Tarnowskie Góry, dn. 02.11.2018 r.

DISTRIBUTION OF THE PRIME NUMBERS

"The mathematical world (and not only) is not ruled by accident"

I dedicate this scientific work to my wonderful wife Karoline
for the huge persistence and patience towards my love for mathematics
and my little son Michael Antoni who from the beginning (the birth)
has become my inspiration and motivation
in searching of an answer to the question regarding the distribution of prime numbers.

1. INTRODUCTION:

In this scientific work I intend to prove that the placement of prime numbers in the space of natural numbers is not so much as regular, but strictly determined by the mathematically able to write order.

2. CLASSIFICATION OF ENTIRE NUMBERS:

Before I going on to the mathematical arguments, I will remind You of some of the important records of my first scientific work[1], and to be more precise - concerning the characteristics of the subsets of integers and the numerical model used. Modestly speaking, I introduced a military order that is close to me.

I created a numerical space model by circulating a numerical axis (a spiral) in the space R^3, i.e. using a mathematical curve called a helix (Fig.1.).

Therefore, for further research I will use the assumptions:

- the set of integers represents the basic helix in the space R^3, i.e. the helix, whose pitch is = 6, and the number of the set appear on the curve as every shift by angle π/3;
- according to this model, the figure 0 is the beginning of the curve, but it is not a prime number – for obvious reasons;
- the number 1 is the first unique prime number[2] and constitutes the second expression of the sequence - the helix determining the set of integers;

[1] "Filtration of prime numbers by means of numerical multiples helices" D. Gołofit, Tarnowskie Góry, 7/8/18.
[2] The first prime number was obtained by deriving the module from the number -1.

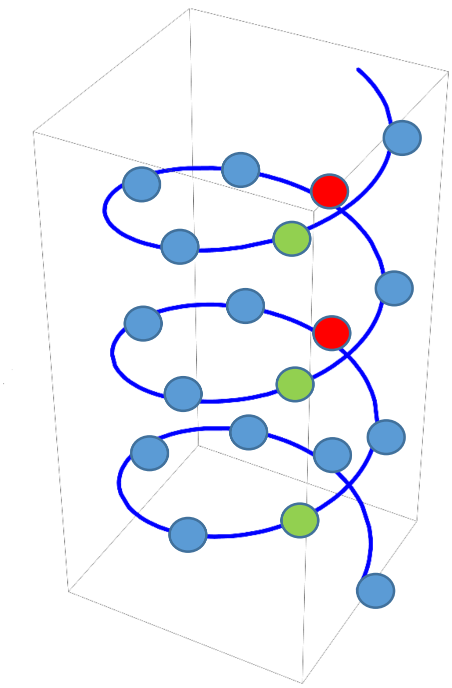

Fig.1. Mathematical curve - helix. Model of the set of integers in the spaceR^3.

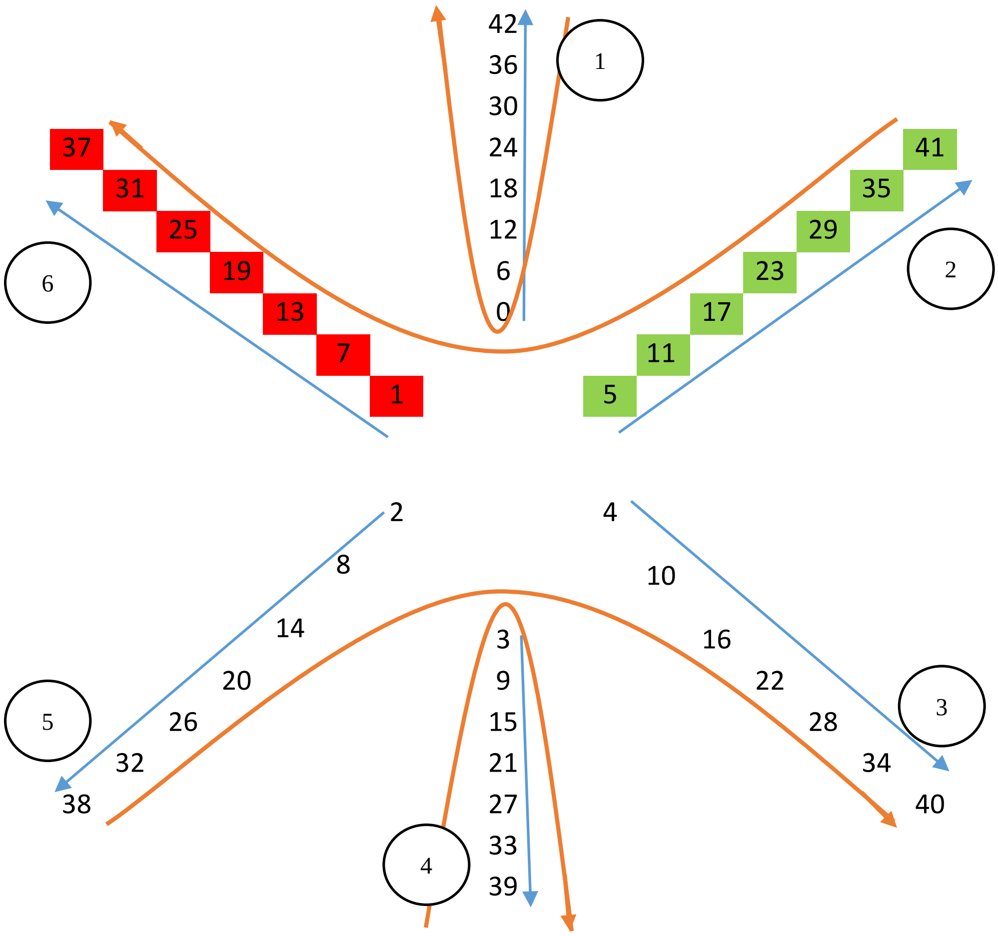

Fig.2. A perspective of a helix that creates a tunnel in the digital space (presentation of six binormal of this helix - forming a cylinder - a tunnel).

I noticed that each of the resulting subsets of numbers is characterized by different and specific (read: exceptional) features;

Example:

- the first subset contains only even numbers divided by the number 2 and 3;
- the second and sixth subsets are just prime numbers "proper" and prime numbers "improper" (so deliberately named by me and constituting the product of two prime numbers proper);
- the fourth subset is only the odd numbers being a multiple of the number 3;
- the third and fifth subsets are only even numbers indivisible by the number 3.

So I stated unequivocally that:

- this division, without any doubt, indicates that the prime numbers are hidden only in the second and sixth subsets;
- between this subsets there is the fifth subset **who posses only the even numbers divided by 2 and 3;**
- the basic helix model narrowed the set of integers three times, i.e. $C/3$ (two out of six subsets contain prime numbers) - intended for further research in the search for dependencies between prime numbers.

After a deeper analysis of the above statements, I noticed that **the sixth subset of integers** is exactly and simply **a continuation of the second subset** in the space of negative integers (see red arrows Fig. 2). Similarly, the third subset is the continuation of the fifth. Elements of the first and fourth subsets in their continuations in the space of negative integers have identical nominal values, but with opposite signs.

Let's go back to subsets - the second and sixth. Their combination gives us a unique collection - which is an excellent object for further mathematical considerations.

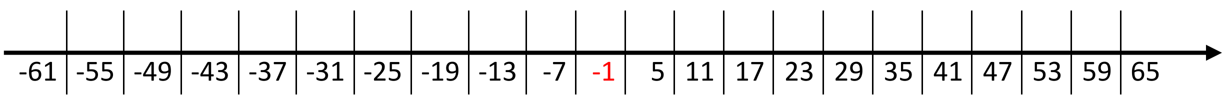

Fig.3. A numerical axis defining a set of proper and improper primes.

In the further part of this scientific work, I will focus primarily on a thorough analysis of a subset of integers obtained in the above way - whose constituent elements are prime numbers or numbers that are the ordinary product of two prime numbers.

3. DESCRIPTION OF INTRODUCED SYMBOLS:

For the purpose of this work, I will introduce a few symbols and markings that help me describe the observed relationships:

$$Z_r - \ a\ set\ of\ integers\ being\ the\ subject\ of\ research \tag{1}$$

$$\begin{gathered} O_r \ = |-1| = 1 \ - the\ first\ prime\ number; \\ (one - piece\ collection) \end{gathered} \tag{2}$$

The number O_r (since it is a one-element set I use a large-letter entry) is the "origin" (read the starting point) of the coordinate system of the test set. It is a unique element of the collection. It is only the beginning and the mother of prime numbers, which will not be included in the collection.

$$\begin{gathered} t_{r1} \ = 2 \ - the\ trivial\ prime\ number; \\ t_{r2} \ = 3 \ - the\ trivial\ prime\ number; \end{gathered} \tag{3}$$

To the set of prime trivial numbers T_r I count only the numbers 2 and 3. These are numbers that satisfy the two fundamental conditions of prime numbers proper, but not belonging to the set Z_r.

$$p_r \ = proper\ prime\ number \tag{4}$$

Elements of the P_r set are the proper primes which are the object of sighs, considerations, research of many modern mathematicians. They are a subset of the Z_r collection.

$$u_r \ = improper\ prime\ number \tag{5}$$

The numbers of the set U_r are prime numbers "improper" (author's own name) that fulfill two fundamental conditions:

- they also elements of the Z_r set;
- they are the product (combination) of the two primes proper p_r;

It is worth noting here that all elements of the Z_r set can be described without any doubt by the expression:

$$Z_r = O_r + P_r + U_r \tag{6}$$

The result of this expression is unequivocally that when we subtract all elements of the subset U_r and the number O_r from the set Z_r we receive a subset P_r consisting only of the proper prime numbers.

When we add the prime trivial numbers to this subset - elements of the set T_r we get the result which is an extremely desirable object (for over 160 years).

4. UNUSUAL CHARACTERISTICS OF THE RESEARCH Z_r:

The most important feature of the Z_r set is the fact that its elements are both arguments (domain are elements of the subset P_r) and values (set of values are elements of the subset U_r) of the searching function - which simply combines and merges them through dependence.

What follows?

If we succeed in deriving the formula of the function sought, whose domain would be based on the numbers p_r, and its values would be the number u_r, we could through this dependence and their final elimination (elements of the subset U_r from the set Z_r) obtain only the subset P_r.

Results:

I have created a pattern (function) whose beginning (read the starting point) has already become the number O_r. This pattern was not created by accident. See Figure 4 and Formula 8.

$$z_{r_k} = O_r + 6\sum_{1}^{k}(-1)^{k+1}k \tag{7}$$

Thanks to the introduction of the above-mentioned alternating sum into the results of the set Z_{r_k} are the numbers appearing once on the one side, once on the other side of the beginning O_r of the established axis and dividing in space are successively:

$Z_{r_1} = 5$	$Z_{r_2} = -7$	$Z_{r_3} = 11$	$Z_{r_4} = -13$	$Z_{r_5} = 17$	$Z_{r_6} = -19$
$Z_{r_7} = 23$	$Z_{r_8} = -25$	$Z_{r_9} = 29$	$Z_{r_{10}} = -31$	$Z_{r_{11}} = 35$	$Z_{r_{12}} = -37$
$Z_{r_{13}} = 41$	$Z_{r_{14}} = -43$	$Z_{r_{15}} = 47$	$Z_{r_{16}} = -49$	$Z_{r_{17}} = 53$	$Z_{r_{18}} = -55$
$Z_{r_{19}} = 59$	$Z_{r_{20}} = -61$	$Z_{r_{21}} = 65$	$Z_{r_{22}} = -67$	$Z_{r_{23}} = 71$	$Z_{r_{24}} = -73$

Tabela.1. A comparison of the first 20 elements of the Z_{r_k}.

Due to the fact that the prime numbers are assumed to be natural numbers, the formula (7) should be modified by an absolute value. However, due to the further need to use this formula in the above form the absolute value was temporarily ignored.

It is also worth noting - which is obvious - that to $|z_{r_7}| = 23$ the numbers obtained are numbers p_r. Numbers p_r are marked in green and u_r in red.

After a closer look at the obtained set Z_r and its display on the X axis, I noticed that the gaps (distances) between the numbers p_r, and their multiples u_r on both sides O_r are the same **and this fact, of course, beautifully refers to the adopted mathematical model, i.e. helix with a pitch equal to 6.**

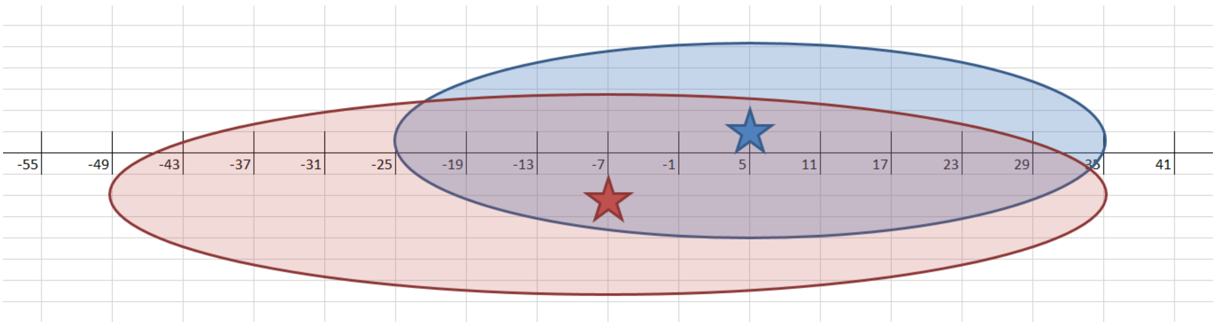

Fig. 4. Distances between numbers p_r, and numbers u_r being their nearest multiples.

Hence, after an even deeper analysis of the problem related to the numbers u_r and their dependencies on p_r and after its repeated multiple checking I derived the formula:

$$u_{r_{kmd}} = \left| p_{r_k} + (-1)^m x\ 6d |p_{r_k}| \right| \quad (8)$$

This formula clearly shows that multiples of $u_{r_{kmd}}$ prime numbers p_{r_k} occur in intervals of $6d|p_{r_k}|$ on both sides of the assumed origin of the coordinate system O_r.

<u>For example:</u>

— let $p_{r_3} = 11$;

— multiples by the formula on $u_{r_{kmd}}$ for:

- $d = 1$ is $u_{r_{311}} = |-55| = 55$ and $u_{r_{321}} = |77| = 77$.
- $d = 2$ is $u_{r_{312}} = |-121| = 121$ and $u_{r_{322}}\ |143| = 143$.
- differences between the above the numbers z_r are $6\ x\ 11 = 66$.

Thus, the overall formula underlying the order in the distribution of prime numbers p_r looks as follows:

$$u_{r_{kmd}} = \left| O_r + 6\sum_{1}^{k}(-1)^{k+1}k + (-1)^m x\, 6d \left| O_r + 6\sum_{1}^{k}(-1)^{k+1}k \right| \right| \qquad (9)$$

— where:
- k — is the ordinal index of the number z_r ($k \in <1, +\infty)$)
- m — determines in which direction the numbers u_r are generated ($m = \{1; 2\}$)
- d — means the degree of multiple ($d \in <1, +\infty)$)

The form of formula (9) explains why the first form (7):

$$z_{r_k} = O_r + 6\sum_{1}^{k}(-1)^{k+1}k \qquad (7a)$$

— is expressed without an absolute value. The answer is simple, because this expression by determining the location of the number p_r with regard to O_r corresponds ultimately to the start of the process of generating consecutive multiples of p_r.

The second form:

$$z_{r_k} = \left| O_r + 6\sum_{1}^{k}(-1)^{k+1}k \right| \qquad (7b)$$

— is placed in the absolute value, because it determines the size of the distance of subsequent numbers u_r with respect to location p_r.

5. STATEMENT OF THE FIRST NUMBER DISTRIBUTION:

"If the distribution of improper prime numbers u_r in the set of numbers Z_r is ordered - because their two features, i.e. value and location in the set Z_r result from the product of two numbers p_r - is the distribution of proper primes p_r also assumes the ordered character by applying the elimination process of the numbers u_r from the set Z_r ". Dariusz GOŁOFIT.

"If we eliminate an ordered subset from the ordered set, we will receive a subset of orderly character" Dariusz GOŁOFIT.

$$Z_r - U_r + T_r = P_r \qquad (10)$$

ULTIMATELY:

"If from the set of numbers $\boldsymbol{Z_r}$ satisfying the dependence (7b) we subtract all elements of the set $\boldsymbol{U_r}$ that fulfill the dependence (9) and add a two-element set $\boldsymbol{T_r}$ (3) we get a set of prime numbers proper $\boldsymbol{P_r}$.

Topics for future consideration:

Another interesting conclusion is the fact that the existence of twin prime numbers according to the mathematical model adopted by me is the effect of reflecting the left side of the subset of Z_r numbers with the beginning of the Cartesian coordinate system, it means 0. On the other hand, in the case of the rebound of the left side of the subset of the numbers Z_r with respect to the assumed beginning of the system of numbers Z_r it means 0_r, the twin numbers overlap.

In fact, the distance between two twin prime numbers is not equal to the number 2 (Cartesian system), only 0 (in the Z_r system), and on the adopted numerical axis the integers (model) their distance is equal to the span between them or the sum of their values.

In the next scientific work I intend to present you a way to calculate the power of the set of prime numbers based on the mathematical relations of this paper.

PREPARED

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