# Toward new thought for the unified theory of electromagnetic field and gravitational field ( Ⅰ )

(starting postulates and Lagrangian for unified theory of field)

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**Abstract:** In this paper, we analyzed the difficulties of Maxwell's electromagnetic theory and Einstein's gravitational theory in detail, built a unified theory of field including consistent nonlinear electromagnetic theory and gravitational theory on the basis of new starting postulates, and extended these results into quantum electrodynamics. In this process, we accepted a new geometrical space in which metric tensor and all main physical functions becomes implicit function, called "KR space" conforming to our starting postulates, and normalization of implicit functions in order to connect all physical functions defined in this space with real world. Here, we naturally unraveled problem of radiation reaction, a historical difficult problem of Maxwell-Lorentz theory, established new quantum electrodynamics without renormalization procedure and also predicted some new theoretical consequences which could not find in traditional theories.

**Keyword**：Unified theory of field, Gravitational field, Electromagnetic field, KR space, Breaking of gauge symmetry, Quantum electrodynamics,

Contents

## Introduction

The classical theory of fields consists of two parts, i.e. Maxwell's theory of electromagnetic field and Einstein's theory of gravitational field (theory of general relativity). Moreover, modern theory of quantum fields stands on the basis of classical theory of fields. This paper is aimed at finding out the internal relation of electromagnetic field and gravitational field, establishing new classical theory of fields with unification of two fields and, based upon it, rebuilding several main things of quantum theory of fields.

For evolution of this theory, the followings are regarded as main principles.

**(1) The conservation law of energy-momentum.**

As a natural fact, this law is, even at any case, an indestructible main law of physics and then other principles (for example, gauge invariance principle, equivalence principle in Einstein's general theory of relativity, etc.), in spite of their importance, possess real meaning only in case of satisfying the law of conservation of the energy and momentum. Here, another important thing for argument of conservation law of the energy-momentum is that physical quantities that characterize energy and momentum should possess real physical meaning in any theory. That is to say, real physical meaning of energy and momentum discussed in a theory is a necessary and enough condition for establishment of conservation formula of the energy and momentum.

**(2) The finiteness of physical quantity.**

Physical quantity that characterizes a finite material system should be always, at any case, finite. Supposing that physical quantity of finite material system is infinite, it is put beyond consideration of physics based upon quantitative analysis. Moreover, this is a premise for establishment of conservation law of energy-momentum. In fact, conservation is meant by conservation of finite quantity and if a finite material system has infinite energy, conservation of energy loses its meaning.

**(3) The consistency of theory**

The principles, axioms consisting of basis of theory, conclusions and laws following from them must not be contradictory each other. This, in a word, is rule of logics to be necessarily observed and introduced in all theories themselves.

**(4) The principle of correspondence in physics.**

According to the principle of correspondence, a new theory should involve some former-old theory as an approximate form or special form. Very natural is that theory established in opposition to principle of correspondence is disqualified as a theory of physics. However, seeing through the present classical theories of fields, we can find several problems that the above-mentioned principles are not enough embodied. Actually, as well known, there are several contradictory problems such as the divergence of energy of electrostatic field and radiation damping in Maxwell's theory, the divergence of scattering matrix in quantum electrodynamics, and absence of physical meaning of energy-momentum tensor of gravitational field in Einstein's general theory of relativity (GR), etc. Until now, for solutions to these divergences and the unification of electromagnetic field and gravitational field, string theory and superstring theory have been widely studied. These theories, concluding that occurrence of infinite quantities is rooted in taking a particle to be a point, were built and studied, based on the starting point that a particle has such finite size as a string but not a point. However, in spite of expending so much time and efforts on these studies, the result is not so satisfactory. At present, many scientists think that the number of string theory known until now are so many and there has not yet been a confirmable theory to be perfect through strict experimental verification, moreover the theories involve some drawbacks. About this, Mandel Sachs described as follows; "Unfortunately, after many years of theoretical studies, the string theory has not yet yielded a mathematically consistent (finite) quantum field theory of matter, which was its original purpose, nor has successfully predicted any observable facts." [1]

A basic reason which the string theory gives these unavoidable problems consists in that the string theory only presented a mathematical form of "string", attempting to fit the theories to mathematical form, without finding a new physical principle that underlies in removing divergence of physical quantities and clarifying internal relation of electromagnetic-gravitation field. In history of physics, appearance of new physical content brought about the birth of new mathematical form compatible to it and in this case, the harmonious combination of new content and form gave birth to miraculous physical conclusions. For example, the principle of invariance of velocity of light, one of starting ideas in Einstein's special theory of relativity led to the introduction of Minkowski space and with appearance of equivalence principle in GR was introduced four dimensional Riemann space, unprecedented in history of physics.

By contrast, for purpose of building of a new physical theory, introduction of new mathematical form about behavior of matter require to discover and receive the new physical content (or principle) which underlies and backs up it. But, though string theory presented new existence form (mathematical form) of matter called "*string*", it has not new physical content (physical principle) which is compatible to it and underlie it. Consequently, while string theory attached importance to mathematical form of string and mathematical formalism with absence of physical principle, it resulted in unavoidable inconsistency and difficulties. That the number of string theories is so many is also caused by not discovering a unified physical principle that underlies model of string. This, in a word, shows that string theory is a not-closed theory without unification between content and form.

Our theory was built on the basis of the starting idea that all divergence does not occur by taking a particle to be a point but is incurred from the difficulties and limitations of Maxwell's theory and General Relativity (GR). That is why our theory is quite different from string theory or superstring theory at starting point.

We, evolving theory based on the above-mentioned principles, obtained following conclusions.

*First, On the basis of the starting idea that the total energy of free particle and fields created by it is equal to* $mc^2$ *(presented in sect. 1), we found out a unified Lagrangian of electromagnetic field and gravitational field and gave theoretical predictions about experimental effects relevant to the dependency of electric field on gravitational field and vice versa.*

*Second, We gave an inartificial solution, with no theoretical inconsistency, to such problems as divergences in electrostatic field, radiation damping and high order terms of scattering matrix of quantum electrodynamics (historical knotty points in physics), and did the consistent theoretical analysis of already known experimental facts, including annihilation and production of particles.*

*Third, We also rebuilt several main things of quantum electrodynamics and made some theoretical predictions of nonlinear quantum effects.*

## 1. Difficulties of Classical Theory of Fields.

Since the presentation of Einstein's GR in 1916, until now, many scientists has put a lot of effort into studying on the unification of classical theories of fields. But these studies have hardly focused on revealing internal physical relation existing objectively between two fields and then mostly tried to discover some unified mathematical means and formalities of space-time geometry which can put two theories in a vessel. The main starting idea of these studies is that two theories of fields are, in classical viewpoint, complete and perfect without any room for touching on and accordingly mathematical means and techniques to combine or unify two theories are the key to solution of all problems. Of course, we know well that all of these attempts led to in failure and conclusions that stood against reality.

Maxwell's theory and Einstein's GR are quite distinguished in its physical contents and mathematical form of description (in view of main principle). It is explicitly impossible to find any physical relations between two fields described by these theories. Actually, in classical theories of fields, electric charge, source of electromagnetic field, has no relation with mass, source of gravitational field. On the other hand, energy of electromagnetic field and energy of gravitational field are conserved separately, and then there can be no argument about unified conservation involving mutual conversion between them.

But now let us make the following assumption. Supposing that electromagnetic field has some physical relation with not only electric charge, source of electro-magnetic field, but also mass, source of gravitation, there appears real conversion of gravitational field into electromagnetic field and vice-versa, and these can be verified experimentally and then universally valid physical arguments for them can be established, what will result in? This presents new unprecedented tasks before classical physics. In a word, it lead to the conclusion that the present classical theories of fields failed to establish some physical relation between two fields should be reconsidered from a critical viewpoint. That one reveals the objective relation between two fields and realizes the unification of two theories is, that is to say, to complete and develop the present classical theories of fields into a new stage, which should be accompanied by clarification and solution of the internal inconsistencies and difficulties of the present classical theories of fields.

## Sect. 1 New reasoning-thinking (starting idea)

Everything starts from an extremely simple thing. In this section is described very simple thinking giving birth to new theory. In the view of classical physics, a charged particle creates electromagnetic field and gravitational field around it. But considering the state of this particle and field in the light of the present classical theory of field, some serious inconsistencies are found.

Let us consider annihilation of particle-antiparticle well known in physics. The non-relativistic approximate formula of energy conservation can be written as follows:

$$m_0c^2 + \frac{1}{2}m_0v_1^2 + m_0c^2 + \frac{1}{2}m_0v_2^2 = 2\hbar\omega \qquad (1-1)$$

where $m_0c^2$ is the energy of free particle, a main result of special theory of relativity (SR). Actually, since the presentation of SR, until now, in physics, $m_0c^2$ has been considered only as energy confined to particle. But, in formula (1-1), it should be surely considered that $m_0c^2$ includes energy of particle, as well as energy of field created by the particle. Why should be viewed like that? It, in a word, is based upon the idea according to which the total energy of particle and field created by it should be always conserved. In case of considering $m_0c^2$ to be energy confined to particle only, the conservation formula of total energy of particle and field can be represented as follows:

$$\left(m_0c^2 + \frac{1}{2}m_0v_1^2 + \varepsilon_1\right) + \left(m_0c^2 + \frac{1}{2}m_0v_2^2 + \varepsilon_2\right) = 2\hbar\omega + \varepsilon \qquad (1-2)$$

where $\varepsilon_1$ is the energy of electromagnetic field and gravitational field created by a particle and $\varepsilon_2$, the energy of two fields created by an antiparticle. From formula (1-1) and (1-2), we have

$$\varepsilon_1 + \varepsilon_2 = \varepsilon \qquad (1-3)$$

where $\varepsilon$ is the energy of another matter newly appeared except photon after annihilation of a system of particle-antiparticle. But, until now has not yet been found any experimental data which, except photon, another matter occurred. Therefore, if there is something except photon after annihilation of particle-antiparticle, it is no alternative but to conclude that only energy of fields remained as it is, as invariant not measured. On the other hand, as long as particle-antiparticle is annihilated, the mass, $m_0$, and charge, $e$, also vanish and accordingly static gravitational field and electric field created by their source - rest mass and charge, respectively, should also disappear. Therefore, the result is

$$\begin{cases} \varepsilon = 0 \\ \varepsilon_1 + \varepsilon_2 > 0 \end{cases} \qquad (1-4)$$

$$\varepsilon_1 + \varepsilon_2 \neq \varepsilon$$

Consequently, considering $m_0c^2$ only as energy confined to particle, we, with occurrence of photon, lead to the conclusion that energy of electromagnetic field and gravitational field should vanish, which obviously stands against conservation law of the energy.

On this context, in this paper, that the total energy of a free particle and fields created by it is just equal to $m_0c^2$ is regarded as a starting point for evolving our theory. From the new starting point is followed the logical conclusion about physical relation between mass and electromagnetic field which in the present classical theories of fields has been considered to have no relation so far, and about mutual conversion of electromagnetic field and gravitational field. In fact, the occurrence of photon as the result of annihilation of particle and antiparticle means occurrence of electromagnetic wave. With occurrence of electromagnetic wave disappears gravitational field of particle-antiparticle. In view of conservation law of energy, this shows that gravitational energy of particle-antiparticle is converted into a part of electromagnetic wave. On the contrary, in case of pair creation, it proves that a part of the energy of electromagnetic wave is converted into the energy of gravitational field.

We can obtain following conclusions, based upon all above-mentioned argument.

*1. When one considers a system of particle and field, the measured mass* $m_0$ *is equivalent to the total energy of particle and all fields (electro-magnetic field, gravitational field and nuclear field created by it) but not energy of particle only.*

*2. The electromagnetic field and gravitational field are mutually converted; accordingly, there exist a unified conservation law of the total energy of particle and electromagnetic-gravitational field, involving mutual conversion.*

This conclusion, of course, was never drawn in terms of abstract assumptions. This conclusion is based on experimental data about annihilation of particle-antiparticle well known and recognized in particle physics, and rooted in conservation law of energy - foundation of physics. But, unfortunately, from the present classical theories of fields cannot be obtained these conclusions, and arguments about these problems leads to contradiction. In this regard, we consider difficulties of classical theories of fields separately according to respective theories and then make a final analysis as a whole.

## Sect. 2 The difficulties of Maxwell's theory of electromagnetic field

The classical theory of electromagnetic field or Maxwell's electrodynamics, as the unique theory of electromagnetic phenomena, until now, has been regarded as a perfect and completed theory. But this theory also involves some unavoidable inconsistencies.

**(1) In Maxwell's theory, divergence of energy of electrostatic field seems to be an unavoidable difficulty, and accordingly the conservation law of total energy of particle-its field is always meaningless.**

The classical theory of field evolves with taking a particle to be a pointis, from demand of Special theory of Relativity in which particle cannot have finite size. But, in case of regarding a particle as a point, the energy of electrostatic field always diverges. The law of energy conservation is based on the idea that energy of a finite material system is always finite. This is because of the fact that conservation law is meaningful only for finite quantity and can be studied quantitatively. That is why, for finite material system with infinite energy, the conservation law of energy leads to absence of meaning. This shows clearly that, in Maxwell's theory, the finiteness of energy - conservation law of energy is not valid and so, not well qualified as a scientific theory.

Understanding Maxwell's theory in the viewpoint of logics, one also can find inconsistency. For building of a consistent closed theory, starting definitions, all conclusions and laws following from them should not be inconsistent each other. But that there is inconsistency between basic definition regarding a particle as a point and conservation law of energy in Maxwell's theory shows that this theory is a not-closed theory with inconsistency

In the past, divergent problem of energy was solved within Maxwell's theory as follows. One artificially removed the term relevant to the divergence in energy of field, regarding it to be absence of physical meaning. Of course, this obviously is in opposition to rule of logics. On the other hand, in case of the divergence of energy of electrostatic field when action radius of electric field approaches zero, confining the applicable region of classical electrodynamics to electron radius, $e^2/m_0c^2$, outside this region of application it was concluded that not classical theory of field but quantum theory of field is meaningful. But, this "measure of solution" is also wrong. Actually, region of application of theory is defined in accordance with what exactly the theory can describe experiments, namely, by applicable limitation in which can give answer to experiment but not by some specific limitation that the theory falls to logical inconsistency. When theory is not closed and has logical inconsistency, we fall to the poor situation that cannot distinguish whether disagreement between some consequences of the theory and experiments is based on internal inconsistency of the theory itself or actual limitation of application of the theory related to that the theory can give no perfect answer to experiments. Consequently, Maxwell's theory stands against the finiteness of energy and conservation law, and accordingly is a not-closed theory that involves contradiction.

This difficulty of Maxwell's theory is represented as the more serious form on the stage of quantum electrodynamics. In this regard, Stephan Weinberg wrote:

"Earlier experience with classical electron theory provided a warning that a point electron will have infinite electromagnetic self-mass. Disappointingly this problem appeared with even greater severity in the early days of quantum field theory, and although greatly ameliorated by subsequent improvements in the theory, it remains with us to the present day.

The problem of infinites in quantum field theory was apparently first noted in the 1929-30 papers of Heisenberg and Pauli. Soon after, the presence of infinites was confirmed in calculations of electromagnetic self-energy of a bound electron by Oppenheimer, and of a free electron by Ivar Waller. …But it had become accepted wisdom in the 1930s, and a point of view especially urged by Oppenheimer, that quantum electrodynamics could not be taken seriously at energies of more than about 100MeV, and that the solution to its problems could be found only in really adventurous new ideas." [2]

It is the obvious fact that even in the quantum electrodynamics in which region of application of theory cannot be limited, the energy of electrostatic field is divergent. The artificial "measure of solution" by what defines limitation which in classical electrodynamics the theory can be applied to can no longer apply to quantum electrodynamics. It is no alternative but to conclude that this difficulty is rooted in the inconsistency of Maxwell's theory.

**(2) In Maxwell's theory, because of the divergence of field energy, one cannot give answer to the experimental fact that the total energy of particle plus its field is equal to the finite quantity, $m_0c^2$**

It was Albert Einstein who gave scientific answer to the relation of mass and field for the first time. He, evolving the theory of gravitational field, proved that in gravitational field of central symmetry the total energy of particle-gravitational field is equivalent to the inertial mass of a system. This shows that equivalence of mass and energy of a particle in SR is more generalized into equivalence of total energy of particle-field and inertial mass of a system. However, this equivalence is confined to within the theory of gravitational field and, until now, the interrelation between total energy of particle-its electromagnetic field and mass has not yet been studied. On the other hand, the experiment formula (1-1) shows explicitly that the total energy of particle and electromagnetic field-gravitational field is equal to the finite quantity, $m_0c^2$. This implies again the serious contrariety of Maxwell's theory.

**(3) In Maxwell's theory, the consideration of radiation damping by radiation of an electric charge leads to a serious difficulty.**

In case of considering electromagnetic wave radiated by a uniformly accelerated electric charge, it was experimentally verified that radiated field reacted on the electric charge. But in Maxwell's theory, the description of radiation damping always leads to a serious difficulty. This problem has been regarded as "the greatest crisis in Maxwell's theory" [3].

We now proceed the discussion of radiation damping in Maxwell's theory. The expansion of power series of four dimensional field potential in $\mathbf{V}/c$ can be written as follows:

$$\varphi = \frac{e}{R} + \frac{e}{2c}\cdot\frac{\partial^2 R}{\partial^2 t} = \varphi^{(1)} + \varphi^{(2)} \qquad (2-1)$$

$$\mathbf{A} = \frac{e}{c}\cdot\frac{\mathbf{V}}{R} - \frac{2}{3c^2}e\dot{\mathbf{V}} = \mathbf{A}^{(1)} + \mathbf{A}^{(2)} \qquad (2-2)$$

where $\varphi^{(3)}$ is zero by the gauge transformation [4]. From this, damping force by radiation is represented as

$$\mathbf{F}^{in} = e\mathbf{E}^{in} = -\frac{1}{c}\dot{\mathbf{A}}^{(2)} = \frac{2}{3c^2}\dddot{\mathbf{d}} \qquad (2-3)$$

$$m\dot{\mathbf{V}} = e\mathbf{E}^{ex} + \frac{e}{c}[\mathbf{V}\cdot\mathbf{H}^{ex}] + \frac{2}{3c^2}\dddot{\mathbf{d}} \qquad (2-4)$$

where $\mathbf{E}^{ex}$ and $\mathbf{H}^{ex}$ are strengths of external fields and $\mathbf{F}^{in} = e\mathbf{E}^{in}$ is the force by field of point charge itself. Consequently, $\mathbf{A}^{(2)}$ was considered only as the additional field that contributes to damping force by radiation. But this argument is followed from the following premise.

1) In the Lagrangian of interaction of particle and field, four dimensional vector $(\mathbf{A}, \varphi)$ of field should be viewed as the sum of external field and field of particle itself (created by particle itself), namely,

$$\mathbf{A} = \mathbf{A}^{ex} + \mathbf{A}^{in} \qquad (2-5)$$

$$\varphi = \varphi^{ex} + \varphi^{in} \qquad (2-6)$$

where $\mathbf{A}^{ex}$ and $\varphi^{ex}$ are external fields and $\mathbf{A}^{in}$ and $\varphi^{in}$ are fields of point charge itself.

2) For finding the force that the field produced by point charge acts on itself, in formula (2-1) and (2-2), radius of action by field should go to zero, namely,

$$\mathbf{A}^{in} = \lim_{R\to 0}\mathbf{A}\ ,\quad \varphi^{in} = \lim_{R\to 0}\varphi \qquad (2-7)$$

But one can easily understand that, under the above mentioned premise, the theory immediately results in inconsistency. Actually, converging radius of action $R$ to zero, (2-1) and (2-2) yield divergence of $\varphi^{(1)}$ and $A^{(1)}$. Consequently, within Maxwell's theory, introduction of field created by electric charge itself necessarily gives divergent terms. This inconsistent conclusion, as mentioned above, is rooted in the fact that the energy of electrostatic field leads to divergence.

In order to overcome this difficulty, in consideration of damping force by radiation, divergent terms, $\varphi^{(1)}$ and $\mathbf{A}^{(1)}$, were artificially subtracted, concluding that they are insignificant terms, and only $\mathbf{A}^{(2)}$ independent of radius of action was regarded as the significant term relevant to radiation damping. Of course, this "measure of solution" is obviously in opposition to logical rule for construction of theory. Many experimental data show that reaction of radiation by electric charge, radiation damping, appears in reality and affects motion of electric charge. But in Maxwell's theory, the fact which introduction of reaction effect by radiation leads to contradiction shows that this theory is not closed one involving inconsistencies.

**(4) Because of the principle of gauge symmetry that underlies Maxwell's theory, the energy of material system loses physical meaning and accordingly energy conservation of material system arrives at absence of its meaning.**

The principle of gauge symmetry that underlies classical electrodynamics, as well as quantum theory of field involves a serious problem to be reconsidered. In SR, the relation among mass of a free particle, its energy and momentum is as follows:

$$m_0c^2 = \frac{E^2}{c^2} - P^2 \qquad (2-8)$$

As referred to in many papers and textbooks, in case of a complex system which consists of elements (or subsystems), formula (2-8) holds [5]. In this case, the energy of the system is

$$E = \sum_i \varepsilon_i + U \qquad (2-9)$$

where $U$ is the interactional energy of constituent particles and $\varepsilon_i = m_ic^2 + T_i$, the sum of rest energy and kinetic energy, and then momentum is

$$\mathbf{P} = \sum_i \mathbf{P}_i \qquad (2-10)$$

Now, if one chooses a coordinate system allowing momentum $\mathbf{P} = 0$ in which inertia center is placed at origin of coordinate system, the result is

$$m_0 = \sum_i m_i + \frac{1}{c^2}\sum_i T_i + \frac{U}{c^2} \qquad (2-11)$$

If $\sum_i T_i \ll |U|$, namely constituent particles maintain relative stability and kinetic energy of particles is supposed to be very small, formula (2-11) arrives at

$$m_0 = \sum_i m_i + \frac{U}{c^2} \qquad (2-12)$$

From this, difference or deficit of mass is as follows:

$$\Delta m = m_0 - \sum_i m_i \qquad (2-13)$$

$$U = \Delta m c^2$$

This conclusion, of course, was verified by many experiments relevant to fission. That is to say, formulas (2-12) and (2-13) are correct results proved by experiments. But in case of applying the principle of gauge invariance (gauge symmetry) to formula (2-12) and (2-13), at once, we arrive at inconsistency. Actually in formula (2-12), as long as the interaction term, $U$, includes potential term of electric interaction, and from the principle of gauge symmetry, any constant can be either added to or subtracted from potential $\varphi$. In this case, the mass and energy of a system cannot be uniquely determined and further by choosing properly a constant included in $\varphi$, the mass and energy of a system can be transformed to zero or even negative value. Consequently, from the principle of gauge symmetry, the energy of material system leads to loss of physical meaning. On the other hand, only when $U < 0$, system becomes stable, but as long as $U$ leads to zero or positive value according to constant chosen, discussion about the criterion of stability and instability is impossible.

In Newton's classical mechanics, the energy of a rest object is not defined uniquely and is positive value or negative value. In contrast, in SR the energy of a free particle is always determined uniquely as positive value and equivalent to rest mass. If one follows the principle of gauge symmetry, the formula (2-12), a main conclusion of SR that has already verified by experiment should be rejected and energy of material system leads to absence of physical meaning. Consequently, one arrives at failure in arguing conservation law of energy. If one receives, as a truth, the equivalence of mass and energy verified experimentally and the fact that the energy of material system can be neither zero nor negative value, the principle of gauge symmetry should be reconsidered.

**(5) Quantum electrodynamics regarding Maxwell's theory as the unique basis raises problem of divergence of scattering matrix within region of large momentum or small area of space.**

The occurrence of divergent terms in approximation of higher order of scattering matrix presents unavoidable knotty points before electrodynamics. In fact, even though first order approximation in scattering theory is well conformed with experimental results, if the approximations of high order diverges, even correctness of first approximation is put into doubt and accordingly this theory leads to loss of qualification as a scientific theory of physics.

As well known, in classical electrodynamics also is raised problem of divergence within classical radius of electron $r_0 = e^2/m_0c^2$ but within small area of Compton wavelength degree, recognizing that quantum theory only is significant, by the way of confining applicable region of classical electrodynamics to Compton wavelength, this inconsistency was overcome. However, as far as quantum electrodynamics considers interaction of particles within any area of space, divergence occurred in some area cannot be solved by the same way as in classical electrodynamics. In this regard, in present quantum electrodynamics, this difficulty was "solved" as follow: At first, one defined boundary momentum $L$ and next, separated infinite quantity from the main expression and then by including these in electric charge and mass, renormalized electric charge and mass to yield finite quantity only. But this cannot certainly be the right measure of solution. In fact, as recognized by many scientists, this measure of solution is very artificial and harm the logical system of the theory.

P.A.M. Dirac was strongly against the procedure of neglecting infinity by renormalization:

"This small correction is interpreted as giving the Lamb shift in the case of the energy levels of hydrogen or an extra magnetic moment of the electron, the anomalous magnetic moment, for an electron in a magnetic field. These calculations do give results in agreement with observation.

Hence most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory, and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation, because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small-not neglecting it just because it is infinitely great and you do not want it!

…

There must be some drastic change introduced into them so that no infinites occur in the theory at all and so that we can carry out the solution of the equations sensibly, according to ordinary rules and without being bothered by difficulties. ⋯ I feel that the change required will be just about as drastic as the passage from the Bohr orbit theory to the quantum mechanics." [6]

Mandel Sachs said as follows:

"While this (method of renormalization) is taken to be a success of the quantum theory, it is still not satisfactory because the renormalization procedures are not mathematically consistent. That is, while some predictions are correct, by changing the method of subtracting the infinities from the divergent series solutions, one may predict any other numbers for the same physical effects! This violates the scientific requirement that there is a unique prediction for any given experimental fact.

Thus it has been my contention, as well as some others in the field (such as one of the original founders of quantum field theory, Paul Dirac) that quantum electrodynamics is not in a satisfactory state as a bona fide theory." [1]

In this paper, we clarified that occurrence of this inconsistency from quantum electrodynamics was rooted in problem of divergence in classical electrodynamics and built a basis of consistent quantum theory without any inconsequence.

## Sect. 3 The difficulties of GR

Difficulties of GR, for the first time in history of physics, was presented by Schrodinger in 1918, and since then, was stated by many physicists like Fock, and discussed collectively in "the relativistic theory of gravitation" (English Edition , 1989) co-written by Logunov and Valssov etc., physicists of former Soviet Union.

With reference to all arguments in preceding research, summarizing difficulties of GR is as follows.

**(1) The energy-momentum tensor defined in Einstein's GR has no physical meaning.**

As demonstrated for the first time by Schrodinger, by choosing properly coordinate system, energy-momentum tensor of gravitational field vanishes outside a ball. From this follows the inconsistent conclusion that the energy-momentum of gravitational field cannot be localized and accordingly energy-momentum density of field existing in any point of space-time cannot be defined and then only total energy-momentum integrated through total space can be well-defined. As stated by many scientists, in this case, propagation of gravitational energy from one place to another is impossible and description of gravitational wave leads to principled inconsistency [8]. Actually, in GR, energy-momentum tensor $\tau^{lm}$ of field is defined as pseudo tensor and in this case, by selecting an appropriate system of coordinates, one can nullify all the components of $\tau^{lm}$ at any point of space [7]. On the other hand, in GR, energy-momentum conservation formula of integral form also possess a limitation. In this theory, energy conservation of matter and field can be written as follows.

$$\partial_n(T_i^n + \tau_i^n) = 0 \qquad (3-1)$$

where $T_i^n$ is energy-momentum density tensor of matter and $\tau_i^n$ energy-momentum density tensor of field. If matter is concentrated only in a volume $V$, Eq (3-1) implies that

$$\frac{d}{dx_0}\int_V \left(T_i^0 + \tau_i^0\right) dV = -\oint \tau_i^\alpha dS_\alpha \qquad (3-2)$$

At present, there exist a whole series of exact solutions to the vacuum Hilbert-Einstein equations for which the stresses, $\tau_0^\alpha$, are everywhere null [9]. Thus, for exact wave solution to Hilbert-Einstein equations that nullifies the components of the energy-momentum pseudo-tensor, Equation (3-2) yields

$$\frac{d}{dx^0}\left\{\int_V \left(T_i^0 + \tau_i^0\right) dV\right\} = 0 \qquad (3-3)$$

that is, the energy of matter and gravitational field inside V is conserved. This means that there is no flow of energy outward from V and, therefore, there can be no action on test bodies placed outside V. And vice versa, in case of absence of gravitational field, i.e. flat space-time, that is, when the metric tensor $g_{ni}$ of the Riemann space-time is equal to the metric tensor, $\gamma_{ni}$, of pseudo-Euclidean space-time, components of energy-momentum pseudo-tensors may not vanish although there is no gravitational field and all components of the curvature tensor are zero. For example, in the spherical system of coordinates of the pseudo-Euclidean spacetime is given following formula

$$R_{klm}^i = 0, \quad g_{00} = 1, \quad g_{rr} = -1, \quad g_{\theta\theta} = -r^2, \quad g_{\varphi\varphi} = -r^2 \qquad (3-4)$$

In this case, component $\tau_0^0$ of Einstein's pseudo-tensor for energy density of the field yields

$$\tau_0^0 = -\frac{1}{8\pi} \qquad (3-5)$$

It is clear that the total energy of gravitational field in this system of coordinates would diverge because of $\tau_0^0 < 0$ . In this case, Landau-Lifshitz pseudo-tensor demonstrates a different energy distribution in space [8].

$$(-g)\tau^{00} = -\frac{r^2}{8\pi}(1 + 4sin^2\theta) < 0 \qquad (3-6)$$

Consequently, In GR, energy-momentum density of field, the main physical quantity characterizing the field is not determined by real field itself but by choice of coordinate system. That is to say, by a suitable choice of coordinate system, the field can vanish in spite of existence of real field or appear even in case of absence of real field. This shows obviously that Einstein's GR failed to have a main character which must possess as scientific theory.

**(2) The principle of equivalence, starting idea of GR is not qualified enough as scientific principle of physics.**

According to principle of equivalence, by choice of system of coordinate, inhomogeneous gravitational field in space-time cannot totally vanish, but in any infinitely small region of space, coordinate system can always be chosen in such a way that the gravitational field in the region vanishes, and accordingly in this region gravitational field can be replaced completely by field of inertia. Just this idea reflected main character of Riemann space in which curved surface, by any choice of coordinate system, cannot transform into flat surface but, for infinitesimal region, into Euclidean infinitesimal space, which just was the main reason that Einstein chose Riemannian space as a form of space-time for evolution of the theory of gravitational field.

But as argued by Logunov, Vlassov and many physicists including Schrodinger and Fock, the above mentioned difficulty (loss of physical meaning of gravitational field) is rooted in equivalence principle. In fact, in GR metric tensor $g_{mn}$ is both metric of space-time and function of field. Therefore, equivalence principle that metric, $g_{mn}$, in any point of space-time can be transformed to Euclidean metric (constant metric) leads us to the inconsistent conclusion that energy-momentum density tensor of field localized in a point of space-time can become zero and gravitational field occurs even in Euclidean spherical system of coordinate (empty space without gravitational field), as long as energy-momentum tensor of field consists of metric tensor $g_{mn}$.

Now let us make the following imaginary thought experiment. Supposing that there is homogeneous and static gravitational field, in this field the particle accelerates to radiate gravitational wave, gravitons. On the other hand, in view of equivalence principle, static and homogeneous field, by an appropriate transformation of coordinate system, can be the state of "null-gravitation". Of course, in this empty space or the state of null-gravitation is followed an inconsistent conclusion that with uniformly and rectilinear motion of a particle vanishes gravitational wave radiated by particle, i.e. graviton. This shows that equivalence principle reflects the inconsistent idea that can either create or remove such objective matter as static field or gravitons. Besides, in GR gravitational mass is not invariant under transformation of three-dimensional spatial coordinate system, and so the descriptions about three effects of gravitation (Red shift of Light, Reflection of Light and Shift of Mercury's Perihelion) have not uniqueness in view of theoretical analysis, and moreover by choice of coordinate system, the radiation strength of gravitational wave can be either zero or negative value. These obviously are inconsistent.

It is our contention, as well as Logunov, that these difficulties are rooted in equivalence principle [8]. The conservation law of energy-momentum is a main idea of physics, whereas the equivalence principle is very essential for building of GR. Sacrificing conservation law of energy, this principle cannot certainly find any foundation for its existence as the principle of physics.

**(3) In GR, the conservation law of the total energy-momentum of matter and field is not based on a main principle of physics relevant to homogeneity and isotropy of space and time, and, moreover, has not physical meaning.**

From Newton's time to now, the relation between conservation law of energy-momentum and homogeneity and isotropy of space-time has been recognized as a main principle in all theories of physics including classical electrodynamics and quantum electrodynamics. But in case of applying this principle to GR, one reaches the inconsistent conclusion. The Lagrangian density of matter and field and action integral formula can be written as follows.

$$L_M = L_M(g_{mn}, \phi_A ), \qquad L_g = \sqrt{-g}R, \qquad S = \int \left(L_M + L_g\right) d\Omega \qquad (3-7)$$

where $L_M$ is Lagrangian density of matter and $L_g$ Lagrangian density of field, $g_{mn}$ metric tensor, and $\phi_A$ field of other matter. In this case, infinitesimal transformation of space-time $x'^i = x^i + \delta x^i$ results in infinitesimal transformation of metric $g'^i = g^i + \delta g^i$, and from $\delta S = 0$ is followed

$$T^{ni}_{(M)} + T^{ni}_{(g)} = 0 \qquad (3-8)$$

where

$$T^{ni}_{(M)} = -2\,\delta L_M / \delta g_{ni}$$

is the symmetric energy-momentum tensor of matter,

$$T^{ni}_{(g)} = -2\,\delta L_g / \delta g_{ni} = -\frac{C^4}{8\pi G} \cdot \sqrt{-g}\left[R^{ni} - \frac{1}{2} g^{ni} R\right]$$

is the energy-momentum tensor of field. Equation (3-8) also implies that all components of the energy-momentum tensor density of the symmetric gravitational field, $T^{ni}_{(g)}$, vanishes everywhere outside matter, Thus, these results imply that the gravitational field in GR does not possess properties inherent in electromagnetic field [8]. Consequently, drawing conservation law of energy-momentum (field plus matter) from general principle of homogeneity and isotropy of space-time naturally, we reach the inconsistent conclusion. If so, why results in the inconsistent conclusion. In GR $g_{mn}$ is both metric of space-time and variables of field, and accordingly obtaining the equation of field from variation of field $\delta g^{mn}$ coincide mathematically with drawing conservation formula of energy-momentum from variation of metric $\delta g^{mn}$ following by variation of space-time $\delta x'$. The conservation formula drawn by this method is invalid as showed in formula (3-8).

In order to avoid this inconsequence, in GR the concept of energy-momentum was defined by the illogic and artificial method as follows. The field equation of Hilbert-Einstein can be written as:

$$-\frac{C^4}{8\pi G}\cdot g\left[R^{ik}-\frac{1}{2}g^{ik}R\right]=-gT^{ik} \qquad (3-9)$$

where $\det g_{ik}=g$ , $R^{ik}$ Ricci tensor and $T^{ik}$ the energy-momentum tensor of matter. Then, the left-hand side can be represented as the sum of two non-covariant quantities

$$-\frac{C^4}{8\pi G}g\left[R^{ik}-\frac{1}{2}g^{ik}R\right]=\frac{\partial h^{ikl}}{\partial x^l}+g\tau^{ik} \qquad (3-10)$$

where $\tau^{ik}=\tau^{ki}$ is the energy-momentum tensor of gravitational field and $h^{ikl}=-\ h^{ilk}$ spin pseudo-tensor. This transforms Hibert-Einstein equations (3-10) into equivalent form

$$-g\left(T^{ik}+\tau^{ik}\right)=\frac{\partial h^{ikl}}{\partial x^l} \qquad (3-11)$$

From the obvious fact that

$$\frac{\partial^2 h^{ikl}}{\partial x^k \partial x^l}=0 \qquad (3-12)$$

Hilbert-Einstein equation yields the following "differential conservation law"

$$\frac{\partial}{\partial \tau^k}\left[-g\left(T^{ik}+\tau^{ik}\right)\right]=0 \qquad (3-13)$$

which formally is similar to the conservation law for energy-momentum in electrodynamics [4]. Of course, this argument is never made on the basis of homogeneity and isotropy of space-time. Moreover, energy-momentum tensor defined in conservation formula (3-13) as pseudo-tensor can vanish by a suitable choice of coordinates system or diverges in Euclidean spherical coordinates, and so is invalid as a physical quantity that characterizes physical field. On the other hand, as showed in formula (3-3), conservation of energy-momentum of integral form also leads to difficulty and in case of discussion of "energy-momentum" of system we also arrives at the inconsistent conclusion which it or inertial mass depends on choice of spatial coordinate [8].

**(4) In case of considering matter as point, in GR the energy of material system also would diverge.**

When one, neglecting macro-character of objects that comprise material system, regarding them as points and calculating energy of the system, in addition to interactional energy dependent on spatial distribution of objects, there appear such divergent terms relevant to self-energy as in Maxwell's theory [4].

**(5) Radiation damping (radiation reaction effect) by gravitational wave created by an accelerated particle arrives at serious choplogic.**

Until now, in spite of so many studies concerning gravitation, radiation damping by gravitational wave has hardly been studied. It is because intensity of gravitational wave is too small to measure and then what accounts for radiation damping effect much smaller than it has no significance. On the other hand, owing to the characteristic of nonlinear equation of gravitational field, it is impossible to obtain the correct solution of the equation and accordingly the strict theoretical consideration of radiation damping effect by radiation wave cannot be given.

But, now let us make an imaginary thought experiment about what radiation damping will result in. In GR, gravitational field is always attraction field and accordingly interactional force between object and field has negative value. Therefore, force of radiation damping, namely interactional force with self-field produced by object also has negative value. Consequently, from this is drawn the inconsistent conclusion that an object, with radiation of gravitational wave, does not lose energy by damping force but by the force in direction of motion comes to obtain energy. For actual approximate calculation, one can get formula for radiation field of gravitational wave by a power series under approximate condition of weak field. In this case, the first term of expansion is proportional to $-1/r$ and as for the interaction with self-field, term of interaction with attraction field has negative infinity. Of course, although this argument is not based on a rigorous calculation and more or less imaginative, the result of the thought experiment presents the main difficulty of GR.

All of these shows that Einstein's GR is also not a closed theory. Until now we considered the main difficulties that Maxwell's theory and Einstein's GR involve. Summarizing all argument above mentioned leads to the following conclusions.

*1. In present classical theory of fields, total energy-momentum conservation law does not has physical meaning*

In case of Maxwell's electrodynamics, the energy of field created by an electric charge always diverges and owing to principle of gauge symmetry, the energy of material system arrives at absence of physical meaning. In Einstein's GR, energy-momentum tensor does not possess physical meaning, and the total energy-momentum conservation formula also leads to inconsequence.

*2. It is impossible that within Maxwell's theory and Einstein's GR give solution to experimental data which total energy of particle-field is equal to* $m_0c^2$.

If one sticks to that the present theories are consistent and closed ones, we should give up main principles of physics for total energy conservation of matter-field and the finiteness of physical quantities, which of course has not objective validity and is never possible.

## 2. Starting Postulates and Lagrangian for Unified Theory of Field (Electromagnetic Field and Gravitational Field)

The unification of the different two theories, i.e. theory of electromagnetic field and theory of gravitational field that have been systemized separately is indeed the establishment of a new consistent classical theory of field. On basis of the analysis of difficulties and inconsistencies (of classical fields), we present the following principles and methodology for solutions to these.

First of all, the principles are as follows:

*In classical and quantum electrodynamics, even in case of considering a particle as a point, theories should be constructed so that do not appear any divergence of physical quantities,.*

*In case of gravitational field theory, limitation of application of equivalence principle should be, to some extent, given and then the theory be constructed so that conservation formula of energy-momentum possesses physical meaning.*

*Theory should be built so that the total energy of particle-fields (electromagnetic field and gravitational field) becomes* $m_0c^2$.

Next, the methodology is as follows:

*The construction of a new theory should base on the assumption which the Lagrangian of Maxwell's electrodynamics is the first approximation of a new Lagrangian.*

*As for gravitation, one should find some application limitation within which equivalence principle is valid, and then construct a new gravitation theory that involves Einstein's GR established as the approximate form within it (application limitation of equivalence principle).*

*In the new Lagrangian, variable of field should include field functions of electromagnetic-gravitation in a unified form, and then Lagrangian should be represented such that can obtain the unified conservation law of electromagnetic-gravitational field.*

### Sect. 4 Starting postulates

The starting postulates of the unified classical theory of electromagnetic-gravitational field or new classical theory of field are as follows.

**1. In all inertial reference frames, all laws of physics are equivalent.**

The inertial reference system regards inertial law as the foundation of self-existence. From this law, in theory of field is drawn conclusion that free particle cannot accelerate automatically to produce wave of radiation, and then damping force by radiation is based upon existence of external field only. This is just understanding of inertia which the theory of matter-field is based and is more generalized expression of mechanical inertial law. On the other hand, as long as the unified theory of field is evolved in inertial system, Lagrangian for matter-field is invariant by Lorentz transformation only. Therefore, in case where, without external field, only self-field of particle is given, we lead to the conclusion that its Lagrangian should be Lagrangian for a free particle in the special theory of relativity (SR).

Though this conclusion seems clear at first glance, it has very important significance. In SR, $mc^2$ is the energy confined to particle only but $mc^2$, in our theory, is the energy which includes not only the energy of a particle but also the energy of electrostatic field. And then it is because in case of introducing term of interaction between particle and its self-field to new Lagrangian for free particle, this term, unlike Maxwell's theory, does not diverge but become zero and the Lagrangian for a free particle in Minkowski space should be obtained.

**2. In all inertial reference frames, the velocity of light is invariant.**

**3. The total energy of matter of free state (or free particle) and all kinds of the fields created by it is equal to measured mass $m$ (rest mass $m_0$, motion mass $m = m_0/\sqrt{1-\beta^2}$) of system consisted of matter-fields multiplied by $c^2$.**

Einstein is the first man who solved equivalence between mass and energy of particle. He, in GR, gave theoretical solution which total energy of matter and static gravitational field created by it is equivalent to inertial mass of matter. Later, this problem was proved theoretically in various forms by many physicists including R. C. Tolman, H. Weyl, and L. D. Landau [10, 11, 4]. That is why, in GR matter and field is considered as an inseperable integrity and total energy of matter-field always is equivalent to the mass of a system. But until now, such equivalence in the physics has been studied only within the theory of gravitational field.

Now let us focus on the following. The electric charge creates not only electromagnetic field but also gravitational field and as shown in experimental data of annihilation and creation of particles (see formula (1-1)), the total energy of matter and all fields created by it is equal to $mc^2$ ($m$; measured mass). Here an important thing to be explicitly emphasized is that a system consisting of particle and field should be considered as an integrity and accordingly a question about what share of $mc^2$ is divided into energy of matter and field, respectively, cannot be given in principle. In a word, the third postulate is the more comprehensive generalization of Einstein's theory and becomes a foundation of unified theory.

**4. The physical field (energy-momentum tensor) existing in any point of space-time, under any transformation of coordinates system and even within infinitely small area of space cannot vanish.**

Since the appearance of Galilean principle of relativity, there were many arguments about transformation of coordinates system. Galilean principle of relativity tacitly reflects an idea according to which although motion state of matter, under any transformation of coordinate system, can change to this way or that way but matter itself can neither vanish nor newly be created. Of course, although Galilei and later other physicists emphasized no more about this, considering transformation of coordinates system, they took it for granted.

The field is also a special form of matter. Nevertheless, supposing that gravitational field can either vanish or be created by an appropriate choice of coordinates system, this necessarily leads to a choplogic basically inconsistent with the idea of matter which is tacitly reflected in transformation theory of coordinates system, argued and inherited from Galilei time to now. In this case, the energy-momentum tensor characterizing physical field arrives at absence of meaning and accordingly the conservation law of total energy also loses its meaning. The energy-momentum conservation law is the most basic and universal idea of physics. At any case, sacrificing this law, one cannot insist reasons for existence of other secondary principles and its justness.

As considered in sect. 3, supposing that this equivalence principle is valid without any restriction, i.e. in any point of space-time and any gravitational field (weak or strong), theory is built in Riemann space in which physical field loses the meaning. This implies that Einstein's principle of equivalence has some limitation and then there exists some application limitation for it to have real meaning. If so, how should such limitation of application be considered? Considering it in the view of origin, Einstein's principle of equivalence is rooted in Newton's theory of gravitation. In this theory, gravitational force acting on an object and acceleration are

$$m\boldsymbol{a} = m\boldsymbol{g}, \qquad \boldsymbol{a} = \boldsymbol{g}$$

From this is followed the conclusion that an object subject to gravitation has identical acceleration irrespective of its mass. We can infer that when Einstein defined equivalence principle, obviously grounded upon Newton's idea of the relation between gravitational force and inertial force of object. Actually, at those days when Einstein defined equivalence principle, there was no another theory and experimental data for gravity except for Newton's theory and experimental data relevant to it. Newton's theory of gravitation is based upon experimental data obtained within nonrelativistic, static and weak field and equivalence principle can also be defined within the scope. There is no experimental and theoretical ground to conclude that this principle can be enough extended to non-static and strong field yet. Nevertheless, Einstein elevated equivalence principle grounded upon experimental results obtained within non-relativistic, static and weak field, by generalization and expansion, to an all-powerful principle of gravitation that can apply even to relativistic, non-static and strong field. As mentioned above repeatedly, unlimited expansion of this principle to any region leads to serious choplogic. Consequently, this implies that as Newton's theory of gravitation is allowed within the non-relativistic, weak and static field, the application limitation of equivalence principle should be also confined to that extent. This is essential to understand interrelation between the new unified theory and Einstein's theory of gravitation.

**5. In unified theory of fields, every physical quantity is expressed as implicit functions (correctly speaking, absolute implicit functions) and the functions that have real physical meaning are determined as explicit ones uniquely corresponded to Finsler space according to rule of normalization.**

This postulate is discussed in sect. 9 and 10.

## Sect. 5 Lagrangian for motion of particle

In this section, we define the Lagrangian of the motion of a particle and consider the motion of a particle in electromagnetic field and gravitational field, respectively. Lagrangian integral formula for the motion of a particle can be defined based on starting postulates as follows:

$$S = -m_0 c \int ds = -m_0 c \int dt\left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right)^{1/2}$$

This formula yields

$$S = \int L ds = -m_0 c \int \left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right)^{\frac{1}{2}} = -m_0 c \int \left[\delta_{\mu\nu}\left(1 + 2\alpha K_\lambda u^\lambda\right)u^\mu u^\nu\right]^{\frac{1}{2}} ds \qquad (5-1)$$

where $L = -m_0 c\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}$, $g_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\alpha K_\lambda u^\lambda\right)$, $\delta_{\mu\nu}$ Minkowski metric tensor, $K_\lambda$ four dimensional field vector ($A_\lambda$ in case of electromagnetic field and $G_\lambda$ in case of gravitational field) and $u^\lambda$ four dimensional velocity and $\alpha$ constant defined from approximate condition ($\alpha = \alpha_{(E)} = e/m_0 c^2$ in case of electromagnetic field and $\alpha = \alpha_{(g)} = m_0/m_0 c^2 = 1/c^2$ in case of gravitational field). Here metric tensor $g_{\mu\nu}$ is non-Euclidean metric dependent on four-dimensional velocity vector and coordinates of space-time.

The Lagrangian integral formula (5-1) is conformed to starting postulates mentioned in sect. 4.

*Firstly, integral of action (5-1) is invariant under Lorentz information and includes the universal constant* $c$.

As easily demonstrated, because in metric tensor $g_{\mu\nu}$, $\delta_{\mu\nu}$ is Minkowski tensor and $K_\lambda u^\lambda$ the scalar product of four dimensional field vector and four dimensional velocity vector, so the formula (5-1) is invariant under Lorentz transformation.

*Secondly, as proved in next sections, from formula (5-1) is derived equivalence between total energy of particle-field and mass of a system.*

*Thirdly, in nonrelativistic, static and week field formula (5-1) is converted to the Lagrangian integral formula in Riemannian space in which Einstein's principle of equivalence is admitted.*

In non-relativistic, static and week field, $G_0 = U$ (gravitational potential) and $G_i = 0$, and ignoring terms of higher order more than $1/c^2$ is ignored, formula (5-1) yields

$$S = -m_0c\int[\delta_{\mu\nu}(1+2\alpha G_0u^0)u^\mu u^\nu]^{\frac{1}{2}}ds = -m_0c\int[\delta_{\mu\nu}(1+2\alpha G_0u^0)dx^\mu dx^\nu]^{\frac{1}{2}}$$

$$= -m_0c\int[(1-\beta^2)(1+2\alpha G_0u^0)]^{\frac{1}{2}}dt = -m_0c\int\left[1-\beta^2+\frac{2}{c^2}U(1-\beta^2)\right]^{\frac{1}{2}}dt$$

$$= -m_0c\int\left[\left(1+\frac{2}{c^2}U\right)-\frac{V^2}{c^2}\right]^{\frac{1}{2}}dt = -m_0c\int\left(g_{\mu\nu}dx^\mu dx^\nu\right)^{1/2} \qquad (5-2)$$

where $g_{00} = 1 + 2U/c^2$, $g_{\alpha\beta} = \delta_{\alpha\beta}$ and $g_{0i} = 0$. The formula (5-2) coincides with Lagrangian integral formula defined in non-relativistic, static and week field in GR. Consequently, the new Lagrangian integral formula, within application limitation in which equivalence principle is valid, coincides with the Lagrangian integral formula in Minkowski space defined in infinitesimal spatiotemporal region in GR. Thus, within just this region of non-relativistic, static and week field is satisfied approximately Einstein's equivalence principle that can eliminate gravitational field in any point of space-time.

*Fourthly, The formula (5-1), under approximation condition $r \gg r_0$ (electron radius,$e^2/m_0c^2$), is transformed into Lagrangian for motion of a particle in Maxwell's electrodynamics.*

The formula (5-1) can be transformed as follows.

$$S = -m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}}\left(1+2\alpha A_\lambda u^\lambda\right)^{\frac{1}{2}} \qquad (5-3)$$

In case of $2\alpha A_\lambda u^\lambda \ll 1$, if one expands formula (5-3) as Taylor series, formula (5-3) can be written as follows,

$$S \approx -m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}} - m_0c\alpha\int A_\lambda \dot{x}^\lambda\, dt$$

When $\alpha = e/m_0c^2$, the following form is obtained

$$dS \approx -m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}} - \frac{e}{c}\int A_\lambda dx^\lambda \qquad (5-4)$$

And then, what is physical meaning implied by the approximate condition $2\alpha A_\lambda u^\lambda \ll 1$? In static and weak field, $u^0 \approx 1$ and $2\alpha A_\lambda u^\lambda \approx 2\alpha\varphi u^0 \approx 2\alpha\varphi$, where $\varphi$ is Coulomb potential. Therefore, $2\alpha A_\lambda u^\lambda$ arrives at

$$2\alpha\varphi = 2\frac{e}{m_0c^2}\cdot\frac{e}{r} = \frac{2e^2}{m_0c^2}\cdot\frac{1}{r} = \frac{r_0}{r} \qquad (5-5)$$

where $r_0 = 2e^2/m_0c^2$ is electron radius (≈10-13cm). Consequently, the approximate condition transformed into Lagrangian for motion of a particle in Maxwell's electrodynamics is corresponded to the case where interaction distance between particles is much farther than electron radius $r_0$. In fact every experiments put in the ground of Maxwell theory were conducted in $r \gg r_0$, i.e., macroscopic region and then, in the process systemizing and generalizing experimental results by inductive method was built Maxwell's electrodynamics. The new electrodynamics is valid in not only region farther than $r_0$ but also neighborhood of $r_0$ or even one smaller than $r_0$.

*Fifthly, formula (5-1) is essential to clarify, in quantum electrodynamics, cause of divergence occurring in approximation of higher order and its natural vanishment (convergence of higher order approximation), and explain, in gravitational theory, three effect verified already by experiments, red shift of light, deflection of light and shift of Mercury's perihelion (considered in next sections).*

From formula (5-1), let us obtain the motion equation of a particle in the field

$$\delta S = -m_0 c\delta \int \left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2} ds = 0 \qquad (5-6)$$

$$\delta \int \left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2} ds =$$

$$= \int \frac{ds}{2\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}} \left[u^\mu u^\nu \left(\frac{\partial g_{\mu\nu}}{\partial x^\lambda} \cdot \delta x^\lambda + \frac{\partial g_{\mu\nu}}{\partial u^\lambda} \cdot \frac{d\delta x^\lambda}{ds}\right) + 2g_{\mu\nu}\frac{dx^\mu}{ds} \cdot \frac{d\delta x^\nu}{ds}\right] = 0$$

Now, by the partial integral of the second term and the third term is obtained

$$\delta \int \left(g_{\lambda\sigma}u^\lambda u^\sigma\right)^{1/2} ds = \int ds\delta x^\lambda \cdot \left[\frac{1}{2} \cdot \frac{\partial g_{\mu\nu}}{\partial x^\lambda} u^\mu u^\nu - \frac{1}{2} \cdot \frac{d}{ds}\left(\frac{\partial g_{\mu\nu}}{\partial u^\lambda} u^\mu u^\nu\right) - \frac{d(g_{\lambda\nu}u^\nu)}{ds}\right]$$

$$= \int \left[\frac{1}{2} \cdot \frac{\partial(2\alpha K_\sigma)}{\partial x^\lambda} u^\sigma u^\mu u^\nu \delta_{\mu\nu} - \frac{1}{2}\frac{d}{ds}\left(2\alpha\delta_{\mu\nu}K_\lambda u^\mu u^\nu\right) - \frac{du_\lambda}{ds}\right] \delta x^\lambda ds = 0$$

where

$$g_{\mu\nu}u^\mu u^\nu = 1, \quad \delta_{\mu\nu}u^\mu u^\nu = \frac{1}{1 + 2\alpha K_\lambda u^\lambda}$$

and then if one rearranges the above formula, the result is

$$m_0 c\frac{du_\lambda}{ds} = \frac{\alpha m_0 c}{1 + 2\alpha K_\mu u^\mu} C_{\lambda\sigma}u^\sigma - K_\lambda \frac{d}{ds}\left(\frac{\alpha m_0 c}{1 + 2\alpha K_\mu u^\mu}\right) \qquad (5-7)$$

where $C_{\lambda\sigma} = \frac{\partial K_\sigma}{\partial x^\lambda} - \frac{\partial K_\lambda}{\partial x^\sigma}$

Let us consider the formula (5-7) for motion of a particle about the cases of electromagnetic field and gravitational field, separately. The motion equation of particle in electromagnetic field is as follows:

$$m_0 c\frac{du_\lambda}{ds} = \frac{1}{c} \cdot \frac{e}{1 + 2\frac{e}{m_0 c^2}A_\mu u^\mu} F_{\lambda\sigma}u^\sigma - \frac{1}{c}A_\lambda \frac{d}{ds}\left(\frac{e}{1 + 2\frac{e}{m_0 c^2}A_\mu u^\mu}\right) \qquad (5-8)$$

Now, let us introduce the so-called *effective electric charge,* $\bar{e}$, dependent on the interaction of particle and electromagnetic field

$$\bar{e} = \frac{e}{1 + 2\frac{e}{m_0 c^2}A_\mu u^\mu} \qquad (5-9)$$

Then formula (5-8) leads to

$$m_0 c\frac{du_\lambda}{ds} = \frac{1}{c}\bar{e}F_{\lambda\sigma}u^\sigma - \frac{1}{c}A_\lambda \frac{d}{ds}(\bar{e}) \qquad (5-10)$$

The motion equation of particle in gravitational field can be written in the following form:

$$m_0 c\frac{du_\lambda}{ds} = \frac{1}{c} \cdot \frac{m_0}{1 + 2\frac{1}{c^2}G_\mu u^\mu} \cdot R_{\lambda\sigma}u^\sigma - \frac{1}{c}G_\lambda \frac{d}{ds}\left(\frac{m_0}{1 + 2\frac{1}{c^2}G_\mu u^\mu}\right) \qquad (5-11)$$

As in formula (5-9), if one introduces effective gravitational mass $\bar{m}_{0g}$, the formula (5-11) arrives at

$$m_0 \frac{du_\lambda}{ds} = \frac{1}{c}\bar{m}_{0g}R_{\lambda\sigma}u^\sigma - \frac{1}{c}G_\lambda \frac{d}{ds}\left(\bar{m}_{0g}\right) \qquad (5-12)$$

where

$$\bar{m}_{0g} = \frac{m_0}{1 + 2\frac{1}{c^2} G_\mu u^\mu} \quad , \qquad R_{\lambda\sigma} = \frac{\partial G_\sigma}{\partial x^\lambda} - \frac{\partial G_\lambda}{\partial x^\sigma}$$

and $G_\mu$ is gravitational potential.

From momentum $P_\lambda = -\partial L / \partial u^\lambda$ follows

$$P_\lambda = m_0 c u_\lambda + \frac{a}{c\left(1 + 2\alpha K_\mu u^\mu\right)} K_\lambda \qquad (5-13)$$

where $a$ is $e$ in case of electromagnetic field and $m_0$ in case of gravitational field. The form of $u_\lambda$ is as follows:

$u_\lambda = g_{\lambda\sigma} u^\sigma =$

$$= \delta_{\lambda\sigma}\left(1 + 2\alpha K_\mu u^\mu\right) \frac{dx^\sigma}{cdt\left[(1-\beta^2)\left(1 + 2\alpha K_\mu u^\mu\right)\right]^{\frac{1}{2}}} = \frac{\dot{x}_\lambda \left(1 + 2\alpha K_\mu u^\mu\right)^{\frac{1}{2}}}{c\sqrt{1-\beta^2}} \qquad (5-14)$$

Now let us introduce effective inertial mass $\bar{m}_0$

$$\bar{m}_0 = m_0 \left(1 + 2\alpha K_\mu u^\mu\right)^{\frac{1}{2}} \qquad (5-15)$$

and effective source $\bar{a}$

$$\bar{a} = \frac{a}{1 + 2\alpha K_\mu u^\mu} \qquad (5-16)$$

(in case of gravitation $a = m_0,\ \bar{a} = \bar{m}_{0g}$ and in case of electromagnetic field, $a = e,\ \bar{a} = \bar{e}$). Thus, formula (5-13) can be written as follows:

$$P_i = \frac{\bar{m}_0 V_i}{\sqrt{1-\beta^2}} + \frac{\bar{a}}{c} K_i \qquad (5-17)$$

$$cP_0 = E = \frac{\bar{m}_0 c^2}{\sqrt{1-\beta^2}} + \frac{\bar{a}}{c} K_0 \qquad (5-18)$$

Next, let us derive the formula for energy of a system, a key to establish new non-linear quantum electrodynamics on the basis of new classical theory. First of all, for the future work on quantum electrodynamics, we confine source $a$ to electric charge $e$. From $g_{\lambda\sigma} = \delta_{\lambda\sigma}\left(1 + 2\alpha K_\mu u^\mu\right)$ and $g_{\lambda\sigma} g^{\lambda\sigma} = 1$, we have

$$g^{\lambda\sigma} = \frac{\delta_{\lambda\sigma}}{1 + 2\alpha A_\mu u^\mu} \qquad (5-19)$$

and

$$g^{\lambda\sigma} u_\lambda u_\sigma = \frac{1}{1 + 2\alpha A_\mu u^\mu} (u_0 u_0 - u_i u_i) = 1$$

$$u_0 u_0 - u_i u_i = 1 + 2\alpha A_\mu u^\mu$$

Consequently, if one uses formula (5-13), (5-17) and (5-18), we obtain the following equation:

$$\left(\frac{E - \bar{e}\varphi}{c^2}\right)^2 - \left(\mathbf{P} - \frac{\bar{e}}{c}\mathbf{A}\right)^2 = \bar{m}_0^2 c^2 \qquad (5-20)$$

$$E = \left[\bar{m}_0^2 c^4 + c^2 \left(\mathbf{P} - \frac{\bar{e}}{c}\mathbf{A}\right)^2\right]^{1/2} + \bar{e}\varphi \qquad (5-21)$$

Finally, let us obtain Hamilton-Jacobi equation of a particle in the field. This equation is obtained by exchanging momentum $\mathbf{P}$ with $\partial S / \partial \mathbf{r}$ and $E$ with $-\partial S / \partial t$. Therefore, from formula (5-20) is derived

$$\left(\mathrm{grad}S-\frac{\bar{a}}{c}\mathbf{K}\right)^2-\frac{1}{c^2}\left(\frac{\partial S}{\partial t}+\bar{a}K_0\right)^2+\bar{m}_0c^2=0 \qquad (5-22)$$

The formula (5-22) is similar to that of former theory. The difference are that $m_0$ and $e$ regarded as constant in Maxwell theory were replaced by effective mass $\bar{m}_0$ and effective electric charge $\bar{e}$, which is very important and significant in the future.

## Sect. 6 Lagrangian for field

This section is devoted to get the equation of field. The Lagrangian for the motion of a particle treated in sect. 5 does not obey principle of linear superposition. Accordingly, in general, the equation of field also becomes a non-linear equation not subject to principle of linear superposition. Of course, the field does not become the arithmetical sum of fields created by individual particles. But, unfortunately, in the paper, owing to mathematical poverty of authors, was not obtained non-linear differential equation, field equation in the most general form (nonlinear differential field equation in most general form) not subject to principle of superposition, whereas we get the approximate equation of field valid within some application limitation.

First of all, let us find new Lagrangian for field by generalizing Maxwell's equation of field to the equation of field established in four dimensional non-Euclidean space and next consider application limitation of the resultant field equation. Generalizing Maxwell's Lagrangian for field to non-Euclidean space, the result is

$$S_f=-\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}d\Omega \qquad (6-1)$$

where as understood easily, $F_{ik}$ is asymmetry tensor and in terms of curved surface geometry, the form of $F_{ik}$ coincides with that of Euclidean space. The only difference is integral volume multiplied by $\sqrt{-g}$. Therefore, the total Lagrangian integral formula can be written as follows:

$$S=-m_0c\int\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}ds-\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}d\Omega \qquad (6-2)$$

or

$$S=-m_0c\int\left(g_{\mu\nu}x^\mu x^\nu\right)^{1/2}dt-\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}d\Omega$$

where $g_{\mu\nu}=\delta_{\mu\nu}\left(1+2\alpha A_\lambda u^\lambda\right)$ and $d\Omega=cdtdxdydz$. If one uses

$$F^{ik}=g^{i\lambda}F_\lambda^k=g^{i\lambda}g^{k\sigma}F_{\lambda\sigma}=-\frac{1}{\sqrt{-g}}F_{ik}$$

and regards the motion of a particle as being given, by variation principle is derived the equation of field

$$\frac{c}{4\pi}\frac{\partial F_{ik}}{\partial x^k}=-\frac{1}{1+2\alpha A_\mu u^\mu}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (6-3)$$

or

$$\frac{\partial F_{ik}}{\partial x^k}=-\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (6-4)$$

In fact, equation (6-4) should include the non-linear component term destroying principle of linear superposition in left-hand side. However, now that we did not find this component, we can ignore the component, supposing that it is extremely small. That is why equation (6-4) is obviously the approximate equation.

Now let us consider application limitation of the equation. If one takes four-dimensional divergence on both sides of equation (6-4), the left-hand side of equation becomes zero and right-hand side leads to

$$\frac{\partial}{\partial x^i}(\bar{e}V^i) = \frac{\partial \bar{e}}{\partial x^i}V^i = \frac{d\bar{e}}{dt} \qquad (6-5)$$

by asymmetry of $F_{ik}$. The formula (6-5) is similar to the law of charge conservation which holds in case of $\bar{e} = e$. But in general effective charge is function of space-time, and then as $d\bar{e}/dt \neq 0$, conservation of effective charge is invalid. Therefore, only in case of

$$\frac{d\bar{e}}{dt} = 0 \qquad (6-6)$$

equation (6-4) is valid. That is to say, the application conditions of (6-4) is determined by formula (6-6) and they are as follows:

*Firstly, when a charge is constant (i.e. in case of a free particle), the formula (6-6) is valid and then equation (6-4) holds.*

*Secondly, in the system of static particles is also satisfied formula (6-4).* Actually in the system of static particles, as interaction term $A_\mu u^\mu$ in the form relevant to effective charge is constant, formula (6-6) holds.

*Thirdly, for average electromagnetic field produced by the system of a particle in which* $\overline{\frac{d\bar{e}}{dt}} = 0$ *is satisfied, the formula (6-4) is valid (where* $\overline{\frac{d\bar{e}}{dt}} = 0$ *is time-average of effective charge).* In case where electric charges moving within finite area possess finite momentum, this motion comes to have stationary characteristics and then we only can consider average electro-magnetic field produced by them. This field depends only on spatial coordinate but not on time coordinate. In this case, (6-4) becomes

$$\frac{\partial \overline{F_{ik}}}{\partial x^k} = -\frac{4\pi}{c}\overline{\bar{e}V^i}\delta(r - r_0) \qquad (6-7)$$

*Fourthly, for field produced by charge moving in external field so that* $\left|\frac{d\bar{e}}{dt}\right| \ll 1$ *is satisfied,* $\frac{d\bar{e}}{dt}$ *can be enough ignored and then formula (6-4) can hold approximately.* Now, let us consider this approximate condition

$$\frac{d\bar{e}}{dt} \to 2\alpha\left(1 + 2\alpha A_\mu u^\mu\right)^{-2}\frac{d}{dt}\left(eA_\mu u^\mu\right) \approx 2\frac{e^2}{mc^2}\frac{d\varphi}{dt} \qquad (6-8)$$

where $e^2/mc^2$ (about $10^{-17}$) is the extremely small quantity and then except when field changes very rapidly, $\frac{d\bar{e}}{dt}$ can be enough ignored. Consequently, under the condition $\left|\frac{d\bar{e}}{dt}\right| \ll 1$, the equation (6-4) has enough validity.

For gravitational field, the field equation is also formally equal to the equation of electromagnetic field.

$$\frac{c}{4\pi}\frac{\partial R_{ik}}{\partial x^k} = -\frac{1}{1 + 2\alpha_{(g)}G_\mu u^\mu}V^i\delta(r - r_0) \qquad (6-9)$$

$$R_{ik} = \frac{\partial G_k}{\partial x^i} - \frac{\partial G_i}{\partial x^k}$$

The equation (6-9) includes Poisson equation for static gravitational field and this yields main theoretical results in the static field, obtained by Einstein.

## Sect. 7 Energy-momentum tensor of particle-field

The total Lagrangian for matter and field can be written as follows:

$$S = \frac{1}{c}\int d\Omega\sqrt{-g}\Lambda = \int d\Omega\left(L_m + L_f\right) =$$

$$= \frac{1}{c}\int d\Omega\sqrt{-g}\left[-\frac{m_0 c}{\sqrt{-g}}\left(g_{\mu\nu}V^\mu V^\nu\right)^{\frac{1}{2}}\delta(\mathbf{r}-\mathbf{r}_0) - \frac{1}{16\pi}C_{ik}C^{ik}\right] \qquad (7-1)$$

where $d\Omega = cdtdxdydz$, $L_m$ Lagrangian of matter, $L_f$ Lagrangian of field, $C_{ik}$ is $F_{ik}$ in case of electromagnetic field and $R_{ik}$ in case of gravitational field. If one introduces infinitesimal transformation $x'^i = x^i + \delta x^i$ and then uses the condition $\delta S = 0$, the well-known formula is obtained

$$\frac{\partial\sqrt{-g}T_i^k}{\partial x^k} = 0 \qquad (7-2)$$

$$T_{ik} = \frac{2}{\sqrt{-g}}\left[\frac{\partial}{\partial x^l}\left(\frac{\sqrt{-g}\Lambda}{\partial\frac{\partial g^{ik}}{\partial x^l}}\right) - \frac{\partial\sqrt{-g}\Lambda}{\partial g^{ik}}\right] = T_{ik}^{(m)} + T_{ik}^{(f)}$$

$$T_{ik}^{(m)} = \frac{1}{\sqrt{-g}}m_0 c u_i u_k\frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0) \qquad (7-3)$$

$$T_{ik}^{(f)} = -\frac{1}{4\pi}\left(C_{i\lambda}C_k^\lambda - \frac{1}{4}C_{lm}C^{lm}g_{ik}\right) \qquad (7-4)$$

where $T_{ik}$ is total energy-momentum tensor and $T_{ik}^{(m)}$ energy-momentum tensor of particle and $T_{ik}^{(f)}$ energy-momentum tensor of field. When obtain formula (7-3) and formula (7-4), the followings were used:

$$C_{i\lambda} = -C_{\lambda i}, \qquad C^{kl} = g^{k\lambda}C_\lambda^l = g^{k\lambda}g^{l\sigma}C_{\lambda\sigma}, \qquad C^{kl} = \frac{-1}{\sqrt{-g}}C_{ki} \qquad (7-5)$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} \qquad (7-6)$$

Now let us prove the conservation of energy-momentum in details. First of all, divergence of the energy-momentum tensor of a particle is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}T_i^{k(m)}\right) = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left[\sqrt{-g}\frac{1}{\sqrt{-g}}m_0 c u_i u^k\frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0)\right] =$$

$$= \frac{1}{\sqrt{-g}}m_0 c\left[\frac{\partial u_i}{\partial x^k}u_k\frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0) + \frac{\partial}{\partial x^k}\left(V^k\delta(\mathbf{r}-\mathbf{r}_0)\right)u_i\right] = \frac{1}{\sqrt{-g}}m_0 c\frac{du_i}{dt}\delta(\mathbf{r}-\mathbf{r}_0)$$

$$(7-7)$$

Next, divergence of energy-momentum tensor of field is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left[\sqrt{-g}\left(-\frac{1}{4\pi}\right)\left(C_{il}C^{kl} - \frac{1}{4}C_{lm}C^{lm}\delta_i^k\right)\right] =$$

$$= -\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}\left[\left(\frac{\partial C_{il}}{\partial x^k}\sqrt{-g}C^{kl} + C_{il}\frac{\partial\left(\sqrt{-g}C^{kl}\right)}{\partial x^k} - \frac{1}{2}\frac{\partial C_{lm}}{\partial x^i}\sqrt{-g}C^{lm}\right)\right]$$

If one uses the identity

$$\frac{\partial C_{lm}}{\partial x^i} = -\frac{\partial C_{mi}}{\partial x^l} - \frac{\partial C_{il}}{\partial x^m}$$

and rearrange the above formula, first term and third term are cancelled. Hence, the following term only remains.

$$-\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{kl}\right)}{\partial x^k}=\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{lk}\right)}{\partial x^k}$$

Therefore, divergence of energy-momentum tensor of field, allowing for formula (7-5) and formula (6-3), is as follows:

$$\nabla_k T_i^{k(f)}\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{lk}\right)}{\partial x^k}=-\frac{1}{c\sqrt{-g}}C_{il}\bar{e}V^l\delta(\mathbf{r}-\mathbf{r}_0)\qquad(7-8)$$

Using formula (7-2), (7-7), (7-8), (5-10) and (6-6), divergence of the total energy-momentum te nsor of field and particle can be written as

$$\frac{\partial\left(\sqrt{-g}T_i^k\right)}{\partial x^k}=\frac{\partial\left(\sqrt{-g}T_i^{k(m)}\right)}{\partial x^k}+\frac{\partial\left(\sqrt{-g}T_i^{k(f)}\right)}{\partial x^k}=$$

$$=m_0c\frac{du_i}{dt}\delta(\boldsymbol{r}-\boldsymbol{r}_0)-C_{ik}\frac{\bar{e}}{c}V^k\delta(\boldsymbol{r}-\boldsymbol{r}_0)=0\qquad(7-9)$$

Thus, in case where the effective charge agrees with static and stationary condition (6-6), i.e. $\frac{d\bar{e}}{dt}=0$, $\overline{\frac{d\bar{e}}{dt}}=0$, and $\left|\frac{d\bar{e}}{dt}\right|\approx 0$, formula (7-9) yields conservation law of energy-momentum.

## Sect. 8 Electromagnetic-gravitational isotopic vector space and unified Lagrangian

In the sect. 6 and 7 was treated the electromagnetic field and gravitational field, separately. In this section, we obtain unified Lagrangian by subjecting electromagnetic field and gravitational field to one metric of space-time. First of all, we, in order to describe unified Lagrangian in a simple and easy mathematical form, introduce the electromagnetic-gravitational isotopic vector space which is essential to understand physical meaning of integral formula.

When electromagnetic field and gravitational field exist together and act on a charged particle, the metric of space-time is given as follows:

$$g_{\lambda\sigma}=\delta_{\lambda\sigma}\left(1+2\alpha^{(E)}A_Ku^K+2\alpha^{(g)}G_Ku^K\right)\qquad(8-1)$$

where clearly $\alpha^{(E)}$ contains electric charge $e$ and $\alpha^{(g)}$ contains mass $m$. Now instead of taking product of source of fields $(e,m)$ and fields $(A_K,G_K)$ to be separate sum as like (8-1), let us write product as the following simple form, i.e. $\alpha\left(\alpha^{(E)},\ \alpha^{(g)}\right)\cdot K^i\left(G^i,A^i\right)$, where $\alpha$ and $K^i$ should be considered as vectors of the electromagnetic-gravitational isotopic vector space.

When a charged particle moves, there exist naturally current of charge and current of mass together and these current, as both sides of motion of particle, are subject to position and velocity of a particle. Likewise, electromagnetic field and gravitational field, as fields of two forms created by a particle, are no more separate and independent and then the total energy of a particle and its field is equivalent to inertial mass and subject to it. In this regard, let us define electromagnetic-gravitational isotopic vector space, based upon the idea that charge and mass are both properties of a particle, and electromagnetic field and gravitational field are both sides of unified field.

Easily speaking, isotropic vector space is two-dimensional space in which $\left(e,A_i,\alpha^{(E)}\right)$ and $\left(m,G_i,\alpha^{(g)}\right)$ are orthogonal each other. Let us carry the more tangible consideration about isotropic vector space.

*1. The unit isotopic vector of electric charge, e, and electromagnetic field, $A_i$, are the same, i.e.*

$$\hat{e}\cdot\hat{A}_i=eA_i,\quad \hat{\alpha}^{(E)}\cdot\hat{A}_i=\frac{\hat{e}}{mc^2}\cdot\hat{A}_i=\frac{e}{mc^2}A_i\qquad(8-2)$$

where ^ denote isotopic vector.

*2. The unit isotopic vector of mass $m$ and gravitational field $G_i$ produced by it are the same.*

$$\widehat{m} \cdot \hat{G}_i = mG_i, \quad \hat{\alpha}^{(g)} \cdot \hat{G}_i = \frac{\widehat{m}}{mc^2} \cdot \hat{G}_i = \frac{1}{c^2} G_i \qquad (8-3)$$

From formula (8-3) follows an important conclusion that the self-interaction of mass and self-field created by it is always positive value and accordingly, with radiation of gravitational wave by radiation damping, matter comes to lose energy. GR does not yield this conclusion. (See sect. 3)

*3. The unit isotopic vector of electric charge* $\hat{e}$ *and electromagnetic field* $\hat{A}_i$ *are orthogonal to the unit isotopic vector of mass* $\widehat{m}$ *and gravitational field* $\hat{G}_i$*.*

$$\hat{e} \cdot \widehat{m} = 0, \quad \hat{e} \cdot \hat{G}_i = 0, \quad \widehat{m} \cdot \hat{A}_i = 0, \quad \hat{A}_i \cdot \hat{G}_i = 0 \qquad (8-4)$$

This reaches the conclusion that electromagnetic field acts only on the charge and gravitational field acts only on mass and then exchange of photon is possible only between charges and exchange of graviton is possible only between masses.

*4. The product of mass* $\widehat{m}$ *and external gravitational field* $\hat{G}_i$ *acting on it is always negative value*

$$\widehat{m} \cdot \hat{G}_i = -mG_i \qquad (8-5)$$

This follows from the fact that gravitational force is always attractive. In this space, source of field is expressed as a single source, $\hat{\alpha}(\hat{e}, \widehat{m})$ and $\hat{\alpha}\left(\hat{\alpha}^{(E)}, \hat{\alpha}^{(g)}\right)$, and field also becomes a unified field $\widehat{K}_i\left(\hat{A}_i, \hat{G}_i\right)$.

The electromagnetic-gravitational isotopic vector space makes it possible to denote Lagrangian of particle-field in a unified form and in very easy, clear form

$$S = -m_0 c \int \left(g_{\mu\nu} u^\mu u^\nu\right)^{1/2} ds - \frac{1}{16\pi c} \int \hat{C}_{ik} \hat{C}^{ik} \sqrt{-g}\, d\Omega \qquad (8-6)$$

where $g_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\hat{\alpha}\widehat{K}_\lambda u^\lambda\right)$ and

$$\hat{\alpha} \to \left(\frac{\hat{e}}{mc^2}, \frac{\widehat{m}}{mc^2}\right)$$

$$\widehat{K}_i \to \left(\hat{A}_i, \hat{G}_i\right)$$

$$\hat{C}_{ik} = \frac{\partial \widehat{K}_k}{\partial x^i} - \frac{\partial \widehat{K}_i}{\partial x^k}$$

From this, the motion equation of particle (5-8) and (5-11) can be written as

$$m_0 c \frac{du_\lambda}{ds} = \frac{1}{c} \cdot \frac{m_0 c \hat{\alpha}}{1 + 2\hat{\alpha}\widehat{K}_i u^i} \cdot \hat{C}_{\lambda\sigma} u^\sigma - \widehat{K}_\lambda \frac{d}{ds}\left(\frac{m_0 c \hat{\alpha}}{1 + 2\hat{\alpha}\widehat{K}_i u^i}\right) \qquad (8-7)$$

On the other hand, the field equation in isotropic vector space is

$$\frac{1}{4\pi} \frac{\partial \hat{C}_{ik}}{\partial x^k} = -\frac{1}{c} \cdot \frac{\hat{e} + \widehat{m}}{1 + 2\hat{\alpha}\widehat{K}_\mu u^\mu} V^i \delta(\mathbf{r} - \mathbf{r}_0) \qquad (8-8)$$

If one writes equation (8-8) separately according to components of isotopic vectors, the formula (6-3) and (6-10) are expressed as follow:

$$\frac{1}{4\pi} \frac{\partial \widehat{F}_{ik}}{\partial x^k} = -\frac{1}{c} \cdot \frac{\hat{e}}{1 + 2\hat{\alpha}\widehat{K}_\mu u^\mu} V^i \delta(\mathbf{r} - \mathbf{r}_0) \qquad (8-9)$$

$$\frac{1}{4\pi} \frac{\partial \widehat{R}_{ik}}{\partial x^k} = -\frac{1}{c} \cdot \frac{\widehat{m}}{1 + 2\hat{\alpha}\widehat{K}_\mu u^\mu} V^i \delta(\mathbf{r} - \mathbf{r}_0) \qquad (8-10)$$

where

$$\widehat{R}_{ik} = \frac{\partial \hat{G}_k}{\partial x^i} - \frac{\partial \hat{G}_i}{\partial x^k}$$

The energy and momentum tensor of electromagnetic-gravitational field is

$$T_{ik}^{(f)} = -\frac{1}{4\pi}\left(\hat{C}_{i\lambda}\hat{C}_k^{\lambda} - \frac{1}{4}\hat{C}_{lm}\hat{C}^{lm}g_{ik}\right) \qquad (8-11)$$

or

$$T_i^{k(f)} = -\frac{1}{4\pi}\left(\hat{C}_{i\lambda}\hat{C}^{k\lambda} - \frac{1}{4}\hat{C}_{lm}\hat{C}^{lm}\delta_i^k\right) \qquad (8-12)$$

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