The Covariant Energy-Momentum Conservation Law in General Relativity, an Overlooked Role of the Equivalence Principle

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Abstract

Reconsideration of overlooked considerations about the Equivalence Principle leads to Covariant Energy-Momentum conservation law in General Theory of Relativity.

The meaning, validity and significance of EP

The study of the Equivalence Principle has many interesting interwoven historical, logical, physical and mathematical aspects:

It can be shown that different versions of the Equivalence Principle which are often stated by the scientific authors can be considered as equivalent statements or implications of two forms:

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The first one which is usually found in the books of popular science states that: In every point in space there is a special coordinate system (freely falling observer) by which the gravitational field can be considered as absent (this is equivalent to another popular statement that accelerating by a rocket in the absence of gravitational field is equivalent to being at rest in a gravitational field).

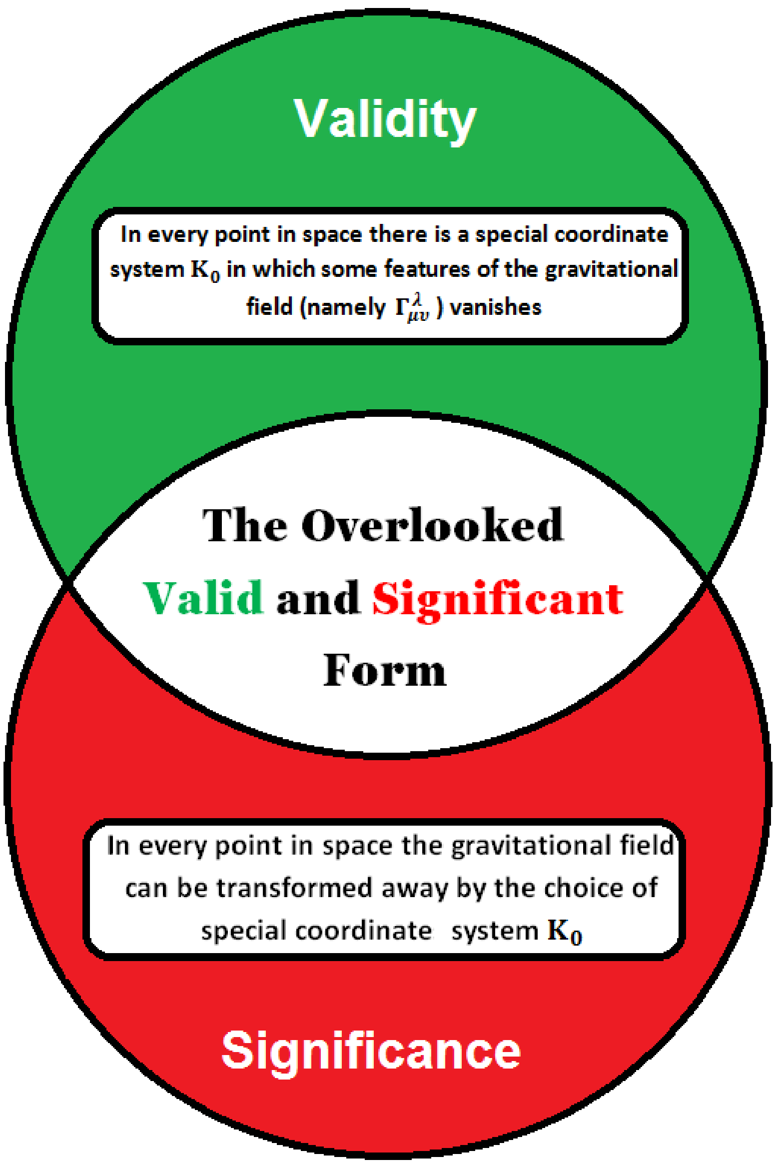
Despite its simplicity and attraction this statement is incorrect because as remarked by many scientist (including Einstein himself), in the theory of relativity either there is a gravitational field or there is none according as the Riemann tensor does not or does vanish which is an absolute property independent from the state of the observer.

The second form states that: in every point in space there is a special coordinate system $\mathbf{K_0}$ in which the first derivatives of the metric tensor vanishes so:

$$\Gamma^{\lambda}_{\mu\upsilon} = 0$$

This is a correct mathematical fact `but its validity comes at the expense of generality and significance

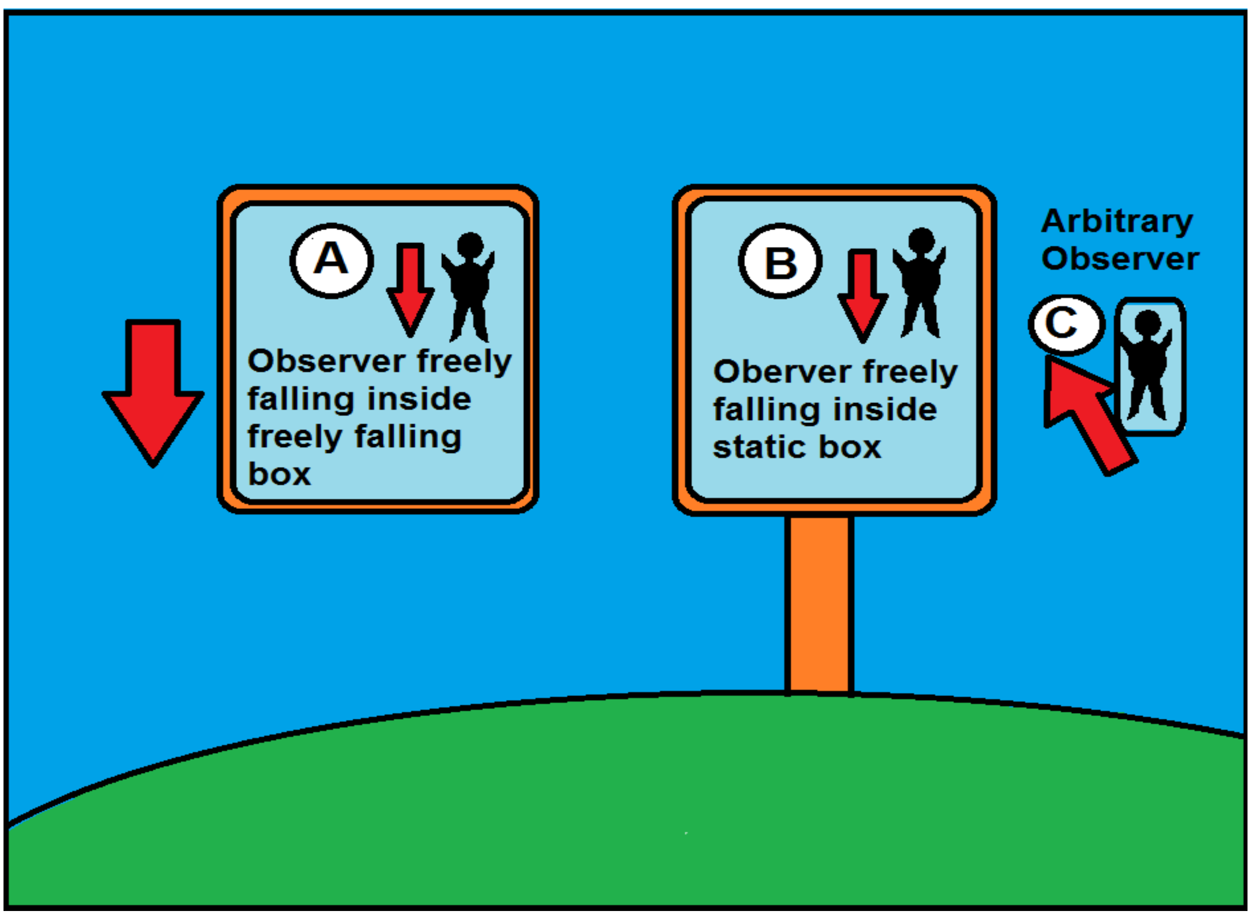
because non-tensor law that transform this quantity make its vanishing in a special coordinate system an internal matter of that system. This does not of course do away with all the significance of this form of the equivalence principle but minimize its importance.



Different forms of the ***Equivalence Principle*** stated by many authors are either invalid or insignificant. This problematic suituation has led some scientist even to describe the principle as meaningless or unnecessary

The Overlooked Consideration

An important consideration which is always overlooked when introducing the equivalence principle is the motion of the region under our study. It is not only the choice of our coordinate system that affect our perception of what is going on in certain region of space, the motion of the region with respect to the source of the gravitational field is also of great importance.



Let us clarify this point by the following example. Let there be two small boxes; one is freely falling under the influence of the gravitational field of the earth and the other box is fixed to the earth. It can be shown as a result of the equivalence principle that the properties of space as observed by a freely falling observer inside the freely falling box (A) and the static one (B) both satisfy the condition:

$$\Gamma^{\lambda}_{\mu\upsilon} = 0$$

However, this similarity of observations of the two regions as observed from the same coordinate system does not mean absolute similarity of the two spaces and it disappears when we compare our observations of the two regions by another arbitrary coordinate system. The static box as seen by this arbitrary observer has nothing special at all and all the quantities which describe its geometry will take certain values according to the gravitational field and the coordinate system (for the quantities dependant on the coordinate system). As for the space inside the freely falling box the geometry is arranged by the geodetic motion of the observed region with respect to the source of gravitational field

so that $\Gamma^{\lambda}_{\mu\upsilon}$ = 0 in all frame of reference, this is what we call the overlooked valid and significant form of the equivalence principle. Unfortunately, the theorist even those who specialize in General Relativity have still not been able to distinguish between these two different concepts i.e. the motion of the coordinate system and the motion of the local part of the space to be observed by the coordinate system (space in static box seen from freely falling observer and space in freely falling box seen from freely falling observer.

The Covariant Energy-Momentum Conservation Law.

The problem of energy-momentum conservation in general relativity was addressed many times in scientific literature and only a passing glance at the titles may reflects the severe confusion about this topic .. (Energy *is Conserved in General Relativity*) and (*Energy is Not Conserved in General Relativity*) both are titles of authentic papers written by true scientist!

Despite much complications associated with the proposed resolutions of those who believe there is a

conservation law in one hand and the complications associated with the proposed justifications of those who think there is no conservation law in the other hand the problem itself is clear and simple:

In gravitational field the coordinate-free divergence $\mathbf{T}^{\mu\nu}_{\ ,\nu}$ (with a comma, should not be confused with the covariance divergence $\mathbf{T}^{\mu\nu}_{\ ;\nu}$ with a semi-colon) does not vanish as required by the conservation law but we have instead:

$$\mathbf{T}^{\mu\nu}_{\ ,\nu} = -\Gamma^{\mu}_{\sigma\upsilon}\mathbf{T}^{\sigma\nu} - \Gamma^{\nu}_{\sigma\upsilon}\mathbf{T}^{\mu\alpha}$$

But according to our understanding of the Equivalence Principle addressed above we can see that because of the vanishing of the christoffel symbols in all coordinate system, $\mathbf{T}^{\mu\nu}$ is conserved in any freely falling region of space. This restriction should not be regarded as a disadvantage of the application of the conservation law in general relativity because the law can be applied everywhere and in all coordinate systems, it is only that we have to let the nature fix the box the way she want

before choosing our coordinate system to investigate the conservation laws in space inside it.