General Schwarzschild Solution in Rindler Space-time

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ABSTRACT

We studied only about Rindler space-time in long time. This knowledge lead that we can combine Schwarzschild solution and an accelerated frame. Our article's aim is that we find the general Schwarzschild solution include an uniformly accelerated frame.

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1. Introduction

In the general relativity theory, Schwarzschild solution is very important solution. In many articles, researchers study this issue in the deep condition. We studied only about Rindler space-time in long time. We think that we combine Schwarzschild solution and an accelerated frame.

Our article's aim is that we find the general Schwarzschild solution with an uniformly accelerated frame.

In the Schwarzschild solution, if mass is zero, it does Minkowski solution. We think if mass is zero, general Schwarzschild solution is an uniformly accelerated frame.

At first, an uniformly accelerated frame is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c}) \tag{1}$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \tag{2}$$

The condition is that it treats the short accelerated time.

$$\xi^0 = \varepsilon_0 \quad , \quad |\varepsilon_0| << 0 \tag{3}$$

Therefore, Eq(1),Eq(2) are in the condition

$$x\,|_{\xi^0=\varepsilon_0} = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\varepsilon_0}{c}) - \frac{c^2}{a_0} \approx \xi^1 \tag{4}$$

$$ct\,|_{\xi^0=\varepsilon_0} = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\varepsilon_0}{c}) \approx (\frac{c^2}{a_0} + \xi^1)\frac{a_0}{c}\varepsilon_0 \approx c(1+\frac{a_0}{c^2}x)\varepsilon_0 \tag{5}$$

The differential transformation is in the condition.

$$\sinh x \approx x \quad , \quad \cosh x \approx 1$$

$$\begin{aligned} cdt\,|_{\xi^0=\varepsilon_0} &= d\xi^1 \sinh(\frac{a_0\varepsilon_0}{c}) + (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\varepsilon_0}{c})\frac{a_0}{c}d\xi^0 \\ &\approx d\xi^1\frac{a_0}{c}\varepsilon_0 + (1+\frac{a_0}{c^2}\xi^1)cd\xi^0 \approx (1+\frac{a_0}{c^2}x)cd\xi^0 \end{aligned} \tag{6}$$

$$\begin{aligned} dx\,|_{\xi^0=\varepsilon_0} &= d\xi^1 \cosh(\frac{a_0\varepsilon_0}{c}) + (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\varepsilon_0}{c})\frac{a_0}{c}d\xi^0 \\ &\approx d\xi^1 + (1+\frac{a_0}{c^2}x)a_0\varepsilon_0 d\xi^0 \approx d\xi^1 \end{aligned} \tag{7}$$

2. General Schwarzschild solution with an accelerated frame

Second, The Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{8}$$

During the accelerated frame's time $\xi^0 = \varepsilon_0$, the Schwarzschild solution is approximately the general Schwarzschild solution with the accelerated frame.

According to the condition Eq(3),

$$r^2 = x^2 + y^2 + z^2 \approx (\xi^1)^2 + (\xi^2)^2 + (\xi^3)^2 = r_\xi{}^2,\ \theta \approx \theta_\xi, \phi \approx \phi_\xi \tag{9}$$

Hence, according to Eq(6),Eq(7), the Schwarzschild solution is approximately the general Schwarzschild solution. Eq(8) is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$\approx (1-\frac{2GM}{r_\xi c^2})(1+\frac{a_0 x}{c^2})(d\xi^0)^2 - \frac{1}{c^2}[\frac{dr_\xi{}^2}{1-\frac{2GM}{r_\xi c^2}} + r_\xi{}^2(d\theta_\xi{}^2 + \sin^2\theta_\xi d\phi_\xi{}^2)] \tag{10}$$

If mass $M = 0$, the general Schwarzschild solution is approximately an accelerated frame.

$$d\tau^2 \approx (1+\frac{a_0 x}{c^2})(d\xi^0)^2 - \frac{1}{c^2}[dr_\xi{}^2 + r_\xi{}^2(d\theta_\xi{}^2 + \sin^2\theta_\xi d\phi_\xi{}^2)]$$

$$= (1+\frac{a_0}{c^2}x)(d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] \tag{11}$$

In this case, in flat space-time, because we need not the condition,Eq(3), in Eq(11), we can replace the coordinate x with the coordinate ξ^1.

The general Schwarzschild solution is

$$d\tau^2 \approx (1-\frac{2GM}{r_\xi c^2})(1+\frac{a_0 x}{c^2})(d\xi^0)^2 - \frac{1}{c^2}[\frac{dr_\xi{}^2}{1-\frac{2GM}{r_\xi c^2}} + r_\xi{}^2(d\theta_\xi{}^2 + \sin^2\theta_\xi d\phi_\xi{}^2)] \tag{12}$$

Einstein gravity field equation is

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br_\xi} + \frac{A'^2}{4AB} = 0 \tag{13}$$

$$R_{rr}=\frac{A''}{2A}-\frac{A'^2}{4A^2}-\frac{A'B'}{4AB}-\frac{B'}{Br_\xi}=0 \qquad (14)$$

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{r_\xi B'}{2B^2}+\frac{r_\xi A'}{2AB}=0 \qquad (15)$$

$R_{\phi\phi}=\sin^2\theta_\xi R_{\theta\theta}$, otherwise $R_{\mu\upsilon}=0$

In this time,

$$-g_{00}=A=(1-\frac{2GM}{r_\xi c^2})(1+\frac{a_0}{c^2}x),\quad g_{11}=B=\frac{1}{1-\frac{2GM}{r_\xi c^2}}$$

$$g_{22}=r_\xi{}^2\ ,\quad g_{33}=r_\xi{}^2\sin^2\theta_\xi \qquad (16)$$

If we calculate Eq(16),

$$\frac{\partial x}{\partial r_\xi}=0,$$

$$A'=\frac{2GM}{r_\xi{}^2c^2}(1+\frac{a_0}{c^2}x)^2,\quad A''=-\frac{4GM}{r_\xi{}^3c^2}(1+\frac{a_0}{c^2}x)^2$$

$$B'=-\frac{2GM}{r_\xi{}^2c^2}\frac{1}{(1-\frac{2GM}{r_\xi c^2})^2},\quad B''=\frac{4GM}{r_\xi{}^3c^2}\frac{1}{(1-\frac{2GM}{r_\xi c^2})^3} \qquad (17)$$

Eq(16), Eq(17) satisfy Einstein gravity filed Equation, Eq(13),Eq(14),Eq(15). Eq(12) is perfect solution in vacuum.

The acceleration of the general Schwarzschild solution is

$$g_{00}=-(1-\frac{2GM}{r_\xi c^2})(1+\frac{a_0}{c^2}x)^2,\ \frac{\partial x}{\partial\xi^1}\approx 1,(i,j,k)=\frac{\vec{r}}{r}$$

$$\vec{a}_\xi=\frac{d^2\vec{\xi}}{(d\xi^0)^2}\approx\frac{c^2}{2}\vec{\nabla}_\xi g_{00}=-\frac{GM}{r_\xi{}^3}(1+\frac{a_0}{c^2}x)^2\vec{r}_\xi-(1-\frac{2GM}{r_\xi c^2})(1+\frac{a_0}{c^2}x)a_0 i \qquad (18)$$

The first right term of Eq(18) is the gravity acceleration. The second right term of Eq(18) is the accelerated frame’s effect.

3. Conclusion

We find the General Schwarzschild solution with uniformly accelerated frame. We expand Schwarzschild solution. According to Eq(3),Eq(6), we think the solution describe the accelerated frame straightforwardly moves in a local gravity space.

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