Constant $e \cdot c/2\pi\,\alpha$ determines magnetic flux quantum in charged leptons

U. Kayser-Herold

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Abstract

The constant $e \cdot c/2\pi\,\alpha$ is a common characteristic of charged leptons (e, μ, τ) resulting from their identical fraction $\hat{m}/\lambda_C$ of magnetons $\hat{m}$ to Compton-wavelengths λ_C, in spite of their largely differing $\hat{m}$ and λ_C. However the physical interpretation of this constant remained uncertain, but now clarified: It is proven that $e \cdot c/2\pi\,\alpha$ is an alternative and equivalent definition of the magnetic flux quantum $h/2\,e$ which makes up the dipole-fields of charged leptons.

1 Introduction, problem and goal

Comprehensive consideration and incorporation of magnetic flux quantization in theories of charged leptons like e, μ, τ has not yet been established. [1]
On the one hand, the magnetic dipole-moments $\hat{m}$ (magnetons) assignable to e, μ and τ belong to the theoretically and experimentally most precisely determined constants. (CODATA) On the other hand, only rough estimates exist for the magnetic flux included in their dipole-fields. However the existence of a presumably constant magnetic flux in the dipole-fields of leptons is reasonable. Moreover, consideration of flux quantization would support the conjecture of magnetic flux conservation in the dipole-fields of charged leptons. Nonetheless current electron-models like the "mathematical" or "dressed" electron don't take into account the magnetic flux included in its dipole-field.
If the principle of magnetic flux quantization was applied to charged leptons the ab-initio postulate that each of their dipole-fields comprises identical magnetic flux amounting at least one magnetic flux quantum $\Phi_0 = h/2e$ would have to be proven. This initial assumption is supported by the finding that the constant $e \cdot c/2\pi\,\alpha$ - being assigned to charged leptons - is an implicit definition of the magnetic flux quantum presumably located in charged leptons. The approach will essentially consist of an analysis of the dipole-fields of charged leptons - focusing on their magnetic flux Φ_l - with the aim to clarify the physical interpretation of the constant $e \cdot c/2\pi\,\alpha$. It can be anticipated that there exists a fundamental relationship among $e \cdot c/2\pi\,\alpha$, the dipole-field of charged leptons,

the "classical-" or "Compton-" radius of leptons and flux quantization. [2]

2 Modeling of the dipole-field of charged leptons

2.1 Frame of reference

For modeling and analysis of the dipole-field of charged leptons the choice of an adequate frame and plane of reference is crucial. Thus a spherical r, Θ, φ and a cartesian $x - y - z$ - frame will be used. Their polar axis coincide with the dipole-axis.

2.2 Equatorial plane of reference p_e

The most convenient plane of reference for determination of the magnetic flux Φ penetrating this plane is the equatorial (azimuthal) $x-y$ plane p_e of the dipole. Let $\vec{n}$ designate the normal unit-vector of p_e. Among the specific advantages of p_e is that it is a symmetry-plane and that the angle of incidence of the induction $\vec{B}$ field into p_e always is perpendicular to p_e, i.e. $\vec{B} \perp p_e$ or $\vec{B} \parallel \vec{n}$.

2.3 Induction-field of a point-like dipole

The induction- or $\vec{B}$- field of a point-like dipole $\hat{m}$ can most conveniently be represented by the $\vec{B}$- field generated by a microscopic circular current-loop of radius r_x located in p_e, where its axis is centered with the z-axis. [3]
Generally, any dipole-field traversing the equatorial plane p_e comprises mutually opposed internal and an external $\vec{B}$- field sections designated $\vec{B}_{int}$ and $\vec{B}_{ext}$, corresponding to magnetic fluxes Φ_{int} and Φ_{ext}. Φ_{int} and Φ_{ext} are delimited by their generating circular current-loop in p_e . The equatorial plane p_e will serve as a reference plane for determination of Φ_{int} and Φ_{ext}.
According to Maxwell's equation $\nabla \cdot \vec{B} = 0 \longrightarrow \Phi_{int} = -\Phi_{ext}$. Hence it will suffice to determine Φ_{ext} so there is no need to dwell with Φ_{int}. Φ_{ext} will be identified with total magnetic flux of a dipole-field.

3 External magnetic flux Φ_{ext} of a dipole-field

The external induction field of a dipole is

$$\vec{B}_{ext} \approx \frac{\mu_0 \, \hat{m}}{4\pi \, r^3} \Big(2 \, cos \, \Theta \, + sin \, \Theta \Big) \qquad (1)$$

where $\mu_0 = 1/\epsilon_0 c^2$ is vacuum permeability, $\hat{m}$ the dipole-moment or magneton, Θ the polar angle and $r > r_c$ the radial distance of a point from the origin.

Generally, the magnetic flux Φ of an induction field $\vec{B}$ penetrating a given surface $\mathcal{A}$ of arbitrary orientation $\vec{n}$ is

$$\Phi = \int_{\mathcal{A}} d\Phi = \int_{\mathcal{A}} \vec{B}\,\vec{n}\,da \tag{2}$$

where da is a surface element differential.

In p_e, the following applies: $\vec{n} \parallel \vec{B}$ or $\vec{n} \cdot \vec{B} = |\vec{B}|$.

For all points in p_e with $r > r_c$:
$\Theta = \pi/2 \rightarrow\ cos\,\Theta = 0,\ sin\,\Theta = 1,\ \vec{B}(\vec{r}) \parallel z$ and $\vec{B}(\vec{r}) \perp p_e$.

Thus the first term in brackets in (1) vanishes and (1) is reduced to

$$B_{ext} = \frac{\mu_0\,\hat{m}_e}{4\,\pi\,r^3} \tag{3}$$

Total external magnetic flux Φ_{ext} through p_e results from integration of (3) over p_e from r_c to ∞.

$$\Phi_{ext} = \int_{r_c}^{\infty} \vec{B}_r da = 2\pi \int_{r_c}^{\infty} \vec{B}_r r dr \tag{4}$$

where

$$da = 2\pi r dr \tag{5}$$

After substitution with (5) and integration of (4)

$$\Phi_{ext} = \frac{\mu_0 \hat{m}_e}{2 r_c} \tag{6}$$

It should be pointed out that (6) reveals how Φ_{ext} is determined by a singular variable r_e (classical electron- or Compton-radius). However r_e should not be misinterpreted as the radius of a classical sphere thus more of a self-determined length-unit allocable to the electron without any geometrical meaning.

3.1 Delimiting "critical" radius r_e

The radius r_c in (4) and (6) is a lower or "critical" integration limit in the dipole-field being identical with the radius of a.m. circular current-loop. As r_c delimits the mutually opposed internal flux Φ_{int} from the external flux Φ_{ext}, r_c is the key variable to be determined. For the electron the author already proved in [4] that r_c is identical with the "classical" or "charge" radius r_e.
r_e can also be expressed as a function of the Compton-wavelength λ_{Ce} thus being a self-defined length-unit of the electron:

$$r_c = r_e = \frac{\alpha \lambda_{Ce}}{2\pi} = \alpha \frac{\hbar}{m_e\, c} \tag{7}$$

where α = fine-structure constant, m_e = electron-mass and λ_{Ce} = electron Compton-wavelength.
Substitution with (7) in (6):

$$\Phi_{ext} = \frac{\mu_0}{2\,\alpha} \cdot \frac{\hat{m}_e}{\lambda_{Ce}} = \frac{\hat{m}_e}{2\,\alpha \cdot \epsilon_0\, c^2\, \lambda_{Ce}} \tag{8}$$

To facilitate interpretation of (8) it can be split into more meaningful factors:

$$\lambda_{C_e} = \frac{h}{m_e\, c} \tag{8 a}$$

$$\alpha = \frac{e^2}{4\,\pi\,\epsilon_0\,\hbar\, c} \tag{8 b}$$

$$\hat{m}_e = \frac{e\,\hbar}{2\, m_c} = \frac{e\, h}{4\,\pi\, m_c} \tag{8 c}$$

By substitution of (8a), (8b) and (8c) in (8) the result reveals

$$\Phi_{ext} = \frac{h}{2\, e} = \Phi_0 \tag{9}$$

where Φ_0 is the magnetic flux quantum!

Remarkably in (8), Φ_{ext} is exclusively determined by the constants ϵ_0, c, α, $\hat{m}_e$, λ_{Ce}.

Let

$$C_\Phi = \frac{\hat{m}_e}{\alpha\, \lambda_{Ce}} = \frac{e \cdot c}{2\pi\, \alpha} \tag{10}$$

Substitution with (10) in (8) yields

$$\Phi_{ext} = \frac{\mu_0}{2} \cdot C_\Phi = \mu_0 \cdot \frac{e \cdot c}{4\pi\, \alpha} = \Phi_0 \tag{11}$$

where Φ_0 is the magnetic flux quantum.

Note that $\Phi_0 = \Phi_{ext} = -\Phi_{int}$.

In p_e, Φ_{ext} is delimited from Φ_{int} by a circle of Compton-radius $\alpha\lambda_C/2\pi$ (7).

3.2 Critical radius of Myons and Tauons

It is evident from (8) that above approach for the electron would also apply for myons and tauons if their individual masses m_l, magnetons $\hat{m}_l$ and Compton-wavelengths λ_{Cl} were used.
Substitution in (8c) with a generalized lepton-mass m_l (instead of m_e) and a general lepton-magneton $\hat{m}_l$ (instead of $\hat{m}_e$) yields

$$\hat{m}_l = \frac{e\, h}{4\pi\, m_l} \tag{12}$$

as well as a generalized Compton-wavelength λ_{C_l}

$$\lambda_{Cl} = \frac{h}{m_l\, c} \tag{13}$$

The ratio (12) to (13) $\times\, \alpha^{-1}$ delivers a constant C_Φ for all charged leptons:

$$C_\Phi = \frac{\hat{m}_l}{\alpha\, \lambda_{Cl}} = \frac{e \cdot c}{2\, \pi\, \alpha} \quad [\frac{A}{m}\, m^2] \tag{14}$$

As (14) is identical with (10) constant magnetic flux Φ_l of all charged leptons can be inferred.
The dimensionality of C_Φ corresponds to a total flux of a magnetic field-strength vector-field $\vec{H}$ through p_e being in proportion to total flux of the induction-field $\vec{B} = \mu_0 \vec{H}$ through p_e.
Substitution in (8) with (10) and (12) yields

$$\Phi_l = \frac{\mu_0}{2} C_\Phi = \frac{h}{2\, e} = \Phi_0 \tag{15}$$

In conclusion (14) and (15) prove that all charged Leptons carry one magnetic flux quantum Φ_0.

4 Summary and Comment

A new commonality among charged leptons e, μ, τ is revealed: Each of their dipole-fields coincides with one magnetic flux quantum being determined by their common constant $e \cdot c/2\pi\alpha$ being given by the fraction $\hat{m}_l/\lambda_{Cl}$ of their individual magnetons $\hat{m}_l$ and Compton-wavelengths λ_{Cl}.
This conclusion is based on analysis of their magnetic dipole-fields which unravels that the constant $e \cdot c/2\pi\, \alpha$ is an equivalent (hidden) definition of the magnetic flux-quantum $h/2\, e$. It denotes identical flux of the leptons dipole fields through their equatorial planes - outside of their Compton-radii - being equivalent to one magnetic flux quantum $h/2\, e$.
The mathematical procedure includes breaking of the fine-structure constant α into its factors (8 b). The analysis further reveals a remarkable and unexpected role of the "classical" or "Compton" electron radius r_e (and its equivalents for myons and tauons), being intimately related to the magnetic flux quantum, Josephson's constant, spin-angular momentum $\hbar/2$ and the constant $e \cdot c/2\, \pi$, even if r_e is regarded as a fictitious length. [5]
The task could also be inverted by asking for a critical radius r_c which delimits one fluxon Φ_0 in p_e following a transformation of (6): $r_c = \mu_0\, \hat{m}_l/2\, \Phi_0$.

It deserves a final remark that (14), (15) would also apply for protons.

References

[1]] F. Gömöry: Superconductor Dynamics (2015)

[2]] R. Heyrovska, S. Narayan: Fine-structure Constant, Anomalous Magnetic Moment, Relativity Factor and the Golden Ratio that Divides the Bohr Radius.

[3] Blinder, S M: Singularity-free electrodynamics for point charges and dipoles: A classical model for electron self-energy and spin. Eur. J. of Phys. 24 (2003) 271-275;

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[5] Kayser-Herold, U: Electron spin 1/2 is "hidden" electromagnetic field angular momentum