}^0 S_{26}^0 S_{28}^0$	S_7^α	$S_{10}^\beta S_{12}^\beta$	–	$\sigma_o S\ \mu_\kappa iZ\ \nu_\iota iP\ \lambda_\jmath E\ \sigma_o iS\ \mu_\kappa Z\ \nu_\iota P\ \lambda_\jmath iE$	Non-Abelian
SZiPiE	Abelian	$\sigma_o\ \gamma_\kappa\ \nu_\iota\ \beta_\jmath$	Non-Abelian	$S_{24}^0 S_{26}^0 S_{28}^0$	$S_2^\alpha S_6^\alpha$	$S_8^\beta S_{11}^\beta$	–	$\sigma_o S\ \gamma_\kappa Z\ \nu_\iota iP\ \beta_\jmath iE\ \sigma_o iS\ \gamma_\kappa iZ\ \nu_\iota P\ \beta_\jmath E$	Non-Abelian
SiZPiE	Abelian	$\sigma_o\ \mu_\kappa\ \delta_\iota\ \beta_\jmath$	Non-Abelian	$S_{24}^0 S_{26}^0 S_{28}^0$	$S_1^\alpha S_4^\alpha$	$S_9^\beta S_{13}^\beta$	–	$\sigma_o S\ \mu_\kappa iZ\ \delta_\iota P\ \beta_\jmath iE\ \sigma_o iS\ \mu_\kappa Z\ \delta_\iota iP\ \beta_\jmath E$	Non-Abelian

Table 6. Unit elements for aligned $\mathbb{M}$ and $\mathbb{T}$ subalgebras with 4 unit elements and complexified aligned $\mathbb{C} \otimes \mathbb{W}$ sub-algebras

Unit elements of $\mathbb{M}$		Unit elements of $\mathbb{T}$ and sedenion-type loop participation						Unit elements of $\mathbb{C} \otimes \mathbb{W}$	
Sub-group	Type	Sub-loop	Type	S_L^0	S_L^α	S_L^β	S_L^γ	Complexified aligned sub-loop	Type
SLiSiL	Abelian	$\sigma_o\ \sigma_\iota\ \alpha_o\ \alpha_\iota$	Non-Abelian	$S_{16}^0S_{17}^0S_{18}^0S_{19}^0S_{22}^0S_{25}^0S_{28}^0$	–	–	–	$\sigma_oS\ \sigma_\iota L\ \alpha_oiS\ \alpha_\iota iL\ \sigma_oiS\ \sigma_\iota iL\ \alpha_oS\ \alpha_\iota L$	Non-Abelian
SMiSiM	Abelian	$\sigma_o\ \sigma_\jmath\ \alpha_o\ \alpha_\jmath$	Non-Abelian	$S_{16}^0S_{17}^0S_{18}^0S_{20}^0S_{23}^0S_{26}^0S_{29}^0$	–	–	–	$\sigma_oS\ \sigma_\jmath M\ \alpha_oiS\ \alpha_\jmath iM\ \sigma_oiS\ \sigma_\jmath iM\ \alpha_oS\ \alpha_\jmath M$	Non-Abelian
SNiSiN	Abelian	$\sigma_o\ \sigma_\kappa\ \alpha_o\ \alpha_\kappa$	Non-Abelian	$S_{16}^0S_{17}^0S_{18}^0S_{21}^0S_{24}^0S_{27}^0S_{30}^0$	–	–	–	$\sigma_oS\ \sigma_\kappa N\ \alpha_oiS\ \alpha_\kappa iN\ \sigma_oiS\ \sigma_\kappa iN\ \alpha_oS\ \alpha_\kappa N$	Non-Abelian
SViSiV	Abelian	$\sigma_o\ \lambda_o\ \alpha_o\ \beta_o$	Non-Abelian	$S_{16}^0S_{19}^0S_{20}^0S_{21}^0S_{25}^0S_{26}^0S_{27}^0$	–	–	–	$\sigma_oS\ \lambda_oV\ \alpha_oiS\ \beta_oiV\ \sigma_oiS\ \lambda_oiV\ \alpha_oS\ \beta_oV$	Non-Abelian
STiSiT	Abelian	$\sigma_o\ \delta_o\ \alpha_o\ \nu_o$	Non-Abelian	$S_{18}^0S_{19}^0S_{20}^0S_{21}^0S_{28}^0S_{29}^0S_{30}^0$	–	–	–	$\sigma_oS\ \delta_oT\ \alpha_oiS\ \nu_oiT\ \sigma_oiS\ \delta_oiT\ \alpha_oS\ \nu_oT$	Non-Abelian
SUiSiU	Abelian	$\sigma_o\ \gamma_o\ \alpha_o\ \mu_o$	Non-Abelian	$S_{17}^0S_{19}^0S_{20}^0S_{21}^0S_{22}^0S_{23}^0S_{24}^0$	–	–	–	$\sigma_oS\ \gamma_oU\ \alpha_oiS\ \mu_oiU\ \sigma_oiS\ \gamma_oiU\ \alpha_oS\ \mu_oU$	Non-Abelian
SXiSiX	Abelian	$\sigma_o\ \gamma_\iota\ \alpha_o\ \mu_\iota$	Non-Abelian	$S_{17}^0S_{19}^0S_{22}^0S_{26}^0S_{27}^0S_{29}^0S_{30}^0$	–	–	–	$\sigma_oS\ \gamma_\iota X\ \alpha_oiS\ \mu_\iota iX\ \sigma_oiS\ \gamma_\iota iX\ \alpha_oS\ \mu_\iota X$	Non-Abelian
SYiSiY	Abelian	$\sigma_o\ \gamma_\jmath\ \alpha_o\ \mu_\jmath$	Non-Abelian	$S_{17}^0S_{20}^0S_{23}^0S_{25}^0S_{27}^0S_{28}^0S_{30}^0$	–	–	–	$\sigma_oS\ \gamma_\jmath Y\ \alpha_oiS\ \mu_\jmath iY\ \sigma_oiS\ \gamma_\jmath iY\ \alpha_oS\ \mu_\jmath Y$	Non-Abelian
SZiSiZ	Abelian	$\sigma_o\ \gamma_\kappa\ \alpha_o\ \mu_\kappa$	Non-Abelian	$S_{17}^0S_{21}^0S_{24}^0S_{25}^0S_{26}^0S_{28}^0S_{29}^0$	–	–	–	$\sigma_oS\ \gamma_\kappa Z\ \alpha_oiS\ \mu_\kappa iZ\ \sigma_oiS\ \gamma_\kappa iZ\ \alpha_oS\ \mu_\kappa Z$	Non-Abelian
SPiSiP	Abelian	$\sigma_o\ \delta_\iota\ \alpha_o\ \nu_\iota$	Non-Abelian	$S_{18}^0S_{19}^0S_{23}^0S_{24}^0S_{26}^0S_{27}^0S_{28}^0$	–	–	–	$\sigma_oS\ \delta_\iota P\ \alpha_oiS\ \nu_\iota iP\ \sigma_oiS\ \delta_\iota iP\ \alpha_oS\ \nu_\iota P$	Non-Abelian
SQiSiQ	Abelian	$\sigma_o\ \delta_\jmath\ \alpha_o\ \nu_\jmath$	Non-Abelian	$S_{18}^0S_{20}^0S_{22}^0S_{24}^0S_{25}^0S_{27}^0S_{29}^0$	–	–	–	$\sigma_oS\ \delta_\jmath Q\ \alpha_oiS\ \nu_\jmath iQ\ \sigma_oiS\ \delta_\jmath iQ\ \alpha_oS\ \nu_\jmath Q$	Non-Abelian
SRiSiR	Abelian	$\sigma_o\ \delta_\kappa\ \alpha_o\ \nu_\kappa$	Non-Abelian	$S_{18}^0S_{21}^0S_{22}^0S_{23}^0S_{25}^0S_{26}^0S_{30}^0$	–	–	–	$\sigma_oS\ \delta_\kappa R\ \alpha_oiS\ \nu_\kappa iR\ \sigma_oiS\ \delta_\kappa iR\ \alpha_oS\ \nu_\kappa R$	Non-Abelian
SDiSiD	Abelian	$\sigma_o\ \lambda_\iota\ \alpha_o\ \beta_\iota$	Non-Abelian	$S_{16}^0S_{19}^0S_{23}^0S_{24}^0S_{25}^0S_{29}^0S_{30}^0$	–	–	–	$\sigma_oS\ \lambda_\iota D\ \alpha_oiS\ \beta_\iota iD\ \sigma_oiS\ \lambda_\iota iD\ \alpha_oS\ \beta_\iota D$	Non-Abelian
SEiSiE	Abelian	$\sigma_o\ \lambda_\jmath\ \alpha_o\ \beta_\jmath$	Non-Abelian	$S_{16}^0S_{20}^0S_{22}^0S_{24}^0S_{26}^0S_{28}^0S_{30}^0$	–	–	–	$\sigma_oS\ \lambda_\jmath E\ \alpha_oiS\ \beta_\jmath iE\ \sigma_oiS\ \lambda_\jmath iE\ \alpha_oS\ \beta_\jmath E$	Non-Abelian
SFiSiF	Abelian	$\sigma_o\ \lambda_\kappa\ \alpha_o\ \beta_\kappa$	Non-Abelian	$S_{16}^0S_{21}^0S_{22}^0S_{23}^0S_{27}^0S_{28}^0S_{29}^0$	–	–	–	$\sigma_oS\ \lambda_\kappa F\ \alpha_oiS\ \beta_\kappa iF\ \sigma_oiS\ \lambda_\kappa iF\ \alpha_oS\ \beta_\kappa F$	Non-Abelian

Table 7. Structure of combinations of 8 element $\mathbb{M}$ subgroups and 8 element $\mathbb{T}$ subloops

$\mathbb{M}$ subgroups and type		Trigintaduonion sub-loop		Sedenion participation				Aligned sub-algebra	Type
Basis elements	Type	Basis elements	Type	S_L^0	S_L^α	S_L^β	S_L^γ	Basis elements	
SiVDiLViDLiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \lambda_\iota\ \alpha_\iota\ \lambda_o\ \beta_\iota\ \sigma_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{19}^0S_{28}^0$	–	–	–	$\sigma_oS\ \beta_oiV\ \lambda_\iota D\ \alpha_\iota iL\ \lambda_oV\ \beta_\iota iD\ \sigma_\iota L\ \alpha_oi$	Split Octonion
SiTPiLTiPLiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \nu_o\ \delta_\iota\ \alpha_\iota\ \delta_o\ \nu_\iota\ \sigma_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{19}^0S_{25}^0$	–	–	–	$\sigma_oS\ \nu_oiT\ \delta_\iota P\ \alpha_\iota iL\ \delta_oT\ \nu_\iota iP\ \sigma_\iota L\ \alpha_oi$	Split Octonion
SXiUiLiXULiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\iota\ \mu_o\ \alpha_\iota\ \mu_\iota\ \gamma_o\ \sigma_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{19}^0S_{22}^0$	–	–	–	$\sigma_oS\ \gamma_\iota X\ \mu_oiU\ \alpha_\iota iL\ \mu_\iota iX\ \gamma_oU\ \sigma_\iota L\ \alpha_oi$	Split Octonion
SXERiXiEiRiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\iota\ \lambda_\jmath\ \delta_\kappa\ \mu_\iota\ \beta_\jmath\ \nu_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{22}^0S_{27}^0S_{29}^0$	–	–	–	$\sigma_oS\ \gamma_\iota X\ \lambda_\jmath E\ \delta_\kappa R\ \mu_\iota iX\ \beta_\jmath iE\ \nu_\kappa iR\ \alpha_oi$	Split Octonion
SXQFiXiQiFiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\iota\ \delta_\jmath\ \lambda_\kappa\ \mu_\iota\ \nu_\jmath\ \beta_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{22}^0S_{26}^0S_{30}^0$	–	–	–	$\sigma_oS\ \gamma_\iota X\ \delta_\jmath Q\ \lambda_\kappa F\ \mu_\iota iX\ \nu_\jmath iQ\ \beta_\kappa iF\ \alpha_oi$	Split Octonion
SiVEiMViEMiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \lambda_\jmath\ \alpha_\jmath\ \lambda_o\ \beta_\jmath\ \sigma_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{20}^0S_{29}^0$	–	–	–	$\sigma_oS\ \beta_oiV\ \lambda_\jmath E\ \alpha_\jmath iM\ \lambda_oV\ \beta_\jmath iE\ \sigma_\jmath M\ \alpha_oi$	Split Octonion
SiTQiMTiQMiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \nu_o\ \delta_\jmath\ \alpha_\jmath\ \delta_o\ \nu_\jmath\ \sigma_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{20}^0S_{26}^0$	–	–	–	$\sigma_oS\ \nu_oiT\ \delta_\jmath Q\ \alpha_\jmath iM\ \delta_oT\ \nu_\jmath iQ\ \sigma_\jmath M\ \alpha_oi$	Split Octonion
SYiUiMiYUMiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\jmath\ \mu_o\ \alpha_\jmath\ \mu_\jmath\ \gamma_o\ \sigma_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{20}^0S_{23}^0$	–	–	–	$\sigma_oS\ \gamma_\jmath Y\ \mu_oiU\ \alpha_\jmath iM\ \mu_\jmath iY\ \gamma_oU\ \sigma_\jmath M\ \alpha_oi$	Split Octonion
SYPFiYiPiFiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\jmath\ \delta_\iota\ \lambda_\kappa\ \mu_\jmath\ \nu_\iota\ \beta_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{23}^0S_{25}^0S_{30}^0$	–	–	–	$\sigma_oS\ \gamma_\jmath Y\ \delta_\iota P\ \lambda_\kappa F\ \mu_\jmath iY\ \nu_\iota iP\ \beta_\kappa iF\ \alpha_oi$	Split Octonion
SYDRiYiDiRiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\jmath\ \lambda_\iota\ \delta_\kappa\ \mu_\jmath\ \beta_\iota\ \nu_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{23}^0S_{27}^0S_{28}^0$	–	–	–	$\sigma_oS\ \gamma_\jmath Y\ \lambda_\iota D\ \delta_\kappa R\ \mu_\jmath iY\ \beta_\iota iD\ \nu_\kappa iR\ \alpha_oi$	Split Octonion
SiVFiNViFNiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \lambda_\kappa\ \alpha_\kappa\ \lambda_o\ \beta_\kappa\ \sigma_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{21}^0S_{30}^0$	–	–	–	$\sigma_oS\ \beta_oiV\ \lambda_\kappa F\ \alpha_\kappa iN\ \lambda_oV\ \beta_\kappa iF\ \sigma_\kappa N\ \alpha_oi$	Split Octonion
SiTRiNTiRNiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \nu_o\ \delta_\kappa\ \alpha_\kappa\ \delta_o\ \nu_\kappa\ \sigma_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{21}^0S_{27}^0$	–	–	–	$\sigma_oS\ \nu_oiT\ \delta_\kappa R\ \alpha_\kappa iN\ \delta_oT\ \nu_\kappa iR\ \sigma_\kappa N\ \alpha_oi$	Split Octonion
SZiUiNiZUNiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\kappa\ \mu_o\ \alpha_\kappa\ \mu_\kappa\ \gamma_o\ \sigma_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{21}^0S_{24}^0$	–	–	–	$\sigma_oS\ \gamma_\kappa Z\ \mu_oiU\ \alpha_\kappa iN\ \mu_\kappa iZ\ \gamma_oU\ \sigma_\kappa N\ \alpha_oi$	Split Octonion
SZDQiZiDiQiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\kappa\ \lambda_\iota\ \delta_\jmath\ \mu_\kappa\ \beta_\iota\ \nu_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{24}^0S_{26}^0S_{28}^0$	–	–	–	$\sigma_oS\ \gamma_\kappa Z\ \lambda_\iota D\ \delta_\jmath Q\ \mu_\kappa iZ\ \beta_\iota iD\ \nu_\jmath iQ\ \alpha_oi$	Split Octonion
SZPEiZiPiEiS	$\mathbb{D}\otimes\mathbb{C}$	$\sigma_o\ \gamma_\kappa\ \delta_\iota\ \lambda_\jmath\ \mu_\kappa\ \nu_\iota\ \beta_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{24}^0S_{25}^0S_{29}^0$	–	–	–	$\sigma_oS\ \gamma_\kappa Z\ \delta_\iota P\ \lambda_\jmath E\ \mu_\kappa iZ\ \nu_\iota iP\ \beta_\jmath iE\ \alpha_oi$	Split Octonion
SLMNiLiMiNiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \alpha_\iota\ \alpha_\jmath\ \alpha_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{17}^0S_{18}^0$	–	–	–	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \alpha_\iota iL\ \alpha_\jmath iM\ \alpha_\kappa iN\ \alpha_oi$	Distributive
SLiEiFiLEFiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \alpha_\iota\ \lambda_\jmath\ \lambda_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{22}^0S_{25}^0$	–	–	–	$\sigma_oS\ \sigma_\iota L\ \beta_\jmath iE\ \beta_\kappa iF\ \alpha_\iota iL\ \lambda_\jmath E\ \lambda_\kappa F\ \alpha_oi$	Distributive
SLiQiRiLQRiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \alpha_\iota\ \delta_\jmath\ \delta_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{22}^0S_{28}^0$	–	–	–	$\sigma_oS\ \sigma_\iota L\ \nu_\jmath iQ\ \nu_\kappa iR\ \alpha_\iota iL\ \delta_\jmath Q\ \delta_\kappa R\ \alpha_oi$	Distributive
SLiYiZiLYZiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \alpha_\iota\ \gamma_\jmath\ \gamma_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{25}^0S_{28}^0$	–	–	–	$\sigma_oS\ \sigma_\iota L\ \mu_\jmath iY\ \mu_\kappa iZ\ \alpha_\iota iL\ \gamma_\jmath Y\ \gamma_\kappa Z\ \alpha_oi$	Distributive
SMiFiDiMFDiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \alpha_\jmath\ \lambda_\kappa\ \lambda_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{23}^0S_{26}^0$	–	–	–	$\sigma_oS\ \sigma_\jmath M\ \beta_\kappa iF\ \beta_\iota iD\ \alpha_\jmath iM\ \lambda_\kappa F\ \lambda_\iota D\ \alpha_oi$	Distributive
SMiRiPiMRPiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \alpha_\jmath\ \delta_\kappa\ \delta_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{23}^0S_{29}^0$	–	–	–	$\sigma_oS\ \sigma_\jmath M\ \nu_\kappa iR\ \nu_\iota iP\ \alpha_\jmath iM\ \delta_\kappa R\ \delta_\iota P\ \alpha_oi$	Distributive
SMiZiXiMZXiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \alpha_\jmath\ \gamma_\kappa\ \gamma_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{26}^0S_{29}^0$	–	–	–	$\sigma_oS\ \sigma_\jmath M\ \mu_\kappa iZ\ \mu_\iota iX\ \alpha_\jmath iM\ \gamma_\kappa Z\ \gamma_\iota X\ \alpha_oi$	Distributive
SNiDiEiNDEiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \alpha_\kappa\ \lambda_\iota\ \lambda_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{18}^0S_{24}^0S_{27}^0$	–	–	–	$\sigma_oS\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \alpha_\kappa iN\ \lambda_\iota D\ \lambda_\jmath E\ \alpha_oi$	Distributive
SNiPiQiNPQiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \alpha_\kappa\ \delta_\iota\ \delta_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{16}^0S_{24}^0S_{30}^0$	–	–	–	$\sigma_oS\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \alpha_\kappa iN\ \delta_\iota P\ \delta_\jmath Q\ \alpha_oi$	Distributive
SNiXiYiNXYiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \alpha_\kappa\ \gamma_\iota\ \gamma_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{17}^0S_{27}^0S_{30}^0$	–	–	–	$\sigma_oS\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \alpha_\kappa iN\ \gamma_\iota X\ \gamma_\jmath Y\ \alpha_oi$	Distributive
SVTUiViTiUiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_o\ \nu_o\ \mu_o\ \alpha_o$	$\mathbb{O}_0^e$	$S_{19}^0S_{20}^0S_{21}^0$	–	–	–	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \beta_oiV\ \nu_oiT\ \mu_oiU\ \alpha_oi$	Distributive
SViPiXiVPXiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_\iota\ \mu_\iota\ \beta_o\ \delta_\iota\ \gamma_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{19}^0S_{29}^0S_{30}^0$	–	–	–	$\sigma_oS\ \lambda_oV\ \nu_\iota iP\ \mu_\iota iX\ \beta_oiV\ \delta_\iota P\ \gamma_\iota X\ \alpha_oi$	Distributive
STiXiDiTXDiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \delta_o\ \mu_\iota\ \beta_\iota\ \nu_o\ \gamma_\iota\ \lambda_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{19}^0S_{26}^0S_{27}^0$	–	–	–	$\sigma_oS\ \delta_oT\ \mu_\iota iX\ \beta_\iota iD\ \nu_oiT\ \gamma_\iota X\ \lambda_\iota D\ \alpha_oi$	Distributive
SUiDiPiUDPiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \mu_o\ \lambda_\iota\ \delta_\iota\ \alpha_o$	$\mathbb{O}_0^e$	$S_{19}^0S_{23}^0S_{24}^0$	–	–	–	$\sigma_oS\ \gamma_oU\ \beta_\iota iD\ \nu_\iota iP\ \mu_oiU\ \lambda_\iota D\ \delta_\iota P\ \alpha_oi$	Distributive
SViQiYiVQYiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_\jmath\ \mu_\jmath\ \beta_o\ \delta_\jmath\ \gamma_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{20}^0S_{28}^0S_{30}^0$	–	–	–	$\sigma_oS\ \lambda_oV\ \nu_\jmath iQ\ \mu_\jmath iY\ \beta_oiV\ \delta_\jmath Q\ \gamma_\jmath Y\ \alpha_oi$	Distributive
STiYiEiTYEiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \delta_o\ \mu_\jmath\ \beta_\jmath\ \nu_o\ \gamma_\jmath\ \lambda_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{20}^0S_{25}^0S_{27}^0$	–	–	–	$\sigma_oS\ \delta_oT\ \mu_\jmath iY\ \beta_\jmath iE\ \nu_oiT\ \gamma_\jmath Y\ \lambda_\jmath E\ \alpha_oi$	Distributive
SUiEiQiUEQiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \mu_o\ \lambda_\jmath\ \delta_\jmath\ \alpha_o$	$\mathbb{O}_0^e$	$S_{20}^0S_{22}^0S_{24}^0$	–	–	–	$\sigma_oS\ \gamma_oU\ \beta_\jmath iE\ \nu_\jmath iQ\ \mu_oiU\ \lambda_\jmath E\ \delta_\jmath Q\ \alpha_oi$	Distributive
SViRiZiVRZiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_\kappa\ \mu_\kappa\ \beta_o\ \delta_\kappa\ \gamma_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{21}^0S_{28}^0S_{29}^0$	–	–	–	$\sigma_oS\ \lambda_oV\ \nu_\kappa iR\ \mu_\kappa iZ\ \beta_oiV\ \delta_\kappa R\ \gamma_\kappa Z\ \alpha_oi$	Distributive
STiZiFiTZFiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \delta_o\ \mu_\kappa\ \beta_\kappa\ \nu_o\ \gamma_\kappa\ \lambda_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{21}^0S_{25}^0S_{26}^0$	–	–	–	$\sigma_oS\ \delta_oT\ \mu_\kappa iZ\ \beta_\kappa iF\ \nu_oiT\ \gamma_\kappa Z\ \lambda_\kappa F\ \alpha_oi$	Distributive
SUiFiRiUFRiS	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \mu_o\ \lambda_\kappa\ \delta_\kappa\ \alpha_o$	$\mathbb{O}_0^e$	$S_{21}^0S_{22}^0S_{23}^0$	–	–	–	$\sigma_oS\ \gamma_oU\ \beta_\kappa iF\ \nu_\kappa iR\ \mu_oiU\ \lambda_\kappa F\ \delta_\kappa R\ \alpha_oi$	Distributive

TABLE 8. Structure of combinations of 8 element $\mathbb{M}$ subgroups and 8 element $\mathbb{T}$ subloops

$\mathbb{M}$ subgroups and type		Trigintaduonion sub-loop		Sedenion participation				Aligned sub-algebra	Type
Basis elements	Type	Basis elements	Type	S_L^0	S_L^α	S_L^β	S_L^γ	Basis elements	
SLMNDEFV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \lambda_\kappa\ \lambda_o$	$\mathbb{O}_0^c$	S_{18}^0	S_1^α	–	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \lambda_\iota D\ \lambda_\jmath E\ \lambda_\kappa F\ \lambda_o$	Distributive
SLiMiNDiEiFV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \beta_\kappa\ \lambda_o$	$\widetilde{O}_0^f$	S_{18}^0	S_5^α	S_9^β	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \lambda_\iota D\ \beta_\jmath iE\ \beta_\kappa iF\ \lambda_o$	Distributive
SiLMiNiDEiFV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \beta_\kappa\ \lambda_o$	$\widetilde{O}_0^f$	S_{18}^0	S_6^α	S_{10}^β	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \beta_\iota iD\ \lambda_\jmath E\ \beta_\kappa iF\ \lambda_o$	Distributive
SiLiMNiDiEFV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \lambda_\kappa\ \lambda_o$	$\widetilde{O}_0^f$	S_{18}^0	S_7^α	S_{11}^β	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \lambda_\kappa F\ \lambda_o$	Distributive
SLMNiDiEiFiV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \beta_\kappa\ \beta_o$	$\widetilde{O}_0^j$	$S_{15}^0S_{18}^0$	–	S_8^β	–	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \beta_\kappa iF\ \beta_o i$	Distributive
SLiMiNiDEFiV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \lambda_\kappa\ \beta_o$	$\widetilde{O}_0^f$	S_{18}^0	S_2^α	S_{12}^β	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \beta_\iota iD\ \lambda_\jmath E\ \lambda_\kappa F\ \beta_o i$	Distributive
SiLMiNDiEFiV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \lambda_\kappa\ \beta_o$	$\widetilde{O}_0^f$	S_{18}^0	S_3^α	S_{13}^β	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \lambda_\iota D\ \beta_\jmath iE\ \lambda_\kappa F\ \beta_o i$	Distributive
SiLiMNDEiFiV	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \beta_\kappa\ \beta_o$	$\widetilde{O}_0^f$	S_{18}^0	S_4^α	S_{14}^β	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \lambda_\iota D\ \lambda_\jmath E\ \beta_\kappa iF\ \beta_o i$	Distributive
SLMNPQRT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \delta_\kappa\ \delta_o$	$\mathbb{O}_0^a$	$S_{15}^0S_{16}^0$	–	–	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \delta_\iota P\ \delta_\jmath Q\ \delta_\kappa R\ \delta_o$	Distributive
SLiMiNPiQiRT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \nu_\kappa\ \delta_o$	$\widetilde{O}_0^f$	S_{16}^0	S_2^α	S_9^β	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \delta_\iota P\ \nu_\jmath iQ\ \nu_\kappa iR\ \delta_o$	Distributive
SiLMiNiPQiRT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \nu_\kappa\ \delta_o$	$\widetilde{O}_0^f$	S_{16}^0	S_3^α	S_{10}^β	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \nu_\iota iP\ \delta_\jmath Q\ \nu_\kappa iR\ \delta_o$	Distributive
SiLiMNiPiQRT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \delta_\kappa\ \delta_o$	$\widetilde{O}_0^f$	S_{16}^0	S_4^α	S_{11}^β	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \delta_\kappa R\ \delta_o$	Distributive
SLMNiPiQiRiT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \nu_\kappa\ \nu_o$	$\widetilde{O}_0^f$	S_{16}^0	S_1^α	S_8^β	–	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \nu_\kappa iR\ \nu_o i$	Distributive
SLiMiNiPQRiT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \delta_\kappa\ \nu_o$	$\widetilde{O}_0^f$	S_{16}^0	S_5^α	S_{12}^β	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \nu_\iota iP\ \delta_\jmath Q\ \delta_\kappa R\ \nu_o i$	Distributive
SiLMiNPiQRiT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \delta_\kappa\ \nu_o$	$\widetilde{O}_0^f$	S_{16}^0	S_6^α	S_{13}^β	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \delta_\iota P\ \nu_\jmath iQ\ \delta_\kappa R\ \nu_o i$	Distributive
SiLiMNPQiRiT	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \nu_\kappa\ \nu_o$	$\widetilde{O}_0^f$	S_{16}^0	S_7^α	S_{14}^β	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \delta_\iota P\ \delta_\jmath Q\ \nu_\kappa iR\ \nu_o i$	Distributive
SLMNXYZU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \gamma_o$	$\widetilde{O}_0^g$	S_{17}^0	–	S_8^β	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \gamma_\kappa Z\ \gamma_o$	Distributive
SLiMiNXiYiZU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \mu_\kappa\ \gamma_o$	$\widetilde{O}_0^i$	S_{17}^0	–	$S_9^\beta S_{12}^\beta$	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \gamma_\iota X\ \mu_\jmath iY\ \mu_\kappa iZ\ \gamma_o$	Distributive
SiLMiNiXYiZU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \mu_\kappa\ \gamma_o$	$\widetilde{O}_0^i$	S_{17}^0	–	$S_{10}^\beta S_{13}^\beta$	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \mu_\iota iX\ \gamma_\jmath Y\ \mu_\kappa iZ\ \gamma_o$	Distributive
SiLiMNiXiYZU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \gamma_\kappa\ \gamma_o$	$\widetilde{O}_0^i$	S_{17}^0	–	$S_{11}^\beta S_{14}^\beta$	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \gamma_\kappa Z\ \gamma_o$	Distributive
SLMNiXiYiZiU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \mu_\kappa\ \mu_o$	$\mathbb{O}_0^b$	$S_{15}^0S_{17}^0$	S_1^α	–	–	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \mu_\kappa iZ\ \mu_o i$	Distributive
SLiMiNiXYZiU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \alpha_\jmath\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \gamma_\kappa\ \mu_o$	$\widetilde{O}_0^h$	S_{17}^0	$S_2^\alpha S_5^\alpha$	–	–	$\sigma_oS\ \sigma_\iota L\ \alpha_\jmath iM\ \alpha_\kappa iN\ \mu_\iota iX\ \gamma_\jmath Y\ \gamma_\kappa Z\ \mu_o i$	Distributive
SiLMiNXiYZiU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \sigma_\jmath\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \gamma_\kappa\ \mu_o$	$\widetilde{O}_0^h$	S_{17}^0	$S_3^\alpha S_6^\alpha$	–	–	$\sigma_oS\ \alpha_\iota iL\ \sigma_\jmath M\ \alpha_\kappa iN\ \gamma_\iota X\ \mu_\jmath iY\ \gamma_\kappa Z\ \mu_o i$	Distributive
SiLiMNXYiZiU	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \alpha_\jmath\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \mu_\kappa\ \mu_o$	$\widetilde{O}_0^h$	S_{17}^0	$S_4^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \alpha_\iota iL\ \alpha_\jmath iM\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \mu_\kappa iZ\ \mu_o i$	Distributive
SVTUDPXL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\iota\ \delta_\iota\ \gamma_\iota\ \sigma_\iota$	$\widetilde{O}_0^g$	S_{19}^0	–	S_9^β	S_0^γ	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \lambda_\iota D\ \delta_\iota P\ \gamma_\iota X\ \sigma_\iota$	Distributive
SViTiUDiPiXL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\iota\ \nu_\iota\ \mu_\iota\ \sigma_\iota$	$\mathbb{O}_0^d$	S_{19}^0	$S_1^\alpha S_5^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \lambda_\iota D\ \nu_\iota iP\ \mu_\iota iX\ \sigma_\iota$	Distributive
SiVTiUiDPiXL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\iota\ \delta_\iota\ \mu_\iota\ \sigma_\iota$	$\widetilde{O}_0^k$	$S_{15}^0S_{19}^0$	S_2^α	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \beta_\iota iD\ \delta_\iota P\ \mu_\iota iX\ \sigma_\iota$	Distributive
SiViTUiDiPXL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \gamma_\iota\ \sigma_\iota$	$\widetilde{O}_0^i$	S_{19}^0	–	$S_8^\beta S_{12}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \beta_\iota iD\ \nu_\iota iP\ \gamma_\iota X\ \sigma_\iota$	Distributive
SVTUiDiPiXiL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\iota\ \nu_\iota\ \mu_\iota\ \alpha_\iota$	$\widetilde{O}_0^i$	S_{19}^0	–	$S_{10}^\beta S_{11}^\beta$	–	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \beta_\iota iD\ \nu_\iota iP\ \mu_\iota iX\ \alpha_\iota i$	Distributive
SViTiUiDPXiL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\iota\ \delta_\iota\ \gamma_\iota\ \alpha_\iota$	$\widetilde{O}_0^h$	S_{19}^0	$S_6^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \beta_\iota iD\ \delta_\iota P\ \gamma_\iota X\ \alpha_\iota i$	Distributive
SiVTiUDiPXiL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\iota\ \nu_\iota\ \gamma_\iota\ \alpha_\iota$	$\widetilde{O}_0^h$	S_{19}^0	$S_3^\alpha S_4^\alpha$	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \lambda_\iota D\ \nu_\iota iP\ \gamma_\iota X\ \alpha_\iota i$	Distributive
SiViTUDPiXiL	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\iota\ \delta_\iota\ \mu_\iota\ \alpha_\iota$	$\widetilde{O}_0^i$	S_{19}^0	–	$S_{13}^\beta S_{14}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \lambda_\iota D\ \delta_\iota P\ \mu_\iota iX\ \alpha_\iota i$	Distributive
SVTUEQYM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\jmath\ \delta_\jmath\ \gamma_\jmath\ \sigma_\jmath$	$\widetilde{O}_0^g$	S_{20}^0	–	S_{10}^β	S_0^γ	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \lambda_\jmath E\ \delta_\jmath Q\ \gamma_\jmath Y\ \sigma_\jmath$	Distributive
SViTiUEiQiYM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\jmath\ \nu_\jmath\ \mu_\jmath\ \sigma_\jmath$	$\mathbb{O}_0^d$	S_{20}^0	$S_1^\alpha S_6^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \lambda_\jmath E\ \nu_\jmath iQ\ \mu_\jmath iY\ \sigma_\jmath$	Distributive
SiVTiUiEQiYM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\jmath\ \delta_\jmath\ \mu_\jmath\ \sigma_\jmath$	$\widetilde{O}_0^k$	$S_{15}^0S_{20}^0$	S_3^α	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \beta_\jmath iE\ \delta_\jmath Q\ \mu_\jmath iY\ \sigma_\jmath$	Distributive
SiViTUiEiQYM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \gamma_\jmath\ \sigma_\jmath$	$\widetilde{O}_0^i$	S_{20}^0	–	$S_8^\beta S_{13}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \beta_\jmath iE\ \nu_\jmath iQ\ \gamma_\jmath Y\ \sigma_\jmath$	Distributive
SVTUiEiQiYiM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\jmath\ \nu_\jmath\ \mu_\jmath\ \alpha_\jmath$	$\widetilde{O}_0^i$	S_{20}^0	–	$S_9^\beta S_{11}^\beta$	–	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \beta_\jmath iE\ \nu_\jmath iQ\ \mu_\jmath iY\ \alpha_\jmath i$	Distributive
SViTiUiEQYiM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\jmath\ \delta_\jmath\ \gamma_\jmath\ \alpha_\jmath$	$\widetilde{O}_0^h$	S_{20}^0	$S_5^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \beta_\jmath iE\ \delta_\jmath Q\ \gamma_\jmath Y\ \alpha_\jmath i$	Distributive
SiVTiUEiQYiM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\jmath\ \nu_\jmath\ \gamma_\jmath\ \alpha_\jmath$	$\widetilde{O}_0^h$	S_{20}^0	$S_2^\alpha S_4^\alpha$	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \lambda_\jmath E\ \nu_\jmath iQ\ \gamma_\jmath Y\ \alpha_\jmath i$	Distributive
SiViTUEQiYiM	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\jmath\ \delta_\jmath\ \mu_\jmath\ \alpha_\jmath$	$\widetilde{O}_0^i$	S_{20}^0	–	$S_{12}^\beta S_{14}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \lambda_\jmath E\ \delta_\jmath Q\ \mu_\jmath iY\ \alpha_\jmath i$	Distributive
SVTUFRZN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \lambda_\kappa\ \delta_\kappa\ \gamma_\kappa\ \sigma_\kappa$	$\widetilde{O}_0^g$	S_{21}^0	–	S_{11}^β	S_0^γ	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \lambda_\kappa F\ \delta_\kappa R\ \gamma_\kappa Z\ \sigma_\kappa$	Distributive
SViTiUFiRiZN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \lambda_\kappa\ \nu_\kappa\ \mu_\kappa\ \sigma_\kappa$	$\mathbb{O}_0^d$	S_{21}^0	$S_1^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \lambda_\kappa F\ \nu_\kappa iR\ \mu_\kappa iZ\ \sigma_\kappa$	Distributive
SiVTiUiFRiZN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \beta_\kappa\ \delta_\kappa\ \mu_\kappa\ \sigma_\kappa$	$\widetilde{O}_0^k$	$S_{15}^0S_{21}^0$	S_4^α	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \beta_\kappa iF\ \delta_\kappa R\ \mu_\kappa iZ\ \sigma_\kappa$	Distributive
SiViTUiFiRZN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \gamma_\kappa\ \sigma_\kappa$	$\widetilde{O}_0^i$	S_{21}^0	–	$S_8^\beta S_{14}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \beta_\kappa iF\ \nu_\kappa iR\ \gamma_\kappa Z\ \sigma_\kappa$	Distributive
SVTUiFiRiZiN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \delta_o\ \gamma_o\ \beta_\kappa\ \nu_\kappa\ \mu_\kappa\ \alpha_\kappa$	$\widetilde{O}_0^i$	S_{21}^0	–	$S_9^\beta S_{10}^\beta$	–	$\sigma_oS\ \lambda_oV\ \delta_oT\ \gamma_oU\ \beta_\kappa iF\ \nu_\kappa iR\ \mu_\kappa iZ\ \alpha_\kappa i$	Distributive
SViTiUiFRZiN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \lambda_o\ \nu_o\ \mu_o\ \beta_\kappa\ \delta_\kappa\ \gamma_\kappa\ \alpha_\kappa$	$\widetilde{O}_0^h$	S_{21}^0	$S_5^\alpha S_6^\alpha$	–	–	$\sigma_oS\ \lambda_oV\ \nu_o iT\ \mu_o iU\ \beta_\kappa iF\ \delta_\kappa R\ \gamma_\kappa Z\ \alpha_\kappa i$	Distributive
SiVTiUFiRZiN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \delta_o\ \mu_o\ \lambda_\kappa\ \nu_\kappa\ \gamma_\kappa\ \alpha_\kappa$	$\widetilde{O}_0^h$	S_{21}^0	$S_2^\alpha S_3^\alpha$	–	–	$\sigma_oS\ \beta_o iV\ \delta_oT\ \mu_o iU\ \lambda_\kappa F\ \nu_\kappa iR\ \gamma_\kappa Z\ \alpha_\kappa i$	Distributive
SiViTUFRiZiN	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \beta_o\ \nu_o\ \gamma_o\ \lambda_\kappa\ \delta_\kappa\ \mu_\kappa\ \alpha_\kappa$	$\widetilde{O}_0^i$	S_{21}^0	–	$S_{12}^\beta S_{13}^\beta$	–	$\sigma_oS\ \beta_o iV\ \nu_o iT\ \gamma_oU\ \lambda_\kappa F\ \delta_\kappa R\ \mu_\kappa iZ\ \alpha_\kappa i$	Distributive

Table 9. Structure of combinations of 8 element $\mathbb{M}$ subgroups and 8 element $\mathbb{T}$ subloops

$\mathbb{M}$ subgroups and type		Trigintaduonion sub-loop		Sedenion participation				Aligned sub-algebra	Type
Basis elements	Type	Basis elements	Type	S_L^0	S_L^α	S_L^β	S_L^γ	Basis elements	
SLiEiFiURQiX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \mu_o\ \delta_\kappa\ \delta_\jmath\ \mu_\iota$	$\widetilde{O}_0^k$	$S_{15}^0 S_{22}^0$	S_5^α	–	–	$\sigma_oS\ \sigma_\iota L\ \beta_\jmath iE\ \beta_\kappa iF\ \mu_o iU\ \delta_\kappa R\ \delta_\jmath Q\ \mu_\iota i$	Distributive
SLEFiUiRiQiX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa\ \mu_o\ \nu_\kappa\ \nu_\jmath\ \mu_\iota$	$\mathbb{O}_0^d$	S_{22}^0	$S_1^\alpha S_2^\alpha$	–	–	$\sigma_oS\ \sigma_\iota L\ \lambda_\jmath E\ \lambda_\kappa F\ \mu_o iU\ \nu_\kappa iR\ \nu_\jmath iQ\ \mu_\iota i$	Distributive
SiLiEFURiQiX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \beta_\jmath\ \lambda_\kappa\ \gamma_o\ \delta_\kappa\ \nu_\jmath\ \mu_\iota$	$\widetilde{O}_0^i$	S_{22}^0	–	$S_{11}^\beta S_{13}^\beta$	–	$\sigma_oS\ \alpha_\iota iL\ \beta_\jmath iE\ \lambda_\kappa F\ \gamma_o U\ \delta_\kappa R\ \nu_\jmath iQ\ \mu_\iota i$	Distributive
SiLEiFUiRQiX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \lambda_\jmath\ \beta_\kappa\ \gamma_o\ \nu_\kappa\ \delta_\jmath\ \mu_\iota$	$\widetilde{O}_0^i$	S_{22}^0	–	$S_{10}^\beta S_{14}^\beta$	–	$\sigma_oS\ \alpha_\iota iL\ \lambda_\jmath E\ \beta_\kappa iF\ \gamma_o U\ \nu_\kappa iR\ \delta_\jmath Q\ \mu_\iota i$	Distributive
SLiEiFUiRiQX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \beta_\jmath\ \beta_\kappa\ \gamma_o\ \nu_\kappa\ \nu_\jmath\ \gamma_\iota$	$\widetilde{O}_0^i$	S_{22}^0	–	$S_8^\beta S_9^\beta$	–	$\sigma_oS\ \sigma_\iota L\ \beta_\jmath iE\ \beta_\kappa iF\ \gamma_o U\ \nu_\kappa iR\ \nu_\jmath iQ\ \gamma_\iota$	Distributive
SLEFURQX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa\ \gamma_o\ \delta_\kappa\ \delta_\jmath\ \gamma_\iota$	$\widetilde{O}_0^g$	S_{22}^0	–	S_{12}^β	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \lambda_\jmath E\ \lambda_\kappa F\ \gamma_o U\ \delta_\kappa R\ \delta_\jmath Q\ \gamma_\iota$	Distributive
SiLiEFiUiRQX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \beta_\jmath\ \lambda_\kappa\ \mu_o\ \nu_\kappa\ \delta_\jmath\ \gamma_\iota$	$\widetilde{O}_0^h$	S_{22}^0	$S_3^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \alpha_\iota iL\ \beta_\jmath iE\ \lambda_\kappa F\ \mu_o iU\ \nu_\kappa iR\ \delta_\jmath Q\ \gamma_\iota$	Distributive
SiLEiFiURiQX	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \lambda_\jmath\ \beta_\kappa\ \mu_o\ \delta_\kappa\ \nu_\jmath\ \gamma_\iota$	$\widetilde{O}_0^h$	S_{22}^0	$S_4^\alpha S_6^\alpha$	–	–	$\sigma_oS\ \alpha_\iota iL\ \lambda_\jmath E\ \beta_\kappa iF\ \mu_o iU\ \delta_\kappa R\ \nu_\jmath iQ\ \gamma_\iota$	Distributive
SMiFiDiUPRiY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \mu_o\ \delta_\iota\ \delta_\kappa\ \mu_\jmath$	$\widetilde{O}_0^k$	$S_{15}^0 S_{23}^0$	S_6^α	–	–	$\sigma_oS\ \sigma_\jmath M\ \beta_\kappa iF\ \beta_\iota iD\ \mu_o iU\ \delta_\iota P\ \delta_\kappa R\ \mu_\jmath i$	Distributive
SMFDiUiPiRiY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \lambda_\kappa\ \lambda_\iota\ \mu_o\ \nu_\iota\ \nu_\kappa\ \mu_\jmath$	$\mathbb{O}_0^d$	S_{23}^0	$S_1^\alpha S_3^\alpha$	–	–	$\sigma_oS\ \sigma_\jmath M\ \lambda_\kappa F\ \lambda_\iota D\ \mu_o iU\ \nu_\iota iP\ \nu_\kappa iR\ \mu_\jmath i$	Distributive
SiMiFDUPiRiY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \beta_\kappa\ \lambda_\iota\ \gamma_o\ \delta_\iota\ \nu_\kappa\ \mu_\jmath$	$\widetilde{O}_0^i$	S_{23}^0	–	$S_9^\beta S_{14}^\beta$	–	$\sigma_oS\ \alpha_\jmath iM\ \beta_\kappa iF\ \lambda_\iota D\ \gamma_o U\ \delta_\iota P\ \nu_\kappa iR\ \mu_\jmath i$	Distributive
SiMFiDUiPRiY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \lambda_\kappa\ \beta_\iota\ \gamma_o\ \nu_\iota\ \delta_\kappa\ \mu_\jmath$	$\widetilde{O}_0^i$	S_{23}^0	–	$S_{11}^\beta S_{12}^\beta$	–	$\sigma_oS\ \alpha_\jmath iM\ \lambda_\kappa F\ \beta_\iota iD\ \gamma_o U\ \nu_\iota iP\ \delta_\kappa R\ \mu_\jmath i$	Distributive
SMiFiDUiPiRY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \beta_\kappa\ \beta_\iota\ \gamma_o\ \nu_\iota\ \nu_\kappa\ \gamma_\jmath$	$\widetilde{O}_0^i$	S_{23}^0	–	$S_8^\beta S_{10}^\beta$	–	$\sigma_oS\ \sigma_\jmath M\ \beta_\kappa iF\ \beta_\iota iD\ \gamma_o U\ \nu_\iota iP\ \nu_\kappa iR\ \gamma_\jmath$	Distributive
SMFDUPRY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \lambda_\kappa\ \lambda_\iota\ \gamma_o\ \delta_\iota\ \delta_\kappa\ \gamma_\jmath$	$\widetilde{O}_0^g$	S_{23}^0	–	S_{13}^β	S_0^γ	$\sigma_oS\ \sigma_\jmath M\ \lambda_\kappa F\ \lambda_\iota D\ \gamma_o U\ \delta_\iota P\ \delta_\kappa R\ \gamma_\jmath$	Distributive
SiMiFDiUiPRY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \beta_\kappa\ \lambda_\iota\ \mu_o\ \nu_\iota\ \delta_\kappa\ \gamma_\jmath$	$\widetilde{O}_0^h$	S_{23}^0	$S_4^\alpha S_5^\alpha$	–	–	$\sigma_oS\ \alpha_\jmath iM\ \beta_\kappa iF\ \lambda_\iota D\ \mu_o iU\ \nu_\iota iP\ \delta_\kappa R\ \gamma_\jmath$	Distributive
SiMFiDiUPiRY	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \lambda_\kappa\ \beta_\iota\ \mu_o\ \delta_\iota\ \nu_\kappa\ \gamma_\jmath$	$\widetilde{O}_0^h$	S_{23}^0	$S_2^\alpha S_7^\alpha$	–	–	$\sigma_oS\ \alpha_\jmath iM\ \lambda_\kappa F\ \beta_\iota iD\ \mu_o iU\ \delta_\iota P\ \nu_\kappa iR\ \gamma_\jmath$	Distributive
SNiDiEiUQPiZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \mu_o\ \delta_\jmath\ \delta_\iota\ \mu_\kappa$	$\widetilde{O}_0^k$	$S_{15}^0 S_{24}^0$	S_7^α	–	–	$\sigma_oS\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \mu_o iU\ \delta_\jmath Q\ \delta_\iota P\ \mu_\kappa i$	Distributive
SNDEiUiQiPiZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \mu_o\ \nu_\jmath\ \nu_\iota\ \mu_\kappa$	$\mathbb{O}_0^d$	S_{24}^0	$S_1^\alpha S_4^\alpha$	–	–	$\sigma_oS\ \sigma_\kappa N\ \lambda_\iota D\ \lambda_\jmath E\ \mu_o iU\ \nu_\jmath iQ\ \nu_\iota iP\ \mu_\kappa i$	Distributive
SiNiDEUQiPiZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \gamma_o\ \delta_\jmath\ \nu_\iota\ \mu_\kappa$	$\widetilde{O}_0^i$	S_{24}^0	–	$S_{10}^\beta S_{12}^\beta$	–	$\sigma_oS\ \alpha_\kappa iN\ \beta_\iota iD\ \lambda_\jmath E\ \gamma_o U\ \delta_\jmath Q\ \nu_\iota iP\ \mu_\kappa i$	Distributive
SiNDiEUiQPiZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \gamma_o\ \nu_\jmath\ \delta_\iota\ \mu_\kappa$	$\widetilde{O}_0^i$	S_{24}^0	–	$S_9^\beta S_{13}^\beta$	–	$\sigma_oS\ \alpha_\kappa iN\ \lambda_\iota D\ \beta_\jmath iE\ \gamma_o U\ \nu_\jmath iQ\ \delta_\iota P\ \mu_\kappa i$	Distributive
SNiDiEUiQiPZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \beta_\iota\ \beta_\jmath\ \gamma_o\ \nu_\jmath\ \nu_\iota\ \gamma_\kappa$	$\widetilde{O}_0^i$	S_{24}^0	–	$S_8^\beta S_{11}^\beta$	–	$\sigma_oS\ \sigma_\kappa N\ \beta_\iota iD\ \beta_\jmath iE\ \gamma_o U\ \nu_\jmath iQ\ \nu_\iota iP\ \gamma_\kappa$	Distributive
SNDEUQPZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ \gamma_o\ \delta_\jmath\ \delta_\iota\ \gamma_\kappa$	$\widetilde{O}_0^g$	S_{24}^0	–	S_{14}^β	S_0^γ	$\sigma_oS\ \sigma_\kappa N\ \lambda_\iota D\ \lambda_\jmath E\ \gamma_o U\ \delta_\jmath Q\ \delta_\iota P\ \gamma_\kappa$	Distributive
SiNiDEiUiQPZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \beta_\iota\ \lambda_\jmath\ \mu_o\ \nu_\jmath\ \delta_\iota\ \gamma_\kappa$	$\widetilde{O}_0^h$	S_{24}^0	$S_2^\alpha S_6^\alpha$	–	–	$\sigma_oS\ \alpha_\kappa iN\ \beta_\iota iD\ \lambda_\jmath E\ \mu_o iU\ \nu_\jmath iQ\ \delta_\iota P\ \gamma_\kappa$	Distributive
SiNDiEiUQiPZ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \lambda_\iota\ \beta_\jmath\ \mu_o\ \delta_\jmath\ \nu_\iota\ \gamma_\kappa$	$\widetilde{O}_0^h$	S_{24}^0	$S_3^\alpha S_5^\alpha$	–	–	$\sigma_oS\ \alpha_\kappa iN\ \lambda_\iota D\ \beta_\jmath iE\ \mu_o iU\ \delta_\jmath Q\ \nu_\iota iP\ \gamma_\kappa$	Distributive
SLiQiRiVZYiD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \beta_o\ \gamma_\kappa\ \gamma_\jmath\ \beta_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_2^α	S_8^β	–	$\sigma_oS\ \sigma_\iota L\ \nu_\jmath iQ\ \nu_\kappa iR\ \beta_o iV\ \gamma_\kappa Z\ \gamma_\jmath Y\ \beta_\iota i$	Distributive
SLQRiViZiYiD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \delta_\jmath\ \delta_\kappa\ \beta_o\ \mu_\kappa\ \mu_\jmath\ \beta_\iota$	$\widetilde{O}_0^j$	$S_{15}^0 S_{28}^0$	–	S_{12}^β	–	$\sigma_oS\ \sigma_\iota L\ \delta_\jmath Q\ \delta_\kappa R\ \beta_o iV\ \mu_\kappa iZ\ \mu_\jmath iY\ \beta_\iota i$	Distributive
SiLiQRVZiYiD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \nu_\jmath\ \delta_\kappa\ \lambda_o\ \gamma_\kappa\ \mu_\jmath\ \beta_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_6^α	S_{11}^β	–	$\sigma_oS\ \alpha_\iota iL\ \nu_\jmath iQ\ \delta_\kappa R\ \lambda_o V\ \gamma_\kappa Z\ \mu_\jmath iY\ \beta_\iota i$	Distributive
SiLQiRViZYiD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \delta_\jmath\ \nu_\kappa\ \lambda_o\ \mu_\kappa\ \gamma_\jmath\ \beta_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_7^α	S_{10}^β	–	$\sigma_oS\ \alpha_\iota iL\ \delta_\jmath Q\ \nu_\kappa iR\ \lambda_o V\ \mu_\kappa iZ\ \gamma_\jmath Y\ \beta_\iota i$	Distributive
SLiQiRViZiYD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ \lambda_o\ \mu_\kappa\ \mu_\jmath\ \lambda_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_1^α	S_9^β	–	$\sigma_oS\ \sigma_\iota L\ \nu_\jmath iQ\ \nu_\kappa iR\ \lambda_o V\ \mu_\kappa iZ\ \mu_\jmath iY\ \lambda_\iota$	Distributive
SLQRVZYD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \delta_\jmath\ \delta_\kappa\ \lambda_o\ \gamma_\kappa\ \gamma_\jmath\ \lambda_\iota$	$\mathbb{O}_0^c$	S_{28}^0	S_5^α	–	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \delta_\jmath Q\ \delta_\kappa R\ \lambda_o V\ \gamma_\kappa Z\ \gamma_\jmath Y\ \lambda_\iota$	Distributive
SiLiQRiViZYD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \nu_\jmath\ \delta_\kappa\ \beta_o\ \mu_\kappa\ \gamma_\jmath\ \lambda_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_4^α	S_{13}^β	–	$\sigma_oS\ \alpha_\iota iL\ \nu_\jmath iQ\ \delta_\kappa R\ \beta_o iV\ \mu_\kappa iZ\ \gamma_\jmath Y\ \lambda_\iota$	Distributive
SiLQiRiVZiYD	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \delta_\jmath\ \nu_\kappa\ \beta_o\ \gamma_\kappa\ \mu_\jmath\ \lambda_\iota$	$\widetilde{O}_0^f$	S_{28}^0	S_3^α	S_{14}^β	–	$\sigma_oS\ \alpha_\iota iL\ \delta_\jmath Q\ \nu_\kappa iR\ \beta_o iV\ \gamma_\kappa Z\ \mu_\jmath iY\ \lambda_\iota$	Distributive
SMiRiPiVXZiE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \beta_o\ \gamma_\iota\ \gamma_\kappa\ \beta_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_3^α	S_8^β	–	$\sigma_oS\ \sigma_\jmath M\ \nu_\kappa iR\ \nu_\iota iP\ \beta_o iV\ \gamma_\iota X\ \gamma_\kappa Z\ \beta_\jmath i$	Distributive
SMRPiViXiZiE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \delta_\kappa\ \delta_\iota\ \beta_o\ \mu_\iota\ \mu_\kappa\ \beta_\jmath$	$\widetilde{O}_0^j$	$S_{15}^0 S_{29}^0$	–	S_{13}^β	–	$\sigma_oS\ \sigma_\jmath M\ \delta_\kappa R\ \delta_\iota P\ \beta_o iV\ \mu_\iota iX\ \mu_\kappa iZ\ \beta_\jmath i$	Distributive
SiMiRPVXiZiE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \nu_\kappa\ \delta_\iota\ \lambda_o\ \gamma_\iota\ \mu_\kappa\ \beta_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_7^α	S_9^β	–	$\sigma_oS\ \alpha_\jmath iM\ \nu_\kappa iR\ \delta_\iota P\ \lambda_o V\ \gamma_\iota X\ \mu_\kappa iZ\ \beta_\jmath i$	Distributive
SiMRiPViXZiE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \delta_\kappa\ \nu_\iota\ \lambda_o\ \mu_\iota\ \gamma_\kappa\ \beta_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_5^α	S_{11}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \delta_\kappa R\ \nu_\iota iP\ \lambda_o V\ \mu_\iota iX\ \gamma_\kappa Z\ \beta_\jmath i$	Distributive
SMiRiPViXiZE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \nu_\kappa\ \nu_\iota\ \lambda_o\ \mu_\iota\ \mu_\kappa\ \lambda_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_1^α	S_{10}^β	–	$\sigma_oS\ \sigma_\jmath M\ \nu_\kappa iR\ \nu_\iota iP\ \lambda_o V\ \mu_\iota iX\ \mu_\kappa iZ\ \lambda_\jmath$	Distributive
SMRPVXZE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \delta_\kappa\ \delta_\iota\ \lambda_o\ \gamma_\iota\ \gamma_\kappa\ \lambda_\jmath$	$\mathbb{O}_0^c$	S_{29}^0	S_6^α	–	S_0^γ	$\sigma_oS\ \sigma_\jmath M\ \delta_\kappa R\ \delta_\iota P\ \lambda_o V\ \gamma_\iota X\ \gamma_\kappa Z\ \lambda_\jmath$	Distributive
SiMiRPiViXZE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \nu_\kappa\ \delta_\iota\ \beta_o\ \mu_\iota\ \gamma_\kappa\ \lambda_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_2^α	S_{14}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \nu_\kappa iR\ \delta_\iota P\ \beta_o iV\ \mu_\iota iX\ \gamma_\kappa Z\ \lambda_\jmath$	Distributive
SiMRiPiVXiZE	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \delta_\kappa\ \nu_\iota\ \beta_o\ \gamma_\iota\ \mu_\kappa\ \lambda_\jmath$	$\widetilde{O}_0^f$	S_{29}^0	S_4^α	S_{12}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \delta_\kappa R\ \nu_\iota iP\ \beta_o iV\ \gamma_\iota X\ \mu_\kappa iZ\ \lambda_\jmath$	Distributive
SNiPiQiVYXiF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \beta_o\ \gamma_\jmath\ \gamma_\iota\ \beta_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_4^α	S_8^β	–	$\sigma_oS\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \beta_o iV\ \gamma_\jmath Y\ \gamma_\iota X\ \beta_\kappa i$	Distributive
SNPQiViYiXiF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \beta_o\ \mu_\jmath\ \mu_\iota\ \beta_\kappa$	$\widetilde{O}_0^j$	$S_{15}^0 S_{30}^0$	–	S_{14}^β	–	$\sigma_oS\ \sigma_\kappa N\ \delta_\iota P\ \delta_\jmath Q\ \beta_o iV\ \mu_\jmath iY\ \mu_\iota iX\ \beta_\kappa i$	Distributive
SiNiPQVYiXiF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \lambda_o\ \gamma_\jmath\ \mu_\iota\ \beta_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_5^α	S_{10}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \nu_\iota iP\ \delta_\jmath Q\ \lambda_o V\ \gamma_\jmath Y\ \mu_\iota iX\ \beta_\kappa i$	Distributive
SiNPiQViYXiF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \lambda_o\ \mu_\jmath\ \gamma_\iota\ \beta_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_6^α	S_9^β	–	$\sigma_oS\ \alpha_\kappa iN\ \delta_\iota P\ \nu_\jmath iQ\ \lambda_o V\ \mu_\jmath iY\ \gamma_\iota X\ \beta_\kappa i$	Distributive
SNiPiQViYiXF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ \lambda_o\ \mu_\jmath\ \mu_\iota\ \lambda_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_1^α	S_{11}^β	–	$\sigma_oS\ \sigma_\kappa N\ \nu_\iota iP\ \nu_\jmath iQ\ \lambda_o V\ \mu_\jmath iY\ \mu_\iota iX\ \lambda_\kappa$	Distributive
SNPQVYXF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \delta_\iota\ \delta_\jmath\ \lambda_o\ \gamma_\jmath\ \gamma_\iota\ \lambda_\kappa$	$\mathbb{O}_0^c$	S_{30}^0	S_7^α	–	S_0^γ	$\sigma_oS\ \sigma_\kappa N\ \delta_\iota P\ \delta_\jmath Q\ \lambda_o V\ \gamma_\jmath Y\ \gamma_\iota X\ \lambda_\kappa$	Distributive
SiNiPQiViYXF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \nu_\iota\ \delta_\jmath\ \beta_o\ \mu_\jmath\ \gamma_\iota\ \lambda_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_3^α	S_{12}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \nu_\iota iP\ \delta_\jmath Q\ \beta_o iV\ \mu_\jmath iY\ \gamma_\iota X\ \lambda_\kappa$	Distributive
SiNPiQiVYiXF	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \delta_\iota\ \nu_\jmath\ \beta_o\ \gamma_\jmath\ \mu_\iota\ \lambda_\kappa$	$\widetilde{O}_0^f$	S_{30}^0	S_2^α	S_{13}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \delta_\iota P\ \nu_\jmath iQ\ \beta_o iV\ \gamma_\jmath Y\ \mu_\iota iX\ \lambda_\kappa$	Distributive
SLiYiZiTFEiP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \nu_o\ \lambda_\kappa\ \lambda_\jmath\ \nu_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_1^α	S_{12}^β	–	$\sigma_oS\ \sigma_\iota L\ \mu_\jmath iY\ \mu_\kappa iZ\ \nu_o iT\ \lambda_\kappa F\ \lambda_\jmath E\ \nu_\iota i$	Distributive
SLYZiTiFiEiP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \nu_o\ \beta_\kappa\ \beta_\jmath\ \nu_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_5^α	S_8^β	–	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \nu_o iT\ \beta_\kappa iF\ \beta_\jmath iE\ \nu_\iota i$	Distributive
SiLiYZTFiEiP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \mu_\jmath\ \gamma_\kappa\ \delta_o\ \lambda_\kappa\ \beta_\jmath\ \nu_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_3^α	S_{11}^β	–	$\sigma_oS\ \alpha_\iota iL\ \mu_\jmath iY\ \gamma_\kappa Z\ \delta_o T\ \lambda_\kappa F\ \beta_\jmath iE\ \nu_\iota i$	Distributive
SiLYiZTiFEiP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \gamma_\jmath\ \mu_\kappa\ \delta_o\ \beta_\kappa\ \lambda_\jmath\ \nu_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_4^α	S_{10}^β	–	$\sigma_oS\ \alpha_\iota iL\ \gamma_\jmath Y\ \mu_\kappa iZ\ \delta_o T\ \beta_\kappa iF\ \lambda_\jmath E\ \nu_\iota i$	Distributive
SLiYiZTiFiEP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ \delta_o\ \beta_\kappa\ \beta_\jmath\ \delta_\iota$	$\widetilde{O}_0^j$	$S_{15}^0 S_{25}^0$	–	S_9^β	–	$\sigma_oS\ \sigma_\iota L\ \mu_\jmath iY\ \mu_\kappa iZ\ \delta_o T\ \beta_\kappa iF\ \beta_\jmath iE\ \delta_\iota$	Distributive
SLYZTEFP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\iota\ \gamma_\jmath\ \gamma_\kappa\ \delta_o\ \lambda_\jmath\ \lambda_\kappa\ \delta_\iota$	$\mathbb{O}_0^c$	S_{25}^0	S_2^α	–	S_0^γ	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \delta_o T\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_\iota$	Distributive
SiLYiZiTiEFP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \gamma_\jmath\ \mu_\kappa\ \nu_o\ \beta_\jmath\ \lambda_\kappa\ \delta_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_7^α	S_{13}^β	–	$\sigma_oS\ \alpha_\iota iL\ \gamma_\jmath Y\ \mu_\kappa iZ\ \nu_o iT\ \beta_\jmath iE\ \lambda_\kappa F\ \delta_\iota$	Distributive
SiLiYZiTEiFP	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\iota\ \mu_\jmath\ \gamma_\kappa\ \nu_o\ \lambda_\jmath\ \beta_\kappa\ \delta_\iota$	$\widetilde{O}_0^f$	S_{25}^0	S_6^α	S_{14}^β	–	$\sigma_oS\ \alpha_\iota iL\ \mu_\jmath iY\ \gamma_\kappa Z\ \nu_o iT\ \lambda_\jmath E\ \beta_\kappa iF\ \delta_\iota$	Distributive
SMiZiXiTDFiQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \nu_o\ \lambda_\iota\ \lambda_\kappa\ \nu_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_1^α	S_{13}^β	–	$\sigma_oS\ \sigma_\jmath M\ \mu_\kappa iZ\ \mu_\iota iX\ \nu_o iT\ \lambda_\iota D\ \lambda_\kappa F\ \nu_\jmath i$	Distributive
SMZXiTiDiFiQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \gamma_\kappa\ \gamma_\iota\ \nu_o\ \beta_\iota\ \beta_\kappa\ \nu_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_6^α	S_8^β	–	$\sigma_oS\ \sigma_\jmath M\ \gamma_\kappa Z\ \gamma_\iota X\ \nu_o iT\ \beta_\iota iD\ \beta_\kappa iF\ \nu_\jmath i$	Distributive
SiMiZXTDiFiQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \mu_\kappa\ \gamma_\iota\ \delta_o\ \lambda_\iota\ \beta_\kappa\ \nu_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_4^α	S_9^β	–	$\sigma_oS\ \alpha_\jmath iM\ \mu_\kappa iZ\ \gamma_\iota X\ \delta_o T\ \lambda_\iota D\ \beta_\kappa iF\ \nu_\jmath i$	Distributive
SiMZiXTiDFiQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \gamma_\kappa\ \mu_\iota\ \delta_o\ \beta_\iota\ \lambda_\kappa\ \nu_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_2^α	S_{11}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \gamma_\kappa Z\ \mu_\iota iX\ \delta_o T\ \beta_\iota iD\ \lambda_\kappa F\ \nu_\jmath i$	Distributive
SMiZiXTiDiFQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \mu_\kappa\ \mu_\iota\ \delta_o\ \beta_\iota\ \beta_\kappa\ \delta_\jmath$	$\widetilde{O}_0^j$	$S_{15}^0 S_{26}^0$	–	S_{10}^β	–	$\sigma_oS\ \sigma_\jmath M\ \mu_\kappa iZ\ \mu_\iota iX\ \delta_o T\ \beta_\iota iD\ \beta_\kappa iF\ \delta_\jmath$	Distributive
SMZXTFDQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\jmath\ \gamma_\kappa\ \gamma_\iota\ \delta_o\ \lambda_\kappa\ \lambda_\iota\ \delta_\jmath$	$\mathbb{O}_0^c$	S_{26}^0	S_3^α	–	S_0^γ	$\sigma_oS\ \sigma_\jmath M\ \gamma_\kappa Z\ \gamma_\iota X\ \delta_o T\ \lambda_\kappa F\ \lambda_\iota D\ \delta_\jmath$	Distributive
SiMZiXiTiFDQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \gamma_\kappa\ \mu_\iota\ \nu_o\ \beta_\kappa\ \lambda_\iota\ \delta_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_5^α	S_{14}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \gamma_\kappa Z\ \mu_\iota iX\ \nu_o iT\ \beta_\kappa iF\ \lambda_\iota D\ \delta_\jmath$	Distributive
SiMiZXiTFiDQ	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\jmath\ \mu_\kappa\ \gamma_\iota\ \nu_o\ \lambda_\kappa\ \beta_\iota\ \delta_\jmath$	$\widetilde{O}_0^f$	S_{26}^0	S_7^α	S_{12}^β	–	$\sigma_oS\ \alpha_\jmath iM\ \mu_\kappa iZ\ \gamma_\iota X\ \nu_o iT\ \lambda_\kappa F\ \beta_\iota iD\ \delta_\jmath$	Distributive
SNiXiYiTEDiR	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \nu_o\ \lambda_\jmath\ \lambda_\iota\ \nu_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_1^α	S_{14}^β	–	$\sigma_oS\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \nu_o iT\ \lambda_\jmath E\ \lambda_\iota D\ \nu_\kappa i$	Distributive
SNXYiTiEiDiR	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \nu_o\ \beta_\jmath\ \beta_\iota\ \nu_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_7^α	S_8^β	–	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \nu_o iT\ \beta_\jmath iE\ \beta_\iota iD\ \nu_\kappa i$	Distributive
SiNiXYTEiDiR	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \delta_o\ \lambda_\jmath\ \beta_\iota\ \nu_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_2^α	S_{10}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \mu_\iota iX\ \gamma_\jmath Y\ \delta_o T\ \lambda_\jmath E\ \beta_\iota iD\ \nu_\kappa i$	Distributive
SiNXiYTiEDiR	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \delta_o\ \beta_\jmath\ \lambda_\iota\ \nu_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_3^α	S_9^β	–	$\sigma_oS\ \alpha_\kappa iN\ \gamma_\iota X\ \mu_\jmath iY\ \delta_o T\ \beta_\jmath iE\ \lambda_\iota D\ \nu_\kappa i$	Distributive
SNiXiYTiEiDR	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ \delta_o\ \beta_\jmath\ \beta_\iota\ \delta_\kappa$	$\widetilde{O}_0^j$	$S_{15}^0 S_{27}^0$	–	S_{11}^β	–	$\sigma_oS\ \sigma_\kappa N\ \mu_\iota iX\ \mu_\jmath iY\ \delta_o T\ \beta_\jmath iE\ \beta_\iota iD\ \delta_\kappa$	Distributive
SNXYTDER	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \sigma_\kappa\ \gamma_\iota\ \gamma_\jmath\ \delta_o\ \lambda_\iota\ \lambda_\jmath\ \delta_\kappa$	$\mathbb{O}_0^c$	S_{27}^0	S_4^α	–	S_0^γ	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \delta_o T\ \lambda_\iota D\ \lambda_\jmath E\ \delta_\kappa$	Distributive
SiNXiYiTiDER	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \gamma_\iota\ \mu_\jmath\ \nu_o\ \beta_\iota\ \lambda_\jmath\ \delta_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_6^α	S_{12}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \gamma_\iota X\ \mu_\jmath iY\ \nu_o iT\ \beta_\iota iD\ \lambda_\jmath E\ \delta_\kappa$	Distributive
SiNiXYiTDiER	$\mathbb{H}\otimes\mathbb{C}$	$\sigma_o\ \alpha_\kappa\ \mu_\iota\ \gamma_\jmath\ \nu_o\ \lambda_\iota\ \beta_\jmath\ \delta_\kappa$	$\widetilde{O}_0^f$	S_{27}^0	S_5^α	S_{13}^β	–	$\sigma_oS\ \alpha_\kappa iN\ \mu_\iota iX\ \gamma_\jmath Y\ \nu_o iT\ \lambda_\iota D\ \beta_\jmath iE\ \delta_\kappa$	Distributive

TABLE 10. Unit elements for aligned $\mathbb{M}$, $\mathbb{T}$ and $\mathbb{W}$ sub-algebras with 16 unit elements

$\mathbb{M}$ sub-group		$\mathbb{T}$ sub-loop		$\mathbb{W}$ sub-loop
Ref.	Unit elements	Ref.	Unit elements	Unit elements
$\mathbb{E}_0$	$SLMNUXYZ$ $VDEFTPQR$	$\mathbb{S}_0^\gamma$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\gamma_o\gamma_\iota\gamma_\jmath\gamma_\kappa$ $\lambda_o\lambda_\iota\lambda_\jmath\lambda_\kappa\delta_o\delta_\iota\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \gamma_oU\ \gamma_\iota X\ \gamma_\jmath Y\ \gamma_\kappa Z$ $\lambda_oV\ \lambda_\iota D\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_oT\ \delta_\iota P\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_1$	$SLMNiViDiEiF$ $iUiXiYiZTPQR$	$\mathbb{S}_1^\alpha$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\beta_o\beta_\iota\beta_\jmath\beta_\kappa$ $\mu_o\mu_\iota\mu_\jmath\mu_\kappa\delta_o\delta_\iota\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \beta_oiV\ \beta_\iota iD\ \beta_\jmath iE\ \beta_\kappa iF$ $\mu_oiU\ \mu_\iota iX\ \mu_\jmath iY\ \mu_\kappa iZ\ \delta_oT\ \delta_\iota P\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_2$	$SLUXiViDiTiP$ $iMiNiYiZEFQR$	$\mathbb{S}_2^\alpha$	$\sigma_o\sigma_\iota\gamma_o\gamma_\iota\beta_o\beta_\iota\nu_o\nu_\iota$ $\alpha_\jmath\alpha_\kappa\mu_\jmath\mu_\kappa\lambda_\jmath\lambda_\kappa\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \gamma_oU\ \gamma_\iota X\ \beta_oiV\ \beta_\iota iD\ \nu_oiT\ \nu_\iota iP$ $\alpha_\jmath iM\ \alpha_\kappa iN\ \mu_\jmath iY\ \mu_\kappa iZ\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_3$	$SMUYiViEiTiQ$ $iLiNiXiZDFPR$	$\mathbb{S}_3^\alpha$	$\sigma_o\sigma_\jmath\gamma_o\gamma_\jmath\beta_o\beta_\jmath\nu_o\nu_\jmath$ $\alpha_\iota\alpha_\kappa\mu_\iota\mu_\kappa\lambda_\iota\lambda_\kappa\delta_\iota\delta_\kappa$	$\sigma_oS\ \sigma_\jmath M\ \gamma_oU\ \gamma_\jmath Y\ \beta_oiV\ \beta_\jmath iE\ \nu_oiT\ \nu_\jmath iQ$ $\alpha_\iota iL\ \alpha_\kappa iN\ \mu_\iota iX\ \mu_\kappa iZ\ \lambda_\iota D\ \lambda_\kappa F\ \delta_\iota P\ \delta_\kappa R$
$\mathbb{E}_4$	$SNUZiViFiTiR$ $iLiMiXiYDEPQ$	$\mathbb{S}_4^\alpha$	$\sigma_o\sigma_\kappa\gamma_o\gamma_\kappa\beta_o\beta_\kappa\nu_o\nu_\kappa$ $\alpha_\iota\alpha_\jmath\mu_\iota\mu_\jmath\lambda_\iota\lambda_\jmath\delta_\iota\delta_\jmath$	$\sigma_oS\ \sigma_\kappa N\ \gamma_oU\ \gamma_\kappa Z\ \beta_oiV\ \beta_\kappa iF\ \nu_oiT\ \nu_\kappa iR$ $\alpha_\iota iL\ \alpha_\jmath iM\ \mu_\iota iX\ \mu_\jmath iY\ \lambda_\iota D\ \lambda_\jmath E\ \delta_\iota P\ \delta_\jmath Q$
$\mathbb{E}_5$	$SLYZiViDiQiR$ $iMiNiUiXEFTP$	$\mathbb{S}_5^\alpha$	$\sigma_o\sigma_\iota\gamma_\jmath\gamma_\kappa\beta_o\beta_\iota\nu_\jmath\nu_\kappa$ $\alpha_\jmath\alpha_\kappa\mu_o\mu_\iota\lambda_\jmath\lambda_\kappa\delta_o\delta_\iota$	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \beta_oiV\ \beta_\iota iD\ \nu_\jmath iQ\ \nu_\kappa iR$ $\alpha_\jmath iM\ \alpha_\kappa iN\ \mu_oiU\ \mu_\iota iX\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_oT\ \delta_\iota P$
$\mathbb{E}_6$	$SMXZiViEiPiR$ $iLiNiUiYDFTQ$	$\mathbb{S}_6^\alpha$	$\sigma_o\sigma_\jmath\gamma_\iota\gamma_\kappa\beta_o\beta_\jmath\nu_\iota\nu_\kappa$ $\alpha_\iota\alpha_\kappa\mu_o\mu_\jmath\lambda_\iota\lambda_\kappa\delta_o\delta_\jmath$	$\sigma_oS\ \sigma_\jmath M\ \gamma_\iota X\ \gamma_\kappa Z\ \beta_oiV\ \beta_\jmath iE\ \nu_\iota iP\ \nu_\kappa iR$ $\alpha_\iota iL\ \alpha_\kappa iN\ \mu_oiU\ \mu_\jmath iY\ \lambda_\iota D\ \lambda_\kappa F\ \delta_oT\ \delta_\jmath Q$
$\mathbb{E}_7$	$SNXYiViFiPiQ$ $iLiMiUiZDETR$	$\mathbb{S}_7^\alpha$	$\sigma_o\sigma_\kappa\gamma_\iota\gamma_\jmath\beta_o\beta_\kappa\nu_\iota\nu_\jmath$ $\alpha_\iota\alpha_\jmath\mu_o\mu_\kappa\lambda_\iota\lambda_\jmath\delta_o\delta_\kappa$	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \beta_oiV\ \beta_\kappa iF\ \nu_\iota iP\ \nu_\jmath iQ$ $\alpha_\iota iL\ \alpha_\jmath iM\ \mu_oiU\ \mu_\kappa iZ\ \lambda_\iota D\ \lambda_\jmath E\ \delta_oT\ \delta_\kappa R$
$\mathbb{E}_8$	$SLMNiTiPiQiR$ $iUiXiYiZVDEF$	$\mathbb{S}_8^\beta$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\nu_o\nu_\iota\nu_\jmath\nu_\kappa$ $\mu_o\mu_\iota\mu_\jmath\mu_\kappa\lambda_o\lambda_\iota\lambda_\jmath\lambda_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \nu_oiT\ \nu_\iota iP\ \nu_\jmath iQ\ \nu_\kappa iR$ $\mu_oiU\ \mu_\iota iX\ \mu_\jmath iY\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\iota D\ \lambda_\jmath E\ \lambda_\kappa F$
$\mathbb{E}_9$	$SLUXiEiFiQiR$ $iMiNiYiZVDTP$	$\mathbb{S}_9^\beta$	$\sigma_o\sigma_\iota\gamma_o\gamma_\iota\beta_\jmath\beta_\kappa\nu_\jmath\nu_\kappa$ $\alpha_\jmath\alpha_\kappa\mu_\jmath\mu_\kappa\lambda_o\lambda_\iota\delta_o\delta_\iota$	$\sigma_oS\ \sigma_\iota L\ \gamma_oU\ \gamma_\iota X\ \beta_\jmath iE\ \beta_\kappa iF\ \nu_\jmath iQ\ \nu_\kappa iR$ $\alpha_\jmath iM\ \alpha_\kappa iN\ \mu_\jmath iY\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\iota D\ \delta_oT\ \delta_\iota P$
$\mathbb{E}_{10}$	$SMUYiDiFiPiR$ $iLiNiXiZVETQ$	$\mathbb{S}_{10}^\beta$	$\sigma_o\sigma_\jmath\gamma_o\gamma_\jmath\beta_\iota\beta_\kappa\nu_\iota\nu_\kappa$ $\alpha_\iota\alpha_\kappa\mu_\iota\mu_\kappa\lambda_o\lambda_\jmath\delta_o\delta_\jmath$	$\sigma_oS\ \sigma_\jmath M\ \gamma_oU\ \gamma_\jmath Y\ \beta_\iota iD\ \beta_\kappa iF\ \nu_\iota iP\ \nu_\kappa iR$ $\alpha_\iota iL\ \alpha_\kappa iN\ \mu_\iota iX\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\jmath E\ \delta_oT\ \delta_\jmath Q$
$\mathbb{E}_{11}$	$SNUZiDiEiPiQ$ $iLiMiXiYVFTR$	$\mathbb{S}_{11}^\beta$	$\sigma_o\sigma_\kappa\gamma_o\gamma_\kappa\beta_\iota\beta_\jmath\nu_\iota\nu_\jmath$ $\alpha_\iota\alpha_\jmath\mu_\iota\mu_\jmath\lambda_o\lambda_\kappa\delta_o\delta_\kappa$	$\sigma_oS\ \sigma_\kappa N\ \gamma_oU\ \gamma_\kappa Z\ \beta_\iota iD\ \beta_\jmath iE\ \nu_\iota iP\ \nu_\jmath iQ$ $\alpha_\iota iL\ \alpha_\jmath iM\ \mu_\iota iX\ \mu_\jmath iY\ \lambda_oV\ \lambda_\kappa F\ \delta_oT\ \delta_\kappa R$
$\mathbb{E}_{12}$	$SLYZiEiFiTiP$ $iMiNiUiXVDQR$	$\mathbb{S}_{12}^\beta$	$\sigma_o\sigma_\iota\gamma_\jmath\gamma_\kappa\beta_\jmath\beta_\kappa\nu_o\nu_\iota$ $\alpha_\jmath\alpha_\kappa\mu_o\mu_\iota\lambda_o\lambda_\iota\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \beta_\jmath iE\ \beta_\kappa iF\ \nu_oiT\ \nu_\iota iP$ $\alpha_\jmath iM\ \alpha_\kappa iN\ \mu_oiU\ \mu_\iota iX\ \lambda_oV\ \lambda_\iota D\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_{13}$	$SMXZiDiFiTiQ$ $iLiNiUiYVEPR$	$\mathbb{S}_{13}^\beta$	$\sigma_o\sigma_\jmath\gamma_\iota\gamma_\kappa\beta_\iota\beta_\kappa\nu_o\nu_\jmath$ $\alpha_\iota\alpha_\kappa\mu_o\mu_\jmath\lambda_o\lambda_\jmath\delta_\iota\delta_\kappa$	$\sigma_oS\ \sigma_\jmath M\ \gamma_\iota X\ \gamma_\kappa Z\ \beta_\iota iD\ \beta_\kappa iF\ \nu_oiT\ \nu_\jmath iQ$ $\alpha_\iota iL\ \alpha_\kappa iN\ \mu_oiU\ \mu_\jmath iY\ \lambda_oV\ \lambda_\jmath E\ \delta_\iota P\ \delta_\kappa R$
$\mathbb{E}_{14}$	$SNXYiDiEiTiR$ $iLiMiUiZVFPQ$	$\mathbb{S}_{14}^\beta$	$\sigma_o\sigma_\kappa\gamma_\iota\gamma_\jmath\beta_\iota\beta_\jmath\nu_o\nu_\kappa$ $\alpha_\iota\alpha_\jmath\mu_o\mu_\kappa\lambda_o\lambda_\kappa\delta_\iota\delta_\jmath$	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \beta_\iota iD\ \beta_\jmath iE\ \nu_oiT\ \nu_\kappa iR$ $\alpha_\iota iL\ \alpha_\jmath iM\ \mu_oiU\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\kappa F\ \delta_\iota P\ \delta_\jmath Q$
$\mathbb{E}_{15}$	$SLMNUXYZ$ $iViDiEiFiTiPiQiR$	$\mathbb{S}_{15}^0$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\gamma_o\gamma_\iota\gamma_\jmath\gamma_\kappa$ $\beta_o\beta_\iota\beta_\jmath\beta_\kappa\nu_o\nu_\iota\nu_\jmath\nu_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \gamma_oU\ \gamma_\iota X\ \gamma_\jmath Y\ \gamma_\kappa Z$ $\beta_oiV\ \beta_\iota iD\ \beta_\jmath iE\ \beta_\kappa iF\ \nu_oiT\ \nu_\iota iP\ \nu_\jmath iQ\ \nu_\kappa iR$
$\mathbb{E}_{16}$	$SLMNUXYZ$ $iSiLiMiNiUiXiYiZ$	$\mathbb{S}_{16}^0$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\gamma_o\gamma_\iota\gamma_\jmath\gamma_\kappa$ $\alpha_o\alpha_\iota\alpha_\jmath\alpha_\kappa\mu_o\mu_\iota\mu_\jmath\mu_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \gamma_oU\ \gamma_\iota X\ \gamma_\jmath Y\ \gamma_\kappa Z$ $\alpha_oiS\ \alpha_\iota iL\ \alpha_\jmath iM\ \alpha_\kappa iN\ \mu_oiU\ \mu_\iota iX\ \mu_\jmath iY\ \mu_\kappa iZ$
$\mathbb{E}_{17}$	$SLMNiViDiEiF$ $iSiLiMiNVDEF$	$\mathbb{S}_{17}^0$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\beta_o\beta_\iota\beta_\jmath\beta_\kappa$ $\alpha_o\alpha_\iota\alpha_\jmath\alpha_\kappa\lambda_o\lambda_\iota\lambda_\jmath\lambda_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \beta_oiV\ \beta_\iota iD\ \beta_\jmath iE\ \beta_\kappa iF$ $\alpha_oiS\ \alpha_\iota iL\ \alpha_\jmath iM\ \alpha_\kappa iN\ \lambda_oV\ \lambda_\iota D\ \lambda_\jmath E\ \lambda_\kappa F$
$\mathbb{E}_{18}$	$SLMNiTiPiQiR$ $iSiLiMiNTPQR$	$\mathbb{S}_{18}^0$	$\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa\nu_o\nu_\iota\nu_\jmath\nu_\kappa$ $\alpha_o\alpha_\iota\alpha_\jmath\alpha_\kappa\delta_o\delta_\iota\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \sigma_\jmath M\ \sigma_\kappa N\ \nu_oiT\ \nu_\iota iP\ \nu_\jmath iQ\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\iota iL\ \alpha_\jmath iM\ \alpha_\kappa iN\ \delta_oT\ \delta_\iota P\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_{19}$	$SLUXiViDiTiP$ $iSiLiUiXVDTP$	$\mathbb{S}_{19}^0$	$\sigma_o\sigma_\iota\gamma_o\gamma_\iota\beta_o\beta_\iota\nu_o\nu_\iota$ $\alpha_o\alpha_\iota\mu_o\mu_\iota\lambda_o\lambda_\iota\delta_o\delta_\iota$	$\sigma_oS\ \sigma_\iota L\ \gamma_oU\ \gamma_\iota X\ \beta_oiV\ \beta_\iota iD\ \nu_oiT\ \nu_\iota iP$ $\alpha_oiS\ \alpha_\iota iL\ \mu_oiU\ \mu_\iota iX\ \lambda_oV\ \lambda_\iota D\ \delta_oT\ \delta_\iota P$
$\mathbb{E}_{20}$	$SMUYiViEiTiQ$ $iSiMiUiYVETQ$	$\mathbb{S}_{20}^0$	$\sigma_o\sigma_\jmath\gamma_o\gamma_\jmath\beta_o\beta_\jmath\nu_o\nu_\jmath$ $\alpha_o\alpha_\jmath\mu_o\mu_\jmath\lambda_o\lambda_\jmath\delta_o\delta_\jmath$	$\sigma_oS\ \sigma_\jmath M\ \gamma_oU\ \gamma_\jmath Y\ \beta_oiV\ \beta_\jmath iE\ \nu_oiT\ \nu_\jmath iQ$ $\alpha_oiS\ \alpha_\jmath iM\ \mu_oiU\ \mu_\jmath iY\ \lambda_oV\ \lambda_\jmath E\ \delta_oT\ \delta_\jmath Q$
$\mathbb{E}_{21}$	$SNUZiViFiTiR$ $iSiNiUiZVFTR$	$\mathbb{S}_{21}^0$	$\sigma_o\sigma_\kappa\gamma_o\gamma_\kappa\beta_o\beta_\kappa\nu_o\nu_\kappa$ $\alpha_o\alpha_\kappa\mu_o\mu_\kappa\lambda_o\lambda_\kappa\delta_o\delta_\kappa$	$\sigma_oS\ \sigma_\kappa N\ \gamma_oU\ \gamma_\kappa Z\ \beta_oiV\ \beta_\kappa iF\ \nu_oiT\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\kappa iN\ \mu_oiU\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\kappa F\ \delta_oT\ \delta_\kappa R$
$\mathbb{E}_{22}$	$SLYZiViDiQiR$ $iSiLiYiZVDQR$	$\mathbb{S}_{22}^0$	$\sigma_o\sigma_\iota\gamma_\jmath\gamma_\kappa\beta_o\beta_\iota\nu_\jmath\nu_\kappa$ $\alpha_o\alpha_\iota\mu_\jmath\mu_\kappa\lambda_o\lambda_\iota\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \beta_oiV\ \beta_\iota iD\ \nu_\jmath iQ\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\iota iL\ \mu_\jmath iY\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\iota D\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_{23}$	$SMXZiViEiPiR$ $iSiMiXiZVEPR$	$\mathbb{S}_{23}^0$	$\sigma_o\sigma_\jmath\gamma_\iota\gamma_\kappa\beta_o\beta_\jmath\nu_\iota\nu_\kappa$ $\alpha_o\alpha_\jmath\mu_\iota\mu_\kappa\lambda_o\lambda_\jmath\delta_\iota\delta_\kappa$	$\sigma_oS\ \sigma_\jmath M\ \gamma_\iota X\ \gamma_\kappa Z\ \beta_oiV\ \beta_\jmath iE\ \nu_\iota iP\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\jmath iM\ \mu_\iota iX\ \mu_\kappa iZ\ \lambda_oV\ \lambda_\jmath E\ \delta_\iota P\ \delta_\kappa R$
$\mathbb{E}_{24}$	$SNXYiViFiPiQ$ $iSiNiXiYVFPQ$	$\mathbb{S}_{24}^0$	$\sigma_o\sigma_\kappa\gamma_\iota\gamma_\jmath\beta_o\beta_\kappa\nu_\iota\nu_\jmath$ $\alpha_o\alpha_\kappa\mu_\iota\mu_\jmath\lambda_o\lambda_\kappa\delta_\iota\delta_\jmath$	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \beta_oiV\ \beta_\kappa iF\ \nu_\iota iP\ \nu_\jmath iQ$ $\alpha_oiS\ \alpha_\kappa iN\ \mu_\iota iX\ \mu_\jmath iY\ \lambda_oV\ \lambda_\kappa F\ \delta_\iota P\ \delta_\jmath Q$
$\mathbb{E}_{25}$	$SLUXiEiFiQiR$ $iSiLiUiXEFQR$	$\mathbb{S}_{25}^0$	$\sigma_o\sigma_\iota\gamma_o\gamma_\iota\beta_\jmath\beta_\kappa\nu_\jmath\nu_\kappa$ $\alpha_o\alpha_\iota\mu_o\mu_\iota\lambda_\jmath\lambda_\kappa\delta_\jmath\delta_\kappa$	$\sigma_oS\ \sigma_\iota L\ \gamma_oU\ \gamma_\iota X\ \beta_\jmath iE\ \beta_\kappa iF\ \nu_\jmath iQ\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\iota iL\ \mu_oiU\ \mu_\iota iX\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_\jmath Q\ \delta_\kappa R$
$\mathbb{E}_{26}$	$SMUYiDiFiPiR$ $iSiMiUiYDFPR$	$\mathbb{S}_{26}^0$	$\sigma_o\sigma_\jmath\gamma_o\gamma_\jmath\beta_\iota\beta_\kappa\nu_\iota\nu_\kappa$ $\alpha_o\alpha_\jmath\mu_o\mu_\jmath\lambda_\iota\lambda_\kappa\delta_\iota\delta_\kappa$	$\sigma_oS\ \sigma_\jmath M\ \gamma_oU\ \gamma_\jmath Y\ \beta_\iota iD\ \beta_\kappa iF\ \nu_\iota iP\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\jmath iM\ \mu_oiU\ \mu_\jmath iY\ \lambda_\iota D\ \lambda_\kappa F\ \delta_\iota P\ \delta_\kappa R$
$\mathbb{E}_{27}$	$SNUZiDiEiPiQ$ $iSiNiUiZDEPQ$	$\mathbb{S}_{27}^0$	$\sigma_o\sigma_\kappa\gamma_o\gamma_\kappa\beta_\iota\beta_\jmath\nu_\iota\nu_\jmath$ $\alpha_o\alpha_\kappa\mu_o\mu_\kappa\lambda_\iota\lambda_\jmath\delta_\iota\delta_\jmath$	$\sigma_oS\ \sigma_\kappa N\ \gamma_oU\ \gamma_\kappa Z\ \beta_\iota iD\ \beta_\jmath iE\ \nu_\iota iP\ \nu_\jmath iQ$ $\alpha_oiS\ \alpha_\kappa iN\ \mu_oiU\ \mu_\kappa iZ\ \lambda_\iota D\ \lambda_\jmath E\ \delta_\iota P\ \delta_\jmath Q$
$\mathbb{E}_{28}$	$SLYZiEiFiTiP$ $iSiLiYiZEFTP$	$\mathbb{S}_{28}^0$	$\sigma_o\sigma_\iota\gamma_\jmath\gamma_\kappa\beta_\jmath\beta_\kappa\nu_o\nu_\iota$ $\alpha_o\alpha_\iota\mu_\jmath\mu_\kappa\lambda_\jmath\lambda_\kappa\delta_o\delta_\iota$	$\sigma_oS\ \sigma_\iota L\ \gamma_\jmath Y\ \gamma_\kappa Z\ \beta_\jmath iE\ \beta_\kappa iF\ \nu_oiT\ \nu_\iota iP$ $\alpha_oiS\ \alpha_\iota iL\ \mu_\jmath iY\ \mu_\kappa iZ\ \lambda_\jmath E\ \lambda_\kappa F\ \delta_oT\ \delta_\iota P$
$\mathbb{E}_{29}$	$SMXZiDiFiTiQ$ $iSiMiXiZDFTQ$	$\mathbb{S}_{29}^0$	$\sigma_o\sigma_\jmath\gamma_\iota\gamma_\kappa\beta_\iota\beta_\kappa\nu_o\nu_\jmath$ $\alpha_o\alpha_\jmath\mu_\iota\mu_\kappa\lambda_\iota\lambda_\kappa\delta_o\delta_\jmath$	$\sigma_oS\ \sigma_\jmath M\ \gamma_\iota X\ \gamma_\kappa Z\ \beta_\iota iD\ \beta_\kappa iF\ \nu_oiT\ \nu_\jmath iQ$ $\alpha_oiS\ \alpha_\jmath iM\ \mu_\iota iX\ \mu_\kappa iZ\ \lambda_\iota D\ \lambda_\kappa F\ \delta_oT\ \delta_\jmath Q$
$\mathbb{E}_{30}$	$SNXYiDiEiTiR$ $iSiNiXiYDETR$	$\mathbb{S}_{30}^0$	$\sigma_o\sigma_\kappa\gamma_\iota\gamma_\jmath\beta_\iota\beta_\jmath\nu_o\nu_\kappa$ $\alpha_o\alpha_\kappa\mu_\iota\mu_\jmath\lambda_\iota\lambda_\jmath\delta_o\delta_\kappa$	$\sigma_oS\ \sigma_\kappa N\ \gamma_\iota X\ \gamma_\jmath Y\ \beta_\iota iD\ \beta_\jmath iE\ \nu_oiT\ \nu_\kappa iR$ $\alpha_oiS\ \alpha_\kappa iN\ \mu_\iota iX\ \mu_\jmath iY\ \lambda_\iota D\ \lambda_\jmath E\ \delta_oT\ \delta_\kappa R$

7. Basis of the notation used for unit elements of $\mathbb{M} \cong M_4(C)$, and its Cayley table

Capital roman letters are used to label real matrix unit elements of $M_4(R)$, as shown in table 11. These labels are combined with i to represent imaginary counterparts. Their Cayley table is shown in table 12.

TABLE 11. Notation used to label 4×4 unit matrices

$$S=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} L=\begin{bmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{bmatrix} M=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{bmatrix} N=\begin{bmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{bmatrix}$$

$$V=\begin{bmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix} D=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} E=\begin{bmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{bmatrix} F=\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}$$

$$iU=\begin{bmatrix}0&0&0&i\\0&0&i&0\\0&-i&0&0\\-i&0&0&0\end{bmatrix} iX=\begin{bmatrix}0&0&i&0\\0&0&0&-i\\i&0&0&0\\0&-i&0&0\end{bmatrix} iY=\begin{bmatrix}0&i&0&0\\i&0&0&0\\0&0&0&i\\0&0&i&0\end{bmatrix} iZ=\begin{bmatrix}i&0&0&0\\0&-i&0&0\\0&0&-i&0\\0&0&0&i\end{bmatrix}$$

$$iT=\begin{bmatrix}0&0&i&0\\0&0&0&-i\\-i&0&0&0\\0&i&0&0\end{bmatrix} iP=\begin{bmatrix}0&0&0&i\\0&0&i&0\\0&i&0&0\\i&0&0&0\end{bmatrix} iQ=\begin{bmatrix}i&0&0&0\\0&-i&0&0\\0&0&i&0\\0&0&0&-i\end{bmatrix} iR=\begin{bmatrix}0&i&0&0\\i&0&0&0\\0&0&0&-i\\0&0&-i&0\end{bmatrix}$$

$$iS=\begin{bmatrix}i&0&0&0\\0&i&0&0\\0&0&i&0\\0&0&0&i\end{bmatrix} iL=\begin{bmatrix}0&i&0&0\\-i&0&0&0\\0&0&0&-i\\0&0&i&0\end{bmatrix} iM=\begin{bmatrix}0&0&i&0\\0&0&0&i\\-i&0&0&0\\0&-i&0&0\end{bmatrix} iN=\begin{bmatrix}0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0\end{bmatrix}$$

$$iV=\begin{bmatrix}0&i&0&0\\-i&0&0&0\\0&0&0&i\\0&0&-i&0\end{bmatrix} iD=\begin{bmatrix}i&0&0&0\\0&i&0&0\\0&0&-i&0\\0&0&0&-i\end{bmatrix} iE=\begin{bmatrix}0&0&0&i\\0&0&-i&0\\0&-i&0&0\\i&0&0&0\end{bmatrix} iF=\begin{bmatrix}0&0&i&0\\0&0&0&i\\i&0&0&0\\0&i&0&0\end{bmatrix}$$

$$U=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{bmatrix} X=\begin{bmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{bmatrix} Y=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} Z=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}$$

$$T=\begin{bmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{bmatrix} P=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix} Q=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix} R=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{bmatrix}$$

Note that some matrix labels have been changed from those used in a previous paper [34].

Table 12. Labels and Cayley table for $\mathbb{M} \cong M_4(C)$

Ref no.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Label	S	L	M	N	V	D	E	F	iU	iX	iY	iZ	iT	iP	iQ	iR	iS	iL	iM	iN	iV	iD	iE	iF	U	X	Y	Z	T	P	Q	R
S	+S	+L	+M	+N	+V	+D	+E	+F	+iU	+iX	+iY	+iZ	+iT	+iP	+iQ	+iR	+iS	+iL	+iM	+iN	+iV	+iD	+iE	+iF	+U	+X	+Y	+Z	+T	+P	+Q	+R
L	+L	−S	+N	−M	−D	+V	−F	+E	+iX	−iU	+iZ	−iY	−iP	+iT	−iR	+iQ	+iL	−iS	+iN	−iM	−iD	+iV	−iF	+iE	+X	−U	+Z	−Y	−P	+T	−R	+Q
M	+M	−N	−S	+L	+E	−F	−V	+D	−iY	+iZ	+iU	−iX	−iQ	+iR	+iT	−iP	+iM	−iN	−iS	+iL	+iE	−iF	−iV	+iD	−Y	+Z	+U	−X	−Q	+R	+T	−P
N	+N	+M	−L	−S	−F	−E	+D	+V	−iZ	−iY	+iX	+iU	+iR	+iQ	−iP	−iT	+iN	+iM	−iL	−iS	−iF	−iE	+iD	+iV	−Z	−Y	+X	+U	+R	+Q	−P	−T
V	+V	−D	+E	−F	−S	+L	−M	+N	+iT	−iP	+iQ	−iR	−iU	+iX	−iY	+iZ	+iV	−iD	+iE	−iF	−iS	+iL	−iM	+iN	+T	−P	+Q	−R	−U	+X	−Y	+Z
D	+D	+V	+F	+E	+L	+S	+N	+M	+iP	+iT	+iR	+iQ	+iX	+iU	+iZ	+iY	+iD	+iV	+iF	+iE	+iL	+iS	+iN	+iM	+P	+T	+R	+Q	+X	+U	+Z	+Y
E	+E	+F	−V	−D	−M	−N	+S	+L	−iQ	−iR	+iT	+iP	+iY	+iZ	−iU	−iX	+iE	+iF	−iV	−iD	−iM	−iN	+iS	+iL	−Q	−R	+T	+P	+Y	+Z	−U	−X
F	+F	−E	−D	+V	+N	−M	−L	+S	−iR	+iQ	+iP	−iT	−iZ	+iY	+iX	−iU	+iF	−iE	−iD	+iV	+iN	−iM	−iL	+iS	−R	+Q	+P	−T	−Z	+Y	+X	−U
iU	+iU	+iX	−iY	−iZ	−iT	−iP	+iQ	+iR	+S	+L	−M	−N	−V	−D	+E	+F	−U	−X	+Y	+Z	+T	+P	−Q	−R	−iS	−iL	+iM	+iN	+iV	+iD	−iE	−iF
iX	+iX	−iU	−iZ	+iY	+iP	−iT	−iR	+iQ	+L	−S	−N	+M	+D	−V	−F	+E	−X	+U	+Z	−Y	−P	+T	+R	−Q	−iL	+iS	+iN	−iM	−iD	+iV	+iF	−iE
iY	+iY	−iZ	+iU	−iX	−iQ	+iR	−iT	+iP	−M	+N	−S	+L	+E	−F	+V	−D	−Y	+Z	−U	+X	+Q	−R	+T	−P	+iM	−iN	+iS	−iL	−iE	+iF	−iV	+iD
iZ	+iZ	+iY	+iX	+iU	+iR	+iQ	+iP	+iT	−N	−M	−L	−S	−F	−E	−D	−V	−Z	−Y	−X	−U	−R	−Q	−P	−T	+iN	+iM	+iL	+iS	+iF	+iE	+iD	+iV
iT	+iT	−iP	−iQ	+iR	+iU	−iX	−iY	+iZ	+V	−D	−E	+F	+S	−L	−M	+N	−T	+P	+Q	−R	−U	+X	+Y	−Z	−iV	+iD	+iE	−iF	−iS	+iL	+iM	−iN
iP	+iP	+iT	−iR	−iQ	−iX	−iU	+iZ	+iY	+D	+V	−F	−E	−L	−S	+N	+M	−P	−T	+R	+Q	+X	+U	−Z	−Y	−iD	−iV	+iF	+iE	+iL	+iS	−iN	−iM
iQ	+iQ	+iR	+iT	+iP	+iY	+iZ	+iU	+iX	−E	−F	−V	−D	−M	−N	−S	−L	−Q	−R	−T	−P	−Y	−Z	−U	−X	+iE	+iF	+iV	+iD	+iM	+iN	+iS	+iL
iR	+iR	−iQ	+iP	−iT	−iZ	+iY	−iX	+iU	−F	+E	−D	+V	+N	−M	+L	−S	−R	+Q	−P	+T	+Z	−Y	+X	−U	+iF	−iE	+iD	−iV	−iN	+iM	−iL	+iS
iS	+iS	+iL	+iM	+iN	+iV	+iD	+iE	+iF	−U	−X	−Y	−Z	−T	−P	−Q	−R	−S	−L	−M	−N	−V	−D	−E	−F	+iU	+iX	+iY	+iZ	+iT	+iP	+iQ	+iR
iL	+iL	−iS	+iN	−iM	−iD	+iV	−iF	+iE	−X	+U	−Z	+Y	+P	−T	+R	−Q	−L	+S	−N	+M	+D	−V	+F	−E	+iX	−iU	+iZ	−iY	−iP	+iT	−iR	+iQ
iM	+iM	−iN	−iS	+iL	+iE	−iF	−iV	+iD	+Y	−Z	−U	+X	+Q	−R	−T	+P	−M	+N	+S	−L	−E	+F	+V	−D	−iY	+iZ	+iU	−iX	−iQ	+iR	+iT	−iP
iN	+iN	+iM	−iL	−iS	−iF	−iE	+iD	+iV	+Z	+Y	−X	−U	−R	−Q	+P	+T	−N	−M	+L	+S	+F	+E	−D	−V	−iZ	−iY	+iX	+iU	+iR	+iQ	−iP	−iT
iV	+iV	−iD	+iE	−iF	−iS	+iL	−iM	+iN	−T	+P	−Q	+R	+U	−X	+Y	−Z	−V	+D	−E	+F	+S	−L	+M	−N	+iT	−iP	+iQ	−iR	−iU	+iX	−iY	+iZ
iD	+iD	+iV	+iF	+iE	+iL	+iS	+iN	+iM	−P	−T	−R	−Q	−X	−U	−Z	−Y	−D	−V	−F	−E	−L	−S	−N	−M	+iP	+iT	+iR	+iQ	+iX	+iU	+iZ	+iY
iE	+iE	+iF	−iV	−iD	−iM	−iN	+iS	+iL	+Q	+R	−T	−P	−Y	−Z	+U	+X	−E	−F	+V	+D	+M	+N	−S	−L	−iQ	−iR	+iT	+iP	+iY	+iZ	−iU	−iX
iF	+iF	−iE	−iD	+iV	+iN	−iM	−iL	+iS	+R	−Q	−P	+T	+Z	−Y	−X	+U	−F	+E	+D	−V	−N	+M	+L	−S	−iR	+iQ	+iP	−iT	−iZ	+iY	+iX	−iU
U	+U	+X	−Y	−Z	−T	−P	+Q	+R	−iS	−iL	+iM	+iN	+iV	+iD	−iE	−iF	+iU	+iX	−iY	−iZ	−iT	−iP	+iQ	+iR	−S	−L	+M	+N	+V	+D	−E	−F
X	+X	−U	−Z	+Y	+P	−T	−R	+Q	−iL	+iS	+iN	−iM	−iD	+iV	+iF	−iE	+iX	−iU	−iZ	+iY	+iP	−iT	−iR	+iQ	−L	+S	+N	−M	−D	+V	+F	−E
Y	+Y	−Z	+U	−X	−Q	+R	−T	+P	+iM	−iN	+iS	−iL	−iE	+iF	−iV	+iD	+iY	−iZ	+iU	−iX	−iQ	+iR	−iT	+iP	+M	−N	+S	−L	−E	+F	−V	+D
Z	+Z	+Y	+X	+U	+R	+Q	+P	+T	+iN	+iM	+iL	+iS	+iF	+iE	+iD	+iV	+iZ	+iY	+iX	+iU	+iR	+iQ	+iP	+iT	+N	+M	+L	+S	+F	+E	+D	+V
T	+T	−P	−Q	+R	+U	−X	−Y	+Z	−iV	+iD	+iE	−iF	−iS	+iL	+iM	−iN	+iT	−iP	−iQ	+iR	+iU	−iX	−iY	+iZ	−V	+D	+E	−F	−S	+L	+M	−N
P	+P	+T	−R	−Q	−X	−U	+Z	+Y	−iD	−iV	+iF	+iE	+iL	+iS	−iN	−iM	+iP	+iT	−iR	−iQ	−iX	−iU	+iZ	+iY	−D	−V	+F	+E	+L	+S	−N	−M
Q	+Q	+R	+T	+P	+Y	+Z	+U	+X	+iE	+iF	+iV	+iD	+iM	+iN	+iS	+iL	+iQ	+iR	+iT	+iP	+iY	+iZ	+iU	+iX	+E	+F	+V	+D	+M	+N	+S	+L
R	+R	−Q	+P	−T	−Z	+Y	−X	+U	+iF	−iE	+iD	−iV	−iN	+iM	−iL	+iS	+iR	−iQ	+iP	−iT	−iZ	+iY	−iX	+iU	+F	−E	+D	−V	−N	+M	−L	+S

Table 13. Labels and Cayley table for $\mathbb{T}$ basis elements

Ref no.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Label	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
σ_o	$+\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$
σ_ι	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\sigma_\jmath$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
σ_κ	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
λ_o	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
λ_ι	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\lambda_\jmath$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
λ_κ	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
μ_o	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
μ_ι	$+\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\mu_\jmath$	$+\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
μ_κ	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
ν_o	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
ν_ι	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\nu_\jmath$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$
ν_κ	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$
α_o	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
α_ι	$+\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$
$\alpha_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$
α_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$
β_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$
β_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$
$\beta_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$
β_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$
γ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$
γ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$
$\gamma_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$
γ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$
δ_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$
δ_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$
$\delta_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$
δ_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$

8. Basis of notation used for unit elements of $\mathbb{T}$ and its Cayley table

Conventional notation for unit elements of $\mathbb{T}$ would use the numbers 0 to 31, standing alone or subscripted as e_0 to e_{31}. In this paper a different form of notation is used based on a modified Moufang loop construction for $\mathbb{T}$.

8.1. Moufang Loop construction for octonions

For Moufang loop construction of octonions, based on quaternion pairs, a dis-association operator, ω, is assigned to the second pair, and a product rule which generates octonions can be defined:
For quaternion pair $(p, \omega p')\times$ quaternion pair $(q, \omega q')$:
$p.q = (pq)$
$p.\omega q' = \omega(p^{-1}q')$
$\omega p'.q = \omega(qp')$
$\omega p'.\omega q' = -(q'p'^{-1})$

8.2. Modified Moufang Loop construction for trigintaduonions

The Moufang loop construction for octonions uses one dis-association operator. An identity operator, σ, and a set of seven dis-association operators, $\lambda, \mu, \nu, \alpha, \beta, \gamma, \delta$, can be applied to quaternions to define trigintaduonions, where products for two quaternions: p and q, are constructed in accordance with table 14.

Table 14. Multiplication procedures for non-associative components

	σq	λq	μq	νq	αq	βq	γq	δq
σp	$+\sigma pq$	$+\lambda qp$	$+\mu qp$	$+\nu qp^{-1}$	$+\alpha qp$	$+\beta qp^{-1}$	$+\gamma qp^{-1}$	$+\delta qp$
λp	$+\lambda pq^{-1}$	$-\sigma q^{-1}p$	$+\nu pq$	$-\mu p^{-1}q$	$+\beta pq$	$-\alpha p^{-1}q$	$-\delta pq$	$+\gamma p^{-1}q$
μp	$+\mu pq^{-1}$	$-\nu qp$	$-\sigma q^{-1}p$	$+\lambda qp^{-1}$	$+\gamma pq$	$+\delta qp$	$-\alpha p^{-1}q$	$-\beta qp^{-1}$
νp	$+\nu pq$	$+\mu q^{-1}p$	$-\lambda pq^{-1}$	$-\sigma p^{-1}q$	$+\delta pq^{-1}$	$-\gamma q^{-1}p$	$+\beta pq^{-1}$	$-\alpha q^{-1}p$
αp	$+\alpha pq^{-1}$	$-\beta qp$	$-\gamma qp$	$-\delta qp^{-1}$	$-\sigma q^{-1}p$	$+\lambda qp^{-1}$	$+\mu qp^{-1}$	$+\nu qp$
βp	$+\beta pq$	$+\alpha q^{-1}p$	$-\delta pq$	$+\gamma p^{-1}q$	$-\lambda pq^{-1}$	$-\sigma p^{-1}q$	$-\nu pq$	$+\mu p^{-1}q$
γp	$+\gamma pq$	$+\delta qp$	$+\alpha q^{-1}p$	$-\beta qp^{-1}$	$-\mu pq^{-1}$	$+\nu qp$	$-\sigma p^{-1}q$	$-\lambda qp^{-1}$
δp	$+\delta pq^{-1}$	$-\gamma q^{-1}p$	$+\beta pq^{-1}$	$+\alpha p^{-1}q$	$-\nu pq$	$-\mu q^{-1}p$	$+\lambda pq^{-1}$	$-\sigma q^{-1}p$

Unit trigintaduonions have been labeled using: $\sigma, \lambda, \mu, \nu, \alpha, \beta, \gamma, \delta$, combined with subscripts: $_o$, $_\iota$, $_\jmath$, $_\kappa$. Each label denotes a unit quaternion and a dis-association operator defining a multiplication procedure. Their Cayley table is shown in table 13.

If, instead of being applied to the quaternions, these dis-association operators and multiplication procedures are applied to the reals, they generate the octonions. If applied to complex numbers, they generate the sedenions. However, when applied to the octonions, they do not generate the sexagintaquatronions. It is possible to apply the dis-association operators to other algebras, such as Clifford algebras, generating different loops. This was described for $Cl_{1,4}(R)$ in a previous paper by the author[35]. Their application to $Cl_6(C) \cong M_8(C)$ will be described in a further paper by this author.

9. Unit elements of $\mathbb{U}$

Table 15. Unit elements of $\mathbb{U}$

σ_oS	σ_oiS	σ_oL	σ_oM	σ_oN	σ_oiL	σ_oiM	σ_oiN	σ_oiV	σ_oV	σ_oU	σ_oiU	σ_oT	σ_oiT	σ_oiD	σ_oiE	σ_oiF	σ_oD	σ_oE	σ_oF	σ_oX	σ_oY	σ_oZ	σ_oiX	σ_oiY	σ_oiZ	σ_oP	σ_oQ	σ_oR	σ_oiP	σ_oiQ	σ_oiR
α_oS	α_oiS	α_oL	α_oM	α_oN	α_oiL	α_oiM	α_oiN	α_oiV	α_oV	α_oU	α_oiU	α_oT	α_oiT	α_oiD	α_oiE	α_oiF	α_oD	α_oE	α_oF	α_oX	α_oY	α_oZ	α_oiX	α_oiY	α_oiZ	α_oP	α_oQ	α_oR	α_oiP	α_oiQ	α_oiR
$\sigma_\iota S$	$\sigma_\iota iS$	$\sigma_\iota L$	$\sigma_\iota M$	$\sigma_\iota N$	$\sigma_\iota iL$	$\sigma_\iota iM$	$\sigma_\iota iN$	$\sigma_\iota iV$	$\sigma_\iota V$	$\sigma_\iota U$	$\sigma_\iota iU$	$\sigma_\iota T$	$\sigma_\iota iT$	$\sigma_\iota iD$	$\sigma_\iota iE$	$\sigma_\iota iF$	$\sigma_\iota D$	$\sigma_\iota E$	$\sigma_\iota F$	$\sigma_\iota X$	$\sigma_\iota Y$	$\sigma_\iota Z$	$\sigma_\iota iX$	$\sigma_\iota iY$	$\sigma_\iota iZ$	$\sigma_\iota P$	$\sigma_\iota Q$	$\sigma_\iota R$	$\sigma_\iota iP$	$\sigma_\iota iQ$	$\sigma_\iota iR$
$\sigma_\jmath S$	$\sigma_\jmath iS$	$\sigma_\jmath L$	$\sigma_\jmath M$	$\sigma_\jmath N$	$\sigma_\jmath iL$	$\sigma_\jmath iM$	$\sigma_\jmath iN$	$\sigma_\jmath iV$	$\sigma_\jmath V$	$\sigma_\jmath U$	$\sigma_\jmath iU$	$\sigma_\jmath T$	$\sigma_\jmath iT$	$\sigma_\jmath iD$	$\sigma_\jmath iE$	$\sigma_\jmath iF$	$\sigma_\jmath D$	$\sigma_\jmath E$	$\sigma_\jmath F$	$\sigma_\jmath X$	$\sigma_\jmath Y$	$\sigma_\jmath Z$	$\sigma_\jmath iX$	$\sigma_\jmath iY$	$\sigma_\jmath iZ$	$\sigma_\jmath P$	$\sigma_\jmath Q$	$\sigma_\jmath R$	$\sigma_\jmath iP$	$\sigma_\jmath iQ$	$\sigma_\jmath iR$
$\sigma_\kappa S$	$\sigma_\kappa iS$	$\sigma_\kappa L$	$\sigma_\kappa M$	$\sigma_\kappa N$	$\sigma_\kappa iL$	$\sigma_\kappa iM$	$\sigma_\kappa iN$	$\sigma_\kappa iV$	$\sigma_\kappa V$	$\sigma_\kappa U$	$\sigma_\kappa iU$	$\sigma_\kappa T$	$\sigma_\kappa iT$	$\sigma_\kappa iD$	$\sigma_\kappa iE$	$\sigma_\kappa iF$	$\sigma_\kappa D$	$\sigma_\kappa E$	$\sigma_\kappa F$	$\sigma_\kappa X$	$\sigma_\kappa Y$	$\sigma_\kappa Z$	$\sigma_\kappa iX$	$\sigma_\kappa iY$	$\sigma_\kappa iZ$	$\sigma_\kappa P$	$\sigma_\kappa Q$	$\sigma_\kappa R$	$\sigma_\kappa iP$	$\sigma_\kappa iQ$	$\sigma_\kappa iR$
$\alpha_\iota S$	$\alpha_\iota iS$	$\alpha_\iota L$	$\alpha_\iota M$	$\alpha_\iota N$	$\alpha_\iota iL$	$\alpha_\iota iM$	$\alpha_\iota iN$	$\alpha_\iota iV$	$\alpha_\iota V$	$\alpha_\iota U$	$\alpha_\iota iU$	$\alpha_\iota T$	$\alpha_\iota iT$	$\alpha_\iota iD$	$\alpha_\iota iE$	$\alpha_\iota iF$	$\alpha_\iota D$	$\alpha_\iota E$	$\alpha_\iota F$	$\alpha_\iota X$	$\alpha_\iota Y$	$\alpha_\iota Z$	$\alpha_\iota iX$	$\alpha_\iota iY$	$\alpha_\iota iZ$	$\alpha_\iota P$	$\alpha_\iota Q$	$\alpha_\iota R$	$\alpha_\iota iP$	$\alpha_\iota iQ$	$\alpha_\iota iR$
$\alpha_\jmath S$	$\alpha_\jmath iS$	$\alpha_\jmath L$	$\alpha_\jmath M$	$\alpha_\jmath N$	$\alpha_\jmath iL$	$\alpha_\jmath iM$	$\alpha_\jmath iN$	$\alpha_\jmath iV$	$\alpha_\jmath V$	$\alpha_\jmath U$	$\alpha_\jmath iU$	$\alpha_\jmath T$	$\alpha_\jmath iT$	$\alpha_\jmath iD$	$\alpha_\jmath iE$	$\alpha_\jmath iF$	$\alpha_\jmath D$	$\alpha_\jmath E$	$\alpha_\jmath F$	$\alpha_\jmath X$	$\alpha_\jmath Y$	$\alpha_\jmath Z$	$\alpha_\jmath iX$	$\alpha_\jmath iY$	$\alpha_\jmath iZ$	$\alpha_\jmath P$	$\alpha_\jmath Q$	$\alpha_\jmath R$	$\alpha_\jmath iP$	$\alpha_\jmath iQ$	$\alpha_\jmath iR$
$\alpha_\kappa S$	$\alpha_\kappa iS$	$\alpha_\kappa L$	$\alpha_\kappa M$	$\alpha_\kappa N$	$\alpha_\kappa iL$	$\alpha_\kappa iM$	$\alpha_\kappa iN$	$\alpha_\kappa iV$	$\alpha_\kappa V$	$\alpha_\kappa U$	$\alpha_\kappa iU$	$\alpha_\kappa T$	$\alpha_\kappa iT$	$\alpha_\kappa iD$	$\alpha_\kappa iE$	$\alpha_\kappa iF$	$\alpha_\kappa D$	$\alpha_\kappa E$	$\alpha_\kappa F$	$\alpha_\kappa X$	$\alpha_\kappa Y$	$\alpha_\kappa Z$	$\alpha_\kappa iX$	$\alpha_\kappa iY$	$\alpha_\kappa iZ$	$\alpha_\kappa P$	$\alpha_\kappa Q$	$\alpha_\kappa R$	$\alpha_\kappa iP$	$\alpha_\kappa iQ$	$\alpha_\kappa iR$
β_oS	β_oiS	β_oL	β_oM	β_oN	β_oiL	β_oiM	β_oiN	β_oiV	β_oV	β_oU	β_oiU	β_oT	β_oiT	β_oiD	β_oiE	β_oiF	β_oD	β_oE	β_oF	β_oX	β_oY	β_oZ	β_oiX	β_oiY	β_oiZ	β_oP	β_oQ	β_oR	β_oiP	β_oiQ	β_oiR
λ_oS	λ_oiS	λ_oL	λ_oM	λ_oN	λ_oiL	λ_oiM	λ_oiN	λ_oiV	λ_oV	λ_oU	λ_oiU	λ_oT	λ_oiT	λ_oiD	λ_oiE	λ_oiF	λ_oD	λ_oE	λ_oF	λ_oX	λ_oY	λ_oZ	λ_oiX	λ_oiY	λ_oiZ	λ_oP	λ_oQ	λ_oR	λ_oiP	λ_oiQ	λ_oiR
γ_oS	γ_oiS	γ_oL	γ_oM	γ_oN	γ_oiL	γ_oiM	γ_oiN	γ_oiV	γ_oV	γ_oU	γ_oiU	γ_oT	γ_oiT	γ_oiD	γ_oiE	γ_oiF	γ_oD	γ_oE	γ_oF	γ_oX	γ_oY	γ_oZ	γ_oiX	γ_oiY	γ_oiZ	γ_oP	γ_oQ	γ_oR	γ_oiP	γ_oiQ	γ_oiR
μ_oS	μ_oiS	μ_oL	μ_oM	μ_oN	μ_oiL	μ_oiM	μ_oiN	μ_oiV	μ_oV	μ_oU	μ_oiU	μ_oT	μ_oiT	μ_oiD	μ_oiE	μ_oiF	μ_oD	μ_oE	μ_oF	μ_oX	μ_oY	μ_oZ	μ_oiX	μ_oiY	μ_oiZ	μ_oP	μ_oQ	μ_oR	μ_oiP	μ_oiQ	μ_oiR
δ_oS	δ_oiS	δ_oL	δ_oM	δ_oN	δ_oiL	δ_oiM	δ_oiN	δ_oiV	δ_oV	δ_oU	δ_oiU	δ_oT	δ_oiT	δ_oiD	δ_oiE	δ_oiF	δ_oD	δ_oE	δ_oF	δ_oX	δ_oY	δ_oZ	δ_oiX	δ_oiY	δ_oiZ	δ_oP	δ_oQ	δ_oR	δ_oiP	δ_oiQ	δ_oiR
ν_oS	ν_oiS	ν_oL	ν_oM	ν_oN	ν_oiL	ν_oiM	ν_oiN	ν_oiV	ν_oV	ν_oU	ν_oiU	ν_oT	ν_oiT	ν_oiD	ν_oiE	ν_oiF	ν_oD	ν_oE	ν_oF	ν_oX	ν_oY	ν_oZ	ν_oiX	ν_oiY	ν_oiZ	ν_oP	ν_oQ	ν_oR	ν_oiP	ν_oiQ	ν_oiR
$\beta_\iota S$	$\beta_\iota iS$	$\beta_\iota L$	$\beta_\iota M$	$\beta_\iota N$	$\beta_\iota iL$	$\beta_\iota iM$	$\beta_\iota iN$	$\beta_\iota iV$	$\beta_\iota V$	$\beta_\iota U$	$\beta_\iota iU$	$\beta_\iota T$	$\beta_\iota iT$	$\beta_\iota iD$	$\beta_\iota iE$	$\beta_\iota iF$	$\beta_\iota D$	$\beta_\iota E$	$\beta_\iota F$	$\beta_\iota X$	$\beta_\iota Y$	$\beta_\iota Z$	$\beta_\iota iX$	$\beta_\iota iY$	$\beta_\iota iZ$	$\beta_\iota P$	$\beta_\iota Q$	$\beta_\iota R$	$\beta_\iota iP$	$\beta_\iota iQ$	$\beta_\iota iR$
$\beta_\jmath S$	$\beta_\jmath iS$	$\beta_\jmath L$	$\beta_\jmath M$	$\beta_\jmath N$	$\beta_\jmath iL$	$\beta_\jmath iM$	$\beta_\jmath iN$	$\beta_\jmath iV$	$\beta_\jmath V$	$\beta_\jmath U$	$\beta_\jmath iU$	$\beta_\jmath T$	$\beta_\jmath iT$	$\beta_\jmath iD$	$\beta_\jmath iE$	$\beta_\jmath iF$	$\beta_\jmath D$	$\beta_\jmath E$	$\beta_\jmath F$	$\beta_\jmath X$	$\beta_\jmath Y$	$\beta_\jmath Z$	$\beta_\jmath iX$	$\beta_\jmath iY$	$\beta_\jmath iZ$	$\beta_\jmath P$	$\beta_\jmath Q$	$\beta_\jmath R$	$\beta_\jmath iP$	$\beta_\jmath iQ$	$\beta_\jmath iR$
$\beta_\kappa S$	$\beta_\kappa iS$	$\beta_\kappa L$	$\beta_\kappa M$	$\beta_\kappa N$	$\beta_\kappa iL$	$\beta_\kappa iM$	$\beta_\kappa iN$	$\beta_\kappa iV$	$\beta_\kappa V$	$\beta_\kappa U$	$\beta_\kappa iU$	$\beta_\kappa T$	$\beta_\kappa iT$	$\beta_\kappa iD$	$\beta_\kappa iE$	$\beta_\kappa iF$	$\beta_\kappa D$	$\beta_\kappa E$	$\beta_\kappa F$	$\beta_\kappa X$	$\beta_\kappa Y$	$\beta_\kappa Z$	$\beta_\kappa iX$	$\beta_\kappa iY$	$\beta_\kappa iZ$	$\beta_\kappa P$	$\beta_\kappa Q$	$\beta_\kappa R$	$\beta_\kappa iP$	$\beta_\kappa iQ$	$\beta_\kappa iR$
$\lambda_\iota S$	$\lambda_\iota iS$	$\lambda_\iota L$	$\lambda_\iota M$	$\lambda_\iota N$	$\lambda_\iota iL$	$\lambda_\iota iM$	$\lambda_\iota iN$	$\lambda_\iota iV$	$\lambda_\iota V$	$\lambda_\iota U$	$\lambda_\iota iU$	$\lambda_\iota T$	$\lambda_\iota iT$	$\lambda_\iota iD$	$\lambda_\iota iE$	$\lambda_\iota iF$	$\lambda_\iota D$	$\lambda_\iota E$	$\lambda_\iota F$	$\lambda_\iota X$	$\lambda_\iota Y$	$\lambda_\iota Z$	$\lambda_\iota iX$	$\lambda_\iota iY$	$\lambda_\iota iZ$	$\lambda_\iota P$	$\lambda_\iota Q$	$\lambda_\iota R$	$\lambda_\iota iP$	$\lambda_\iota iQ$	$\lambda_\iota iR$
$\lambda_\jmath S$	$\lambda_\jmath iS$	$\lambda_\jmath L$	$\lambda_\jmath M$	$\lambda_\jmath N$	$\lambda_\jmath iL$	$\lambda_\jmath iM$	$\lambda_\jmath iN$	$\lambda_\jmath iV$	$\lambda_\jmath V$	$\lambda_\jmath U$	$\lambda_\jmath iU$	$\lambda_\jmath T$	$\lambda_\jmath iT$	$\lambda_\jmath iD$	$\lambda_\jmath iE$	$\lambda_\jmath iF$	$\lambda_\jmath D$	$\lambda_\jmath E$	$\lambda_\jmath F$	$\lambda_\jmath X$	$\lambda_\jmath Y$	$\lambda_\jmath Z$	$\lambda_\jmath iX$	$\lambda_\jmath iY$	$\lambda_\jmath iZ$	$\lambda_\jmath P$	$\lambda_\jmath Q$	$\lambda_\jmath R$	$\lambda_\jmath iP$	$\lambda_\jmath iQ$	$\lambda_\jmath iR$
$\lambda_\kappa S$	$\lambda_\kappa iS$	$\lambda_\kappa L$	$\lambda_\kappa M$	$\lambda_\kappa N$	$\lambda_\kappa iL$	$\lambda_\kappa iM$	$\lambda_\kappa iN$	$\lambda_\kappa iV$	$\lambda_\kappa V$	$\lambda_\kappa U$	$\lambda_\kappa iU$	$\lambda_\kappa T$	$\lambda_\kappa iT$	$\lambda_\kappa iD$	$\lambda_\kappa iE$	$\lambda_\kappa iF$	$\lambda_\kappa D$	$\lambda_\kappa E$	$\lambda_\kappa F$	$\lambda_\kappa X$	$\lambda_\kappa Y$	$\lambda_\kappa Z$	$\lambda_\kappa iX$	$\lambda_\kappa iY$	$\lambda_\kappa iZ$	$\lambda_\kappa P$	$\lambda_\kappa Q$	$\lambda_\kappa R$	$\lambda_\kappa iP$	$\lambda_\kappa iQ$	$\lambda_\kappa iR$
$\gamma_\iota S$	$\gamma_\iota iS$	$\gamma_\iota L$	$\gamma_\iota M$	$\gamma_\iota N$	$\gamma_\iota iL$	$\gamma_\iota iM$	$\gamma_\iota iN$	$\gamma_\iota iV$	$\gamma_\iota V$	$\gamma_\iota U$	$\gamma_\iota iU$	$\gamma_\iota T$	$\gamma_\iota iT$	$\gamma_\iota iD$	$\gamma_\iota iE$	$\gamma_\iota iF$	$\gamma_\iota D$	$\gamma_\iota E$	$\gamma_\iota F$	$\gamma_\iota X$	$\gamma_\iota Y$	$\gamma_\iota Z$	$\gamma_\iota iX$	$\gamma_\iota iY$	$\gamma_\iota iZ$	$\gamma_\iota P$	$\gamma_\iota Q$	$\gamma_\iota R$	$\gamma_\iota iP$	$\gamma_\iota iQ$	$\gamma_\iota iR$
$\gamma_\jmath S$	$\gamma_\jmath iS$	$\gamma_\jmath L$	$\gamma_\jmath M$	$\gamma_\jmath N$	$\gamma_\jmath iL$	$\gamma_\jmath iM$	$\gamma_\jmath iN$	$\gamma_\jmath iV$	$\gamma_\jmath V$	$\gamma_\jmath U$	$\gamma_\jmath iU$	$\gamma_\jmath T$	$\gamma_\jmath iT$	$\gamma_\jmath iD$	$\gamma_\jmath iE$	$\gamma_\jmath iF$	$\gamma_\jmath D$	$\gamma_\jmath E$	$\gamma_\jmath F$	$\gamma_\jmath X$	$\gamma_\jmath Y$	$\gamma_\jmath Z$	$\gamma_\jmath iX$	$\gamma_\jmath iY$	$\gamma_\jmath iZ$	$\gamma_\jmath P$	$\gamma_\jmath Q$	$\gamma_\jmath R$	$\gamma_\jmath iP$	$\gamma_\jmath iQ$	$\gamma_\jmath iR$
$\gamma_\kappa S$	$\gamma_\kappa iS$	$\gamma_\kappa L$	$\gamma_\kappa M$	$\gamma_\kappa N$	$\gamma_\kappa iL$	$\gamma_\kappa iM$	$\gamma_\kappa iN$	$\gamma_\kappa iV$	$\gamma_\kappa V$	$\gamma_\kappa U$	$\gamma_\kappa iU$	$\gamma_\kappa T$	$\gamma_\kappa iT$	$\gamma_\kappa iD$	$\gamma_\kappa iE$	$\gamma_\kappa iF$	$\gamma_\kappa D$	$\gamma_\kappa E$	$\gamma_\kappa F$	$\gamma_\kappa X$	$\gamma_\kappa Y$	$\gamma_\kappa Z$	$\gamma_\kappa iX$	$\gamma_\kappa iY$	$\gamma_\kappa iZ$	$\gamma_\kappa P$	$\gamma_\kappa Q$	$\gamma_\kappa R$	$\gamma_\kappa iP$	$\gamma_\kappa iQ$	$\gamma_\kappa iR$
$\mu_\iota S$	$\mu_\iota iS$	$\mu_\iota L$	$\mu_\iota M$	$\mu_\iota N$	$\mu_\iota iL$	$\mu_\iota iM$	$\mu_\iota iN$	$\mu_\iota iV$	$\mu_\iota V$	$\mu_\iota U$	$\mu_\iota iU$	$\mu_\iota T$	$\mu_\iota iT$	$\mu_\iota iD$	$\mu_\iota iE$	$\mu_\iota iF$	$\mu_\iota D$	$\mu_\iota E$	$\mu_\iota F$	$\mu_\iota X$	$\mu_\iota Y$	$\mu_\iota Z$	$\mu_\iota iX$	$\mu_\iota iY$	$\mu_\iota iZ$	$\mu_\iota P$	$\mu_\iota Q$	$\mu_\iota R$	$\mu_\iota iP$	$\mu_\iota iQ$	$\mu_\iota iR$
$\mu_\jmath S$	$\mu_\jmath iS$	$\mu_\jmath L$	$\mu_\jmath M$	$\mu_\jmath N$	$\mu_\jmath iL$	$\mu_\jmath iM$	$\mu_\jmath iN$	$\mu_\jmath iV$	$\mu_\jmath V$	$\mu_\jmath U$	$\mu_\jmath iU$	$\mu_\jmath T$	$\mu_\jmath iT$	$\mu_\jmath iD$	$\mu_\jmath iE$	$\mu_\jmath iF$	$\mu_\jmath D$	$\mu_\jmath E$	$\mu_\jmath F$	$\mu_\jmath X$	$\mu_\jmath Y$	$\mu_\jmath Z$	$\mu_\jmath iX$	$\mu_\jmath iY$	$\mu_\jmath iZ$	$\mu_\jmath P$	$\mu_\jmath Q$	$\mu_\jmath R$	$\mu_\jmath iP$	$\mu_\jmath iQ$	$\mu_\jmath iR$
$\mu_\kappa S$	$\mu_\kappa iS$	$\mu_\kappa L$	$\mu_\kappa M$	$\mu_\kappa N$	$\mu_\kappa iL$	$\mu_\kappa iM$	$\mu_\kappa iN$	$\mu_\kappa iV$	$\mu_\kappa V$	$\mu_\kappa U$	$\mu_\kappa iU$	$\mu_\kappa T$	$\mu_\kappa iT$	$\mu_\kappa iD$	$\mu_\kappa iE$	$\mu_\kappa iF$	$\mu_\kappa D$	$\mu_\kappa E$	$\mu_\kappa F$	$\mu_\kappa X$	$\mu_\kappa Y$	$\mu_\kappa Z$	$\mu_\kappa iX$	$\mu_\kappa iY$	$\mu_\kappa iZ$	$\mu_\kappa P$	$\mu_\kappa Q$	$\mu_\kappa R$	$\mu_\kappa iP$	$\mu_\kappa iQ$	$\mu_\kappa iR$
$\delta_\iota S$	$\delta_\iota iS$	$\delta_\iota L$	$\delta_\iota M$	$\delta_\iota N$	$\delta_\iota iL$	$\delta_\iota iM$	$\delta_\iota iN$	$\delta_\iota iV$	$\delta_\iota V$	$\delta_\iota U$	$\delta_\iota iU$	$\delta_\iota T$	$\delta_\iota iT$	$\delta_\iota iD$	$\delta_\iota iE$	$\delta_\iota iF$	$\delta_\iota D$	$\delta_\iota E$	$\delta_\iota F$	$\delta_\iota X$	$\delta_\iota Y$	$\delta_\iota Z$	$\delta_\iota iX$	$\delta_\iota iY$	$\delta_\iota iZ$	$\delta_\iota P$	$\delta_\iota Q$	$\delta_\iota R$	$\delta_\iota iP$	$\delta_\iota iQ$	$\delta_\iota iR$
$\delta_\jmath S$	$\delta_\jmath iS$	$\delta_\jmath L$	$\delta_\jmath M$	$\delta_\jmath N$	$\delta_\jmath iL$	$\delta_\jmath iM$	$\delta_\jmath iN$	$\delta_\jmath iV$	$\delta_\jmath V$	$\delta_\jmath U$	$\delta_\jmath iU$	$\delta_\jmath T$	$\delta_\jmath iT$	$\delta_\jmath iD$	$\delta_\jmath iE$	$\delta_\jmath iF$	$\delta_\jmath D$	$\delta_\jmath E$	$\delta_\jmath F$	$\delta_\jmath X$	$\delta_\jmath Y$	$\delta_\jmath Z$	$\delta_\jmath iX$	$\delta_\jmath iY$	$\delta_\jmath iZ$	$\delta_\jmath P$	$\delta_\jmath Q$	$\delta_\jmath R$	$\delta_\jmath iP$	$\delta_\jmath iQ$	$\delta_\jmath iR$
$\delta_\kappa S$	$\delta_\kappa iS$	$\delta_\kappa L$	$\delta_\kappa M$	$\delta_\kappa N$	$\delta_\kappa iL$	$\delta_\kappa iM$	$\delta_\kappa iN$	$\delta_\kappa iV$	$\delta_\kappa V$	$\delta_\kappa U$	$\delta_\kappa iU$	$\delta_\kappa T$	$\delta_\kappa iT$	$\delta_\kappa iD$	$\delta_\kappa iE$	$\delta_\kappa iF$	$\delta_\kappa D$	$\delta_\kappa E$	$\delta_\kappa F$	$\delta_\kappa X$	$\delta_\kappa Y$	$\delta_\kappa Z$	$\delta_\kappa iX$	$\delta_\kappa iY$	$\delta_\kappa iZ$	$\delta_\kappa P$	$\delta_\kappa Q$	$\delta_\kappa R$	$\delta_\kappa iP$	$\delta_\kappa iQ$	$\delta_\kappa iR$
$\nu_\iota S$	$\nu_\iota iS$	$\nu_\iota L$	$\nu_\iota M$	$\nu_\iota N$	$\nu_\iota iL$	$\nu_\iota iM$	$\nu_\iota iN$	$\nu_\iota iV$	$\nu_\iota V$	$\nu_\iota U$	$\nu_\iota iU$	$\nu_\iota T$	$\nu_\iota iT$	$\nu_\iota iD$	$\nu_\iota iE$	$\nu_\iota iF$	$\nu_\iota D$	$\nu_\iota E$	$\nu_\iota F$	$\nu_\iota X$	$\nu_\iota Y$	$\nu_\iota Z$	$\nu_\iota iX$	$\nu_\iota iY$	$\nu_\iota iZ$	$\nu_\iota P$	$\nu_\iota Q$	$\nu_\iota R$	$\nu_\iota iP$	$\nu_\iota iQ$	$\nu_\iota iR$
$\nu_\jmath S$	$\nu_\jmath iS$	$\nu_\jmath L$	$\nu_\jmath M$	$\nu_\jmath N$	$\nu_\jmath iL$	$\nu_\jmath iM$	$\nu_\jmath iN$	$\nu_\jmath iV$	$\nu_\jmath V$	$\nu_\jmath U$	$\nu_\jmath iU$	$\nu_\jmath T$	$\nu_\jmath iT$	$\nu_\jmath iD$	$\nu_\jmath iE$	$\nu_\jmath iF$	$\nu_\jmath D$	$\nu_\jmath E$	$\nu_\jmath F$	$\nu_\jmath X$	$\nu_\jmath Y$	$\nu_\jmath Z$	$\nu_\jmath iX$	$\nu_\jmath iY$	$\nu_\jmath iZ$	$\nu_\jmath P$	$\nu_\jmath Q$	$\nu_\jmath R$	$\nu_\jmath iP$	$\nu_\jmath iQ$	$\nu_\jmath iR$
$\nu_\kappa S$	$\nu_\kappa iS$	$\nu_\kappa L$	$\nu_\kappa M$	$\nu_\kappa N$	$\nu_\kappa iL$	$\nu_\kappa iM$	$\nu_\kappa iN$	$\nu_\kappa iV$	$\nu_\kappa V$	$\nu_\kappa U$	$\nu_\kappa iU$	$\nu_\kappa T$	$\nu_\kappa iT$	$\nu_\kappa iD$	$\nu_\kappa iE$	$\nu_\kappa iF$	$\nu_\kappa D$	$\nu_\kappa E$	$\nu_\kappa F$	$\nu_\kappa X$	$\nu_\kappa Y$	$\nu_\kappa Z$	$\nu_\kappa iX$	$\nu_\kappa iY$	$\nu_\kappa iZ$	$\nu_\kappa P$	$\nu_\kappa Q$	$\nu_\kappa R$	$\nu_\kappa iP$	$\nu_\kappa iQ$	$\nu_\kappa iR$

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