

The Joy of Pi

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Abstract. This note presents a logarithmic integral for Pi.

- The number π is defined by:

$$\pi = 4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots \right) = 3.141592\dots$$

- Logarithmic integral for π :

$$\pi = \int_0^1 \ln \left(\frac{1 + \sqrt{1-x^4} + \sqrt{2+2\sqrt{1-x^4}}}{1 - \sqrt{1-x^4} + \sqrt{2-2\sqrt{1-x^4}}} \right) dx$$

- A related integral:

$$2\pi = \int_0^1 \frac{4 + (3-x)\sqrt{2+2x} + (3+x+2\sqrt{2+2x})\sqrt{2-2x}}{\sqrt[4]{1-x^2} (1+x+\sqrt{2+2x})(1-x+\sqrt{2-2x})} dx$$

References

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