Black Hole Mass Lost By Both Thermodynamics and Hydrodynamics Effects*

R. T. Ndongmo,[1] S. Mahamat,[2] T. B. Bouetou,[1,3] and T. C. Kofane[1,4]

[1] Department of Physics, Faculty of Science, University of Yaounde I, P.O. Box. 812, Yaounde, Cameroon,
[2] Department of Physics, Higher Teachers Training College, University of Maroua, P.O. Box 55, Maroua, Cameroon,
[3] Ecole Nationale Suprieure Polytechnique, University of Yaounde I, P.O. Box. 8390, Yaounde, Cameroon
[4] The African Center of Excellence in Information and Communication Technologies, Yaounde, Cameroon

We purpose a new approach of black hole mass decreasing, which takes into account thermodynamics and hydrodynamics processes, in the presence of quintessence and phantom dark energies. Accordingly, we insert some terms into the Schwarzschild metric in order to obtain a new expression of the black hole mass. Further, we show that by the thermodynamics process, the second-order phase transition does not occur when we take into account phantom energy, except for complex entropies or relativistic time. At the end, we show a new principle to analyze black holes coalescence into a space-time, dilated by dark energy.

PACS numbers: 04.70.s, 95.30.Lz, 04.70.Dy, 98.80.Cq

Introduction. —The discovery in the early 1970s that black holes radiate as black bodies has radically affect our understanding of general relativity, and offer us some early hints about the nature of quantum gravity. Since the seminal works of Hawking[1] and Bekenstein[2], it is understood that black holes behave as thermodynamic objects, with characteristic temperature and entropy.

Accelerated expansion of the universe is the most recent fascinating result of observational cosmology. To explain the accelerated expansion of the universe, it is proposed that the universe is regarded as being dominated by an exotic scalar field with a large negative pressure called "dark energy", which constitutes about 70% of the total energy of the universe[3]. Thus it is the major component of the universe. There are several candidates for dark energy. "Quintessence" is one among them. It is characterized by a parameter ϵ, the ratio of the pressure to energy density of the dark energy, and the value of ϵ falls in the range $-1 \leq \epsilon \leq -\frac{1}{3}$[3–5]. Another one is "Phantom"[6] and is characterized by $\epsilon < -1$, so we can not talk about the thermodynamics and hydrodynamics of a black hole without knowing the influence of dark energy on it.

Several works have been done on the black hole phase transition[7–9]. Already in 1983, the classic paper by Hawking and Page[10] on black hole phase transitions appeared. The black hole phase transition can be studied theoretically in the light of the expression of its heat capacity[3].

Tharanath et *al.*[11] have computed the mass of Schwarzschild black hole in the presence of quintessence, without including the possibility of its decreasing. Nevertheless, Babichev et *al.* have introduced and developed the theory of black hole mass decreasing[12]. In their paper, due to hydrodynamical analysis, they have shown that accretion of phantom energy is accompanied by the gradual decrease of the black hole mass. Since that, many investigations have been done about the possibility of black hole to loose its mass, due to phantom dark energy [13–17]. Now, we aim to investigate how could the black hole behave either we take into account hydrodynamics and thermodynamics effect due to quintessence and phantom energy.

Here, we shall give more precisions about process of black holes mass decreasing, and give a model of superposition of thermodynamics and hydrodynamics effects around them, thus redefining the metrics of the Schwarzschild black hole. Afterwards, we shall present a new principle to analyze black holes coalescence into a space-time, dilated by the presence of dark energy.

Modified metrics. —Kiselev[18] has derived a static spherically symmetric exact solution of Einstein equations for a black hole surrounded by quintessence, with the energy momentum tensor, which satisfies the conditions of additivity and linearity. The geometry of this black hole can be expressed using spherical coordinates $(r,\ \theta,\ \phi)$ as

$$ds^2 = g(r)dt^2 - \frac{1}{g(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (1)$$

where

$$g(r) = 1 - \frac{2M}{r} - \frac{a}{r^{3\epsilon+1}} \qquad (2)$$

Here, M is the black hole mass and a is the normalization parameter, which is positive, depending on the energy density of quintessence[11], like

$$\rho_q = -\frac{3a\epsilon}{2r^{3(\epsilon+1)}}. \qquad (3)$$

Now, using a phenomenological approach, if we take into account thermodynamics and hydrodynamics effects, we could add into the expression of the metrics, some terms due to hydrodynamics process, thus we get the form

$$g(r) = 1-\frac{2M}{r}-\frac{a}{r^{3\epsilon+1}} - \frac{2}{r}\left\{\beta M_i(1-t\{4\pi AM_i[\rho_\infty+p(\rho_\infty)]\})-M_i\right\}. \quad (4)$$

Here, the expression $1-\frac{2M}{r}-\frac{a}{r^{3\epsilon+1}}$ is obtained due to thermodynamics way and $\frac{2}{r}\left\{\beta M_i(1-t\{4\pi AM_i[\rho_\infty+p(\rho_\infty)]\})-M_i\right\}$ is added to include hydrodynamics effects[12]. t is the relativistic time, depending on the gravitational characteristics in the black hole space-time.

The event horizon characteristics of the black hole can be found by using the following equation[19]

$$g(r_h)=0. \quad (5)$$

This equation leads us to establish the black hole mass evolution that way:

$$M(t) = M_i + \kappa(M_{wide} - \alpha M_{thermo}) - \beta M_{hydro}. \quad (6)$$

Because of the slightness of thermodynamics evolution, we input the scale factor κ which takes into account the scale of the initial black hole mass M_i. We will deal with the case of a stellar black hole, $M_i = 4\times M_{sun} = 7.9564\times 10^{30}kg$. M_{wide} represents the thermodynamics evolution without dark energy, M_{thermo} the perturbation due to quintessence dark energy in the case of thermodynamics process and M_{hydro} represents the hydrodynamics evolution due to the presence of phantom dark energy. They are expressed as

$$M_{wide} = \frac{1}{2}\left(\sqrt{\frac{t}{\pi}}\right), \quad (7)$$

$$M_{thermo} = \frac{1}{2}\left(a\left(\frac{\pi}{t}\right)^{\frac{3\epsilon}{2}}\right), \quad (8)$$

$$M_{hydro} = M_i\left(1-t\{4\pi AM_i[\rho_\infty+p(\rho_\infty)]\}\right). \quad (9)$$

Here, due to the second law of black holes thermodynamics, $S\propto t$, allowing us to substitute entropy S by the relativistic time t. Further we have taken the value of ρ_∞ as the cosmological constant density $\rho_\infty = \rho_\Lambda = 81\times 10^{-12}ev$.

Now, we can deduce the temperature T and heat capacity C from the above expression of mass in terms of t with their formula given by[11, 20, 21]

$$T = \left(\frac{\partial M}{\partial t}\right), \qquad C = T\left(\frac{\partial t}{\partial T}\right). \quad (10)$$

Due to these formula, we get about Schwarzschild black hole surrounded by dark energy($\alpha=1$),

$$T = \kappa\left(\frac{1}{4\sqrt{\pi}t^{\frac{1}{2}}} + \frac{3a\epsilon\pi^{\frac{3\epsilon}{2}}}{4t^{\frac{3\epsilon}{2}+1}}\right) + M_i\left(4\pi AM_i[\rho_\infty+p(\rho_\infty)]\right). \quad (11)$$

Taking into account hydrodynamics effects, we derive this expression of heat capacity

$$C = \frac{-2t\kappa\left[1+3a\epsilon\left(\frac{t}{\pi}\right)^{-\frac{3\epsilon+1}{2}}\right]+8\pi^{\frac{1}{2}}t^{\frac{3}{2}}M_i(4\pi AM_i[\rho_\infty+p(\rho_\infty)])}{1+3a\epsilon(3\epsilon+2)\left(\frac{t}{\pi}\right)^{-\frac{3\epsilon+1}{2}}}. \quad (12)$$

Phase transition occurs when the heat capacity contains a discontinuity. In this expression, we can compute that for quintessence dark energy $\epsilon \in]-1,-\frac{1}{3}]$, the denominator has a zero point and this shows the presence of second-order phase transition. For $\epsilon=-0.66$ with $a = 1.02\times 10^{-4}$, this phenomenon occurs for

$$t = 3.191651693\times 10^{11}u, \qquad u \text{ is the time unity.} \quad (13)$$

However, for a phantom dark energy $\epsilon<-1$, the denominator doesn't become null, unless if relativistic time has a complex value. For example, if we take $\epsilon=-1.1$, we have

$$t = \{-1.395558167\times 10^{12} + 6.654546312\times 10^{11}\times I, - 1.395558167\times 10^{12} - 6.654546312\times 10^{11}\times I\}u. \quad (14)$$

Here, the parameter a is obtained from the following equation, due to Eq.(2).

$$a \propto r_h^{3\epsilon+1} = \left(\frac{2GM}{c^2}\right)^{3\epsilon+1}. \quad (15)$$

Neglecting hydrodynamics effects, the heat capacity gets the form

$$C = -\kappa\left[\frac{16t^{3\epsilon+5}+48a\epsilon\pi^{\frac{3\epsilon+1}{2}}t^{\frac{3\epsilon+9}{2}}}{8t^{3\epsilon+4}+\left(72a\epsilon^2\pi^{\frac{3\epsilon+1}{2}}+48a\epsilon\pi^{\frac{3\epsilon+1}{2}}\right)t^{\frac{3\epsilon+7}{2}}}\right], \quad (16)$$

which can be simplified to[22]

$$C = -2t\kappa\left[\frac{1+3a\epsilon\left(\frac{t}{\pi}\right)^{-\frac{3\epsilon+1}{2}}}{1+3a\epsilon(3\epsilon+2)\left(\frac{t}{\pi}\right)^{-\frac{3\epsilon+1}{2}}}\right]. \quad (17)$$

Hawking et *al.*[23] have presented that the diminishing of black hole mass is caused by the violation of the energy domination condition $\rho+p(\rho)\geq 0$, which is the assumption of the classical black hole non diminishing theorems. This theorem was used in many case[24, 25] . Further, in the quintessence background, black hole respects this theorem then it does not get its mass decreased. Now, For these assumptions, we could give a values of α and β inside the expression of mass.

$$\alpha = 1 \text{ and } \beta = \begin{cases} 0 \;\; for & \text{quintessence dark energy,} \\ 1 \;\; for & \text{phantom dark energy.} \end{cases} \quad (18)$$

The numerical values of mass in our model is shown in Fig.1. It puts out well the impact of dark energy (quintessence or phantom) in the vicinity of the Schwarzchild black hole and we can see that after its birth, it grows its mass before having decreased it. Moreover, the behavior for $(\epsilon, a) = (-0.48, 0.02)$ is more pronounced than $(\epsilon, a) = (-0.50, 9.19 \times 10^{-3})$ meaning that the black hole reaches a higher maximum value of mass.

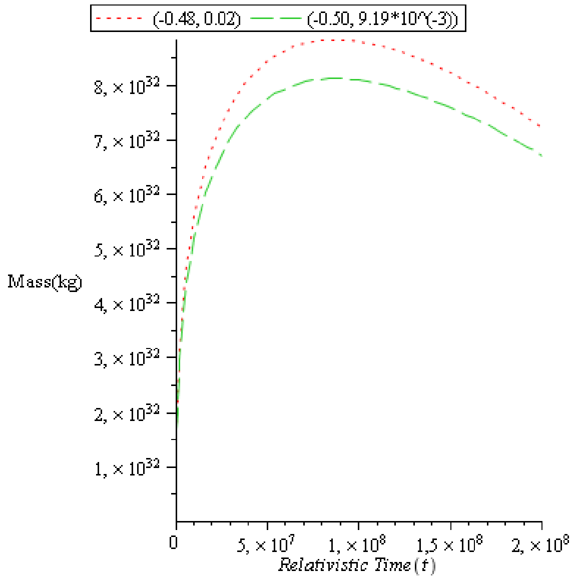

FIG. 1: Variation of the black hole mass in the presence of quintessence with characteristics (ϵ, a), which are $(-0.48, 0.02)$ in red dots and $(-0.50, 9.19 \times 10^{-3})$ in green dash in term of relativistic time t.

Now, let us analyze the case of two black holes in coalescence, in the presence of dark energy.

Power and time from coalescence within dark energy.—Gravitational waves are propagating fluctuations of gravitational fields that is ripples in space-time, generated mainly by moving bodies. These distortions of space-time travel at the speed of light [26, 27].

The weakness of the gravitational field is once again expressed as our ability to decompose the metric into the flat Minkowski metric plus a small perturbation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \text{ where } |h_{\mu\nu}| \ll 1. \tag{19}$$

We will restrict ourselves to coordinates in which $\eta_{\mu\nu}$ takes its canonical form, $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$.

Let us consider two black holes designated by a and b of mass M in a circular orbit in the $x^1 - x^2$ plane, at distance r from their common center of mass and compute the power P during the coalescence. We will treat their motions in the Newtonian approximation. In term of the angular frequency orbit $\Omega = (\frac{GM}{4r^3})^{1/2}$, we can write down the explicit path of black holes as [26]

$$\begin{cases} x_a^1 &= r\cos(\Omega t),\ x_a^2 = r\sin(\Omega t), \\ x_b^1 &= -r\cos(\Omega t),\ x_b^2 = -r\sin(\Omega t). \end{cases} \tag{20}$$

Here, (x_a^1, x_a^2) and (x_b^1, x_b^2) are respectively the coordinates of the two black holes a and b.

Basing on the cosmological principle in the $x^1 - x^2$ plane, namely, that apart from local irregularities, the universe presents the same general aspect at every point[28] and in relation to the expansion of universe, meaning that everywhere in the universe, the expansion takes place in such a way to produce a dilatation at the same rate in all directions. Thus, Sys. (15) can be modified to

$$\begin{cases} x_a^1 &= r\cos(\Omega t) + \alpha,\ x_a^2 = r\sin(\Omega t) + \alpha, \\ x_b^1 &= -r\cos(\Omega t) - \alpha,\ x_b^2 = -r\sin(\Omega t) - \alpha. \end{cases} \tag{21}$$

Then, the term responsible of the dark energy action would be α. For reasons of homogeneity for our known about quintessence type of dark energy, we assume by Eq.(2) that

$$\alpha \propto a^{\frac{1}{3\epsilon+1}}, \tag{22}$$

The corresponding energy-momentum density can be computed as

$$\begin{aligned} T^{00}(t,x) &= M\delta(x^3)\times \\ & \{\delta(x^1 - r\cos(\Omega t) - \alpha)\delta(x^2 - r\sin(\Omega t) - \alpha) \\ &+ \delta(x^1 + r\cos(\Omega t) + \alpha)\delta(x^1 + r\sin(\Omega t) + \alpha\}. \end{aligned} \tag{23}$$

Before computing the power P, we have first to find the expression of Q_{ij}, the traceless part of the quadrupole momentum q_{ij}, which can be computed by the relation:

$$Q_{ij} = q_{ij} - \frac{1}{2}\delta_{ij}\delta^{kl}q_{kl}.$$

The quadrupole momentum tensor is given by [29]

$$q_{ij}(t) = 3\int y^i y^j T^{00}(t, \vec{y})\ d^3y, \tag{24}$$

and the formula of power is given by [29]

$$P = \frac{G}{45}\left[\frac{d^3Q^{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3}\right]_{tr}. \tag{25}$$

Thus after calculus, we get this expression

$$P = \frac{75}{45}M^2\Omega^6\left[16r^4 + 2\alpha^2r^2 + 8\alpha r^3(\cos(\Omega t) + \sin(\Omega t))\right]. \tag{26}$$

Here we can see that introducing dark energy leads to get a superposition of quadratic and trigonometric form.

The loss of energy can be described with its relation to the power of gravitational radiation as [30]

$$P = \frac{dE}{dt} = -\frac{GM^2}{2r^2}\frac{dr}{dt}. \quad (27)$$

With the power computed above, we get the differential nonlinear equation which traduces the evolution of the distance between black holes

$$\frac{dr}{dt} = -B\frac{[16r^2 + \alpha + 8\alpha r(\cos\Omega t + \sin\Omega t)]}{r^5}, \quad (28)$$

$$\text{with} \quad B = \frac{G^2M^3}{20}.$$

This differential equation could lead us to have more precisions about the time needed to two black holes when the are still colliding, meaning that $r(t) \to 0$.

Summary and conclusion. —In summary, we have studied the influence of dark energy on two cases. The first is the Schwarzschild black hole and the second is the mechanics of two black holes in coalescence. In the first system, we have first putted out a new formula of the Schwarzschild metric in the dark energy background. We have taken into account both thermodynamics and hydrodynamics effects caused by quintessence and phantom energy. This led us to get the expression of mass, temperature and heat capacity in term of relativistic time. Moreover, we have shown that associating these effects permits us to appreciate the decrease of mass. Then, we putted out that phantom dark energy does not permit the black hole undergoing a second-order phase transition.

For the second case, we calculated the power of the gravitational radiation that could be emitted by two black holes in coalescence and the time necessary for it. Our results showed that, introducing dark energy leads to a superposition of quadratic and trigonometric forms inside its expression, and we have found a differential equation which could give the evolution of distance between black holes.

Our conclusion is that dark energy, would have a great goal in our universe as well for black holes as gravitational waves, observed by the decrease of black hole mass some modifications of power and the distance evolution of two black holes during a coalescence.

Acknowledgment —We wish to thank R. R. Caldwell for useful comments about the aim of dark energy in the vicinity of black holes and encouragement.

* Corresponding author: nragilbrand@gmail.com

[1] S. W. Hawking, Nature **248**, 30 (1974).
[2] J. D. Bekeinstein, Phys. Rev. D **7**, 2333 (1973).
[3] V. C. Kuriakose and R. Tharanath, Mod. Phys. Lett. A **28**, 1350003 (2013).
[4] M. Shahjalal, Nucl. Phys. B (2019).
[5] W. Javed and R. Babar, Advances in High Energy Physics **2019** (2019).
[6] R. Cadwell, Phys. Lett. B **545**, 23 (2002).
[7] R. Banerjee and D. Roychowdhury, JHEP **11**, 14 (2011).
[8] M. Appels, R. Gregory, and D. Kubizňák, Phys. Rev. Lett. **117**, 131303 (2016).
[9] R. A. Hennigar, R. B. Mann, and E. Tjoa, Phys. Rev. Lett. **118**, 021301 (2017).
[10] S. W. Hawking and D. Page, Com. Math. Phys. **87**, 577 (1983).
[11] R. Tharanath, N. Varghese, and V. C. Kuriakose, Mod. Phys. Lett. A **29**, 1450057 (2014).
[12] E. Babichev, V. Dokuchaev, and Y. Eroshenko, Phys. Rev. Lett. **93**, 021102 (2004).
[13] E. Babichev, V. Dokuchaev, and Y. N. Eroshenko, JETP **100**, 528 (2005).
[14] Cai, Rong-Gen, and A. Wang, Phys. Rev. D **73**, 063005 (2006).
[15] A. V. Yurov, P. Martín-Moruno, and P. F. González-Díaz, Nucl. Phys. B **759**, 320 (2006).
[16] J. J. Madrid and P. F. Gonzalez-Diaz, Gravitation and Cosmology **14**, 213 (2008).
[17] M. Jamil, Eur. Phys. Jour. C **62**, 609 (2009).
[18] V. V. Kiselev, Class.Quantum Grav **20**, 1187 (2003).
[19] B. B. Thomas, M. Saleh, and T. C. Kofane, General Relativity and Gravitation **44**, 2181 (2012).
[20] R.-G. Cai, L.-M. Cao, and N. Ohta, Phys. Lett. B **679**, 504 (2009).
[21] B. Majeed, M. Jamil, and P. Pradhan, AHEP **2015** (2015).
[22] K. K. Kamiko, M. Saleh, B. B. Thomas, and T. C. Kofane, arXiv preprint arXiv:1808.03474 (2018).
[23] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Vol. 1 (Cambridge university press, 1973).
[24] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).
[25] N. Engelhardt and A. C. Wall, Phys. Rev. Lett. **121**, 211301 (2018).
[26] S. M. Carroll, arXiv preprint gr-qc/9712019 (1997).
[27] B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).
[28] E. Mörtsell and S. Dhawan, arXiv preprint arXiv:1801.07260 (2018).
[29] P. C. Peters and J. Mathews, Phys. Rev. **131**, 435 (1963).
[30] C. Cutler and E. E. Flanagan, Phys. Rev. D **49**, 2658 (1994).