The G-connections

A.Balan

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Abstract

We define the notion of G-connections over vector fiber bundles.

1 The usual connections

A connection of Koszul is ∇ which verifies:

$$\nabla_{fX}(s) = f\nabla_X(s)$$

$$\nabla_X(fs) = (Xf)s + f\nabla_X(s)$$

X is a vector field, s is a section and f is a function.

2 The G-connections

2.1 Definition

We suppose that we have a vector fiber bundle E over a manifold M and an action of a Lie group G over it.
A G-connection is ∇ which verifies:

$$\nabla_{fX}(s) = f\nabla_X(s)$$

$$\nabla_X(g.s) = Xg.s + g.\nabla_X(s)$$

with X a vector field, s a section and g in the gauge group $\mathcal{G} = \Gamma(M, G)$.

2.2 Properties

The curvature of ∇ commutes with the Lie action of G.

3 Bibliography

S.Gallot, D.Hulin, J.Lafontaine, "Riemannian Geometry", Springer, Berlin, 2004.
J.Jost, "Riemannian Geometry and Geometric Analysis", Springer, Berlin, 2008.