The title

Proof to the twin prime conjecture

Abstract

The elementary proof to the twin prime conjecture.

The content of the article Let p_s denote the s'th prime and $P_s$the product of the first s primes.

Define A_s to be the set of all positive integers less than P_s which are relatively prime to P_s

1. Each A_s, for $s \geq 3$, contains two elements which differ by 2.

2. Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$. There exists $m1$ in two different arithmetic progression such that $a1 + m1P_s$ and $a2 + m1P_s$ are prime numbers.

3. Combining 1) and 2), there is always a pair of twin primes which are relatively prime to P_s, and therefore infinitely many twin primes.

For every pair of values a, b in A_s differing by d, there exist at least $p_{s+1}-2$ pairs of values in A_{s+1} differing by d. (And exactly that many when d is not divisible by p_{s+1}).

Given this, the claim follows using induction with $d = 2$, noting for the base case that $11, 13$ are both in A_3.

The proof to 1 is as follows: Suppose a and b are in A_s, with $b - a = d$. Consider the set of values $a + mP_s$,

where $0 \leq m < p_{s+1}$. These are all less than P_{s+1}, and since P_s is relatively prime to p_{s+1},

there is a unique value $m1$ with $a + m1P_s$ divisible by p_{s+1}.

Similarly, there is a unique value $m2$ with $b + m2P_s$ divisible by p_{s+1}.

Furthermore, if $m1 = m2$, then $(b + m2P_s) - (a + m1P_s) = d$ would be divisible by p_{s+1}.

So when d is not divisible by p_{s+1}, for the $p_{s+1} - 2$ values of $0 \leq m < p_{s+1}$ which are not equal to $m1$ or $m2$, the pair $(a + mP_s, b + mP_s)$ are a pair in A_{s+1} differing by d.

The proof to 2 is as follows:

Consider the finite arithmetic progression $\{a + mP_s\}$, where a is in A_s and $0 \leq m < P_s$.

If $a1 + m1P_s$ is divisable by f then $m1$ is the only unique value from the interval $\geq o$ and $< o + f$ that $a1 + m1P_s$ is divisable by f.

All non-prime numbers greater than 1 must be divisable by an odd number greater than or equal to 3 and less than or equal to $P_s - 1$.

Consider two finite arithmetic progression in $\{a + mP_s\}$, where a is in A_s and $a \neq 1$ and $0 \leq m < P_s$.

If $a1 + m1P_s$ is divisable by f then $m1$ is the only unique value from the interval $\geq o$ and $< o + f$ that $a1 + m1P_s$ is divisable by f.

If $a2 + m2P_s$ is divisable by f then $m2$ is the only unique value from the interval $\geq o$ and $< o + f$ that $a2 + m2P_s$ is divisable by f.

To count the number of maximum non-prime numbers generated per unique m in either $a1 + m1P_s$ or $a2 + m2P_s$ assume if number is divisable by f that number is non-prime and assume if $a1 + m1P_s$ and $a2 + m2P_s$ is divisable by f that $m1 \neq m2$

Base case is $f = P_s - 1$ and $0 \leq m < f$ There exist $f - 2$ different values of m such that $a1 + mP_s$ or $a2 + mP_s$ is not divisable by f

Induction is count the number of elements divisable by the next smaller odd number where next smaller odd number is $f - 2$ and assume $a1 + mP_s$ or $a2 + mP_s$ is not divisable by f when $0 \leq m < f - 2$.

Stop after counting number of elements divisable by 3.

Therefore there exists $m5$ in two different arithmetic progression such that $a1 + m5P_s$ and $a2 + m5P_s$ are prime numbers.